INTERNATIONAL STANDARD

ISO 18314-2

First edition
2015-06-01

Analytical colorimetry —

Part 2: Saunderson correction, solutions of the Kubelka-Munk equation, tinting strength, hiding power

Analyse colorimétrique —

Partie 2: Correction de Saunderson, solutions de l'équation de Kubelka-Munk, force colorante, pouvoir couvrant

Reference number
ISO 18314-2:2015(E)

ISO copyright office
Ch. de Blandonnet 8 • CP 401
CH-1214 Vernier, Geneva, Switzerland
Tel. +41 22 749 01 11
Fax +41 22 749 09 47
copyright@iso.org
www.iso.org

Contents

Page

Foreword ... **iv**

1 Scope ... **1**

2 Terms, definitions, symbols, and abbreviated terms ... **1**
- 2.1 Terms and definitions ... 1
- 2.2 Symbols and abbreviated terms ... 2

3 Saunderson correction ... **4**
- 3.1 General ... 4
- 3.2 Incidence diffuse, observation 0° (d/0°) ... 4
- 3.3 Incidence 45°, observation 0° (45°: 0°) ... 4

4 Solution of the Kubelka-Munk equations ... **5**

5 Determination of relative tinting strength and residual colour difference of coloured pigments ... **6**
- 5.1 General ... 6
- 5.2 Principle ... 6
- 5.3 Procedure ... 6
  - 5.3.1 General ... 6
  - 5.3.2 Evaluation of absorption at the absorption maximum ... 7
  - 5.3.3 Evaluation of the weighted ***K*/*S*** sum ... 7
  - 5.3.4 Evaluation by equalizing the tristimulus value, ***Y*** ... ***8***
  - 5.3.5 Evaluation by equalizing the smallest of the tristimulus values ***X***, ***Y***, and ***Z*** ... ***8***
  - 5.3.6 Evaluation by equalizing the shade depth ... 9

6 Determination of hiding power of pigmented media ... **10**
- 6.1 General ... 10
- 6.2 Example for white or light coloured paints with a contrast ratio of 0,98 as hiding power criterion ... 11

7 Repeatability and reproducibility ... **12**

8 Test report ... **12**

Annex A (normative) **Tables of coefficients for calculating a(φ) values (standard illuminant D65 and 10° standard observer)** ... **13**

Annex B (normative) **Tables of coefficients for calculating a(φ) values (standard illuminant C and 2° standard observer)** ... **15**

Bibliography ... **17**

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 256, *Pigments, dyestuffs and fillers*.

ISO 18314 consists of the following parts, under the general title *Analytical colorimetry*:

— *Part 1: Practical colour measurement*

— *Part 2: Saunderson correction, solutions of the Kubelka-Munk equation, tinting strength, hiding power*

— *Part 3: Special indices*

Analytical colorimetry —

Part 2:
Saunderson correction, solutions of the Kubelka-Munk equation, tinting strength, hiding power

1 Scope

This part of ISO 18314 specifies the Saunderson correction for different measurement geometries and the solutions of the Kubelka-Munk equation for hiding and transparent layers. It also specifies methods for the calculations of the tinting strength including the residual colour difference with different criteria and of the hiding power.

The procedures for preparing the samples for these measurements are not part of this part of ISO 18314. They are agreed between the contracting parties or are described in other national or International Standards.

2 Terms, definitions, symbols, and abbreviated terms

2.1 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

2.1.1
tinting strength
measure of the ability of a colorant, based on its absorption, to impart colour to other materials

2.1.2
relative tinting strength
C_{rel}
percentage ratio of those mass fractions of the coloured pigment reference and test samples (m_r and m_t, respectively) that cause the particular tinting strength criterion used to have identical values for the reference and test samples

2.1.3
tinting strength criterion
parameter that describes the colouring effect of a colorant, based on its absorption

Note 1 to entry: The tinting strength criteria used in this part of ISO 18314 are the following:

— value of the Kubelka-Munk function at the absorption maximum;

— weighted sum of the Kubelka-Munk function values;

— tristimulus value Y;

— the smallest of the tristimulus values X, Y, Z;

— shade depth parameter B.

Examples of other tinting strength parameters not used in this part of ISO 18314 are the following:

— unweighted sum of the Kubelka-Munk function values;

— chromaticity given by the three colour coordinates (L*, a*, b*);

— reflectance factor at the absorption maximum.

2.1.4
residual colour difference
colour difference that remains between the white reductions of the reference and test samples when the tinting strength criterion values are the same or have been equalized

EXAMPLE Given by ΔE^*.

2.1.5
standard shade depth
shade depth
measure of the intensity of a colour sensation, which increases with increasing chroma and decreases with increasing lightness

Note 1 to entry: Standard shade depths are values set by convention. For colourimetric purposes, the standard shade depth is defined by the shade depth parameter B = 0, which is calculated from the tristimulus value, *Y*, and the chromaticity coordinates, *x* and *y*.

2.1.6
hiding power
ability of a pigmented medium to hide the colour or the colour differences of a substrate

2.2 Symbols and abbreviated terms

a constant

a^* CIELAB colour coordinate

$a(\varphi)$ factor

$a(\lambda)$ auxiliary variable

b^* CIELAB colour coordinate

$b(\lambda)$ auxiliary variable

B shade depth parameter

C_{rel} relative tinting strength

D_m hiding power value indicating the area of the contrast substrate concerned, in m^2, which can be coated with 1 kg

D_v hiding power value indicating the area of the contrast substrate concerned, in m^2, which can be coated with 1 l

$F(\lambda)$ Kubelka-Munk function

$F'(\lambda)$ modified Kubelka-Munk function

$g(\lambda)$ weighting function (defined as the sum of the colour matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ for a 10° standard observer)

h thickness

K coefficient

$K(\lambda)$	absorption coefficient
$(K/S)_r$	Kubelka-Munk value of reference sample
$(K/S)_t$	Kubelka-Munk value of test sample
L^*	CIELAB lightness
m_r	mass fraction of coloured pigment reference sample
m_t	mass fraction of coloured pigment test sample
n	refractive index
r_0	reflection coefficient at the surface for directional light incident perpendicular from outside
$\overline{r}_0$	reflection coefficient at the surface for directional light incident parallel under 45° from outside
r_2	reflection coefficient for light incident diffusely from the inside of the specimen
$R(\lambda)$	reflectance spectrum
$R(\lambda)_\infty$	reflectance of infinitely thick layer
$R(\lambda)^*$	reflectance of the sample
$R(\lambda)_{ob}^*$	Saunderson-corrected reflectance of the black substrate
$R(\lambda)_{ow}^*$	Saunderson-corrected reflectance of the white substrate
$R(\lambda)_b^*$	Saunderson-corrected reflectance of the sample on black substrate
$R(\lambda)_w^*$	Saunderson-corrected reflectance of the sample on white substrate
$R'(\lambda)$	modified reflectance spectrum including surface effects
s	saturation
$S(\lambda)$	scattering coefficient
T	weighted sum
x, y	chromaticity coordinates
X, Y, Z	tristimulus values
ΔE^*	residual colour difference

ΔE^*_{ab} CIELAB colour difference

φ hue angle

φ_0 closest angle in the table below the hue angle

3 Saunderson correction

3.1 General

For colourimetric calculation it is necessary to account for surface phenomena to obtain viable results. The formulas are known as Saunderson correction, their derivation can be found in References [1] and [2] The necessary coefficients are solutions of the Fresnel formulae[3] depending on the index of refraction for the given binder.

The formulae are derived assuming an ideal surface, a perfectly hiding layer and a perfectly diffuse scattering of light inside the interior of the specimen. Any deviation from these assumptions shall lead to consideration of the usefulness of the following calculations.

The formulae given here are for two of the most widespread geometries: diffuse incidence, 0° observation (d/0°) and 45° incidence, 0° observation (45°/0°). In nearly every colourimeter used, the measurement angle is not 0° but 8°. This deviation is not considered problematic.

The constants necessary for the calculation are the following:

r_0: reflection coefficient at the surface for directional light incident perpendicular from outside. For n = 1,5 r_0 = 0,040.

$\bar{r}_0$: reflection coefficient at the surface for directional light incident parallel under 45° from outside. For n = 1,5, $\bar{r}_0$ = 0,050.

r_2: reflection coefficient for light incident diffusely from the inside of the specimen. For n = 1,5, r_2 = 0,596.

3.2 Incidence diffuse, observation 0° (d/0°)

The constant a = 1 if a gloss trap is closed and a = 0 if the gloss trap is open and the specular reflection is excluded.

$$R(\lambda) = ar_0 + \frac{(1-r_0)(1-r_2)R(\lambda)*}{1-r_2R(\lambda)*} \quad (1)$$

$$\text{for a} = 1: \quad R(\lambda)^* = \frac{R(\lambda)-r_0}{1-r_0-r_2\left[1-R(\lambda)\right]} \quad (2)$$

$$\text{for a} = 1: \quad R(\lambda)^* = \frac{R(\lambda)}{1 - r_0 - r_2 + r_2\left[r_0 + R(\lambda)\right]} \quad (3)$$

3.3 Incidence 45°, observation 0° (45°: 0°)

$$R(\lambda) = \frac{(1-r_0)(1-\bar{r}_0)\frac{1}{n^2}R(\lambda)^*}{1-r_2R(\lambda)^*} \quad (4)$$

$$R(\lambda)^{*} = \frac{n^{2} R(\lambda)}{1 - r_0 - \bar{r}_0 + r_0\bar{r}_0 + n^{2} r_2 R(\lambda)} \tag{5}$$

4 Solution of the Kubelka-Munk equations

The Kubelka-Munk theory describes the reflection of a pigmented layer by two constants: absorption [$K(\lambda)$] and scattering [$S(\lambda)$]. It is based on the following assumptions:

a) ideally diffuse radiation distribution on the irradiation side;

b) ideally diffuse radiation distribution in the interior of the layer;

c) no consideration of surface phenomena resulting from the discontinuity in refractive index.

For an infinitely thick, respectively hiding layer with a reflectance of $R(\lambda)_\infty$, the following solutions are found, which allow the determination of the relation between the scattering and the absorption coefficient:

$$\frac{K(\lambda)}{S(\lambda)} = \frac{\left(1 - R(\lambda)_\infty\right)^{2}}{2\,R(\lambda)_\infty} \equiv F\left(R(\lambda)_\infty\right) \tag{6}$$

respectively the inverse:

$$R(\lambda)_\infty = 1 + \frac{K(\lambda)}{S(\lambda)} - \sqrt{2\left(\frac{K(\lambda)}{S(\lambda)}\right) + \left(\frac{K(\lambda)}{S(\lambda)}\right)^{2}} \tag{7}$$

For the determination of the scattering and absorption coefficient two different methods can be applied (the Saunderson correction shall be used):

Method 1 Measurement of the reflectance of an infinite thick (respectively hiding) layer and the reflectance $R(\lambda)^*$ of a coating of the thickness, h, on a substrate of the reflection $R(\lambda)_0^*$.

$$a(\lambda) = \frac{1}{2}\left(\frac{1}{R(\lambda)_\infty^{*}} + R(\lambda)_\infty^{*}\right) \tag{8}$$

$$b(\lambda) = a(\lambda) - R(\lambda)_\infty^{*} = \frac{1}{2}\left(\frac{1}{R(\lambda)_\infty^{*}} - R(\lambda)_\infty^{*}\right) \tag{9}$$

$$S(\lambda) = \frac{1}{b(\lambda)\,h}\,\mathrm{Arcoth}\,\frac{1 - a(\lambda)\left[R(\lambda)^{*} - R(\lambda)_0^{*}\right] + R(\lambda)^{*}\,R(\lambda)_0^{*}}{b(\lambda)\left[R(\lambda)^{*} - R(\lambda)_0^{*}\right]} \tag{10}$$

$$K(\lambda) = S(\lambda)\left[a(\lambda) - 1\right] \tag{11}$$

Method 2 This method applies two layers of equal thickness (h) on black and white substances. After the determination of the auxiliary variables $a(\lambda)$, $b(\lambda)$ according to Formulae (12) and (13), either Formula (14) or (15) may be used to calculate the scattering coefficient $S(\lambda)$. The possibility with the least experimental uncertainty should be chosen.

$$a(\lambda) = \frac{\left[1 + R(\lambda)_w^{*} R(\lambda)_{ow}^{*}\right]\left[R(\lambda)_b^{*} - R(\lambda)_{ob}^{*}\right] + \left[1 + R(\lambda)_b^{*} R(\lambda)_{ob}^{*}\right]\left[R(\lambda)_{ow}^{*} - R(\lambda)_w^{*}\right]}{2\left[R(\lambda)_b^{*} R(\lambda)_{ow}^{*} - R(\lambda)_w^{*} R(\lambda)_{ob}^{*}\right]} \tag{12}$$

$$b(\lambda) = \sqrt{a(\lambda)^2 - 1} \tag{13}$$

$$S(\lambda) = \frac{1}{b(\lambda)\ h} \text{Arcoth} \frac{1 - a(\lambda)[R(\lambda)_b^* - R(\lambda)_{ob}^*] + R(\lambda)_b^*\ R(\lambda)_{ob}^*}{b(\lambda)\ [R(\lambda)_b^* - R(\lambda)_{ob}^*]} \tag{14}$$

$$S(\lambda) = \frac{1}{b(\lambda)\ h} \text{Arcoth} \frac{1 - a(\lambda)[R(\lambda)_w^* - R(\lambda)_{ow}^*] + R(\lambda)_w^*\ R(\lambda)_{ow}^*}{b(\lambda)\ [R(\lambda)_w^* - R(\lambda)_{ow}^*]} \tag{15}$$

$$R(\lambda)^* = \frac{1 - R(\lambda)_o^*\ [a(\lambda) - b(\lambda)\ \coth\{b(\lambda)\ S(\lambda)\, h\}]}{a(\lambda) - R(\lambda)_o^* + b(\lambda)\ \coth\{b(\lambda)\ S(\lambda)\, h\}} \tag{16}$$

NOTE The formulation of the Kubelka-Munk theory leads to a system of differential equations. The solution can be stated in different ways either by the use of the trigonometric functions used here or by the use of logarithmic functions. They are mathematically equivalent.

5 Determination of relative tinting strength and residual colour difference of coloured pigments

5.1 General

All the methods specified here presuppose, at least approximately, a linear relationship between the concentration of the colorant and the Kubelka-Munk function.

It is assumed that the scattering by the draw-downs being measured is dominated by the white pigment and the absorption by the coloured pigment. All these conditions shall be met to ascertain correct results of the methods described here. The Kubelka-Munk function for the white paste can be neglected in most cases.

5.2 Principle

The reference and test samples are incorporated into white pastes. The corresponding reflectance spectra are measured on opaque draw-downs of the resulting coloured pastes. The appropriate tinting strength criterion is calculated from the measured values.

If the tinting strength criterion values for the reference and test samples differ, the mass fraction of the sample is increased or decreased until the values become equal. This adjustment may be performed either experimentally or mathematically.

If the tinting strength criterion values for the reference and test samples are the same, or after they have been equalized, the residual colour difference between the white reductions of the reference and test samples is calculated from the corresponding reflectance spectra.

A spectrophotometer with d:8° or 8°:d measuring geometry with or without gloss trap, or instruments with 45°:0° or 0°:45° measuring geometry are recommended.

5.3 Procedure

5.3.1 General

The reflectance of an opaque draw-down of the white reduction of the reference sample and the corresponding reflectance of the test sample are measured in the visible spectral range.

5.3.2 Evaluation of absorption at the absorption maximum

The tinting strength criterion is the maximum Kubelka-Munk value. Prerequisite for this method are equal concentrations of reference and test pigments in the white pastes.

Determine the wavelength in the reflectance spectra of the white reductions at which the reflectance is a minimum. From the minimum Saunderson-corrected reflectance R_r* and R_t*, calculate the Kubelka-Munk values $(K/S)_r$ and $(K/S)_t$ for this wavelength by means of Formula (6). The relative tinting strength C_{rel} is then obtained from:

$$C_{rel} = \left[\frac{\left(\frac{K}{S}\right)_t}{\left(\frac{K}{S}\right)_r} \right] \cdot 100 \tag{17}$$

NOTE This method does not involve any explicit equalization of the tinting strength criterion. Because of the assumption of linearity between the Kubelka-Munk function and the concentration, equalization is implicit in the formalism of

$$\frac{\left(\frac{K}{S}\right)_t}{\left(\frac{K}{S}\right)_r} = \frac{m_r}{m_t} \tag{18}$$

Consequently, Formula (17) can be transformed into the defining Formula (19).

$$C_{rel} = \frac{m_r}{m_t} \cdot 100 \tag{19}$$

5.3.3 Evaluation of the weighted *K*/*S* sum

The tinting strength criterion is the weighted *K*/*S* sum. From the spectra of the Saunderson-corrected reflectance $R(\lambda)$* for the test and reference samples, calculate the corresponding Kubelka-Munk values $F(\lambda) = (K/S)(\lambda)$ and in each case generate the following weighted sum:

$$T = \sum_{(400\text{-}700\text{nm})} g(\lambda) \cdot \mathrm{F}(\lambda) \tag{20}$$

The function $g(\lambda)$ is a weighting function, defined as the sum of the colour matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ for a 10° standard observer (see Reference [4]). This weighting function is an empirical function, but without any theoretical foundation.

The relative tinting strength is calculated from the weighted sums and the mass fractions of the test and reference samples:

$$C_{rel} = \left[\frac{(T_t \cdot m_r)}{(T_r \cdot m_t)} \right] \cdot 100 \tag{21}$$

$$= \frac{\left(\frac{m_r}{T_r}\right)}{\left(\frac{m_t}{T_t}\right)} \cdot 100$$

NOTE This method does not involve any explicit equalization of the tinting strength criterion. Because of the assumption of linearity between the Kubelka-Munk function and the concentration, and hence also between the Kubelka-Munk function and the tinting strength criterion *T*, equalization is implicit in the formalism of Formula (21).

If the difference between the tinting strength criterion of the reference sample T_r and that of the test sample T_t is greater than 15 %, the mass fraction of the test sample should be varied accordingly.

To obtain the residual colour difference, the Kubelka-Munk function of the test sample is modified as follows:

$$F_t'(\lambda) = F_t(\lambda) \cdot \frac{T_r}{T_t} \quad (22)$$

Solving Formula (6) for R [as done in Formula (7)], calculate a modified reflectance spectrum $R_r{}^{*\prime}(\lambda)$ for the test sample from its modified Kubelka-Munk function $F_t'(\lambda)$ and then subject it to an inverse Saunderson correction (see Clause 3) to obtain a modified $R_t'(\lambda)$ that includes surface effects. This spectrum yields the colour coordinates of the white reduction of the test sample after equalizing the tinting strength. Calculate the residual colour difference from the reflectance spectrum $R_r(\lambda)$ of the white reduction of the reference sample and the modified reflectance spectrum $R_t'(\lambda)$.

5.3.4 Evaluation by equalizing the tristimulus value, *Y*

The tinting strength criterion is the tristimulus value, Y. From the reflectance spectra, $R(\lambda)$, for the white reductions of the test and reference samples, calculate the tristimulus value Y_r for the reference sample and Y_t for the test sample (see Reference [5]). The contracting parties agree on the standard illuminant and standard observer used.

From Formula (6) and the Saunderson-corrected reflectance $R_t{}^*(\lambda)$ for the white reduction of the test sample, calculate the corresponding Kubelka-Munk function $F_t(\lambda)$ for the test sample. Adjust Y_t to the value for the reference sample by varying m_t and then use the resulting value of m_t to determine a modified Kubelka-Munk function $F_t'(\lambda)$ for the test sample:

$$F_t'(\lambda) = F_t(\lambda) \cdot \frac{m_t'}{m_t} \quad (23)$$

Solving Formula (6) for R, calculate a modified reflectance spectrum $R_t{}^{*\prime}(\lambda)$ for the test sample from its modified Kubelka-Munk function $F_t'(\lambda)$ and subject it to an inverse Saunderson correction to obtain a modified $R_t'(\lambda)$ that includes surface effects. From this spectrum determine Y_t'.

Vary the mass fraction of the test sample until the tinting strength criterion has been equalized, i.e.:

$$Y_r = Y_t' \quad (24)$$

This variation of m_t is best carried out by an iterative mathematical procedure, but may also be done experimentally.

Calculate the relative tinting strength from the mass fraction m_t' of the test sample that results in equalization of the tinting strength:

$$C_{rel} = \left[\frac{m_r}{m_t'}\right] \cdot 100 \quad (25)$$

Calculate the residual colour difference from the reflectance spectrum $R_r(\lambda)$ of the white reduction of the reference sample and the modified reflectance spectrum $R_t'(\lambda)$ of the white reduction of the test sample.

5.3.5 Evaluation by equalizing the smallest of the tristimulus values *X*, *Y*, and *Z*

The tinting strength criterion is the smallest of the tristimulus values X, Y, and Z. From the reflectance spectra $R(\lambda)$ for the white reductions of the test and reference samples, calculate the corresponding tristimulus values X_r, Y_r, and Z_r for the reference sample and X_t, Y_t, and Z_t for the test sample (see Reference [5]). The contracting parties agree on the standard illuminant and standard observer used.

The tinting strength criterion is defined to be the tristimulus value that has the smallest numerical value for the test and reference samples.

The subsequent procedure is as described in 5.3.4, but replacing the tristimulus value Y by the smallest of the calculated tristimulus values (X, Y, or Z).

5.3.6 Evaluation by equalizing the shade depth

The tinting strength criterion is the shade depth parameter B. From the reflectance spectra $R(\lambda)$ for the white reductions of the test and reference samples, calculate the tristimulus values Y and the chromaticity coordinates x and y (see Reference [5]). This may be carried out with standard illuminant D65 and a 10° standard observer or with standard illuminant C and a 2° standard observer. The contracting parties agree on the parameters used.

From the values of Y, x, and y for the test and reference samples, calculate the shade depth parameter B of the white reductions for a standard shade depth of 1/1, 1/3, 1/9, 1/25, or 1/200. The standard shade depth used should preferably be that which yields the smallest value of B. However, it may also be a shade depth agreed on by the contracting parties. The standard shade depth used is quoted together with the result for the relative tinting strength.

The shade depth parameters for the various standard shade depths are calculated as follows:

$$B_{1/1} = \sqrt{Y} \cdot \left(s \cdot a\left(\varphi\right)_{1/1} - 10\right) + 19 \qquad (26)$$

$$B_{1/3} = \sqrt{Y} \cdot \left(s \cdot a\left(\varphi\right)_{1/3} - 10\right) + 29 \qquad (27)$$

$$B_{1/9} = \sqrt{Y} \cdot \left(s \cdot a\left(\varphi\right)_{1/9} - 10\right) + 41 \qquad (28)$$

$$B_{1/25} = \sqrt{Y} \cdot \left(s \cdot a\left(\varphi\right)_{1/25} - 10\right) + 56 \qquad (29)$$

$$B_{1/200} = \sqrt{Y} \cdot \left(s \cdot a\left(\varphi\right)_{1/200} - 10\right) + 73 \qquad (30)$$

where

s is the saturation: 10 times the linear distance between the chromaticity coordinates x, y and the achromatic point, i.e.:

$$s = 10 \cdot \sqrt{\left(x - x_0\right)^2 + \left(y - y_0\right)^2} \qquad (31)$$

with x_0 = 0,3138 and y_0 = 0,3310 for standard illuminant D65 and a 10° standard observer and x_0 = 0,3101 and y_0 = 0,3162 for standard illuminant C and a 2° standard observer

φ is the hue angle (in degrees):

$$\varphi = \frac{180°}{\pi} \cdot \arctan \frac{y - y_0}{x - x_0} \qquad (32)$$

with

$0° < \varphi < 90°$ for $y - y_0 > 0$ and $x - x_0 > 0$

$90° < \varphi < 180°$ for $y - y_0 > 0$ and $x - x_0 < 0$

$180° < \varphi < 270°$ for $y - y_0 < 0$ and $x - x_0 < 0$

$270° < \varphi < 360°$ for $y - y_0 < 0$ and $x - x_0 > 0$

$a(\varphi)$ are factors:

$$a(\varphi) = a(\varphi_0) + K_1 \cdot W + K_2 \cdot W^2 + K_3 \cdot W^3 \qquad (33)$$

Tables of coefficients K_1, K_2, and K_3 for standard shade depths 1/1, 1/3, 1/9, 1/25, and 1/200 are given in Annex A for standard illuminant D65 and a 10° standard observer and in Annex B for standard illuminant C and a 2° standard observer.

$$W = \frac{\varphi - \varphi_0}{100°} \qquad (34)$$

φ_0 is the closest angle in the table below the hue angle φ (in degrees). The coefficients K_1, K_2, and K_3 of φ_0 are used in Formula (33).

From Formula (6) and the Saunderson-corrected reflectance $R(\lambda)^*$ for the test sample, calculate the corresponding Kubelka-Munk functions $F(\lambda)$ for the test and reference samples. Adjust the B values for the white reductions of the test and reference samples to the specified standard shade depth by varying the mass fractions m_t and m_r, and use the resulting values of m_t and m_r to determine modified Kubelka-Munk functions $F'(\lambda)$ as follows:

$$F_t'(\lambda) = F_t(\lambda) \cdot \frac{m_t'}{m_t} \qquad (35)$$

$$F_r'(\lambda) = F_r(\lambda) \cdot \frac{m_r'}{m_r} \qquad (36)$$

Solving Formula (6) for R, calculate modified reflectance spectra $R'(\lambda)^*$ from these modified Kubelka-Munk functions $F'(\lambda)$ and subject the spectra to an inverse Saunderson correction (see Clause 3) to obtain modified $R(\lambda)$ that include surface effects. From these spectra calculate the B values for the white reductions of the test and reference samples.

Vary the mass fractions m_t and m_r independently of one another, preferably by an iterative mathematical procedure, until the shade depth of the white reductions of the test and reference samples equals that of the specified standard shade depth, i.e. until:

$B_t = 0$ for m_t' and

$B_r = 0$ for m_r'

Then calculate the relative tinting strength from:

$$C_{rel} = \left[\frac{m_r'}{m_t'}\right] \cdot 100 \qquad (37)$$

Calculate the residual colour difference (see Reference [6]) from those reflectance spectra $R_t'(\lambda)$ and $R_r'(\lambda)$ for the test and reference samples that result in equalization of the shade depth.

6 Determination of hiding power of pigmented media

6.1 General

The hiding power value indicates what area, in square metres, of a given contrast substrate can be coated with the unit of quantity of the sample in such a manner that a specified hiding criterion is achieved. The hiding criterion shall be an agreed colour difference between the two contrasting areas of the coated contrast substrate. A $\Delta E^*_{ab} = 1$ is commonly applied. For achromatic coatings, a contrast ratio of 0,98 is used to take into account the 2 % threshold value for brightness perception by the human eye.

The substrates have of course to be standardized for an exact method of determination. Values of reflectance close to zero for black and 0,8 for white substrates are employed.

The parameter determined by all methods is the minimal film thickness h_D necessary to fulfill the criteria used. The reciprocal of this parameter is equivalent to hiding power value $D = 1/h_D$ .The hiding power value can be determined and specified in the following ways:

— hiding power value D_v in square metres per litre;

— hiding power value D_m, in square metres per kilogram.

D_v and D_m each indicate the area of the contrast substrate concerned, in square metres, which can be coated with 1 l or 1 kg of the pigmented medium so as to ensure hiding (in the sense of the hiding criterion).

Colourimetric methods for the determination of the hiding power use the general solutions for the Kubelka-Munk equation in combination with an iterative program to determine the necessary film thickness to fulfill the hiding power criteria chosen. This is the most straightforward way leading to the best results.

It is possible to work with two non hiding layers on white and black substances or with a hiding layer in combination with a non hiding one on a black or white substrate. The method should preferably be determined by reviewing the data available.

6.2 Example for white or light coloured paints with a contrast ratio of 0,98 as hiding power criterion

The following equations specify a method applicable for white and light-coloured paints where the colour differences between black and white substrates is determined by the lightness differences. In such cases $R(\lambda)$ can be replaced by $R(Y)$ with $R(Y)$ being the tristimulus value Y divided by 100.

The following equations give the reflectance over a black $R(Y)_B$* and white $R(Y)$* substrate:

$$R(Y)_B^* = \frac{1}{\mathrm{a}(Y) - R(Y)_0^* + \mathrm{b}(Y)\,\coth\{\mathrm{b}(Y)\,S(Y)h\}} \tag{38}$$

$$R(Y)^* = \frac{1 - R(Y)_0^*[\mathrm{a}(Y) - \mathrm{b}(Y)\coth\{\mathrm{b}(Y)S(Y)h\}]}{\mathrm{a}(Y) - R(Y)_0^* + \mathrm{b}(Y)\coth\{\mathrm{b}(Y)S(Y)h\}} \tag{39}$$

A reflectance of 0,8 is assumed as $R(Y)_0^*$. So $R(Y)_B^* / R(Y)^*$ is with a contrast ratio of 0,98:

$$\frac{R(Y)_B^*}{R(Y)^*} = \frac{a(Y) - 0{,}8 + \mathrm{b}(Y)\coth\{\mathrm{b}(Y)S(Y)h_{0,98}\}}{[a(Y) + b(Y)\coth\{\mathrm{b}(Y)S(Y)h_{0,98}\}][1 - 0{,}8(a(Y) + b(Y)\coth\{b(Y)S(Y)h_{0,98}\})]} = 0{,}98 \tag{40}$$

this can be solved to

$$h_{0,98} = \frac{1}{b(Y)S(Y)}\,\mathrm{arccoth}\left(\frac{0{,}02 + \sqrt{V(Y)}}{1{,}568\,\mathrm{b}(Y)}\right) \tag{41}$$

with

$$V(Y) = 3{,}136\, a(Y)\left[1 - 0{,}98\left\{1 - 0{,}8a(Y)\right\}\right] - 2{,}5084 \tag{42}$$

The equations above give a result of

$$D(Y) = \frac{\mathrm{b}(Y)S(Y)}{\operatorname{arccoth}\left(\frac{0{,}02 + \sqrt{V(Y)}}{1{,}568\, b(Y)}\right)} \tag{43}$$

$a(Y)$, $b(Y)$, and $S(Y)$ have the same meaning as defined in Formulae 12 to 15.

7 Repeatability and reproducibility

Typically the choice of the correct sampling and the preparation of the test specimen have influences on the result of the measurement far superior to those of the calculation applied. It shall be ascertained by suitable measurements and checked by mathematical analysis (error propagation) that the parameter determined are valid.

8 Test report

The equations outlined in the text above shall be properly cited in the national and international standards in which they are used so that the influence on the test results can be investigated in case of interest.

Annex A
(normative)

Tables of coefficients for calculating a(φ) values (standard illuminant D65 and 10° standard observer)

The coefficients for different standard colour depths (SD) are given in Tables A.1 to A.5 for standard illuminant D65 and 10° standard observer.

Table A.1 — 1/1 SD

Range	φ_0	$a(\varphi_0)$	K_1	K_2	K_3
1	0,0	2,250	1,850 34	8,681 15	-11,632 8
2	60,0	3,973	1,512 13	-6,235 96	3,898 74
3	156,0	3,127	-1,218 51	5,390 38	-8,918 09
4	224,0	1,987	-4,006 96	-47,812 5	284,023
5	240,0	1,285	7,426 88	-17,554 7	8,955 08
6	268,0	2,185	0,822 426	0,741 241	-2,732 09
7	340,0	2,142	-1,032 000	8,056 27	-0,828 125

Table A.2 — 1/3 SD

Range	φ_0	$a(\varphi_0)$	K_1	K_2	K_3
1	0,0	2,040	1,801 64	9,156 25	-12,686 5
2	52,0	3,669	1,445 9	-3,590 46	2,000 06
3	140,0	3,524	1,218 93	-8,803 59	7,202 39
4	196,0	2,710	-0,562 195	-9,452 64	1,992 19
5	236,0	1,101	-4,187 35	16,971 7	118,672
6	252,0	1,351	7,984 62	-9,832 03	-14,277 3
7	276,0	2,504	1,549 35	-3,910 61	1,621 25
8	340,0	2,319	-2,578 89	-4,038 09	50,031 2

Table A.3 — 1/9 SD

Range	φ_0	$a(\varphi_0)$	K_1	K_2	K_3
1	0,0	2,345	-0,634 155	11,219 2	-7,883 79
2	52,0	3,94	2,405 70	-4,966 34	2,477 12
3	148,0	3,864	-1,289 06	1,005 37	-3,893 8
4	224,0	1,756	-4,939 51	-20,88 96	188,117
5	244,0	1,438	5,801 15	-1,049 80	-12,962 9
6	280,0	2,785	1,678 51	-3,350 52	1,463 81
7	336,0	2,932	-1,5271	-15,6045	49,1836