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Analytical colorimetry —

Part 4: Metamerism index for pairs of samples for change of illuminant

Analyse colorimétrique —

Partie 4: Indice de métamérisme de paires d'échantillons pour changement d'illuminant

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Foreword

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This second edition cancels and replaces the first edition (ISO 18314-4:2020), which has been technically revised.

The main changes are as follows:

— a brief introduction about differentiation between metamerism and paramerism has been added in 8.1;

— Formula (1) has been updated to align with Formulae (2) and (4) to (24);

— the key of Figure A.1 has been updated.

A list of all parts in the ISO 18314 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document distinguishes three kinds of metamerism of pairs of samples:

a) Illuminant metamerism occurs if both of the object colours of a pair of samples are perceived as being the same only under a specific illuminant (e.g. under illuminant D65), while they differ under a different illuminant (e.g. illuminant A).

b) Observer metamerism occurs if the object colours of a pair of samples are perceived as being the same by one observer, while a different observer perceives a colour difference under the same illuminant and the same reference conditions.

NOTE 1 The observer metamerism is caused by differences between the distributions of spectral colour matching functions of different observers.

c) Field-size metamerism occurs if both of the object colours of a pair of samples are perceived as being the same on the retina for a size of an observation field (e.g. determined by the 2° standard observer), while they differ for a different observation field on the retina (e.g. 10°).

NOTE 2 The reason for field-size metamerism is based on the existent colour matching functions of an observer during an observation situation. The colour matching functions change with the size of the observation field on the retina. Such change of the observation field can also occur if, for example, the pair of samples is examined from different distances.

Analytical colorimetry —

Part 4:
Metamerism index for pairs of samples for change of illuminant

1 Scope

This document specifies a formalism for the calculation of the illuminant metamerism of solid surface colours. It cannot be applied to colours of effect coatings without metrical adaptation.

This document only covers the phenomenon of metamerism for change of illuminant, which has the greatest meaning in practical application. In the case where chromaticity coordinates of a pair of samples under reference conditions do not exactly match, this document gives guidance on which correction measures to take. Regarding the reproduction of colours, the metamerism index is used as a measure of quality in order to specify tolerances for colour differences between a colour sample and a colour match under different illumination conditions.

The quantification of the illuminant metamerism of pairs of samples is formally performed by a colour difference assessment, for which tolerances that are common for the evaluation of residual colour differences can be used.

NOTE In the colorimetric literature and textbooks, the term geometric metamerism is sometimes used for the case where two colours appear to be the same under a specific geometry for visual assessment and selected standard observer and standard illuminant pair, but are perceived as two different colours at changed observation geometry. The term geometric metamerism is different to metamerism described in this document.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/CIE 11664-1, *Colorimetry — Part 1: CIE standard colorimetric observers*

ISO/CIE 11664-2, *Colorimetry — Part 2: CIE standard illuminants*

ISO/CIE 11664-4, *Colorimetry — Part 4: CIE 1976 L*a*b* colour space*

CIE 015, *Colorimetry*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
metamerism
property of spectrally different colour stimuli that have the same tristimulus values in a specified colorimetric system

[SOURCE: CIE S 017:2020, 17-23-006]

3.2
paramerism
characteristic of a pair of samples with spectral colour stimulus functions which have different fundamental colour stimulus functions as well as different residuals or metameric black values within the visible spectral range

Note 1 to entry: Parameric objects are characterized by the fact that they reflect colour stimuli of different spectral power distribution functions under a specified standard illuminant, which cause approximately the same colour perception under the selected observation conditions.

3.3
colour difference
ΔE
difference between two colour stimuli, defined as a distance between the points representing them in a specified colour space

3.4
reference illuminant
illuminant with which other illuminants are compared

[SOURCE: CIE S 017:2022, 17-22-108]

3.5
test illuminant
illuminant, for which the *colour difference* (3.3) between the two samples to be tested is assessed

3.6
metamerism index for change in illuminant
M_t
colour difference (3.3), ΔE, between the two samples under *test illuminant* (3.5) if $\Delta E = 0$ is observed under the *reference illuminant* (3.4)

3.7
correction method
algorithm for theoretically eliminating a *colour difference* (3.3) of the pair of samples under the *reference illuminant* (3.4)

4 Symbols

For the application of this document, the symbols given in Table 1 apply.

Table 1 — Symbols

Symbol	Identification
X, Y, Z	standard tristimulus values of a measured object colour
X_n, Y_n, Z_n	standard tristimulus values of the used illuminant
$\bar{x}, \bar{y}, \bar{z}$	colour-matching functions
L^*, a^*, b^*	basic coordinates of the CIELAB system
$\Delta L^*, \Delta a^*, \Delta b^*$	differences between basic coordinates of the CIELAB system
M_t	metamerism index for change in illuminant

Table 1 *(continued)*

Symbol	Identification
$\vec{N}, \vec{N}_f, \vec{N}_r$	vector of the radiometric function of a sample with associated fundamental colour stimulus (f) and metameric black (r)
λ	wavelength
S	relative spectral distribution function of an illuminant
$\vec{W}$	vector of the standard tristimulus values
w	integration weights for the calculation of the standard tristimulus values
A	matrix of the integration weights w for the calculation of the standard tristimulus values
R	projection matrix
I	identity matrix
Index spl	sample
Index std	standard
Index t	colour under test illuminant
Index corr	corrected value
Index multipl	multiplicative correction
Index f	fundamental colour stimulus
Index r	metameric black values (residuals)
Index ref	reference illuminant
Index T	transposed matrix

5 Reference illuminant

The standard illuminant D65 shall be chosen as reference illuminant in accordance with ISO/CIE 11664-2. Other reference illuminants required in special cases shall be specified.

6 Test illuminant

The selection of the test illuminant depends on the application. If the test illuminants are not particularly specified, standard illuminant A in accordance with ISO/CIE 11664-2 and/or illuminants of the fluorescent lamp type, such as FL11 in accordance with CIE 015, shall be selected. The test illuminant used shall be indicated as an index to M, e.g. M_A or M_{FL11}.

When calculating the standard tristimulus values X, Y, Z under the selected test illuminants, the basic raster of wavelengths shall comply with those given in ISO/CIE 11664-2 or CIE 015 for A and D65, and in CIE 015 for FL11 and FL2. In cases of missing measuring values of the standard or sample for these wavelengths, these values shall be interpolated and/or extrapolated.

7 CIELAB coordinates L^*, a^*, b^*

The metamerism index, M_t, is based on the CIELAB coordinates L^*, a^*, b^* of samples 1 and 2 which are compared. L^*, a^*, b^* shall be calculated in accordance with ISO/CIE 11664-4 from the standard tristimulus values X, Y, Z. These values are derived from the sample for the CIE 1964 10° standard observer in accordance with ISO/CIE 11664-1 for the reference illuminant and the selected test illuminant. If calculating L^*, a^*, b^* under the test illuminant, the respective standard tristimulus values X_n, Y_n, Z_n of the entirely matt white surface shall be used in accordance with CIE 015. For the standard illuminants A and D65 or for the illuminant recommendation FL11, the standard tristimulus values X_n, Y_n, Z_n of the entirely matt white surface apply in accordance with Table 2.

Table 2 specifies standard tristimulus values for the frequently used standard illuminants D65 and A as well as illuminant FL11 and both of the standard observers according to CIE 015.

Table 2 — Standard tristimulus values

Standard tristimulus values	2° standard observer			10° standard observer		
	Illuminant					
	D65	A	FL11	D65	A	FL11
X_n	95,04	109,85	100,96	94,81	111,14	103,86
Y_n	100,00	100,00	100,00	100,00	100,00	100,00
Z_n	108,88	35,58	64,35	107,32	35,20	65,61

For fluorescent samples, the illuminant used for measurement shall be adjusted as close as possible to that illuminant for which the standard tristimulus values are determined.

NOTE In contrast to non-fluorescent samples, the calculation of metamerism indices for fluorescent samples is erroneous if the samples are measured only under one illuminant.

8 Metamerism index for change in illuminant

8.1 General calculation methods

Metamerism implies no colour difference under the reference illuminant. The colour difference under the test illuminant is used as metamerism index. This index is described in Formula (1):

$$M_t = \sqrt{\left(\Delta L_t^*\right)^2 + \left(\Delta a_t^*\right)^2 + \left(\Delta b_t^*\right)^2} \quad (1)$$

where

t is the colour under test illuminant;

ΔL_t^* $= L_{spl,corr,t}^* - L_{std,t}^*$;

Δa_t^* $= a_{spl,corr,t}^* - a_{std,t}^*$;

Δb_t^* $= b_{spl,corr,t}^* - b_{std,t}^*$.

In case of a small colour difference already present under reference illuminant conditions, the colour difference at change of illuminant is called paramerism. To eliminate the effect of the difference under reference illuminant, a mathematically corrected virtual sample is created, having no remaining colour difference under the reference illuminant.

Three different correction methods for calculating a metamerism index in the case of paramerism have been proposed in References [6] to [13]. All methods assume that, for practical cases, there can already be a small difference between the colours of the sample and the standard, even under the reference illuminant from the very beginning, due to problems of fabrication. In the case of two methods, called the additive and the multiplicative correction, these inherent colour differences often merge with the difference introduced by the change of the illuminant. The third method, the spectral correction, works more fundamentally by the separation of inherent colour differences under the reference illuminant from those introduced by the change of the illuminant.

NOTE Annex A includes calculation examples.

8.2 Basic calculation of the metamerism index from colour differences

After this correction (see 8.1) leading to the virtual sample, the common formula for a metamerism index at change in illuminant, expressed in CIELAB coordinates for the test illuminant (t), is given by Formula (2):

$$M_{\mathrm{t}}(x)=\sqrt{\left(\Delta L_{\mathrm{corr}}^{*}\right)^{2}+\left(\Delta a_{\mathrm{corr}}^{*}\right)^{2}+\left(\Delta b_{\mathrm{corr}}^{*}\right)^{2}} \tag{2}$$

where

t is the colour under test illuminant;

$\Delta L_{\mathrm{corr}}^{*}$ $= L_{\mathrm{spl,corr,t}}^{*} - L_{\mathrm{std,t}}^{*}$;

$\Delta a_{\mathrm{corr}}^{*}$ $= a_{\mathrm{spl,corr,t}}^{*} - a_{\mathrm{std,t}}^{*}$;

$\Delta b_{\mathrm{corr}}^{*}$ $= b_{\mathrm{spl,corr,t}}^{*} - b_{\mathrm{std,t}}^{*}$;

x nominates the correction method.

Formulae (1) and (2) are provided as examples if using the CIELAB colour space.

Analogous equations apply for other Euclidian colour spaces such as DIN 99o as specified in DIN 6176. In non-Euclidian colour spaces such as CIE 94 or CIEDE2000,[6] the specific colour differences are provided with colour-space dependent weight functions and, in regard to the latter case, are expanded by an additional rotation term. The CIELAB metric used in this document is an example and should be replaced in practical applications by one of the more recent metrics mentioned (e.g. CIE 94, CIEDE2000, DIN 99o), which are significantly more uniform than the CIELAB model.

8.3 Correction methods

8.3.1 Additive correction

When using the additive correction, the differences of colorimetric coordinates between the standard (std) and the sample (spl) under the reference illuminant (ref) are added to the respective differences between the standard and the sample under the test illuminant (t). The resulting calculation for the metamerism index M_{t} (add), expressed in differences of CIELAB coordinates, is then given by Formula (3):

$$M_{\mathrm{t(add)}}=\sqrt{\left(\Delta L_{\mathrm{corr}}^{*}\right)^{2}+\left(\Delta a_{\mathrm{corr}}^{*}\right)^{2}+\left(\Delta b_{\mathrm{corr}}^{*}\right)^{2}} \tag{3}$$

where

$\Delta L_{\mathrm{corr}}^{*}$ $= L_{\mathrm{spl,t}}^{*} - L_{\mathrm{std,t}}^{*} - \Delta L_{\mathrm{ref}}^{*}$;

$\Delta L_{\mathrm{ref}}^{*}$ $= L_{\mathrm{spl,ref}}^{*} - L_{\mathrm{std,ref}}^{*}$.

Analogous relationships apply for Δa^{*} and Δb^{*}. It should be noted that slightly different results are to be expected, if the correction is applied to standard tristimulus values prior to transformation into a uniform colour space such as CIELAB or DIN 99o.

8.3.2 Multiplicative correction

NOTE The multiplicative correction is specified in CIE 015 as the correction method.

When using the multiplicative correction, the standard tristimulus values of the sample (spl), which are observed under test conditions (t) are multiplied with the quotient of the standard tristimulus values of standard (std) and sample (spl), which are obtained under reference conditions (ref). The resulting calculation is given in Formula (4):

$$Y_{\text{spl,corr,t}} = Y_{\text{spl,t}} \frac{Y_{\text{std,ref}}}{Y_{\text{spl,ref}}} \tag{4}$$

in which case, again, analogous combinations for X_{corr} and Z_{corr} apply. Subsequently, a transformation into a uniform colour space (e.g. CIELAB) takes place and results in Formula (5):

$$M_{\text{t}}(\text{multipl}) = \sqrt{\left(\Delta L^*_{\text{corr}}\right)^2 + \left(\Delta a^*_{\text{corr}}\right)^2 + \left(\Delta b^*_{\text{corr}}\right)^2} \tag{5}$$

with

$$\Delta L^*_{\text{corr}} = L^*_{\text{spl,corr,t}} - L^*_{\text{std,t}} \ .$$

Analogous relationships apply for the two remaining specific differences Δa^*_{corr} and Δb^*_{corr} .

8.3.3 Spectral correction

The spectral method considers that under the reference illuminant, minor differences between the tristimulus values of the sample and the standard can already exist, which are not relevant for the metamerism characteristics. In order to first mathematically compensate them and only determine the effective component of metamerism at change in illuminant of sample pairs with given spectral reflectance, it is possible to mathematically split a spectral reflectance into two additive components.

One component describes only the function that is effective for the formation of the colour stimulus under the reference illuminant and the other component describes a function, which does not lead to a contribution to the colour stimulus when integrating via the stimulus under the reference illuminant.

This function necessarily includes positive and negative components. The fundamental colour stimulus function results from the first component of the spectral reflectance under the reference illuminant. This is effective for the formation of the colour. The respective second part of the colour stimulus function leads to a metameric black of the decomposition (residue), i.e. an invisible contribution with a resulting colour stimulus identical to zero.

The compensation of the deviations of the colour stimuli of a test sample from the standard sample, which are non-effective for metamerism characteristics, is realized by replacing the fundamental colour stimulus of the sample by that of the standard. The component that is effective for the metamerism characteristic remains unchanged, i.e. a new colour stimulus function of the sample is generated from the sum of the replaced fundamental colour stimulus and the unchanged second component. The sum determines the metamerism at change in illuminant with regard to the standard.

The mathematical description of the method of spectral correction starts with the general definition of the spectral reflection function of a sample in Formula (6) and the definition of a matrix of spectral weights [Formula (9)] to calculate the expected tristimulus values. This matrix of weights is composed from the spectral illuminant and the spectral matching functions of the observer in Formula (8). The product of the matrix of weights with the spectral reflection function results in the tristimulus values in Formula (11), which appear under the illuminant considered.

As specified in this document by Formulae (12) to (15), a decomposition into visible and invisible parts of colour stimuli uses the splitting of the spectral reflectance function under the defined reference illuminant into a “fundamental reflection function” [Formula (13)], and a “black reflection function” [Formula (14)]. These parts lead to the visible fundamental colour stimulus and the invisible black colour stimulus functions for the illuminant considered. So, it should always be noted that this method is only valid under the assumption that these components of the spectral reflection function describe the visual effects in combination with the spectral distribution of the reference illuminant (D65 in this document). Consequently, the distribution function of the reference illuminant is inherently included in the decomposition of the reflectance functions.

In order to highlight this connection, the components are additionally marked in the text by "for the reference illuminant ", when decomposing the reflectance function of a colour in Formulae (16) to (18). These formulae describe differences and resulting correction terms for a pair of samples.

In the model of the spectral decomposition developed by Cohen and Kappauf,[9],[11] the spectral reflectance of an object colour obtained in the visible spectral range is summarized in the vector in Formula (6):

$$\vec{N} = \begin{pmatrix} \rho(\lambda_1) \\ \rho(\lambda_2) \\ \vdots \\ \rho(\lambda_n) \end{pmatrix} \tag{6}$$

The components of the vector $\vec{N}$ are the reflectance values $\rho(\lambda_i)(i=1,2,\ldots,n)$ of the examined colour, which are discretely present on n intervals. For the calculation of the respective standard tristimulus values X, Y, Z from the reflectance functions, the product based on the supporting points of the distribution function of the used illuminant $S(\lambda)$, the respective standard colour-matching function [see Formula (7)], the distance of supporting points $\Delta\lambda$ and a normalization constant k shall be determined.

$$\bar{\alpha}(\lambda_i) = [\bar{x}(\lambda_i), \bar{y}(\lambda_i), \bar{z}(\lambda_i)] \tag{7}$$

The components $\rho(\lambda_i)$ of the vector $\vec{N}$ are the reflectances of the examined colour. Considering the illuminant $S(\lambda_i)$, the standard tristimulus components *X, Y, Z* are calculated from the sum of the products $S(\lambda_i)\rho(\lambda_i)\bar{\alpha}(\lambda_i)\Delta\lambda$ k with the normalization constant k. The term $\bar{\alpha}(\lambda_i) = [\bar{x}(\lambda_i), \bar{y}(\lambda_i), \bar{z}(\lambda_i)]$ describes the colour matching functions. The constant k is determined from $\text{k} = 1/\sum_i^n S(\lambda_i)\bar{y}(\lambda_i)\Delta\lambda$ for the *Y*-component of the illuminant considered.

These products mentioned above are introduced as weights in Formula (8):

$$w_{\bar{\alpha}}(\lambda_i) = kS(\lambda_i)\bar{\alpha}(\lambda_i)\Delta\lambda \tag{8}$$

All weights of all supporting points for the standard colour-matching curves are given in matrix form in a $n \times 3$ matrix in Formula (9):

$$A = \begin{pmatrix} w_{\bar{x}}(\lambda_1) w_{\bar{y}}(\lambda_1) w_{\bar{z}}(\lambda_1) \\ w_{\bar{x}}(\lambda_2) w_{\bar{y}}(\lambda_2) w_{\bar{z}}(\lambda_2) \\ \vdots \quad \vdots \quad \vdots \\ w_{\bar{x}}(\lambda_n) w_{\bar{y}}(\lambda_n) w_{\bar{z}}(\lambda_n) \end{pmatrix} \tag{9}$$

The transposed vector of the tristimulus values in Formula (10):

$$\vec{W} = \{X, Y, Z\}^T \tag{10}$$

results in accordance with Formula (11):

$$\vec{W} = A^T \cdot \vec{N} \tag{11}$$

from the matrix multiplication of the transposed weighting matrix A^T with the radiometric function $\vec{N}$.

From the matrix A of the integration weights, Cohen and Kappauf[9],[11] constructed an orthogonal $n \times n$ matrix shown in Formula (12):

$$R = A\left(A^T \cdot A\right)^{-1} A^T. \tag{12}$$

The application of the matrix in Formula (13):

$$R \cdot \vec{N} = \vec{N}_{\mathrm{f}} \tag{13}$$

to the radiometric function $\vec{N}$ of an object colour isolates its fundamental reflectance function for the reference illuminant $\vec{N}_{\mathrm{f}}$, which actively forms the process of colour perception.

The tristimulus value $\vec{W}_{\mathrm{r}}$ assigned to the difference in Formula (14):

$$\vec{N}_{\mathrm{r}} = \vec{N} - \vec{N}_{\mathrm{f}} \tag{14}$$

is calculated from Formula (15):

$$\vec{W}_{\mathrm{r}} = A^T \cdot \vec{N}_{\mathrm{r}} \tag{15}$$

and results in zero.

This contribution, identified as metameric black value or residuals, does not bear any colour information and does not become visible during the colour-perception process under the reference illuminant. $\vec{N}_{\mathrm{r}}$ contributes positively as well as negatively. Also, regarding the obtained fundamental colour stimuli, negative vector elements can occur. It is essential that $\vec{N}_i \geq 0 \forall i$ applies for the reflectance function, which is composed of fundamental reflectance function and the metameric black values, in order to be physically realized. The designations of fundamental or black parts of reflectance function consider their action in combination with the reference illuminant, which is included in the weights according to Formula (8). The $n \times n$ projection matrix R depends on the standard observer (2°, 10°) and the used standard illuminant.

Fairman[10],[11],[12],[16] proposed a model of residual colour difference of parameric pairs of samples with minor colour difference. This model was based on the described decomposition of a spectral reflectance function into a fundamental reflectance function for the reference illuminant and the residuals as metameric black values of the reflectance function.

The vectors $\vec{N}_{std}$ and $\vec{N}_{spl}$ describe the reflectance functions of a parameric pair of samples, the *n* elements of which represent the measured reflectance values of the respective object colour within the visible spectral range. They can be decomposed into their fundamental reflectance functions for the reference illuminant and metameric black functions (r) for the reference illuminant by means of the Cohen-Kappauf decomposition in Formula (16) and Formula (17):

$$\vec{N}_{std} = \vec{N}_{f,std} + \vec{N}_{r,std} \tag{16}$$

$$\vec{N}_{spl} = \vec{N}_{f,spl} + \vec{N}_{r,spl}\,. \tag{17}$$

By addition of the residuals of the sample (indicated by the index r,spl) and the fundamental reflectance function of the standard for the reference (indicated by the index f,std), a virtual sample with corrected reflectance function $\vec{N}_{spl,corr}$ is generated. This composition does not show any residual colour difference with regard to the standard under the reference illuminant, but still has the same metameric characteristics compared to the standard. The resulting Formula (18) is:

$$\vec{N}_{spl,corr} = R \cdot \vec{N}_{std} + (I - R) \cdot \vec{N}_{spl} \tag{18}$$

where *I* is the *n* × *n* identity matrix and *R* is the decomposition or projection matrix developed by Cohen and Kappauf. For the calculation of a specific metamerism index for the illuminant metamerism, the experimental reflectance function of sample $\vec{N}_{spl}$ is replaced by the reflectance function $\vec{N}_{spl,corr}$, which has been modified by means of the spectral correction.

NOTE 1 Annex A includes calculation examples.

In summary, the calculation of the metamerism index for the spectral correction method requires the following steps.

1) Measure the radiometric functions $\vec{N}_{std}$ and $\vec{N}_{spl}$ of the reference and the sample, respectively.

2) Calculate the weighting matrix *A* for the reference illuminant and the respective weighting matrix A_t for the test illuminant;

3) Calculate the Cohen–Kappauf matrix *R* for the reference illuminant as shown in Formula (19):

$$R = A\left(A^T \cdot A\right)^{-1} A^T \tag{19}$$

4) Calculate the radiometric function of the corrected sample, according to Formula (20):

$$\vec{N}_{spl,corr} = R \cdot \vec{N}_{std} + (I - R) \cdot \vec{N}_{spl} \tag{20}$$

where *I* is the identity matrix;

5) Calculate the tristimulus values for the reference illuminant in Formula (21) and Formula (22):

$$\vec{W}_{std} = A^{T} \cdot \vec{N}_{std} \tag{21}$$

$$\vec{W}_{spl,corr} = A^{T} \cdot \vec{N}_{spl,corr} \tag{22}$$

and for the test illuminant in Formula (23) and Formula (24):

$$\vec{W}_{std,t} = A_t^{T} \cdot \vec{N}_{std} \tag{23}$$

$$\vec{W}_{spl,corr,t} = A_t^T \cdot \vec{N}_{spl,corr} \qquad (24)$$

NOTE 2 $\vec{W}_{std} = \vec{W}_{spl,corr}$ under the reference illuminant and the fundamental parts of $\vec{W}_{std,t}$ and $\vec{W}_{spl,corr,t}$ under the test illuminant are equalized. Any differences between $\vec{W}_{spl,corr,t}$ and $\vec{W}_{std,t}$ are introduced only by the residual part of $\vec{N}_{spl,corr}$ that appears black for the reference illuminant and changed for the test illuminant.

6) Calculate the colour differences between $W_{std,t}$ and $W_{spl,corr,t}$ using any appropriate system in mind and calculate the metamerism index. For the CIELAB-colour space, for example, the resulting formulae are shown as Formulae (25) to (28):

$$M_t\,(\text{spectr}) = \sqrt{\left(\left(\Delta L^*_{corr}\right)^2 + \left(\Delta a^*_{corr}\right)^2 + \left(\Delta b^*_{corr}\right)^2\right)} \qquad (25)$$

$$\Delta L^*_{corr} = L^*_{spl,corr,t} - L^*_{std,t} \qquad (26)$$

$$\Delta a^*_{corr} = a^*_{spl,corr,t} - a^*_{std,t} \qquad (27)$$

$$\Delta b^*_{corr} = b^*_{spl,corr,t} - b^*_{std,t} \,. \qquad (28)$$

8.4 Test report

In the test report, the used correction method for the residual colour difference under reference conditions shall be indicated in the formula of the metamerism index as argument. In the identifier $M_t\,(x)$, the index “t” is for the used test illuminant (e.g. t = A for test illuminant A or FL11 for test illuminant FL11, etc.) and the argument “x” is for the correction method (x = add for the additive correction, x = multipl for the multiplicative correction, and x = spectr for the spectral Cohen-Kappauf correction). For example, M_A (spectr) is for a metamerism index for the test illuminant A using the spectral Cohen-Kappauf correction.

Further, colour differences should not exceed the range recommended in the respective standards of the colour difference formulae (for CIELAB $\Delta E^*_{ab} < 5$ should be fulfilled).

Annex A
(informative)

Calculation examples

The following calculation examples serve to verify the programmed implementation of the correction methods recommended in this document. As examples, pairs of green samples Green-1, Green-2 and Green-3 have been chosen. Their spectral reflectance factors are given as a function of wavelength in the second and fifth column in Tables A.2 and A.4, respectively. The wavelength ranges from 400 nm to 700 nm with data each 10 nm, based on illumination D65, 10° CIE 1964 observer. Green-1 serves as the standard and Green-2 and Green-3 as the samples, respectively. The colorimetric data of all samples had been calculated in the range of 400 nm to 700 nm using the weighting functions given in ASTM E308-22, Table 5.19. Values at wavelengths below 400 nm and above 700 nm are summed up and included in the results of the 400 nm and 700 nm values. All colour differences are calculated in the CIELAB ΔE^*_{ab}. For both test examples of green pairs under reference conditions (illuminant D65, CIE 1964 10° standard observer), there is a residual colour difference of about $\Delta E^*_{ab} = 3$ units.

All relevant data of both examples are summarized in Table A.2 to Table A.5.

In a first step, the projection matrix R is calculated for the selected reference conditions (illuminant D65, CIE 1964 10° standard observer) in accordance with 8.3. The result of this calculation is given in Figure A.2 and in Table A.1.

Applying this matrix R to the reflectance functions of standard ($\vec{N}_{std}$) and sample ($\vec{N}_{spl}$) leads to the vectors for the fundamental reflectance functions $\vec{N}_{f,std}$ and $\vec{N}_{f,spl}$, the black parts of the reflectance functions (residuals $\vec{N}_{r,std}$ and $\vec{N}_{r,spl}$) and to the summarized spectrally corrected reflectance function of sample $\vec{N}_{spl,corr}$. These are given for the colours Green-1 as standard and Green-2 as sample (see Table A.2) as well as for the colours Green-1 as standard and Green-3 as sample (see Table A.4). In accordance with this method, the corrected functions do not show residual colour differences any more with regard to the standard (sample Green-1) under the reference illuminant D65.

Figure A.1 a) shows the reflectance functions of the standard Green-1 and of the sample Green-2 and, in comparison, the corrected reflectance function of the sample Green-2. This insignificantly differs from the original reflectance function, particularly at short wavelengths. Figure A.1 b) shows the corresponding fundamental reflectance functions of standard Green-1 and of the sample (Green-2) which differ insignificantly.

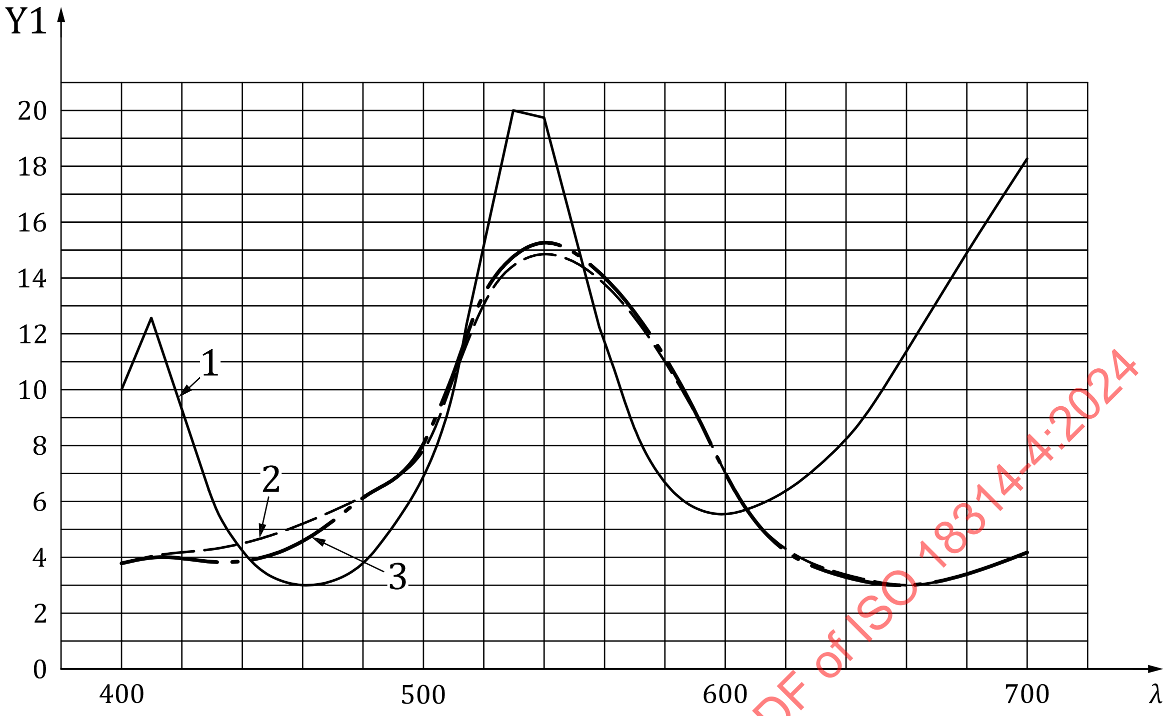
Both the reflectance functions illustrate the components that actively create the visible colours under the reference illuminant D65. Figure A.1 c) shows the residuals of the standard and the sample under the reference illuminant D65, demonstrating the black parts of the reflectance functions, which do not lead to visible colours. These functions necessarily also include negative values, since their integrals, assessed by the spectral power distribution of the reference illuminant and the spectral matching functions of the observer are identical to zero.

Further calculation provides a change of illuminant D65 to illuminant A. All colorimetric data of the pair of samples Green-1 and Green-2 for the reference illuminant and illuminant A are summarized in Table A.3. For illuminant D65, the corrected reflectance function of the sample Green-2, in accordance with the specification, does not lead to a colour difference compared to Green-1, while the parameric residual colour difference of the original sample Green-2 compared to the standard Green-1 is given in column D65/Green-2/spl and equals 3,005 1.

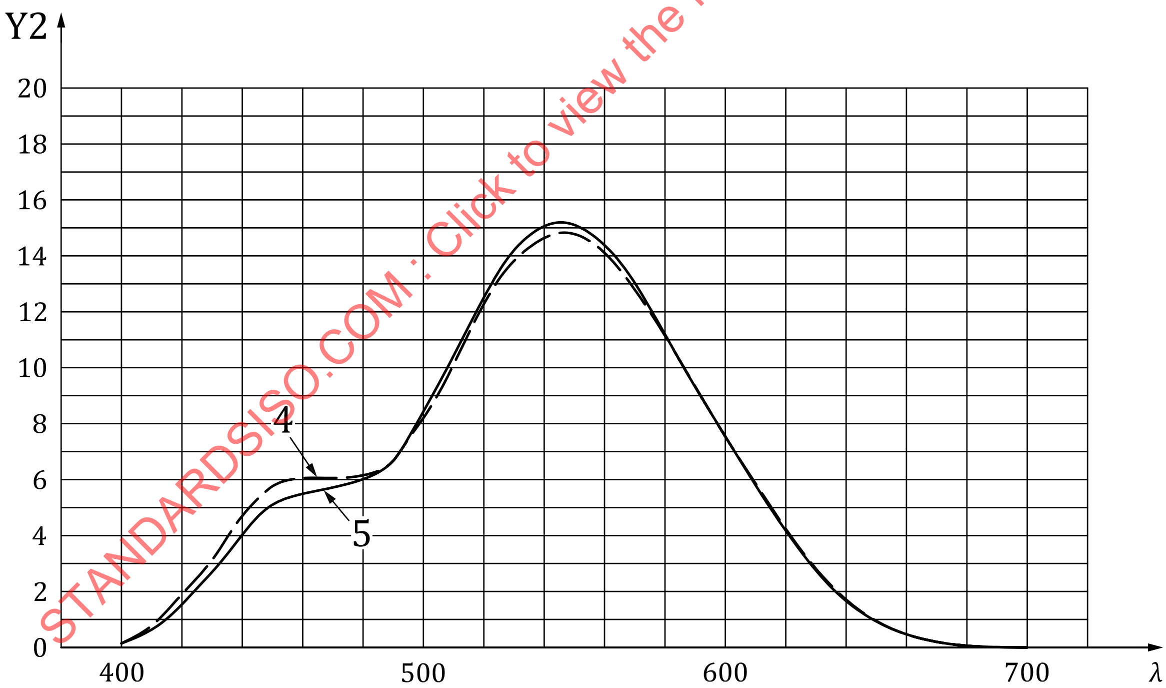
Under illuminant A, the colour calculated from the corrected reflectance function also shows a difference compared to the colour of the standard Green-1. Accordingly, the metamerism index is 3,541 7 in this case. For comparison, the data resulting from the additive and the multiplicative correction and the respective metamerism indices (3,79) are also given in Table A.3. In this case, these data give slightly higher values for the respective metamerism index, since in their definitions the existing colour difference related to paramerism is not removed, thereby spectrally matching the fundamental parts of the reflectance functions.

In the example for colour Green-3, an exception was selected. The colour calculated from the corrected reflectance function coincidentally also does not give any colour difference for illuminant A compared to the calculated colour of standard Green-1 under illuminant A. Accordingly, the metamerism index for the change of illuminant D65 to illuminant A is identical to zero in this case. This characteristic does not become apparent from the definitions of metamerism indices of the additive or multiplicative correction, since both give finite values. Thus, these definitions do not clarify that in this case the colour difference compared to the standard is a mere residual colour difference.

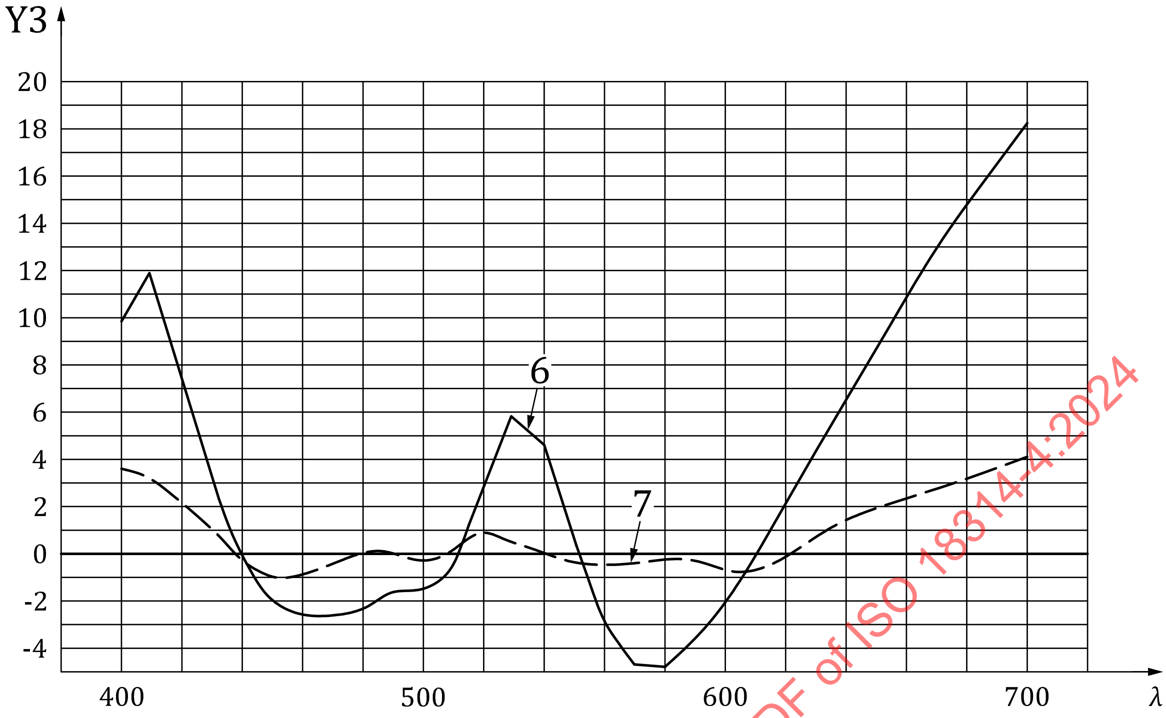
For the calculation of the examples, reflectances below 400 nm and above 700 nm are ignored since the weighting functions applied already include an approximation of rolled up values at 400 nm and 700 nm and, hence, are specified for the range from 400 mm to 700 nm only.



a) Reflectance functions of the standard Green-1, the sample Green-2 and corrected reflectance function of the sample Green-2 under the reference illuminant D65



b) Fundamental reflectance function and corrected reflectance function of the sample Green-2 for the decomposition under the reference illuminant D65



c) Metameric black values (residuals) of the reflectance functions of standard Green-1 and of the sample Green-2 for the decomposition under the reference illuminant D65

Key

λ wavelength, in nm

Y1 spectral reflectance functions, in %

Y2 fundamental reflectance ratios, in %

Y3 metameric black values of the reflectance functions (residuals), in %

1 reference function of the standard, std, Green-1,

2 reference function of the sample, spl, Green-2

3 corrected function of the sample, spl,corr, Green-2

4 fundamental part of the reflectance function of the sample, spl, Green-2

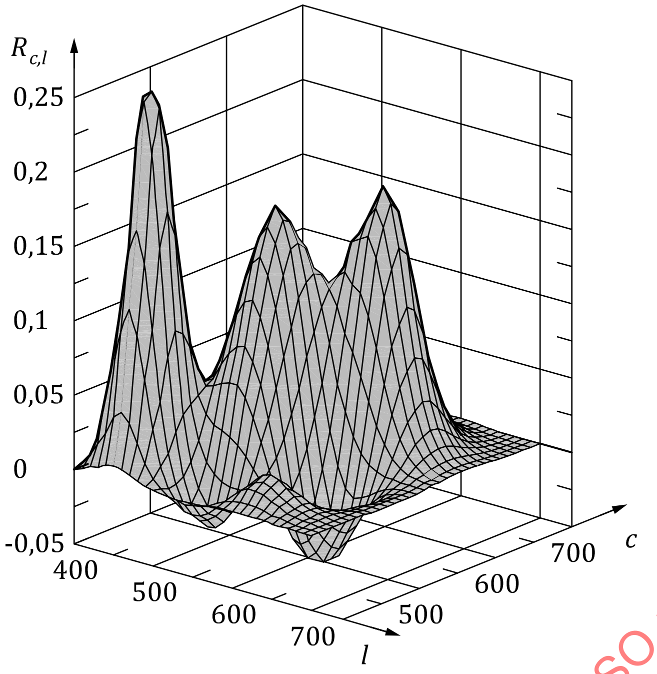
5 corrected fundamental part of the reflectance function of the sample Green-2

6 black part (residual) of the reflectance function of standard Green-1

7 black part (residual) of the reflectance function of the sample Green-2

NOTE The curves correspond to the data compiled in Table A.2 (std – standard Green-1, spl – sample Green-2, spl,corr - corrected sample Green-2).

Figure A.1 — Diagrams of the reflectance functions for the CIE 1964 10° standard colorimetric observer



Key

l	wavelength λ (line), in nm
$R_{c,l}$	intensity
c	wavelength λ (column), in nm

Figure A.2 — Diagram of the orthogonal projection matrix R for the CIE 1964 10° standard colorimetric observer and illuminant D65

Table A.1 — Elements of the orthogonal projection matrix R for the CIE 1964 10° standard colorimetric observer and illuminant D65

R_{ij}	**400**	**410**	**420**	**430**	**440**	**450**	**460**	**470**
400	0,000 300	0,001 446	0,003 529	0,005 560	0,00 7987	0,008 931	0,007 738	0,005 339
410	0,001 446	0,006 936	0,017 004	0,026 799	0,038 523	0,043 113	0,037 416	0,025 920
420	0,003 529	0,017 004	0,041 538	0,065 498	0,094 205	0,105 526	0,091 732	0,063 802
430	0,005 560	0,026 799	0,065 498	0,103 342	0,148 764	0,166 856	0,145 397	0,101 719
440	0,007 987	0,038 523	0,094 205	0,148 764	0,214 407 2	0,240 916	0,210 648	0,148 578
450	0,008 931	0,043 113	0,105 526	0,166 856	0,240 916	0,271 448	0,238 578	0,170 364
460	0,007 738	0,037 416	0,091 732	0,145 397	0,210 648	0,238 578	0,211 737	0,154 646
470	0,005 339	0,025 920	0,063 802	0,101 719	0,148 578	0,170 364	0,154 646	0,118 709
480	0,002 657	0,013 056	0,032 522	0,052 752	0,078 900	0,093 642	0,090 205	0,077 755
490	0,000 739	0,003 854	0,010 146	0,017 731	0,029 100	0,038 887	0,044 374	0,048 931
500	−0,000 370	−0,001 441	−0,002 663	−0,002 132	0,001 277	0,009 076	0,020 796	0,036 582
510	−0,001 046	−0,004 663	−0,010 435	−0,014 101	−0,015 273	−0,008 221	0,007 942	0,031 454
520	−0,001 393	−0,006 336	−0,014 510	−0,020 424	−0,024 058	−0,017 408	0,001 181	0,028 963
530	−0,001 491	−0,006 858	−0,015 883	−0,022 730	−0,027 540	−0,021 456	−0,002 425	0,026 455
540	−0,001 355	−0,006 296	−0,014 733	−0,021 351	−0,026 333	−0,021 322	−0,004 196	0,022 145
550	−0,001 039	−0,004 912	−0,011 674	−0,017 196	−0,021 622	−0,018 118	−0,004 731	0,016 243
560	−0,000 572	−0,002 829	−0,006 985	−0,010 669	−0,013 915	−0,012 295	−0,004 242	0,008 901
570	−0,000 021	−0,000 355	−0,001 378	−0,002 789	−0,004 480	−0,004 934	−0,003 106	0,000 813
580	0,000 512	0,002 046	0,004 080	0,004 907	0,004 771	0,002 330	−0,001 906	−0,007 009
590	0,000 925	0,003 919	0,008 371	0,011 006 0	0,012 166	0,008 217	−0,000 803	−0,013 094
600	0,001 196	0,005 155	0,011 218	0,015 073	0,017 125	0,012 200	−0,000 001	−0,017 101
610	0,001 256	0,005 456	0,0119 689	0,016 230	0,018 647	0,013 560	0,000 489	−0,018 043
620	0,001 106	0,004 826	0,010 631	0,014 482	0,016 731	0,012 288	0,000 656	−0,015 933
630	0,000 838	0,003 663	0,008 085	0,011 040	0,012 791	0,009 441	0,000 585	−0,012 082
640	0,000 573	0,002 509	0,005 547	0,007 586 9	0,008 807	0,006 522	0,000 442	−0,008 273
650	0,000 352	0,001 544	0,003 417	0,004 679	0,005 438	0,004 037	0,000 290	−0,005 089
660	0,000 202	0,000 886	0,001 961	0,002 686	0,003 124	0,002 321	0,000 170	−0,002 919
670	0,000 110	0,000 481	0,001 065	0,001 459	0,001 697	0,001 262	0,000 094	−0,001 584
680	0,000 053	0,000 231	0,000 512	0,000 702	0,000 817	0,000 607	0,000 045	−0,000 762
690	0,000 023	0,000 103	0,000 227	0,000 311	0,000 362	0,000 269	0,000 020	−0,000 338
700	0,000 021	0,000 091	0,000 203	0,000 277	0,000 323	0,000 240	0,000 018	−0,000 301

Table A.1 *(continued)*

R_{ij}	480	490	500	510	520	530	540	550
400	0,002 657	0,000 739	−0,000 370	−0,001 046	−0,001 393	−0,001 491	−0,001 355	−0,001 039
410	0,013 056	0,003 854	−0,001 441	−0,004 663	−0,006 336	−0,006 858	−0,006 296	−0,004 912
420	0,032 522	0,010 146	−0,002 663	−0,010 435	−0,014 510	−0,015 883	−0,014 733	−0,011 674
430	0,052 752	0,017 731	−0,002 132	−0,014 101	−0,020 424	−0,022 730	−0,021 351	−0,017 196
440	0,078 900	0,029 100	0,001 277	−0,015 273	−0,024 058	−0,027 540	−0,026 333	−0,021 622
450	0,093 642	0,038 887	0,009 076	−0,008 221	−0,017 408	−0,021 456	−0,021 322	−0,018 118
460	0,090 205	0,044 374	0,020 796	0,007 942	0,001 181	−0,002 425	−0,004 196	−0,004 731
470	0,077 755	0,048 931	0,036 582	0,031 454	0,028 963	0,026 455	0,022 145	0,016 243
480	0,062 926	0,053 052	0,053 172	0,056 664	0,059 075	0,058 100	0,051 415	0,040 106
490	0,053 052	0,056 995	0,066 416	0,076 526	0,082 899	0,083 445	0,075 312	0,060 250
500	0,053 172	0,066 416	0,083 068	0,098 939	0,109 094	0,111 318	0,101 934	0,083 334
510	0,056 664	0,076 526	0,098 939	0,119 827	0,133 634	0,137 917	0,128 058	0,106 998 6
520	0,059 075	0,082 899	0,109 094	0,133 634	0,150 588	0,157 347	0,148 439	0,127 150
530	0,058 100	0,083 445	0,111 318	0,137 917	0,157 347	0,166 984	0,160 695	0,141 847
540	0,051 415	0,075 312	0,101 934	0,128 058	0,148 439	0,160 695	0,158 503	0,144 952
550	0,040 106	0,060 250	0,083 334	0,106 998 6	0,127 150	0,141 847	0,144 952	0,138 990
560	0,024 691	0,039 026	0,056 479	0,075 812	0,094 518	0,111 281	0,120 529	0,123 964
570	0,006 995	0,014 301	0,024 846	0,038 650	0,055 030	0,073 407	0,089 039	0,102 660
580	−0,010 289	−0,009 986	−0,006 424	0,001 630	0,015 277	0,034 664	0,055 992	0,079 037
590	−0,024 027	−0,029 544	−0,031 968	−0,029 134	−0,018 523	0,000 604	0,025 445	0,054 995
600	−0,033 206	−0,042 722	−0,049 337	−0,050 275	−0,042 071	−0,023 584	0,003 159	0,036 623
610	−0,035 890	−0,047 013	−0,055 608	−0,058 770	−0,052 763	−0,036 298	−0,010 752	0,022 154
620	−0,032 085	−0,042 402	−0,050 730 1	−0,054 514	−0,050 419	−0,037 267	−0,015 927	0,012 035
630	−0,024 482	−0,032 499	−0,039 104	−0,042 363	−0,039 732	−0,030 305	−0,014 602	0,006 169
640	−0,016 835	−0,022 415	−0,027 073	−0,029 486	−0,027 907	−0,021 706	−0,011 177	0,002 845
650	−0,010 387	−0,013 859	−0,016 783	−0,018 348	−0,017 473	−0,013 771	−0,007 392	0,001 145
660	−0,005 965	−0,007 965	−0,009 656	−0,010 572	−0,010 092	−0,007 994	−0,004 358	0,000 518
670	−0,003 240	−0,004 328	−0,005 250	−0,005 752	−0,005 499	−0,004 368	−0,002 402	0,000 239
680	−0,001 559	−0,002 083	−0,002 527 2	−0,002770	−0,002 648	−0,002 105	−0,001 159	0,000 111
690	−0,000 691	−0,000 923	−0,001 121	−0,001 229	−0,001 176	−0,000 936	−0,000 518	0,000 044
700	−0,000 616	−0,000 823	−0,000 997	−0,001 092	−0,001 043	−0,000 826	−0,000 451	0,000 053

Table A.1 *(continued)*

R_{ij}	560	570	580	590	600	610	620	630
400	−0,000 572	−0,000 021	0,000 512	0,000 925	0,001 196	0,001 256	0,001 106	0,000 838
410	−0,002 829	−0,000 355	0,002 046	0,003 919	0,005 155	0,005 456	0,004 826	0,003 663
420	−0,006 985	−0,001 378	0,004 080	0,008 371	0,011 218	0,011 969	0,010 631	0,008 084 6
430	−0,010 669	−0,002 789 1	0,004 907	0,011 006	0,015 073	0,016 230	0,014 482	0,011 040
440	−0,013 915	−0,004 480	0,004 771	0,012 166	0,017 125	0,018 647	0,016 731	0,012 791
450	−0,012 295	−0,004 934	0,002 330	0,008 217	0,012 200	0,013 560	0,012 288	0,009 441 1
460	−0,004 242	−0,003 106	−0,001 906	−0,000 803	−0,000 001	0,000 489	0,000 656	0,000 585
470	0,008 901	0,000 813	−0,007 009	−0,013 094	−0,017 101	−0,018 043	−0,015 933	−0,012 082
480	0,024 691	0,006 995	−0,010 289	−0,024 027	−0,033 206	−0,035 890	−0,032 085	−0,024 482
490	0,039 026	0,014 301	−0,009 986	−0,029 544	−0,042 722	−0,047 013	−0,042 402	−0,032 499 0
500	0,056 479	0,024 846	−0,006 424	−0,031 968	−0,049 337	−0,055 608	−0,050 730 1	−0,039 104
510	0,075 812	0,038 650	0,001 630	−0,029 134	−0,050 275	−0,058 770	−0,054 514	−0,042 363
520	0,094 518	0,055 030	0,015 277	−0,018 523	−0,042 071	−0,052 763	−0,050 419	−0,039 732
530	0,111 281	0,073 407	0,034 664	0,000 604	−0,023 584	−0,036 298	−0,037 267	−0,030 305
540	0,120 529	0,089 039	0,055 992	0,025 445	0,003 159	−0,010 752	−0,015 927	−0,014 602
550	0,123 964	0,102 660	0,079 037	0,054 995	0,036 623	0,022 154	0,012 035	0,006 169
560	0,121 013	0,113 085	0,102 062	0,087 157 2	0,074 505	0,060 190	0,044 744	0,030 624
570	0,113 085	0,120 156	0,123 398 5	0,119 106	0,113 225	0,099 636	0,078 935	0,056 298
580	0,102 062	0,123 398 5	0,140 313	0,146 390	0,147 221	0,134 737	0,109 583	0,079 399
590	0,087 157 2	0,119 106	0,146 390	0,160 716	0,167 060	0,156 198	0,128 776	0,094 047
600	0,074 505	0,113 225	0,147 221	0,167 060	0,177 166	0,167 713	0,139 335	0,102 206
610	0,060 190	0,099 636	0,134 737	0,156 198	0,167 713	0,159 958	0,133 498	0,098 178
620	0,044 744	0,078 935	0,109 583	0,128 776	0,139 335	0,133 498	0,111 721	0,082 290
630	0,030 624	0,056 298	0,079 399	0,094 047	0,102 206	0,098 178	0,082 290	0,060 665
640	0,019 429	0,036 891	0,052 645	0,062 719	0,068 377	0,065 804	0,055 217	0,040 732
650	0,011 275	0,021 964	0,031 626	0,037 843	0,041 355	0,039 853	0,033 469	0,024 700
660	0,006 312	0,012 431	0,017 967	0,021 537	0,023 558	0,022 716	0,019 083	0,014 086
670	0,003 378	0,006 696	0,009 698	0,011 637	0,012 736	0,012 285	0,010 322	0,007 620
680	0,003 217	0,004 662	0,005 595	0,006 124	0,005 908	0,004 964	0,003 665	0,001 621 3
690	0,000 713	0,001 419	0,002 059	0,002 472	0,002 707	0,002 612	0,002 195	0,001 620
700	0,000 651	0,001 283	0,001 855	0,002 223	0,002 432	0,002 345	0,001 970	0,001 454

Table A.1 *(continued)*

R_{ij}	**640**	**650**	**660**	**670**	**680**	**690**	**700**
400	0,000 573	0,000 352	0,000 202	0,000 110	0,000 053	0,000 023	0,000 021
410	0,002 509	0,001 544	0,000 886	0,000 481	0,000 231	0,000 103	0,000 091
420	0,005 547	0,003 417	0,001 961	0,001 065	0,000 512	0,000 227	0,000 203
430	0,007 587	0,004 679	0,002 686	0,001 459	0,000 702	0,000 311	0,000 277
440	0,008 807	0,005 438	0,003 124	0,001 697	0,000 817	0,000 362	0,000 323
450	0,006 522	0,004 037	0,002 321	0,001 262	0,000 607	0,000 269	0,000 240
460	0,000 442	0,000 290	0,000 170	0,000 094	0,000 045	0,000 020	0,000 018
470	−0,008 273	−0,005 089	−0,002 919	−0,001 584	−0,000 762	−0,000 338	−0,000 301
480	−0,016 835	−0,010 387	−0,005 965	−0,003 240	−0,001 559	−0,000 691	−0,000 616
490	−0,022 415	−0,013 859	−0,007 965	−0,004 328	−0,002 083	−0,000 923	−0,000 823
500	−0,027 073	−0,016 783	−0,009 656	−0,005 250	−0,002 527 2	−0,001 121	−0,000 997
510	−0,029 486	−0,018 348	−0,010 5717	−0,005 752	−0,002 770	−0,001 229	−0,001 092
520	−0,027 907	−0,017 473	−0,010 092	−0,005 499	−0,002 648	−0,001 176	−0,001 043
530	−0,021 706	−0,013 771	−0,007 994	−0,004 368	−0,002 105	−0,000 936	−0,000 826
540	−0,011 177	−0,007 392	−0,004 358	−0,002 402	−0,001 159	−0,000 518	−0,000 451
550	0,002 845	0,001 145	0,000 518	0,000 239	0,000 111	0,000 044	0,000 053
560	0,019 429	0,011 275	0,006 312	0,003 378	0,001 621 3	0,000 713	0,000 651
570	0,036 891	0,021 964	0,012 431	0,006 696	0,003 217	0,001 419	0,001 283
580	0,052 645	0,031 626	0,017 967	0,009 698	0,004 662	0,002 059	0,001 855
590	0,062 719	0,037 843	0,021 537	0,011 637	0,005 595	0,002 472	0,002 223
600	0,068 377	0,041 355	0,023 558	0,012 736	0,006 124	0,002 707	0,002 432
610	0,065 804	0,039 853	0,022 716	0,012 285	0,005 908	0,002 612	0,002 345
620	0,055 217	0,033 469	0,019 083	0,010 322	0,004 964	0,002 195	0,001 970
630	0,040 732	0,024 700	0,014 086	0,007 620	0,003 665	0,001 620	0,001 454
640	0,027 360	0,016 597	0,009 466	0,005 121	0,002 463	0,001 089	0,000 977
650	0,016 597	0,010 070	0,005 744	0,003 108	0,001 495	0,000 661	0,000 593
660	0,009 466	0,005 744	0,003 277	0,001 773	0,000 853	0,000 377	0,000 338
670	0,005 121	0,003 108	0,001 773	0,000 959	0,000 461	0,000 204	0,000 183
680	0,002 463	0,001 495	0,000 853	0,000 461	0,000 222	0,000 098	0,000 088
690	0,001 089	0,000 661	0,000 377	0,000 204	0,000 098	0,000 043	0,000 039
700	0,000 977	0,000 593	0,000 338	0,000 183	0,000 088	0,000 039	0,000 035
The elements in this table are calculated for the wavelength range 400 nm ≤ λ ≤ 700 nm and a distance of supporting points of λ = 10 nm.							