
**Optics and optical instruments —
Field procedures for testing geodetic
and surveying instruments —**

**Part 9:
Terrestrial laser scanners**

*Optique et instruments d'optique — Méthodes d'essai sur site des
instruments géodésiques et d'observation —*

Partie 9: Scanners laser terrestres

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

A list of all parts in the ISO 17123 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document specifies field procedures for adoption when determining and evaluating the uncertainty of measurement results obtained by geodetic instruments and their ancillary equipment, when used in building and surveying measuring tasks. Primarily, these tests are intended to be field verifications of suitability of a particular instrument for the immediate task. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposely designed to minimize atmospheric influences.

The definition and concept of uncertainty as a quantitative attribute to the final result of measurement was developed mainly in the last two decades, even though error analysis has already long been a part of all measurement sciences. After several stages, the CIPM (Comité Internationale des Poids et Mesures) referred the task of developing a detailed guide to ISO. Under the responsibility of the ISO Technical Advisory Group on Metrology (TAG 4), and in conjunction with six worldwide metrology organizations, a guidance document on the expression of measurement uncertainty was compiled with the objective of providing rules for use within standardization, calibration, laboratory accreditation and metrology services. ISO/IEC Guide 98-3 was first published in 1995.

With the introduction of uncertainty in measurement in ISO 17123 (all parts), it is intended to finally provide a uniform, quantitative expression of measurement uncertainty in geodetic metrology with the aim of meeting the requirements of customers.

ISO 17123 (all parts) provides not only a means of evaluating the precision (experimental standard deviation) of an instrument, but also a tool for defining an uncertainty budget, which allows for the summation of all uncertainty components, whether they are random or systematic, to a representative measure of accuracy, i.e. the combined standard uncertainty.

ISO 17123 (all parts) therefore provides, for defining for each instrument investigated by the procedures, a proposal for additional, typical influence quantities, which can be expected during practical use. The customer can estimate, for a specific application, the relevant standard uncertainty components in order to derive and state the uncertainty of the measuring result.

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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 9: Terrestrial laser scanners

1 Scope

This document specifies field procedures for determining and evaluating the precision (repeatability) of terrestrial laser scanners and their ancillary equipment when used in building, civil engineering and surveying measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand, and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

This document can be thought of as one of the first steps in the process of evaluating the uncertainty of measurements (more specifically of measurands).

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

3 Terms and definitions

For the purpose of this document, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, ISO/IEC Guide 98-3 and ISO/IEC Guide 99 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

4 Symbols and subscripts

4.1 Symbols

Symbol	Quantity	Unit
c	sensitivity coefficient	—
d	calculated distance	m and mm
\bar{d}	calculated mean distance	m and mm
d_{obs}	measured distance	m
Δ	distance difference	m and mm
e	eccentricity	mm
F	F-distribution	—
l	turning axis error	°
k	coverage factor	—
k_0	zero point error	m and mm
z_0	zero point error	m and mm
ν	degree of freedom	—
Ω	sum of squared residual	m ² and mm ²
θ	tilting angle	°
φ	turning angle	°
r	residual calculated by means of single distances	m and mm
\bar{r}	residuals calculated by means of the mean distances	m and mm
S	instrument station	—
\hat{s}, s	experimental standard deviation for a precision measure	m and mm
\hat{s}_0	experimental standard deviation for an accuracy measure	m and mm
σ	theoretical standard deviation of a population	m and mm
T	target point	—
u	uncertainty	various
U	expanded uncertainty with coverage factor k	various
x, y, z	Cartesian coordinate	m
χ	Chi-Square distribution	—
ζ_v	Index error of tilting axis	°
ζ	resolution	mm

4.2 Subscripts

Subscript	Term
c	collimation axis error
cen	centring of targets
d	calculated distance
\bar{d}	calculated mean distance
Δ	difference
diff	diffusion of the measuring beam
ec	eccentricity of the collimation axis
l	turning axis error
ia	incident angle
ij	index for target
ISO-TLS	of standard uncertainty of the TLS (type A)
k_0	zero point error
m	maximum
ms	manufacture specification
m_0	scale factor
n	index for station
φ	turning angle
p	typical influence quantities for the TLS measurements
pr	pressure
pri	primary rotation axis
r	measured range
rc	roughness
rh	relative humidity
S	instrument station
se	sighting axis deviation
sec	secondary rotation axis
sta	stability setup
T	target point
temp	temperature
θ	tilting angle
v_0	tumbling deviation
w	set number or repetition number
xyz	3D point
x,y,z	cartesian coordinate
ζ_v	index of tilting axis
ζ_θ	resolution tilting angle
ζ_φ	resolution turning angle

5 Requirements and recommendations

Before commencing the measurements, the operator shall ensure that the precision in use of the measuring equipment is appropriate for the intended measuring task.

The laser scanner and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's handbook.

The coordinates are considered as observables because of modern laser scanners they are the standard output quantities. All coordinates shall be measured on the same day. The instrument need not, but may be levelled.

Meteorological data shall be recorded during the data acquisition in order to derive atmospheric corrections. If possible the option for meteorological corrections within the software of the laser scanner should be used. If the systematic deviations, created by the non-consideration of the atmospheric corrections are too significant, and the automatic correction is not possible, then the raw distances shall be corrected manually.

The operator should note the actual weather conditions at the time of measurement and the type of surface on which the measurements are made. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high, and therefore they are not practicable for most users. In addition, laboratory tests yield precisions much higher than those that can be obtained under field conditions.

This document describes one field procedure with two different amounts of work as given in [Clauses 7](#) and [8](#). If enough time is available, the full test procedure according to [Clause 8](#) is recommended. It allows a more refined and reliable judgement of the instrument.

6 Test principle

6.1 General

As raw observation values of laser scanners the x-, y- and z-coordinates of single points are treated. In contradiction to other geodetic instruments, for example total stations, these coordinates do not have a representative geometrical meaning. Furthermore, single points cannot be reproduced by repetition. The quality of the scans can only be derived from estimated geometrical elements, like planes, spheres or cylinders.

In the proposed test procedures the targets and the software for the target centre detection, which are both important parts of the standard laser scanner equipment, shall be used as key elements for the evaluation of the achievable precision. The 3D distances between the targets serve as indicator for the quality of the measurements. The distances are chosen as datum independent measures for levelled and non-levelled instruments.

Other targets, like spheres, may be used instead of the standard targets, which are recommended by the manufacturer.

6.2 Procedure 1: Simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given laser scanner equipment is within the specified permitted deviation in accordance with ISO 4463-1.

The simplified test procedure is based on a limited number of measurements. This test procedure relies on measurements of x-, y- and z-coordinates in a test field without nominal values.

An accurate standard deviation cannot be obtained. If a more precise assessment of the laser scanner under field conditions is required, the more rigorous full test procedure as given in [Clause 8](#), should be used.

6.3 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision and partly accuracy of a laser scanner and its ancillary equipment under field conditions within an acceptable time. The geometry of the test field is identical to the geometry of the simplified test

procedure. In this test procedure three series of measurements are taken instead of one series as in the simplified test procedure. In addition, the statistical tests are applicable only for this test procedure.

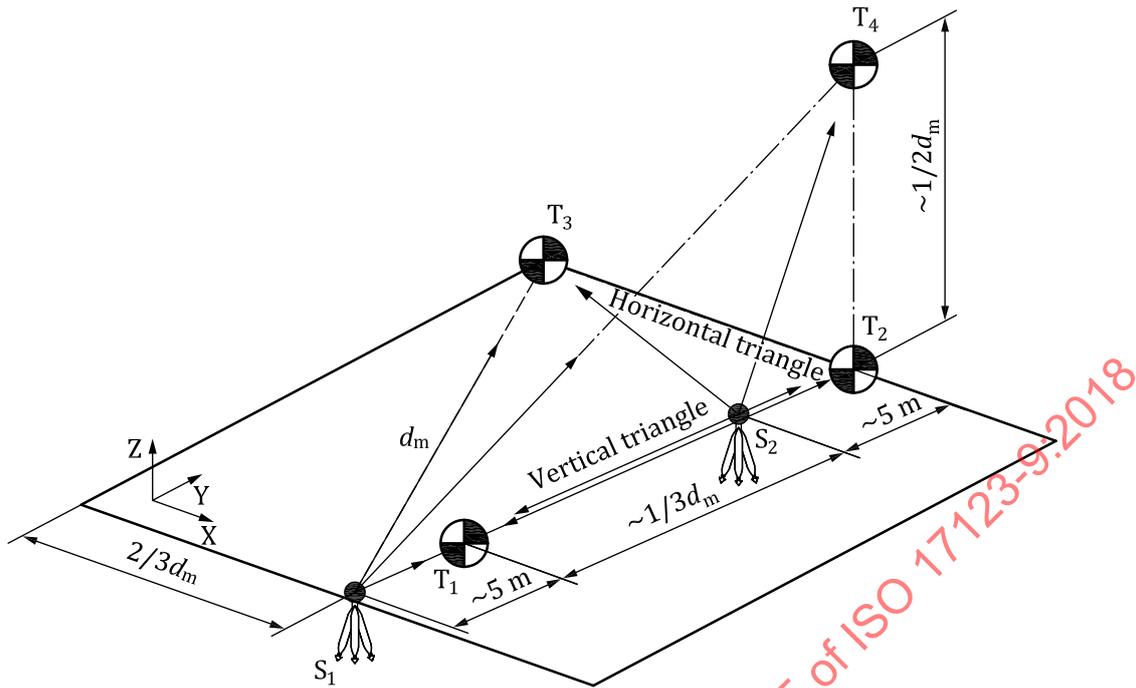
7 Simplified test procedure

7.1 Configuration of the test field

In total two instrument positions and four target marks, also called targets, are arranged in a horizontal and a vertical triangle. The measurement setup is shown in [Figure 1](#) and [Figure 2](#). Both triangles share one edge. The two instrument stations S_1 and S_2 as well as the two targets T_1 and T_2 are aligned on the shared edge. This is necessary in particular for the determination of a systematic distance deviation. The dimensions of the triangles and also the distance between the two instrument stations are determined essentially by the range of the examined TLS and by the maximum distance for capturing the targets as recommended by the manufacturer.

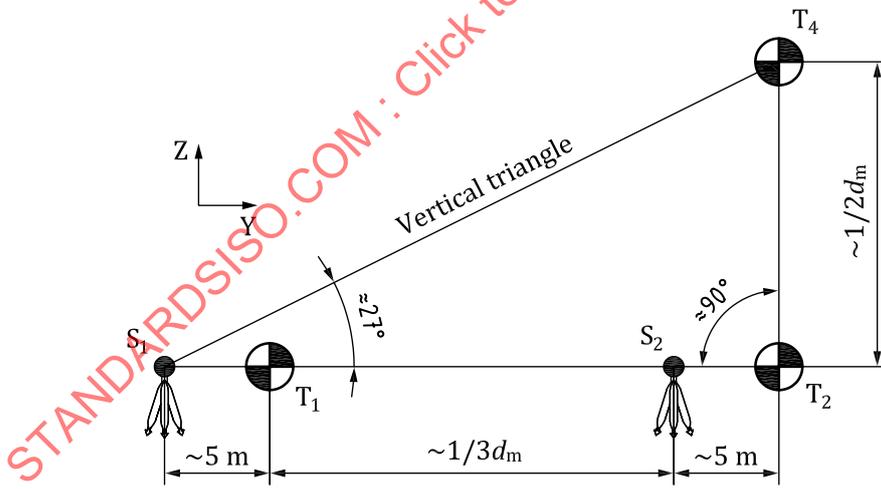
The following recommendations concerning the measurement setup shall be taken into account:

- The two instrument stations S_1 and S_2 as well as the two targets T_1 and T_2 shall be aligned on a line in space.
- Both the horizontal and the vertical triangle shall be realized as right-angled triangles (each with a right angle at target T_2).
- The hypotenuse S_1T_3 of the horizontal triangle shall match the maximum recommended distance for target capturing. This distance will be called maximum distance d_m in the following.
- The distance T_2T_4 of the vertical triangle should be made as long as the local conditions permit and it shall however be at least one third of the maximum distance. Moreover, the target T_4 shall be observed in steep sighting. The minimum value of 27° for the tilting angle, under which T_4 shall be observed from S_1 , is recommended (see [Figure 2](#)). [Table 1](#) gives some examples for possible configuration of the test field.
- The desirable ratio of the cathetus in the vertical triangle is 1:1. If possible, a ratio of 2:1 shall not be exceeded, however only in case the site allows for placing T_4 sufficiently high.



Key
 S₁, S₂ instrument station
 T₁, T₂, T₃, T₄ target point
 d_m maximum distance

Figure 1 — Configuration of the test field



Key
 S₁, S₂ instrument station
 T₁, T₂, T₃, T₄ target point
 d_m maximum distance

Figure 2 — Vertical plane of the test field

Table 1 — Examples of distances for the testfield-setup based on the maximum distance d_m

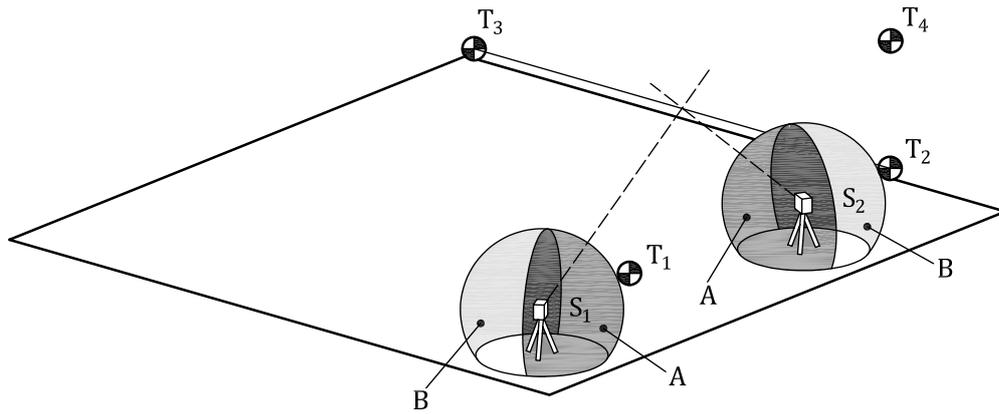
$ S_1T_3 ^a$ m	$ S_2T_1 ^b$ m	$ S_1T_4 ^c$ m	$ S_2T_3 ^d$ m	$ S_2T_4 ^e$ m	Elev. Angle on S_1 to T_4 °
20,00	6,67	19,44	11,06	10,00	30,964
25,00	8,33	22,19	17,00	12,50	34,287
30,00	10,00	25,00	22,36	15,00	36,870
35,00	11,67	27,85	27,49	17,50	38,928
40,00	13,33	30,73	32,49	20,00	40,601
45,00	15,00	33,63	37,42	22,50	41,987
50,00	16,67	36,55	42,30	25,00	43,152
$a \quad d_m$ $b \quad 1/3 d_m$ $c \quad \sqrt{ T_2T_4 ^2 + (5m+ S_2T_1 +5m)^2}$ $d \quad \sqrt{d_m^2 - (5m+ S_2T_1 +5m)^2}$ $e \quad 1/2 d_m$					

- Several laser scanners deflect the laser beam by rotations about two orthogonal axes, one slowly rotating axis (primary rotation axis) and one fast rotating axis (secondary rotating axis). This type of laser scanners typically can scan the complete surrounding by turning only by 180° about the slowly rotating axis, while the fast rotating axis deflects the laser beam to the front side (face I) as well as to the back (face II) of the laser scanner. In order to detect systematic deviations (e.g. axis misalignments) of the laser scanner and reliably check the instrument the following orientation rule is required.

On station S_1 as well as on S_2 the face in which T_3 is scanned shall be different from the face in which T_2 and T_4 are scanned. This means, in case the targets are scanned in a full-dome scan, the “seam line” of the full-dome scan always shall run between T_3 and T_2/T_4 . On instrument station S_2 the targets T_2 , T_3 and T_4 shall be scanned in a different face than on S_1 (see [Figure 3](#) and [Figure 4](#) dark grey hemisphere and bright grey hemisphere of the dome).

7.2 Example 1: Target scan by full dome scan

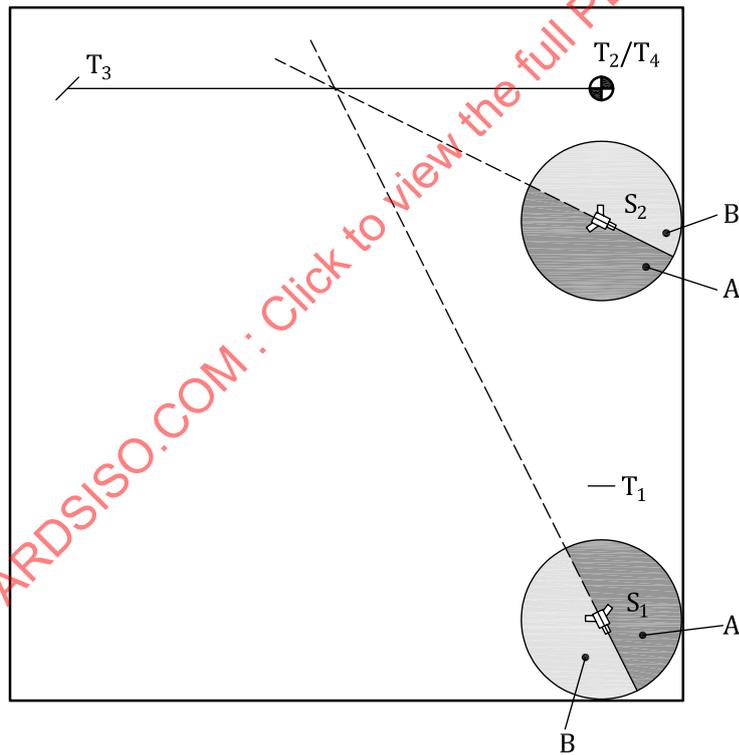
The targets will be scanned by a single full-dome scan on each station. On instrument station S_1 the TLS instrument is oriented in a way that the first vertical scan line will run between T_3 and T_2/T_4 . The targets T_1 , T_2 and T_4 will be scanned in face I and T_3 will be scanned in face II. On position S_2 the instrument will again be oriented in a way that the first vertical scan line will run between T_3 and T_2/T_4 , but now T_2 and T_4 are scanned in face II, while T_1 and T_3 are scanned in face I.



Key

- A face I
- B face II
- S₁, S₂ instrument station
- T₁, T₂, T₃, T₄ target point

Figure 3 — Instrument orientations on both positions (side view)



Key

- A face I
- B face II
- S₁, S₂ instrument station
- T₁, T₂, T₃, T₄ target point

Figure 4 — Instrument orientation on both positions (top view)

The seam line of the full-dome scan needs to run between T₃ and T₂/T₄. The faces on both instrument positions shall be inverted but can also be vice versa. Dark grey colour indicates the face I and the bright grey one the face II.

7.3 Example 2: Two face target scan

The TLS instrument offers target scanning functionality in both faces. If all targets will be scanned in both faces a selection process shall be carried out. When evaluating the measurements on instrument station S₁ for T₁, T₂ and T₄ the target scans in face I will be considered, while for T₃ the target scan in face II shall be considered. When evaluating the measurements on S₂ it shall be vice versa: For T₂ and T₄ the target scans in face II shall be considered, while for T₁ and T₃ the target scan in face I shall be considered.

7.4 Measurements

Before beginning the measurements the instrument shall become acclimatised to the ambient temperature (if not stated else by the manufacturer in the user manual, use 2 min/°C difference for acclimatization). All coordinates shall be measured within a 24 hour period.

After completing the measurements, point clouds of the targets in form of three-dimensional Cartesian coordinates are available, from which the centre coordinates, e.g. of spheres or chessboard targets shall be determined. For this work step, the corresponding processing software of the manufacturer should be used. It shall be ensured that irregular pixels are eliminated before determining the target centers. After this work step, three-dimensional Cartesian coordinates for the centre points of the targets T₁ through T₄ are available.

The four targets are scanned once from each station. The results are local 3D coordinates for each target. Each station is defining a locale Cartesian coordinate system. The resulting coordinates are listed in [Table 2](#).

Table 2 — Coordinates of the local scans from S₁ and S₂

Station S _n	Target T _j	x _{n,j}	y _{n,j}	z _{n,j}
S ₁	T ₁	x _{1,1}	y _{1,1}	z _{1,1}
	T ₂	x _{1,2}	y _{1,2}	z _{1,2}
	T ₃	x _{1,3}	y _{1,3}	z _{1,3}
	T ₄	x _{1,4}	y _{1,4}	z _{1,4}
S ₂	T ₁	x _{2,1}	y _{2,1}	z _{2,1}
	T ₂	x _{2,2}	y _{2,2}	z _{2,2}
	T ₃	x _{2,3}	y _{2,3}	z _{2,3}
	T ₄	x _{2,4}	y _{2,4}	z _{2,4}

7.5 Calculation

The distances between the targets are calculated for instrument stations S₁ and S₂ according to [Formulae \(1\)](#) and [\(2\)](#):

$$d_{S1,j,i} = \sqrt{(x_{S1,i} - x_{S1,j})^2 + (y_{S1,i} - y_{S1,j})^2 + (z_{S1,i} - z_{S1,j})^2} \quad (1)$$

$$d_{S2,j,i} = \sqrt{(x_{S2,i} - x_{S2,j})^2 + (y_{S2,i} - y_{S2,j})^2 + (z_{S2,i} - z_{S2,j})^2} \quad (2)$$

with $i = 2, 3, 4$ and $j = 1, 2, 3$

The resulting distances are summarized in [Table 3](#).

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Table 3 — Calculations for the distances between the targets

Distances for Station S_1	Distances for Station S_2
$d_{S1,1,2} = \sqrt{(x_{S1,2} - x_{S1,1})^2 + (y_{S1,2} - y_{S1,1})^2 + (z_{S1,2} - z_{S1,1})^2}$	$d_{S2,1,2} = \sqrt{(x_{S2,2} - x_{S2,1})^2 + (y_{S2,2} - y_{S2,1})^2 + (z_{S2,2} - z_{S2,1})^2}$
$d_{S1,1,3} = \sqrt{(x_{S1,3} - x_{S1,1})^2 + (y_{S1,3} - y_{S1,1})^2 + (z_{S1,3} - z_{S1,1})^2}$	$d_{S2,1,3} = \sqrt{(x_{S2,3} - x_{S2,1})^2 + (y_{S2,3} - y_{S2,1})^2 + (z_{S2,3} - z_{S2,1})^2}$
$d_{S1,1,4} = \sqrt{(x_{S1,4} - x_{S1,1})^2 + (y_{S1,4} - y_{S1,1})^2 + (z_{S1,4} - z_{S1,1})^2}$	$d_{S2,1,4} = \sqrt{(x_{S2,4} - x_{S2,1})^2 + (y_{S2,4} - y_{S2,1})^2 + (z_{S2,4} - z_{S2,1})^2}$
$d_{S1,2,3} = \sqrt{(x_{S1,3} - x_{S1,2})^2 + (y_{S1,3} - y_{S1,2})^2 + (z_{S1,3} - z_{S1,2})^2}$	$d_{S2,2,3} = \sqrt{(x_{S2,3} - x_{S2,2})^2 + (y_{S2,3} - y_{S2,2})^2 + (z_{S2,3} - z_{S2,2})^2}$
$d_{S1,2,4} = \sqrt{(x_{S1,4} - x_{S1,2})^2 + (y_{S1,4} - y_{S1,2})^2 + (z_{S1,4} - z_{S1,2})^2}$	$d_{S2,2,4} = \sqrt{(x_{S2,4} - x_{S2,2})^2 + (y_{S2,4} - y_{S2,2})^2 + (z_{S2,4} - z_{S2,2})^2}$
$d_{S1,3,4} = \sqrt{(x_{S1,4} - x_{S1,3})^2 + (y_{S1,4} - y_{S1,3})^2 + (z_{S1,4} - z_{S1,3})^2}$	$d_{S2,3,4} = \sqrt{(x_{S2,4} - x_{S2,3})^2 + (y_{S2,4} - y_{S2,3})^2 + (z_{S2,4} - z_{S2,3})^2}$

The distance differences $\Delta_{i,j}$ with $i = 1, 2, 3$ and $j = 2, 3, 4$ resulting from the two different scan positions are calculated as follows:

$$\begin{aligned}\Delta_{1,2} &= d_{S1,1,2} - d_{S2,1,2} \\ \Delta_{1,3} &= d_{S1,1,3} - d_{S2,1,3} \\ \Delta_{1,4} &= d_{S1,1,4} - d_{S2,1,4} \\ \Delta_{2,3} &= d_{S1,2,3} - d_{S2,2,3} \\ \Delta_{2,4} &= d_{S1,2,4} - d_{S2,2,4} \\ \Delta_{3,4} &= d_{S1,3,4} - d_{S2,3,4}\end{aligned}\tag{3}$$

The values $\Delta_{1,2}$, $\Delta_{1,3}$, $\Delta_{1,4}$, $\Delta_{2,3}$, $\Delta_{2,4}$, $\Delta_{3,4}$ are indicators for the geometrical quality of the scans. In the ideal case they can be neglected. They should be within the specified permitted deviations $\pm U_{\Delta}$ (see also 7.6.2 for deriving the permitted deviations).

$\Delta_{1,2}$ is sensitive to a constant distance offset (i.e. zero point error) of the measured distances of the laser scanner. For all other differences ($\Delta_{1,3}$, $\Delta_{1,4}$, $\Delta_{2,3}$, $\Delta_{2,4}$, $\Delta_{3,4}$) deviations of the measured distances, turning angles and tilting angles are superposing each other.

If the differences $\Delta_{i,j}$ are too large for the intended measurement task, it is necessary to continue further in order to identify the main sources of the deviations. The presence of systematic measurement deviations can be inferred from these differences if they differ significantly from zero. This evaluation requires a quantitative assessment of the measurement uncertainty.

7.6 Derivation of a reference quantity for computing permitted deviations

7.6.1 Introduction

In this subclause, a reference quantity U_{Δ} for assessment of the distance differences calculated in Formula (3) will be derived (see 7.6.2 and 7.6.3). The same derivation holds also for the full test procedure and will be applied to calculate the permitted deviations (see 8.5 and 8.6).

7.6.2 Determination of measurement uncertainty of the target centers

To assess the significance of the distance differences from Formula (3), it is essential to make an appropriate assumption regarding the standard uncertainty u_T , with which the target centers can be determined by means of the laser scanner and the processing software. A simple option for determining and fixing u_T in the simplified test procedure is the use of available manufacturer information.

However, to arrive at an approximation of the standard uncertainty u_T of the target centers which is best representative for the true value, the greatest possible number of independent influence factors should be incorporated into the uncertainty measure. One way of merging multiple independent influence factors to a total uncertainty is offered by the ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty* (GUM, 2008), whose application is recommended also in ISO 17123-1.

In the case of an uncertainty quantity u_T for the target centers, for instance the following quantities can be used:

- Uncertainty measure by statistical analysis of series of observations (Type A): An uncertainty measure for the target centre precision can be derived by means of an empirical standard deviation of repeated target scans.

- The manufacturer specifications (Type B): The data sheets of some manufacturer can contain a 3D accuracy for capturing the recommended target marks, which can be incorporated as an uncertainty quantity of Type B directly into an uncertainty measure u_T for the target centers. Alternatively, the 3D accuracy of the target capture can be derived from the specifications of the angle and distance accuracies in case both the number of points used for measuring the targets and the modelling accuracy regarding the target centers are additionally taken into account. Some manufacturers provide both systematic measurement deviations and a measurement noise as a function of distance and reflectivity. These specifications can also be included.
- Values stemming from one's own experience (Type B): Influence factors that are known from one's own experience to affect the uncertainty of the target centers may be taken into account as well.

The derivation of a combined measurement uncertainty is demonstrated in general in ISO 17123-1, and ISO 17123-5 for the example of total station measurements. [Annexes B](#) and [C](#) also give examples.

7.6.3 Derivation of the permitted deviation for the simple test procedure

In case the uncertainty quantity u_T for the target centers is available, the reference value u_Δ for assessing the distance differences determined in [Formula \(3\)](#) shall be derived. Given u_T , first the uncertainty u_d of the computed distances is obtained with the uncertainty propagation law as

$$u_d = u_T \times \sqrt{2} \quad (4)$$

Using u_d , the uncertainty u_Δ of the distance differences

$$u_\Delta = u_d \times \sqrt{2} = 2u_T \quad (5)$$

can be computed by applying the uncertainty propagation law again. The expanded measurement uncertainty

$$U_\Delta = k \cdot u_\Delta \quad (6)$$

indicates an area containing a majority of the measurements that appear to be probable measurement values.

NOTE A frequent choice is $k = 2$, which corresponds in case of a correctly assumed normal distribution to a confidence interval of 95 %.

The expanded measurement uncertainty U_Δ with $k = 2$ for distance differences results in

$$U_\Delta = k \cdot u_\Delta = k \times 2 \times u_T = 4 u_T \quad (7)$$

The uncertainty U_Δ shall be used in the following as the reference quantity for assessing and judging the distance differences.

7.7 Quantification of measurement deviations and judgement of the instrument for the simple test procedure

7.7.1 Analysis of distance measurements

To detect a constant distance offset, the distance between T_1 and T_2 is analysed. This distance is not influenced by a constant distance offset when the measurements of instrument station S_1 are used, whereas the influence of a constant distance offset is doubled when the measurements of station S_2 are used, see [Figure 1](#) and [Figure 2](#). Forming the difference $\Delta_{1,2}$ given in [Formula \(3\)](#) in analogy to the

"Partially Distance Method" yields twice the value of a constant distance offset. The value $\Delta_{1,2}$ shall be compared with the expanded measurement uncertainty U_{Δ} from [Formula \(7\)](#). In case of

$$|\Delta_{1,2}| > U_{\Delta} \quad (8)$$

a significant systematic deviation of the distance measurement is assumed. In this case, an adjustment by the manufacturer or, if the TLS permits, the input and storage of the determined deviation as a correction value in the device followed by a control measurement is recommended. It shall be avoided to consider further deviations $\Delta_{i,j}$ since they are superimposed by the influence of the systematic deviation of the distance measurement. In case of

$$|\Delta_{1,2}| \leq U_{\Delta} \quad (9)$$

no significant systematic deviation of the distance measurement can be ascertained, and it is then reasonable to analyse the influence of systematic deviations in the further distance differences.

NOTE For more information concerning the distance deviation see Deumlich and Staiger^[1].

7.7.2 Remarks on the scale problem

It should be remarked that the described testing configuration does not allow for an analysis of the instrument's scale since this would require reference values for the target distances. In case these reference values are known, e.g. from independent measurements with superior accuracy, the instrument's scale can be determined. Measurement uncertainties such as centering deviations or insufficient consideration of the meteorological conditions shall be avoided by all means, because otherwise they are misinterpreted as a scale correction. For instance, a centering deviation of 1 mm causes, on an 80 m long distance, a value of 12,5 ppm for a supposed scale correction. Therefore, it is more advisable to determine the instrument's scale by means of a fixed test bay, in which the reference distances were determined from tacheometric network measurements.

NOTE For more information concerning the scale problem by TLS see Feldmann^[2] as well as Staiger and Heister^[5]

7.7.3 Analysis of further distance differences

Concerning the analysis of direction and angle measurements, it should be pointed out first that deviations in the axis system from its reference geometry (deviations of the sighting and tilting axes) have an influence on the direction of the turning circle that depends on the steepness of the sighting.

NOTE See Stahlberg^[4] as well as Deumlich and Staiger^[1].

To detect deviations, it is therefore indispensable to include the high target T_4 in the following investigations. For this reason, all distance combinations between targets T_1 , T_2 , T_3 and T_4 are calculated. Using [Formula \(3\)](#), the differences for these distances are: $\Delta_{1,3}$, $\Delta_{1,4}$, $\Delta_{2,3}$, $\Delta_{2,4}$, and $\Delta_{3,4}$.

The reason for considering all distance differences is that, depending on the magnitude and the sign of a systematic deviation both in the direction of the turning circle and in the tilting angle, the effects show up either in horizontal plane and/or in vertical plane. The computed differences shall be compared with the expanded measurement uncertainty from [Formula \(7\)](#). In case of

$$|\Delta_{1,3}| > U_{\Delta} \text{ or } |\Delta_{1,4}| > U_{\Delta} \text{ or } |\Delta_{2,3}| > U_{\Delta} \text{ or } |\Delta_{2,4}| > U_{\Delta} \text{ or } |\Delta_{3,4}| > U_{\Delta} \quad (10)$$

other significant systematic deviations of the instrument are assumed. Before making this final decision, different error sources should be identified and eliminated, e.g.:

- The targets are stable and set up on solid ground.
- The instrument was set up on a stable tripod on solid ground.

- Instrument had been acclimatised to the ambient temperature before the measurements were started.
- The used equipment is in good condition (tripods, tribrachs, adapters).
- Check measurements for gross errors.

In case no error source can be identified and the significant deviation is confirmed, it shall be assumed that there is a significant systematic deviation either of the direction angle of the turning circle or of the tilting angle, or that there are deviations of all sources. In such a case, the execution of the full test procedure is recommended. If the full test procedure cannot be carried out, then the simplified test procedure shall be repeated with a slightly different configuration of the test field, e.g. by changing the locations of the instrument stations slightly. If this test also fails, an adjustment by the manufacturer is recommended.

With the presented testing procedure, it is not possible for the user to determine and take into account the individual effects, because effects, e.g. of axis deviations, interfere with each other or partly compensate each other. It should be remarked that the instrument's deviations lead indeed to systematic deviations of the three-dimensional coordinates of the considered points. This fact, however, cannot be exploited to carry out a more differentiated examination of the individual deviations since the analyses are based on distances in space, and thus on relative quantities.

A schematic overview of the TLS testing procedure presented here is given in [Figure 5](#) in form of a flow chart diagram. [Annex A](#) provides an example intended to use the simple test procedure described throughout [Clause 7](#).

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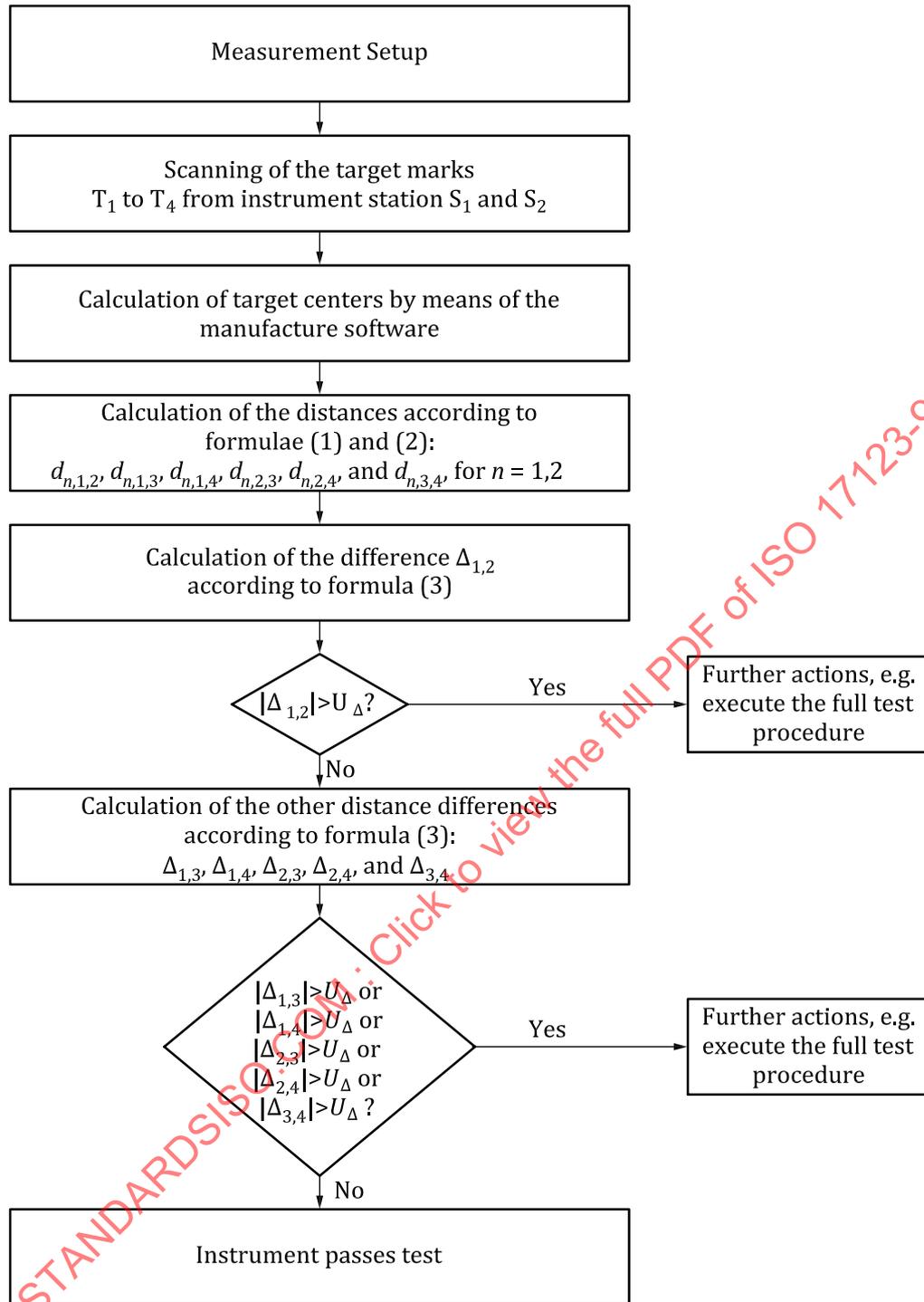


Figure 5 — Flow chart diagram of the TLS simple test procedure

8 Full test procedure

8.1 Configuration of the test field

The test field is identical to the test field for the simplified procedure, see [Figure 1](#) to [Figure 4](#). The size of the test field can be modified depending on the capabilities of the scanner and the actual size of the object for which the scanner will be used.

8.2 Measurements

Before commencing the measurements, the instruments shall be adjusted as specified by the manufacturer. Furthermore, the instrument shall become acclimatized to the ambient temperature (if not stated by the manufacturer in the user manual, use 2 min/°C). All coordinates shall be measured within a 24 h period (see 7.4).

Three series of measurements ($w = 1, 2, 3$) are carried out on each instrument station, which means that each of the four targets is scanned three times from each instrument station. The sequence is as follows: All targets are scanned three times from the first station S_1 . Once the first instrument station is completed, the same procedure will be repeated at the second instrument station S_2 by considering the different face orientation (see 7.1 to 7.3). The resulting coordinates are listed in Table 4 and Table 5.

Table 4 — Coordinates of the local scans from S_1

Station S_n	Target T_j	Set w	$x_{n,j,w}$	$y_{n,j,w}$	$z_{n,j,w}$
S_1	T ₁	1	$x_{1,1,1}$	$y_{1,1,1}$	$z_{1,1,1}$
	T ₂		$x_{1,2,1}$	$y_{1,2,1}$	$z_{1,2,1}$
	T ₃		$x_{1,3,1}$	$y_{1,3,1}$	$z_{1,3,1}$
	T ₄		$x_{1,4,1}$	$y_{1,4,1}$	$z_{1,4,1}$
	T ₁	2	$x_{1,1,2}$	$y_{1,1,2}$	$z_{1,1,2}$
	T ₂		$x_{1,2,2}$	$y_{1,2,2}$	$z_{1,2,2}$
	T ₃		$x_{1,3,2}$	$y_{1,3,2}$	$z_{1,3,2}$
	T ₄		$x_{1,4,2}$	$y_{1,4,2}$	$z_{1,4,2}$
	T ₁	3	$x_{1,1,3}$	$y_{1,1,3}$	$z_{1,1,3}$
	T ₂		$x_{1,2,3}$	$y_{1,2,3}$	$z_{1,2,3}$
	T ₃		$x_{1,3,3}$	$y_{1,3,3}$	$z_{1,3,3}$
	T ₄		$x_{1,4,3}$	$y_{1,4,3}$	$z_{1,4,3}$

Table 5 — Coordinates of the local scans from S_2

Station S_n	Target T_j	Set w	$x_{n,j,w}$	$y_{n,j,w}$	$z_{n,j,w}$
S_2	T ₁	1	$x_{2,1,1}$	$y_{2,1,1}$	$z_{2,1,1}$
	T ₂		$x_{2,2,1}$	$y_{2,2,1}$	$z_{2,2,1}$
	T ₃		$x_{2,3,1}$	$y_{2,3,1}$	$z_{2,3,1}$
	T ₄		$x_{2,4,1}$	$y_{2,4,1}$	$z_{2,4,1}$
	T ₁	2	$x_{2,1,2}$	$y_{2,1,2}$	$z_{2,1,2}$
	T ₂		$x_{2,2,2}$	$y_{2,2,2}$	$z_{2,2,2}$
	T ₃		$x_{2,3,2}$	$y_{2,3,2}$	$z_{2,3,2}$
	T ₄		$x_{2,4,2}$	$y_{2,4,2}$	$z_{2,4,2}$
	T ₁	3	$x_{2,1,3}$	$y_{2,1,3}$	$z_{2,1,3}$
	T ₂		$x_{2,2,3}$	$y_{2,2,3}$	$z_{2,2,3}$
	T ₃		$x_{2,3,3}$	$y_{2,3,3}$	$z_{2,3,3}$
	T ₄		$x_{2,4,3}$	$y_{2,4,3}$	$z_{2,4,3}$

8.3 Calculation

Calculate all possible distances between targets T₁, T₂, T₃ and T₄ by measured coordinates, given in Table 4 and Table 5.

$$d_{S_n,i,j,w} = \sqrt{(x_{S_n,j,w} - x_{S_n,i,w})^2 + (y_{S_n,j,w} - y_{S_n,i,w})^2 + (z_{S_n,j,w} - z_{S_n,i,w})^2} \tag{11}$$

with S_n (n = 1, 2) indicating the station number, i (=1, 2, 3) and j (=2, 3, 4) indicating the target number and w (=1, 2, 3) indicating the repetition number.

The resulting mean distances and their standard deviations are given in Table 6:

Table 6 — Calculation scheme for the mean distances between the targets

S _n	Distance	Mean distances \bar{d}	Standard deviation s _d for a single observed distance
S ₁	1-2	$\bar{d}_{S1,1,2} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S1,1,2,w}$	$s_{d,S1,1,2} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S1,1,2} - d_{S1,1,2,w})^2 / 2}$
	1-3	$\bar{d}_{S1,1,3} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S1,1,3,w}$	$s_{d,S1,1,3} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S1,1,3} - d_{S1,1,3,w})^2 / 2}$
	1-4	$\bar{d}_{S1,1,4} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S1,1,4,w}$	$s_{d,S1,1,4} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S1,1,4} - d_{S1,1,4,w})^2 / 2}$
	2-3	$\bar{d}_{S1,2,3} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S1,2,3,w}$	$s_{d,S1,2,3} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S1,2,3} - d_{S1,2,3,w})^2 / 2}$
	2-4	$\bar{d}_{S1,2,4} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S1,2,4,w}$	$s_{d,S1,2,4} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S1,2,4} - d_{S1,2,4,w})^2 / 2}$
	3-4	$\bar{d}_{S1,3,4} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S1,3,4,w}$	$s_{d,S1,3,4} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S1,3,4} - d_{S1,3,4,w})^2 / 2}$

Table 6 (continued)

S_n	Distance	Mean distances \bar{d}	Standard deviation s_d for a single observed distance
S_2	1-2	$\bar{d}_{S2,1,2} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S2,1,2,w}$	$s_{d,S2,1,2} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S2,1,2} - d_{S2,1,2,w})^2 / 2}$
	1-3	$\bar{d}_{S2,1,3} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S2,1,3,w}$	$s_{d,S2,1,3} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S2,1,3} - d_{S2,1,3,w})^2 / 2}$
	1-4	$\bar{d}_{S2,1,4} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S2,1,4,w}$	$s_{d,S2,1,4} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S2,1,4} - d_{S2,1,4,w})^2 / 2}$
	2-3	$\bar{d}_{S2,2,3} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S2,2,3,w}$	$s_{d,S2,2,3} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S2,2,3} - d_{S2,2,3,w})^2 / 2}$
	2-4	$\bar{d}_{S2,2,4} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S2,2,4,w}$	$s_{d,S2,2,4} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S2,2,4} - d_{S2,2,4,w})^2 / 2}$
	3-4	$\bar{d}_{S2,3,4} = \frac{1}{3} \cdot \sum_{w=1}^3 d_{S2,3,4,w}$	$s_{d,S2,3,4} = \sqrt{\sum_{w=1}^3 (\bar{d}_{S2,3,4} - d_{S2,3,4,w})^2 / 2}$

The mean distance differences resulting from the two different scan positions are calculated as follows:

$$\begin{aligned}
 \bar{\Delta}_{1,2} &= \bar{d}_{S1,1,2} - \bar{d}_{S2,1,2} \\
 \bar{\Delta}_{1,3} &= \bar{d}_{S1,1,3} - \bar{d}_{S2,1,3} \\
 \bar{\Delta}_{1,4} &= \bar{d}_{S1,1,4} - \bar{d}_{S2,1,4} \\
 \bar{\Delta}_{2,3} &= \bar{d}_{S1,2,3} - \bar{d}_{S2,2,3} \\
 \bar{\Delta}_{2,4} &= \bar{d}_{S1,2,4} - \bar{d}_{S2,2,4} \\
 \bar{\Delta}_{3,4} &= \bar{d}_{S1,3,4} - \bar{d}_{S2,3,4}
 \end{aligned} \tag{12}$$

The presence of systematic measurement deviations can be inferred from these differences if they differ significantly from zero. This evaluation requires a quantitative assessment of the measurement uncertainty.

The residuals are calculated for both stations S_1 and S_2 as follows:

$$r_{S1,i,j,w} = \bar{d}_{S1,i,j} - d_{S1,i,j,w} \quad \text{and} \quad r_{S2,i,j,w} = \bar{d}_{S2,i,j} - d_{S2,i,j,w} \tag{13}$$

where $i = 1, 2, 3$ and $j = 2, 3, 4$ are the target numbers and $w = 1, 2, 3$ is indicating the repetition number. The mean distances $\bar{d}_{S1,i,j}$ and $\bar{d}_{S2,i,j}$ are given in [Table 6](#) and the $d_{S1,i,j,w}$ and $d_{S2,i,j,w}$ are calculated according to [Formula \(11\)](#).

By means of the sum of squares of the residuals for each station and by means of its degree of freedom (v_{S1} and v_{S2}) the experimental standard deviation of S_1 ($\hat{s}_{0,1}$) and of S_2 ($\hat{s}_{0,2}$) for a single observed distance are calculated as follows:

$$\hat{s}_{0,1} = \sqrt{\frac{\sum_{w=1}^3 r_{S1,1,2,w}^2 + r_{S1,1,3,w}^2 + r_{S1,1,4,w}^2 + r_{S1,2,3,w}^2 + r_{S1,2,4,w}^2 + r_{S1,3,4,w}^2}{v_{S1}}} = \sqrt{\frac{\Omega_{S1}}{v_{S1}}} \quad (14)$$

$$\hat{s}_{0,2} = \sqrt{\frac{\sum_{w=1}^3 r_{S2,1,2,w}^2 + r_{S2,1,3,w}^2 + r_{S2,1,4,w}^2 + r_{S2,2,3,w}^2 + r_{S2,2,4,w}^2 + r_{S2,3,4,w}^2}{v_{S2}}} = \sqrt{\frac{\Omega_{S2}}{v_{S2}}} \quad (15)$$

with degree of freedom in [Formulae \(14\)](#) and [\(15\)](#) for $v_{S1} = v_{S2} = 18 - 6 = 12$ (18 is the number of the calculated distances at every instrument station and 6 is the number of unknown parameters, here the mean distances), and Ω_{S1} , Ω_{S2} are the sum of squared residual of the instrument stations (S_1 and S_2).

In the case that both experimental variances $\hat{s}_{0,1}^2$ and $\hat{s}_{0,2}^2$ belong to the same theoretical variances σ^2 (see statistical tests, [8.4](#), testing two variances), then they should be calculated as follows:

$$\hat{s}_0 = \sqrt{\frac{\Omega_{S1} + \Omega_{S2}}{v_{S1} + v_{S2}}} \quad (16)$$

In case that the experimental standard deviations $\hat{s}_{0,1}^2$ and $\hat{s}_{0,2}^2$ differs significantly then the experimental standard deviation of the differences calculated in [Formula \(12\)](#) results in

$$\hat{s}_0 = \frac{\hat{s}_{0,1} + \hat{s}_{0,2}}{2} \quad (17)$$

The experimental standard deviations $\hat{s}_{0,1}$, $\hat{s}_{0,2}$ and \hat{s}_0 given in [Formula \(14\)](#) to [\(17\)](#) are measures for the precision of distances measured by the instrument. If the precision for a 3D point is required, the values $\hat{s}_{0,1}$, $\hat{s}_{0,2}$ and \hat{s}_0 have to be divided by $\sqrt{2}$.

In order to derive a representative standard deviation for the instrument under real conditions, mean values for the distances for both instrument stations are introduced:

$$\begin{aligned} \bar{d}_{1,2} &= (\bar{d}_{S1,1,2} + \bar{d}_{S2,1,2}) / 2 \\ \bar{d}_{1,3} &= (\bar{d}_{S1,1,3} + \bar{d}_{S2,1,3}) / 2 \\ \bar{d}_{1,4} &= (\bar{d}_{S1,1,4} + \bar{d}_{S2,1,4}) / 2 \\ \bar{d}_{2,3} &= (\bar{d}_{S1,2,3} + \bar{d}_{S2,2,3}) / 2 \\ \bar{d}_{2,4} &= (\bar{d}_{S1,2,4} + \bar{d}_{S2,2,4}) / 2 \\ \bar{d}_{3,4} &= (\bar{d}_{S1,3,4} + \bar{d}_{S2,3,4}) / 2 \end{aligned} \quad (18)$$

The residuals calculated by means of the mean distances (refer to [Formula \(18\)](#)) for both instrument stations are:

$$\bar{r}_{S1,i,j,w} = \bar{d}_{i,j} - d_{S1,i,j,w} \text{ and } \bar{r}_{S2,i,j,w} = \bar{d}_{i,j} - d_{S2,i,j,w} \quad (19)$$

where $\bar{d}_{i,j}$ are the mean distances calculated in [Formula \(18\)](#) and $d_{S1,i,j,w}$, $d_{S2,i,j,w}$ are the distances calculated in [Formula \(11\)](#). The resulting experimental standard deviation \hat{s}_0 for the instrument then follows by:

$$\hat{s}_0 = \sqrt{\frac{\sum_{i=1}^2 \sum_{w=1}^3 \bar{r}_{Si,1,2,w}^2 + \bar{r}_{Si,1,3,w}^2 + \bar{r}_{Si,1,4,w}^2 + \bar{r}_{Si,2,3,w}^2 + \bar{r}_{Si,2,4,w}^2 + \bar{r}_{Si,3,4,w}^2}{\nu}} \quad (20)$$

$$\hat{s}_0 = \sqrt{\frac{\bar{\Omega}_{S1} + \bar{\Omega}_{S2}}{\nu}} \quad (21)$$

with $\nu = 36 - 6 = 30$ is the degree of freedom (36 is the number of all calculated distances and 6 is the number of unknown parameters, here mean distances). Finally, the standard uncertainty of the TLS is obtained by:

$$s_{\text{ISO-TLS}} = \frac{\hat{s}_0}{\sqrt{2}} \quad (22)$$

$$u_{\text{ISO-TLS}} = s_{\text{ISO-TLS}} \quad (23)$$

It can be expected that \hat{s}_0 is larger than the experimental standard deviations calculated in [Formulae \(14\)](#), [\(15\)](#), [\(16\)](#) and [\(17\)](#), since the instrumental deviations already affect the distances between the two laser scanner stations and therefore the calculation of \hat{s}_0 , the figure $u_{\text{ISO-TLS}}$ is a standard uncertainty for a 3D point measured by the laser scanner.

8.4 Statistical tests

8.4.1 General description

The statistical tests are applicable for the full test procedure only. The statistical tests are applied only to verify if the measurements from both instrument stations S_1 and S_2 have the same precision. The final judgement if the accuracy of the instruments fulfils the requirements will be treated in [8.6](#). The standard deviations $\hat{s}_{0,1}$, $\hat{s}_{0,2}$ and \hat{s}_0 , estimated in [Formulae \(14\)](#), [\(15\)](#), [\(16\)](#) and [\(17\)](#) are the measures for the precision of measured distances of the instrument. The combined standard uncertainty u_{xyz} (refer to [8.5](#)) is a quantitative measure for the accuracy of a 3D point measured by the instrument. Therefore, the following questions have to be answered:

- Is the calculated experimental standard deviation $\hat{s}_0 / \sqrt{2}$ smaller than or equal to a corresponding value, σ_0 stated by the manufacturer (if given) or smaller than another predetermined (theoretical) value, σ_0 ?
- The two experimental standard deviations $\hat{s}_{0,1}$ and $\hat{s}_{0,2}$, as determined from two different samples of measurements and (assuming that both samples have the same number of degrees of freedom

$v_{S1} = v_{S2}$), belong to the same population? The experimental standard deviation, $\hat{s}_{0,1}$ and $\hat{s}_{0,2}$, can be obtained from, e.g. two samples of measurements by the same instrument at different times and stations (from station S_1 and S_2).

For the tests, a confidence level of $1 - \alpha = 0,95$ and, according to the design of measurements given in 8.3, a number of degrees of freedom of $v_{S1} = v_{S2} = 12$ for test performed in a), $v = 24$ for test performed in b) are assumed.

Table 7 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$\frac{\hat{s}_0}{\sqrt{2}} \leq \sigma_0$	$\frac{\hat{s}_0}{\sqrt{2}} > \sigma_0$
b)	$\hat{s}_{0,1} = \hat{s}_{0,2}$	$\hat{s}_{0,1} \neq \hat{s}_{0,2}$

8.4.2 Question a)

The null hypothesis stating that the experimental standard deviation $\hat{s}_0 / \sqrt{2}$ is smaller than or equal to a theoretical or predetermined value σ_0 , is not rejected if the following condition is fulfilled:

$$\frac{\hat{s}_0}{\sqrt{2}} \leq \sigma_0 \sqrt{\frac{\chi^2_{1-\alpha}(v)}{v}} \tag{24}$$

$$\frac{\hat{s}_0}{\sqrt{2}} \leq \sigma_0 \sqrt{\frac{\chi^2_{0,95}(24)}{24}} \tag{25}$$

where $\frac{\hat{s}_0}{\sqrt{2}}$ and σ_0 are standard deviations (as measure for the precision) for a 3D point. Because of $\chi^2_{0,95}(24) = 36,42$ the null hypothesis will be accepted if:

$$\hat{s}_0 / \sqrt{2} \leq \sigma_0 \times \sqrt{1,52} = \sigma_0 \times 1,23 \tag{26}$$

Otherwise, the null hypothesis is rejected. If the null hypothesis is rejected, the precision of the instrument does not meet the requirements.

8.4.3 Question b)

In the case of two different samples, a test should decide whether the experimental standard deviation $\hat{s}_{0,1}$ and $\hat{s}_{0,2}$, belong to the same population. The corresponding null hypothesis $\hat{s}_{0,1} = \hat{s}_{0,2}$ is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(v_{S1}, v_{S2})} \leq \frac{\hat{s}_{0,1}^2}{\hat{s}_{0,2}^2} \leq F_{1-\alpha/2}(v_{S1}, v_{S2}) \tag{27}$$

$$\frac{1}{F_{0,975}(12,12)} \leq \frac{\hat{s}_{0,1}^2}{\hat{s}_{0,2}^2} \leq F_{0,975}(12,12) \quad (28)$$

And because of $F_{0,975}(12,12) = 3,28$, the null hypothesis will be accepted if the following holds:

$$0,31 \leq \frac{\hat{s}_{0,1}^2}{\hat{s}_{0,2}^2} \leq 3,28 \quad (29)$$

In such a case the total experimental variance of the instrument will be calculated according to [Formula \(16\)](#). Otherwise, the null hypothesis is rejected and the full measurement procedure shall be repeated according to [8.2](#). In this case the total experimental standard deviation is calculated according to [Formula \(17\)](#). Reasons are that measurement conditions have been changed, or targets or the instrument station have been unstable during the repeated measurements at one station

8.5 Derivation of a reference quantity for computing permitted deviation

8.5.1 Determination of measurement uncertainty of the target centre

The combined standard uncertainty u_T of a target can be obtained based on different cases. In order to estimate the combined standard uncertainty u_T of a target, we distinguish between three cases:

- **Case A:** The uncertainty is known or can be given by means of manufacturer specifications:

$$u_T = u_{ms} \quad (30)$$

where u_{ms} is the combined standard uncertainty of a target centre by the manufacturer.

- **Case B:** The uncertainty is calculated based on the following Formula:

$$u_T = \sqrt{u_{ISO-TLS}^2 + u_p^2} \quad (31)$$

where $u_{ISO-TLS}$ is calculated according to [Formulae \(22\)](#) and [\(23\)](#) and u_p is derived as type B uncertainty from the typical influence quantities for the TLS measurements. A detailed example for the derivation of u_p is given in the [Annex C](#). If knowledge about u_p is available it should be considered.

- **Case C:** The combined standard uncertainty u_T is approximated with $u_{ISO-TLS}$ by setting u_p in [Formula \(31\)](#) to zero

$$u_T \approx \sqrt{u_{ISO-TLS}^2} \quad \text{with } u_p = 0 \quad (32)$$

8.5.2 Derivation of the permitted deviation for the full test procedure

After deriving the combined standard uncertainty u_T of a target for one of the above cases A, B and C (see [8.5.1](#)), we assume that this uncertainty represents in particular the centre target uncertainty

u_T mentioned in 7.6.2. Based on this value, the expanded measurement uncertainty U_Δ with $k = 2$ is determined according to the Formulae (4) to (7):

$$U_\Delta = k \cdot u_\Delta = k \times 2 \times u_T = 4 u_T \tag{33}$$

Based on the above mentioned expanded measurement uncertainty, the permitted deviation for the full test procedure is given by $U_\Delta / \sqrt{3}$. The deviation by $\sqrt{3}$ here is due to considering the averaging of the three measured distances in the full test procedure.

8.6 Quantification of measurement deviations and judgement of the instrument for the full test procedure

Analogous to the simple test procedure (see 7.7.1), firstly the mean distance between T₁ and T₂ is analysed in order to detect if a significant constant distance offset is existing:

$$|\bar{\Delta}_{1,2}| > \frac{U_\Delta}{\sqrt{3}} \tag{34}$$

Secondly, all other computed mean differences in Formula (12) have to be compared with the aforementioned permitted deviation:

$$|\bar{\Delta}_{1,3}| > \frac{U_\Delta}{\sqrt{3}} \text{ or } |\bar{\Delta}_{1,4}| > \frac{U_\Delta}{\sqrt{3}} \text{ or } |\bar{\Delta}_{2,3}| > \frac{U_\Delta}{\sqrt{3}} \text{ or } |\bar{\Delta}_{2,4}| > \frac{U_\Delta}{\sqrt{3}} \text{ or } |\bar{\Delta}_{3,4}| > \frac{U_\Delta}{\sqrt{3}} \tag{35}$$

If the computed mean distances are larger than the permitted deviation the error sources listed in 7.7.3 after Formula (10) may be responsible for the deviations and shall be investigated. If these error sources can be excluded, then the full test procedure shall be repeated with a slightly different configuration of the test field, e.g. by changing the locations of the instrument stations or of the targets slightly. If that test also fails, an adjustment by the manufacturer is recommended. The division by $\sqrt{3}$ in Formula (34) is due to considering the averaging of the three measured distances in the full test procedure. A schematic overview of the full test procedure presented here is given in Figure 6 in form of a flow chart diagram. Annex B provides an example intended to use the full test procedure described in Clause 8.

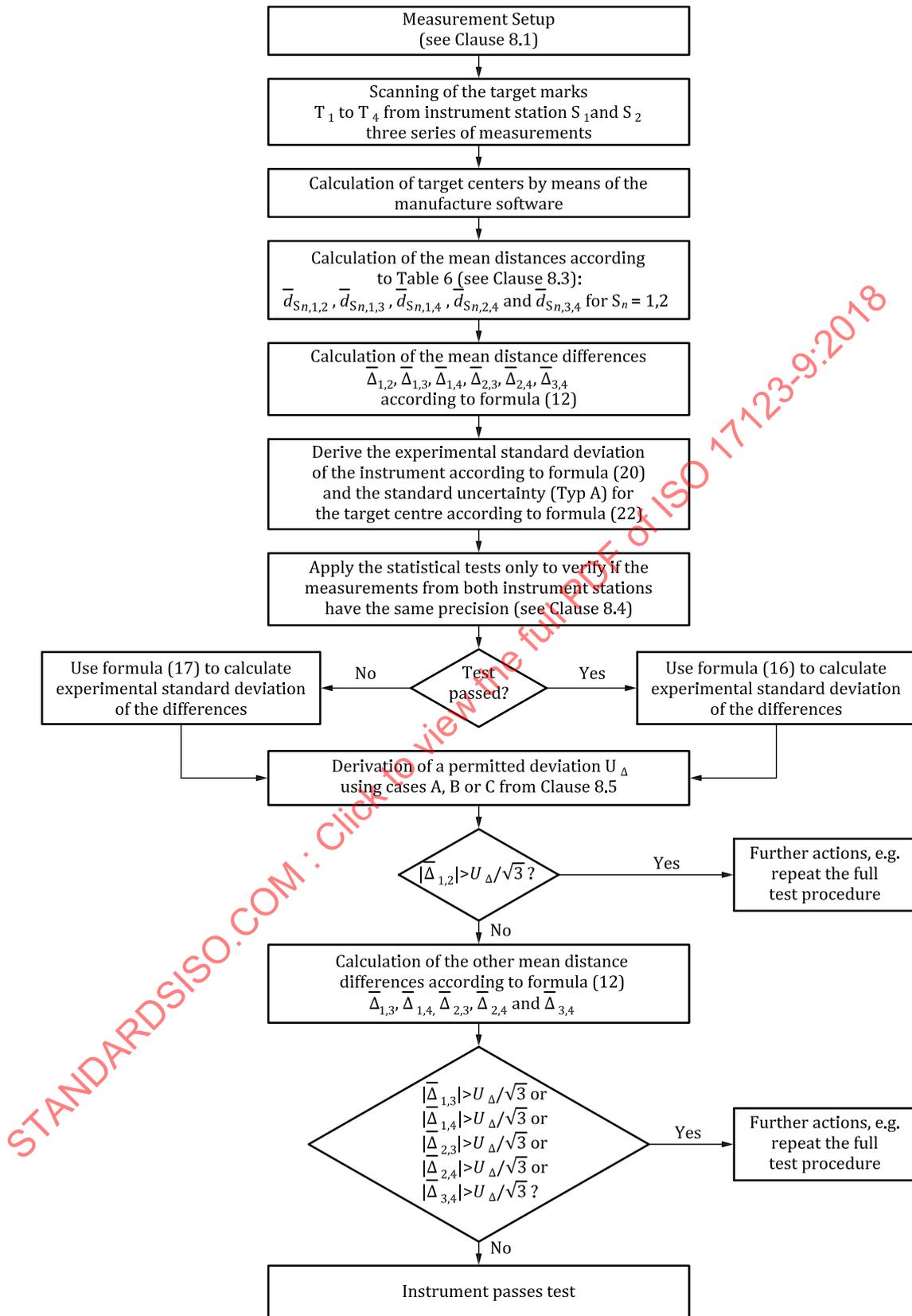


Figure 6 — Flow chart diagram of the TLS full test procedure

Annex A (informative)

Example for the simplified test procedure

A.1 Measurements

In [Table A.1](#) all measurements are carried out according to the observation scheme given in [Table 1](#).

Table A.1 — Measurements

Dimensions in metres

Instrument station S_n	Target T_j	x	y	z
S₁	T ₁	0,962 8	4,903 0	-0,099 6
	T ₂	8,618 0	43,878 5	0,053 4
	T ₃	-30,628 4	51,588 9	0,097 1
	T ₄	8,614 3	43,878 1	19,998 3
S₂	T ₁	17,885 7	29,755 4	-0,034 1
	T ₂	-2,573 4	-4,280 7	0,097 9
	T ₃	31,703 4	-24,889 9	0,128 2
	T ₄	-2,571 4	-4,298 1	20,043 9

A.2 Calculation

According to [Formulae \(1\)](#) and [\(2\)](#), the distances are given in [Table A.2](#):

Table A.2 — Determined distances

Dimensions in metres

Station	Distance from... to...	Target T ₁	Target T ₂	Target T ₃
S₁	Target T ₂	39,721 5	—	—
	Target T ₃	56,371 2	39,996 7	—
	Target T ₄	44,515 3	19,944 9	44,671 1
S₂	Target T ₂	39,712 1	—	—
	Target T ₃	56,365 5	39,995 5	—
	Target T ₄	44,511 4	19,946 0	44,670 2

Table A.3 — Determined distance differences

Dimensions in mm

Distance differences Δ_{ij}	Target T ₁	Target T ₂	Target T ₃
Target T ₂	9,4	—	—
Target T ₃	5,7	1,2	—
Target T ₄	3,9	-1,1	0,9

In accordance of the previously given [Table A.3](#) we notice the distances differences according to [Formula \(3\)](#):

$$\begin{aligned}\Delta_{1,2} &= 9,4 \text{ mm} \\ \Delta_{1,3} &= 5,7 \text{ mm} \\ \Delta_{1,4} &= 3,9 \text{ mm} \\ \Delta_{2,3} &= 1,2 \text{ mm} \\ \Delta_{2,4} &= -1,1 \text{ mm} \\ \Delta_{3,4} &= 0,9 \text{ mm}\end{aligned}\tag{A.1}$$

A.3 Derivation of a reference quantity for computing permitted deviation

We assume that the uncertainty quantity u_T for the target centers (for example from manufacturer specifications) is

$$u_T = 1 \text{ mm}\tag{A.2}$$

According to [Formula \(4\)](#), the uncertainty u_d of the computed distances is obtained with the uncertainty propagation law from the uncertainty quantity u_T :

$$u_d = 1 \text{ mm} \times \sqrt{2} = \sqrt{2} \text{ mm}\tag{A.3}$$

Recalling furthermore [Formula \(5\)](#), the uncertainty u_Δ of the distance differences becomes:

$$u_\Delta = 1 \text{ mm} \times \sqrt{2} = \sqrt{2} \text{ mm}\tag{A.4}$$

Due to [Formula \(6\)](#) the expanded measurement uncertainty with $k = 2$ for distance differences results in:

$$U_\Delta = 1 \text{ mm} \times 2 \times \sqrt{2} = 4 \text{ mm}\tag{A.5}$$

A.4 Quantification of measurement deviations and judgement of the instrument for the simple test procedure

To analyse a constant distance offset, the distance between T_1 and T_2 is considered. We notice that:

$$|\Delta_{1,2}| = 9,4 \text{ mm} > U_\Delta\tag{A.6}$$

A systematic deviation of the distance measurement can be assumed, because the difference between $d_{S1,1,2}$ and $d_{S2,1,2}$ is around twice of the permitted deviation. In such a case, the execution of the full test procedure is recommended. If the full test procedure cannot be carried out, then the simplified test procedure shall be repeated with a slightly different configuration of the test field, e.g. by changing the locations of the instrument stations slightly. If that test also fails, an adjustment by the manufacturer is recommended.

Annex B (informative)

Example for the full test procedure

B.1 Measurements

[Table B.1](#) contains an example of observed data taken in accordance with the full test procedure.

Table B.1 — Measurements

Dimensions in metres

Instrument station S_n	Target point j	Set w	x	y	z
S_1	1	1	0,961 5	4,906 5	-0,100 1
	2		8,598 9	43,886 0	0,051 9
	3		-30,655 2	51,576 6	0,093 7
	4		8,570 5	43,891 0	40,001 0
	1	2	0,961 4	4,905 7	-0,100 0
	2		8,598 9	43,885 0	0,049 6
	3		-30,655 0	51,579 4	0,094 1
	4		8,569 5	43,889 6	40,002 4
	1	3	0,961 4	4,905 8	-0,099 7
	2		8,595 8	43,887 9	0,052 2
	3		-30,656 6	51,575 5	0,099 4
	4		8,569 1	43,891 7	40,000 3
S_2	1	1	17,873 0	29,767 1	-0,033 6
	2		-2,574 8	-4,287 6	0,097 9
	3		31,718 7	-24,880 3	0,126 7
	4		-2,534 1	-4,329 7	40,046 5
	1	2	17,872 7	29,767 1	-0,034 6
	2		-2,573 4	-4,286 5	0,097 7
	3		31,718 4	-24,877 5	0,125 5
	4		-2,534 7	-4,330 6	40,047 5
	1	3	17,874 7	29,764 9	-0,031 1
	2		-2,573 6	-4,286 1	0,097 9
	3		31,715 3	-24,880 1	0,126 2
	4		-2,534 6	-4,331 4	40,047 6

B.2 Calculation

According to [Formula \(11\)](#) the distances in all combinations between the targets T₁, T₂, T₃ and T₄ are:

Table B.2 — Calculated distances

Dimensions in metres

Instrument station S ₁			Instrument station S ₂		
Set w	$d_{S1,i,j,w}$	Value	Set w	$d_{S2,i,j,w}$	Value
1	$d_{S1,1,2,1}$	39,721 0	1	$d_{S2,1,2,1}$	39,722 2
	$d_{S1,1,3,1}$	56,371 5		$d_{S2,1,3,1}$	56,374 4
	$d_{S1,1,4,1}$	56,442 8		$d_{S2,1,4,1}$	56,439 8
	$d_{S1,2,3,1}$	40,000 4		$d_{S2,2,3,1}$	40,001 3
	$d_{S1,2,4,1}$	39,949 1		$d_{S2,2,4,1}$	39,948 6
	$d_{S1,3,4,1}$	56,482 9		$d_{S2,3,4,1}$	56,472 8
2	$d_{S1,1,2,2}$	39,720 8	2	$d_{S2,1,2,2}$	39,720 4
	$d_{S1,1,3,2}$	56,374 2		$d_{S2,1,3,2}$	56,371 6
	$d_{S1,1,4,2}$	56,443 2		$d_{S2,1,4,2}$	56,441 9
	$d_{S1,2,3,2}$	40,000 9		$d_{S2,2,3,2}$	39,999 0
	$d_{S1,2,4,2}$	39,952 8		$d_{S2,2,4,2}$	39,949 8
	$d_{S1,3,4,2}$	56,483 3		$d_{S2,3,4,2}$	56,473 1
3	$d_{S1,1,2,3}$	39,722 9	3	$d_{S2,1,2,3}$	39,719 3
	$d_{S1,1,3,3}$	56,372 0		$d_{S2,1,3,3}$	56,370 8
	$d_{S1,1,4,3}$	56,442 8		$d_{S2,1,4,3}$	56,439 3
	$d_{S1,2,3,3}$	39,998 2		$d_{S2,2,3,3}$	39,998 0
	$d_{S1,2,4,3}$	39,948 1		$d_{S2,2,4,3}$	39,949 7
	$d_{S1,3,4,3}$	56,478 1		$d_{S2,3,4,3}$	56,471 4

The corresponding mean distances and the standard deviation for a single observed distance are calculated for both station numbers S_1 and S_2 as follows:

Table B.3 — Calculated mean distances and their standard deviations

Dimensions in metres

S_n	Distance	Mean distances \bar{d}	Standard deviation s_d for a single observed distance
S_1	$\bar{d}_{S1,1,2}$	39,721 6	0,001 2
	$\bar{d}_{S1,1,3}$	56,372 6	0,001 4
	$\bar{d}_{S1,1,4}$	56,442 9	0,000 2
	$\bar{d}_{S1,2,3}$	39,999 8	0,001 5
	$\bar{d}_{S1,2,4}$	39,950 0	0,002 5
	$\bar{d}_{S1,3,4}$	56,481 4	0,002 9
S_2	$\bar{d}_{S2,1,2}$	39,720 6	0,001 5
	$\bar{d}_{S2,1,3}$	56,372 3	0,001 9
	$\bar{d}_{S2,1,4}$	56,440 4	0,001 4
	$\bar{d}_{S2,2,3}$	39,999 4	0,001 7
	$\bar{d}_{S2,2,4}$	39,949 4	0,000 7
	$\bar{d}_{S2,3,4}$	56,472 4	0,000 9

According to the values from previous table we calculate the differences of the mean distances based on [Formula \(12\)](#):

$$\bar{\Delta}_{1,2} = 1,0 \text{ mm}$$

$$\bar{\Delta}_{1,3} = 3,0 \text{ mm}$$

$$\bar{\Delta}_{1,4} = 2,5 \text{ mm}$$

$$\bar{\Delta}_{2,3} = 4,0 \text{ mm}$$

$$\bar{\Delta}_{2,4} = 0,6 \text{ mm}$$

$$\bar{\Delta}_{3,4} = 9,0 \text{ mm}$$

The calculated residuals for determining the precision of the distance of each instrument station are given in the [Table B.4](#), using [Formula \(13\)](#):

Table B.4 — Calculated residuals according to [Formula \(13\)](#)

Dimensions in metres

Instrument station S ₁			Instrument station S ₂		
Set w	$r_{S1,i,j,w}$	Value	Set w	$r_{S2,i,j,w}$	Value
1	$r_{S1,1,2,1}$	0,000 6	1	$r_{S2,1,2,1}$	-0,001 6
	$r_{S1,1,3,1}$	0,001 0		$r_{S2,1,3,1}$	-0,002 1
	$r_{S1,1,4,1}$	0,000 1		$r_{S2,1,4,1}$	0,000 5
	$r_{S1,2,3,1}$	-0,000 6		$r_{S2,2,3,1}$	-0,001 9
	$r_{S1,2,4,1}$	0,000 8		$r_{S2,2,4,1}$	0,000 8
	$r_{S1,3,4,1}$	-0,001 4		$r_{S2,3,4,1}$	-0,000 3
2	$r_{S1,1,2,2}$	0,000 7	2	$r_{S2,1,2,2}$	0,000 2
	$r_{S1,1,3,2}$	-0,001 6		$r_{S2,1,3,2}$	0,000 6
	$r_{S1,1,4,2}$	-0,000 3		$r_{S2,1,4,2}$	-0,001 5
	$r_{S1,2,3,2}$	-0,001 1		$r_{S2,2,3,2}$	0,000 5
	$r_{S1,2,4,2}$	-0,002 8		$r_{S2,2,4,2}$	-0,000 4
	$r_{S1,3,4,2}$	-0,001 9		$r_{S2,3,4,2}$	-0,000 7
3	$r_{S1,1,2,3}$	-0,001 4	3	$r_{S2,1,2,3}$	0,001 3
	$r_{S1,1,3,3}$	0,000 6		$r_{S2,1,3,3}$	0,001 5
	$r_{S1,1,4,3}$	0,000 1		$r_{S2,1,4,3}$	0,001 0
	$r_{S1,2,3,3}$	0,001 7		$r_{S2,2,3,3}$	0,001 4
	$r_{S1,2,4,3}$	0,001 9		$r_{S2,2,4,3}$	-0,000 3
	$r_{S1,3,4,3}$	0,003 3		$r_{S2,3,4,3}$	0,001 0
	$\Omega_{S1} = \sum r_{S1,i,j,k}^2$	0,000 040 24 m ²		$\Omega_{S2} = \sum r_{S2,i,j,k}^2$	0,000 023 26 m ²

The experimental standard deviation $\hat{s}_{0,1}$ and $\hat{s}_{0,2}$ for a distance of S₁ and of S₂ are calculated according to [Formulae \(14\)](#) and [\(15\)](#):

$$\hat{s}_{0,1} = \sqrt{\frac{0,000\,040\,24\text{ m}^2}{12}} = 0,0018\text{ m} = 1,8\text{ mm} \quad (\text{B.1})$$

$$\hat{s}_{0,2} = \sqrt{\frac{0,000\,023\,26\text{ m}^2}{12}} = 0,001\,4\text{ m} = 1,4\text{ mm} \tag{B.2}$$

The experimental standard deviation for both instrument stations is calculated with [Formula \(16\)](#) because $\hat{s}_{0,1}$ and $\hat{s}_{0,2}$ belong to the same population according to the tests in [B.4](#):

$$\hat{s}_0 = \sqrt{\frac{0,000\,063\,50\text{ m}^2}{24}} = 0,001\,6\text{ m} = 1,6\text{ mm} \tag{B.3}$$

The mean values for the distances for both instrument stations are calculated with [Formula \(18\)](#):

$$\begin{aligned} \bar{d}_{1,2} &= 39,721\,1\text{ m}, \bar{d}_{1,3} = 56,372\,4\text{ m}, \bar{d}_{1,4} = 56,441\,6\text{ m}, \\ \bar{d}_{2,3} &= 39,999\,6\text{ m}, \bar{d}_{2,4} = 39,949\,7\text{ m}, \bar{d}_{3,4} = 56,476\,9\text{ m} \end{aligned} \tag{B.4}$$

According to [Formula \(19\)](#), the residuals calculated by means of the mean distances for both instrument stations are:

Table B.5 — Calculated residuals according to [Formula \(19\)](#)

Dimensions in metres

Instrument station S ₁			Instrument station S ₂		
Set <i>w</i>	$\bar{r}_{S1,i,j,w}$	Value	Set <i>w</i>	$\bar{r}_{S2,i,j,w}$	Value
1	$\bar{r}_{S1,1,2,1}$	0,000 1	1	$\bar{r}_{S2,1,2,1}$	-0,001 1
	$\bar{r}_{S1,1,3,1}$	0,000 9		$\bar{r}_{S2,1,3,1}$	-0,001 9
	$\bar{r}_{S1,1,4,1}$	-0,001 1		$\bar{r}_{S2,1,4,1}$	0,001 8
	$\bar{r}_{S1,2,3,1}$	-0,000 8		$\bar{r}_{S2,2,3,1}$	-0,001 7
	$\bar{r}_{S1,2,4,1}$	0,000 6		$\bar{r}_{S2,2,4,1}$	0,001 1
	$\bar{r}_{S1,3,4,1}$	-0,005 9		$\bar{r}_{S2,3,4,1}$	0,004 2
2	$\bar{r}_{S1,1,2,2}$	0,000 3	2	$\bar{r}_{S2,1,2,2}$	0,000 7
	$\bar{r}_{S1,1,3,2}$	-0,001 8		$\bar{r}_{S2,1,3,2}$	0,000 8
	$\bar{r}_{S1,1,4,2}$	-0,001 6		$\bar{r}_{S2,1,4,2}$	-0,000 3
	$\bar{r}_{S1,2,3,2}$	-0,001 3		$\bar{r}_{S2,2,3,2}$	0,000 7
	$\bar{r}_{S1,2,4,2}$	-0,003 1		$\bar{r}_{S2,2,4,2}$	-0,000 1
	$\bar{r}_{S1,3,4,2}$	-0,006 4		$\bar{r}_{S2,3,4,2}$	0,003 8

Table B.5 (continued)

Instrument station S ₁			Instrument station S ₂		
Set w	$\bar{r}_{S1,i,j,w}$	Value	Set w	$\bar{r}_{S2,i,j,w}$	Value
3	$\bar{r}_{S1,1,2,3}$	-0,001 8	3	$\bar{r}_{S2,1,2,3}$	0,001 8
	$\bar{r}_{S1,1,3,3}$	0,000 4		$\bar{r}_{S2,1,3,3}$	0,001 6
	$\bar{r}_{S1,1,4,3}$	-0,001 1		$\bar{r}_{S2,1,4,3}$	0,002 3
	$\bar{r}_{S1,2,3,3}$	0,001 5		$\bar{r}_{S2,2,3,3}$	0,001 6
	$\bar{r}_{S1,2,4,3}$	0,001 6		$\bar{r}_{S2,2,4,3}$	-0,000 03
	$\bar{r}_{S1,3,4,3}$	-0,001 2		$\bar{r}_{S2,3,4,3}$	0,005 5
	$\bar{\Omega}_{S1} = \sum \bar{r}_{S1,i,j,k}^2$	0,000 107 0 m ²		$\bar{\Omega}_{S2} = \sum \bar{r}_{S2,i,j,k}^2$	0,000 090 1 m ²

The experimental standard deviation of the distance for both instrument stations is calculated with [Formula \(20\)](#) and [\(21\)](#):

$$\hat{s}_0 = \sqrt{\frac{0,000\ 197\ 13\ \text{m}^2}{30}} = 0,002\ 56\ \text{m} = 2,56\ \text{mm} \quad (\text{B.5})$$

Finally, the standard uncertainty of the TLS for a point is (refer to [Formula \(23\)](#)):

$$u_{\text{ISO-TLS}} = \frac{\hat{s}_0}{\sqrt{2}} = 1,8\ \text{mm} \quad (\text{B.6})$$

B.3 Statistical tests

— Statistical test according to Question b):

$$\hat{s}_{0,1} = 1,8\ \text{mm}$$

$$\hat{s}_{0,2} = 1,4\ \text{mm}$$

$$\nu_{S1} = \nu_{S2} = 12$$

$$0,31 \leq \frac{\hat{s}_{0,1}^2}{\hat{s}_{0,2}^2} \leq 3,28$$

$$0,31 \leq 1,73 \leq 3,28$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations $\hat{s}_{0,1}$ and $\hat{s}_{0,2}$ belong to the same population is not rejected at the confidence level of 95 %.

B.4 Determination of measurement uncertainty of the target centre

Case A: Assuming that the uncertainty given by the manufacturer is $u_{ms} = 3$ mm, the uncertainty of the 3D point results, according to [Formula \(30\)](#), in $u_T = 3$ mm

Case B: Due to [Formula \(31\)](#), the combined standard uncertainty is calculated based on the standard uncertainty $u_{ISO-TLS} = 1,8$ mm of type A and based on the type B uncertainty $u_p = 2,9$ mm as derived from the typical influence quantities (refer to [C.3](#)):

$$u_T = \sqrt{0,000\ 003\ 3\ m^2 + 0,000\ 008\ 6\ m^2} = \sqrt{0,000\ 011\ 9\ m^2} = 0,003\ 4\ m = 3,4\ mm \quad (B.7)$$

Case C: According to [Formula \(32\)](#), the uncertainty is approximated based only on the standard uncertainty $u_{ISO-TLS} = 1,8$ mm of type A:

$$u_T \approx \sqrt{0,000\ 003\ 3\ m^2} = 0,001\ 8\ m = 1,8\ mm \quad (B.8)$$

B.5 Derivation of the permitted deviation for the full test procedure

After determination of the uncertainty of the target centers, we derive the permitted deviation for this procedure for all three cases. Note, that the expanded measurement uncertainty U_Δ with $k = 2$ is determined according to [8.5.2](#) by $U_\Delta = 4\ u_T$.

Case A: Due to calculation in [B.4](#) (case A), the uncertainty of the target centre is $u_T = 3,0$ mm. The expanded measurement uncertainty for distance differences results in:

$$U_\Delta = 4 \times u_T = 12\ mm \quad (B.9)$$

The permitted deviation results in this case to $U_\Delta / \sqrt{3} = 7$ mm .

Case B: Due to calculation in [B.4](#) (case B), the uncertainty of the target centre is $u_T = 3,4$ mm. The expanded measurement uncertainty for distance differences results in:

$$U_\Delta = 4 \times u_T = 13,6\ mm \quad (B.10)$$

The permitted deviation results in this case to $U_\Delta / \sqrt{3} = 7,8$ mm .

Case C: Due to calculation in [B.4](#) (case C), the uncertainty of target center is $u_T = 1,8$ mm. The expanded measurement uncertainty for distance differences results in:

$$U_\Delta = 4 \times u_T = 7,2\ mm \quad (B.11)$$

The permitted deviation results in this case to $U_\Delta / \sqrt{3} = 4,2$ mm .