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**Mechanical vibration and shock —  
Signal processing —**

**Part 3:  
Methods of time-frequency analysis**

*Vibrations et chocs mécaniques — Traitement du signal —*

*Partie 3: Méthodes d'analyses en temps et fréquence et par échelle de temps*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 108, *Mechanical vibration, shock and condition monitoring*.

ISO 18431 consists of the following parts, under the general title *Mechanical vibration and shock — Signal processing*:

- *Part 1: General introduction*
- *Part 2: Time domain windows for Fourier Transform analysis*
- *Part 3: Methods of time-frequency analysis*
- *Part 4: Shock-response spectrum analysis*

## Introduction

Time-frequency analysis is used to quantitatively display a vibration or shock in terms of time and frequency. This is useful for analysing vibrations in a machine at varying speeds, e.g. in an automobile at varying engine rotational frequencies. Time-frequency analysis is also used to quantitatively display impulsive responses from machinery, e.g. the response of an impact. The duration of the impact as well as the frequency response is displayed. The frequency response can be displayed in terms of frequency, RPM, or octaves. The four methods included in this part of ISO 18431 are the short-time Fourier transform, the Wigner-Ville transform, the Choi-Williams transform, and the wavelet transform. When any of these methods is used with the correctly specified parameters, time and frequency components of shocks and vibrations are displayed quantitatively. Quantitative display enables quantitative specification of the machinery.

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# Mechanical vibration and shock — Signal processing —

## Part 3: Methods of time-frequency analysis

### 1 Scope

This part of ISO 18431 specifies methods for the digital calculation of a time-frequency analysis of a given sampled measurement of a physical or engineering quantity, such as acceleration, force, or displacement, over an interval of time. Several mathematical formulations of time-frequency transformations are given with requirements for recording of parameters and recommendations.

The data can be obtained experimentally from measurements of a mechanical structure or obtained from numerical simulation of a mechanical structure. This category of data is very broad because there is a wide variety of mechanical structures, e.g. microscopic instruments, musical instruments, automobiles, manufacturing machines, buildings, and civil structures. The data can determine the response of machines or of humans to mechanical vibration and shock.

### 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041, *Mechanical vibration, shock and condition monitoring — Vocabulary*

ISO 18431-1, *Mechanical vibration and shock — Signal processing — Part 1: General introduction*

### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2041, ISO 18431-1, and the following apply.

#### 3.1

##### **short-time Fourier transform**

Fourier-related transform used to determine the time dependence of the sinusoidal frequency and phase content of a time-varying vibration

#### 3.2

##### **Wigner-Ville distribution**

quadratic time-frequency description of a vibration based on the autocorrelation of a signal

#### 3.3

##### **generalized Wigner-Ville distribution**

time-frequency description of a vibration based on a filtered autocorrelation of a signal

#### 3.4

##### **Choi-Williams transform**

generalized Wigner-Ville distribution of a vibration using a specific kernel

#### 3.5

##### **wavelet transform**

time-frequency description of a vibration based on the scaled frequency transformation of a signal

## 4 Symbols

$f_s$	= $1/\Delta t$ , sampling frequency; the inverse of the sampling period
$K$	number of samples in window $w(k)$
$m$	index of scaled frequency
$N$	data block length; the number of sampled points that are transformed
$n$	index of time
$S_x(m,n)$	short-time Fourier transform of sampled data $x(n)$
$T$	= $N\Delta t$ , total time of a block of sampled data
$t$	= $n\Delta t$ , time
$u$	units of the sampled signal, e.g. displacement or velocity
$V(n,m)$	Cohen class filter for smoothing the Wigner distribution
$X(m)$	discrete Fourier transform of $x(n)$
$\tilde{X}(m)$	namely $\tilde{X}(m) = 0$ for $N/2 \leq m$
$x(n)$	sampled physical data in the time domain
$\tilde{x}(n)$	analytic sampled signal
$\Delta f$	frequency resolution
$\Delta n$	increment of index of time
$\Delta t$	sample period
$E_x(m,n)$	Choi-Williams distribution
$\sigma$	filter parameter of Choi-Williams distribution
$\Omega_x(m,n)$	Wigner distribution of signal $x(n)$
$O_x(k,m)$	wavelet transform of $x(n)$

## 5 Time-frequency transforms

### 5.1 Short-time Fourier transform

The short-time Fourier transform of a signal is the Fourier transform of segments shorter than a given data block. It shows how the spectrum changes with time. It is defined by Formula (1) and detailed in Reference [3].

$$S_x(m,n) = \frac{1}{f_s} \sum_{k=0}^{K-1} x(n+k)w(k) \exp\left(-i2\pi \frac{mk}{K}\right) \quad (1)$$

for

$$0 \leq m < K/2 \text{ and } 0 \leq n < N - K$$

The following parameters shall be specified: the short-time integration window,  $K$ , window function  $w(k)$ ,  $0 \leq k \leq K - 1$ , and the increment of the index of time,  $\Delta n$ .

The quantity  $|S_x(m,n)|^2$  is plotted for quantitative analysis. The units are  $u^2/\text{Hz}^2$ .

NOTE 1 The increment of the index of time is often 1,  $K/2$ , or  $K$ .

NOTE 2 The value of  $K$  depends on the time and frequency dependence of the signal to be analysed. Larger values of  $K$  show finer frequency details and smaller values of  $K$  show finer time details.

## 5.2 Generalized Wigner-Ville transform

### 5.2.1 Wigner-Ville transform

The Wigner-Ville time-frequency transform of a signal is defined by Formula (2).

$$\Omega_x(m,n) = \frac{f_s}{N} \sum_{k=0}^r \tilde{X}(m+k) \tilde{X}^*(m-k) \exp\left(+i \frac{2\pi}{N} 2nk\right) \quad (2)$$

for

$$0 \leq m \leq (N/2) - 1, 0 \leq n \leq N, r = \min[(N/2) - 1 - m, n]$$

The absolute value of  $\Omega_x(m,n)$  is plotted for quantitative analysis. The units are  $u^2 \text{ s}$ .

NOTE 1 The Wigner-Ville transform is also expressed in terms of the autocorrelation with Formula (3).

$$\Omega_x(m,n) = \begin{cases} \frac{2}{f_s} \text{Re} \left[ \sum_{k=0}^{2n} x^* \left( n - \frac{k}{2} \right) x \left( n + \frac{k}{2} \right) \exp \left( -i 2\pi \frac{mk}{n} \right) \right] & \text{for } m < n, 0 \leq n < \frac{N}{2} \\ \frac{2}{f_s} \text{Re} \left[ \sum_{k=0}^{2(N-n)} x^* \left( n - \frac{k}{2} \right) x \left( n + \frac{k}{2} \right) \exp \left( -i 2\pi \frac{mk}{N-n} \right) \right] & \text{for } m < (N-n), \frac{N}{2} \leq n < N \end{cases} \quad (3)$$

NOTE 2 The Wigner distribution is derived from the autocorrelation of the complete data block. This allows greater accuracy in the resolution of time and frequency components, but also generates spurious components in the distribution.

NOTE 3 The sample period of time in the Wigner distribution is twice that of the sample period of the original data  $x(n)$  because the time increments of the autocorrelation have twice the period. Therefore, the time corresponding to sample  $n$  is  $t = n2\Delta t$ .

NOTE 4 An example of the resolution and spurious components are shown in [Annex A](#).

### 5.2.2 Choi-Williams transform

The Choi-Williams transform is defined by Formulae (4) and (5).

$$\Xi_x(m,n) = \frac{2}{f_s} \sum_{k=0}^{N-1} \exp\left(-i 4\pi \frac{mk}{N}\right) \sum_{l=k}^{N-k-1} V(k,l,n;\sigma) \tilde{x}(l+k) \tilde{x}^*(l-k) \quad (4)$$

where

$$V(k,l,n;\sigma) = \frac{1}{\sqrt{4\pi k^2/\sigma}} \exp\left[-\frac{(l-n)^2}{4k^2/\sigma}\right] \quad \text{and} \quad V(0,l,n;\sigma) = 0 \quad (5)$$

The value of parameter  $\sigma$  is specified. This parameter is dimensionless.

The absolute values of  $\mathcal{E}_x(m,n)$  are plotted for quantitative analysis. The units of  $|\mathcal{E}_x(m,n)|$  are  $u^2$  s.

NOTE This transform decreases the specific types of spurious components.

### 5.3 Wavelet transform

#### 5.3.1 Definition of continuous wavelet transform

The continuous wavelet transform is defined by Formula (6).

$$O_x(k,m) = \frac{1}{f_s} 2^{m/2} \sum_{n=0}^r \tilde{x}(n) \Psi(2^m n - k) \quad (6)$$

for

$$r \leq 2^{-m}(N - 1)$$

where

$\Psi(n)$  is the mother wavelet used in the analysis.

The specific mother wavelet shall be stated. The absolute value squared  $|O_x(k,m)|^2$  is plotted. The units are  $u^2/\text{Hz}^2$ .

NOTE The wavelet transform is based on performing analysis over scaled frequencies. Therefore, the frequency resolution is coarser at lower frequencies and finer at higher frequencies. The time resolution is less at lower frequencies and greater at higher frequencies.

#### 5.3.2 Definition of mother wavelet

The mother wavelet,  $\Psi(n)$ , determines the appearance of the transform. The choice of the proper mother wavelet is based on the time and frequency characteristics of the signal to be analysed. The definition of the Gaussian enveloped sinusoid is defined by Formula (7).

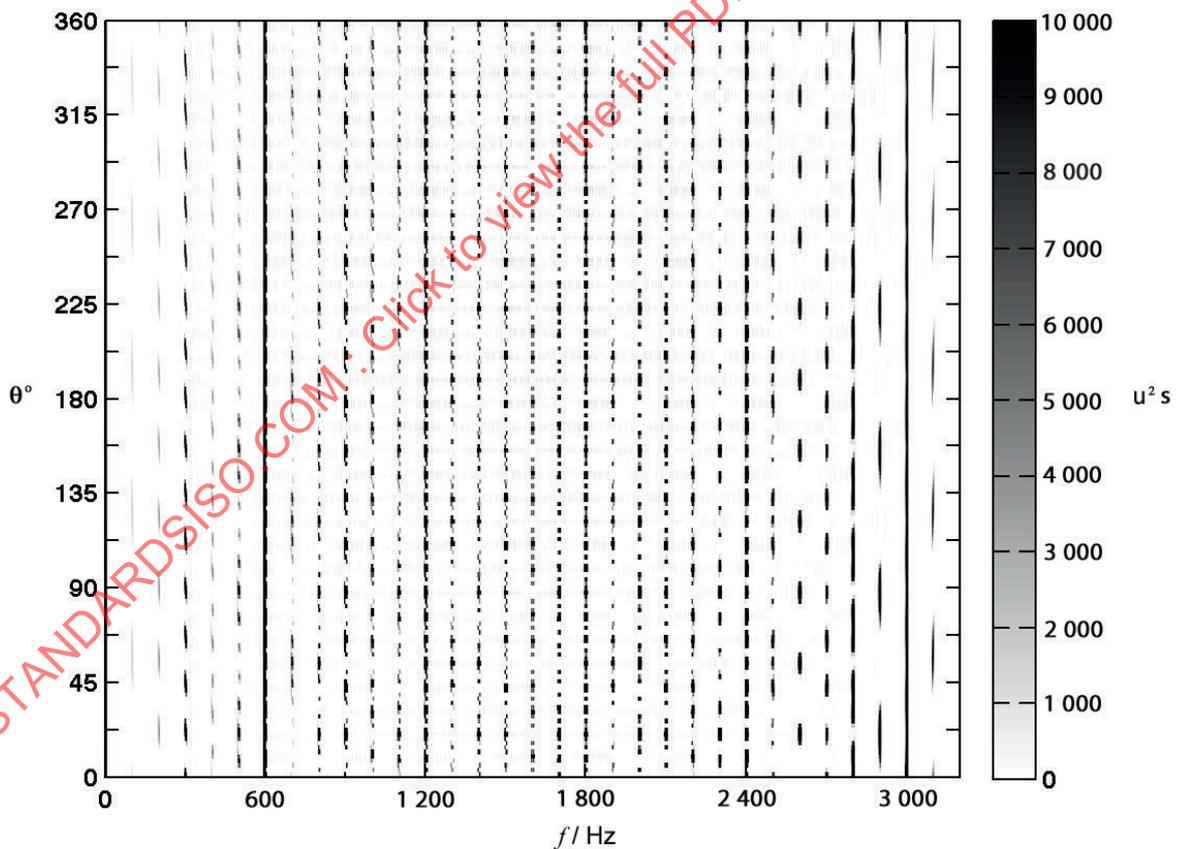
$$\Psi(n) = c \exp(-\sigma^2 n^2) \exp\left(-i2\pi \frac{f_0}{f_s} n\right) \quad (7)$$

where the constant  $c$  is imaginary so that the real part of  $\Psi$  is real with a magnitude that normalizes  $\Psi$  over the range of the summation.

## Annex A (informative)

### Analysis of gear tooth fault using Wigner distribution

For the example of a pinion gear in Reference [6], we consider an input gear with 24 teeth and a pinion gear with 16 teeth. The rotation frequency of the pinion gear is 37,5 Hz, resulting in a mesh frequency of 600 Hz. A gear signal is numerically simulated with 2 048 samples with a sampling frequency of 6 600 Hz. The upper time half of the signal is set to zero to prevent aliasing in the time domain. The analytic signal representation of the signal is used in Formula (2) to prevent aliasing the frequency domain. The case of perfect gear teeth is modeled as a periodic series of unit impulses with period  $1/600$  s or 11 samples. Therefore,  $x(n) = [1000000000010\dots]r$ . The absolute value of the Wigner-Ville transform of the perfect case is shown in Figure A.1. The horizontal axis of the plot is frequency and the vertical axis is pinion rotation angle. The harmonics of the mesh frequency are seen at multiples of 600 Hz. The positions of the gear meshes occur at multiples of  $22,5^\circ$ . The greyscale is chosen so that large values are darker and the differences between the perfect and defective pinion gears are apparent.



#### Key

$\theta$  pinion angle

$f$  frequency

**Figure A.1 — The Wigner transform of an impulse sequence simulating the vibration of gears meshing — Perfect pinion**