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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 8: GNSS field measurement systems in real-time kinematic (RTK)

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 8: Systèmes de mesure GNSS sur site en temps réel cinématique

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This second edition cancels and replaces the first edition (ISO 17123-8:2007), which has been technically revised.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*:

— *Part 1: Theory*

— *Part 2: Levels*

— *Part 3: Theodolites*

— *Part 4: Electro-optical distance meters (EDM measurements to reflectors)*

— *Part 5: Total stations*

— *Part 6: Rotating lasers*

— *Part 7: Optical plumbing instruments*

— *Part 8: GNSS field measurement systems in real-time kinematic (RTK)*

Annex A, Annex B, and Annex C of this part of ISO 17123 are for information only.

Introduction

This part of ISO 17123 specifies field procedures for adoption when determining and evaluating the uncertainty of measurement results obtained by geodetic instruments and their ancillary equipment, when used in building and surveying measuring tasks. Primarily, these tests are intended to be field verifications of suitability of a particular instrument for the immediate task. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

The definition and concept of uncertainty as a quantitative attribute to the final result of measurement was developed mainly in the last two decades, even though error analysis has already long been a part of all measurement sciences. After several stages, the CIPM (Comité Internationale des Poids et Mesures) referred the task of developing a detailed guide to ISO. Under the responsibility of the ISO Technical Advisory Group on Metrology (TAG 4) and in conjunction with six worldwide metrology organizations, a guidance document on the expression of measurement uncertainty was compiled with the objective of providing rules for use within standardization, calibration, laboratory, accreditation, and metrology services. ISO/IEC Guide 98-3 was first published as the Guide to the Expression of Uncertainty in Measurement (GUM) in 1995.

With the introduction of uncertainty in measurement in ISO 17123 (all parts), it is intended to finally provide a uniform, quantitative expression of measurement uncertainty in geodetic metrology with the aim of meeting the requirements of customers.

ISO 17123 (all parts) provides not only a means of evaluating the precision (experimental standard deviation) of an instrument but also a tool for defining an uncertainty budget, which allows for the summation of all uncertainty components, whether they are random or systematic, to a representative measure of accuracy, i.e. the combined standard uncertainty.

ISO 17123 (all parts) therefore provides, for defining for each instrument investigated by the procedures, a proposal for additional, typical influence quantities, which can be expected during practical use. The customer can estimate, for a specific application, the relevant standard uncertainty components in order to derive and state the uncertainty of the measuring result.

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 8: GNSS field measurement systems in real-time kinematic (RTK)

1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of Global Navigation Satellite System (GNSS) field measurement systems (this includes GPS, GLONASS, as well as the future systems like GALILEO) in real-time kinematic (GNSS RTK) and their ancillary equipment when used in building, surveying, and industrial measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the required application at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 12858-2, *Optics and optical instruments — Ancillary devices for geodetic instruments — Part 2: Tripods*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

ISO 17123-2, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 2: Levels*

ISO 17123-5, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 5: Total stations*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM: 1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 9849, ISO 17123-1, ISO 17123-2, ISO 17123-5, ISO/IEC Guide 98-3, and ISO/IEC Guide 99 apply.

4 General

4.1 Preamble

The real-time kinematic positioning method is a relative measuring procedure using reference (base) and moving (rover) receivers. For utilization of network RTK applications, a separate reference receiver is not required. Both receivers perform the observations simultaneously and merge their results by wireless transmission. Thus, the rover can display the instantaneous coordinates of the antenna in any appropriate datum, e.g. International Terrestrial Reference Frame (ITRF). For practical use, they are transformed to horizontal coordinates and ellipsoidal heights. Subsequently, only these types of coordinate are treated as original observables.

4.2 Requirements

Before commencing surveying, it is important for the operator to ensure that the equipment, the GNSS receiver and antenna, has sufficient precision for the task required.

The test should apply typically to a set of GNSS receivers and antennae listed in the manufacturer's reference manual. In case of using network RTK, consistency of antenna models (e.g. antenna correction parameters) shall be ensured.

The receiver, antenna, and their ancillary equipment for rover points shall be checked to be in acceptable condition according to the methods specified in the reference manual.

The operator shall follow the guidelines in the manufacturer's reference manual for positioning requirements such as the minimum number of satellites, maximum Position Dilution Of Precision (PDOP) value, minimum observation time, and possibly other required pre-conditions.

The operator shall initialize the receiver by resetting its power prior to every measurement and collect the data after the integer ambiguity fixing has been completed.

The following is the guideline for achievable centring precision expressed in standard deviation:

— centring: 1 mm;

— measuring the antenna height: 1 mm.

The results of the test are influenced by several factors, such as satellite configuration visible at the points, ionospheric and tropospheric conditions, multipath environment around the points, precision of the equipment, and quality of the software running in the rover equipment or in the system generating the data transmitted from the base point.

This part of ISO 17123 describes two different field procedures, namely the simplified test procedure and the full test procedure, as given in Clauses 5 and 6, respectively. Therefore, the observation time of test procedure shall be so arranged to cover such variations.

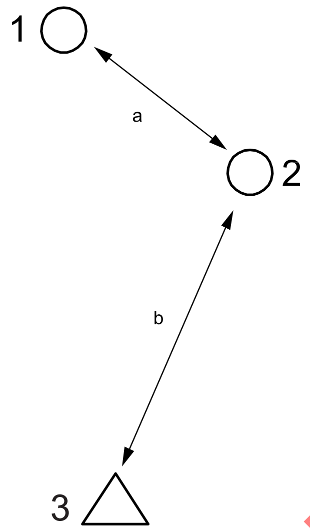
The operator shall choose the procedure that is most appropriate to the requirements of the project.

4.3 Concept of the test procedures

The test field consists of a base point and two rover points. The location of the rover points shall be close to the area of the task concerned. The separation of two rover points shall be a minimum of 2 m and shall not exceed 20 m. The positions of two rover points may be selected at convenience in the field (see Figure 1).

The horizontal distance and height difference between two rover points shall be determined by methods with precision better than 3 mm other than RTK. These values are considered as nominal values and are used in the first step of both test procedures. The horizontal distances and height differences calculated from the measured coordinates in each set of measurements shall be compared with these values in order to ensure that the measurements are free from any outlier. However, the nominal values are not used in the statistical tests.

A series of measurements consists of five sets. Each set of measurements consists of successive measurements at rover points 1 and 2.



Key

1 rover point 1

2 rover point 2

3 base point

a Minimum 2 m; shall not exceed 20 m.

b Corresponding distance according to the task concerned.

Figure 1 — Configuration of the field test network

The time lag between successive sets shall be approximately 5 min. This requirement makes the span of a series of measurements to be about 25 min and five sets of measurements at both rover points shall be uniformly distributed in this span. Due to the fact that the variation cycle of a typical multipath influence is about 20 min, the measuring procedure will mostly cover the period of this influence factor.

The start time for each successive series shall be separated by at least 90 min. Thus, multiple series of measurements tend to reflect influences such as changes in satellite configuration and variations in the ionospheric and tropospheric conditions.

The standard deviations calculated over all measurements will therefore represent a quantitative measure of precision in use including most of the typical influences in satellite positioning.

The simplified test procedure contains only one series of measurements and therefore only deals with the outlier detection and with no statistical evaluation. Conversely, the full test procedure consists of three series and additionally enables the estimation of standard deviations and statistical tests.

4.4 Procedure 1: Simplified test procedure

The simplified test procedure consists of a single series of measurements and provides an estimate as to whether the precision of the equipment in use is within a specified allowable deviation.

The simplified test procedure is based on a limited number of measurements. Therefore, a significant standard deviation cannot be obtained and the statistical tests are not applied. If a more precise assessment of the equipment is required, it is recommended to adopt the more rigorous full test procedure as given in 4.5.

4.5 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of the equipment in use.

The full test procedure consists of three series of measurements.

The full test procedure is intended for determining the experimental standard deviation for a single position and height measurement.

Further, this procedure may be used to determine the following:

— the measure of the precision of equipment under given conditions (including typical short- and long-term influences);

— the measure of the precision of equipment used in different periods of time or under different conditions (multiple samples);

— the measure of the capability of comparison between different precision of equipment achievable under similar conditions.

Statistical tests shall be applied to determine whether the sample from the experiment belongs to the same population as the one giving the theoretical standard deviation and to determine whether two samples from different experiments belong to the same population.

5 Simplified test procedure

5.1 Measurements

For the simple test procedure, one series of measurements shall be taken, in which the observer shall obtain five sets of measurements at two rover points. The sequence of the measurements is shown in Table 1 in which the column labelled "Seq. No." explicitly indicates the sequence of the measurement.

Table 1 — Sequence of the measurements for one series

Seq. No.	Series	Set	Rover point	Measurement		
	i	j	k	x	y	h
1	1	1	1	$x_{1,1,1}$	$y_{1,1,1}$	$h_{1,1,1}$
2	1	1	2	$x_{1,1,2}$	$y_{1,1,2}$	$h_{1,1,2}$
3	1	2	1	$x_{1,2,1}$	$y_{1,2,1}$	$h_{1,2,1}$
4	1	2	2	$x_{1,2,2}$	$y_{1,2,2}$	$h_{1,2,2}$
5	1	3	1	$x_{1,3,1}$	$y_{1,3,1}$	$h_{1,3,1}$
6	1	3	2	$x_{1,3,2}$	$y_{1,3,2}$	$h_{1,3,2}$
7	1	4	1	$x_{1,4,1}$	$y_{1,4,1}$	$h_{1,4,1}$
8	1	4	2	$x_{1,4,2}$	$y_{1,4,2}$	$h_{1,4,2}$
9	1	5	1	$x_{1,5,1}$	$y_{1,5,1}$	$h_{1,5,1}$
10	1	5	2	$x_{1,5,2}$	$y_{1,5,2}$	$h_{1,5,2}$

A specific set of measurements is expressed as $x_{i,j,k}$, $y_{i,j,k}$, and $h_{i,j,k}$ where *x*, *y*, and *h* are coordinates of a local coordinate system. The index *i* stands for the series number, the index *j* for the set number, and the index *k* for the rover point number. For example, $x_{1,3,2}$ is the *x* component of the third set of measurement at rover point 2 in the first series.

The sequence of measurements should follow Table 1 in the full test procedure (see 6.1).

5.2 Calculation

The individual measurements are compared directly with the nominal values available so as to detect any measurement with gross error.

For each set j (= 1,...,5) in the series i (= 1), calculate the horizontal distance and height difference between two rover points. Subsequently, calculate their deviations from the nominal values.

$$\begin{aligned} D_{i,j} &= \sqrt{(x_{i,j,2} - x_{i,j,1})^2 + (y_{i,j,2} - y_{i,j,1})^2} \\ \Delta h_{i,j} &= h_{i,j,2} - h_{i,j,1} \\ \varepsilon_{D\,i,j} &= D_{i,j} - D^* \\ \varepsilon_{h\,i,j} &= h_{i,j} - h^* \end{aligned} \qquad i = 1,\ j = 1,\ldots,5 \tag{1}$$

where

$x_{i,j,k}$, $y_{i,j,k}$, $h_{i,j,k}$	are x, y, and h measurements, respectively, in the set j at rover point k in series i;
$D_{i,j}$, $\Delta h_{i,j}$	are the calculated horizontal distance and height difference, respectively, in the set j in series i;
D^*, h^*	are nominal values of the horizontal distance and height difference, respectively;
$\varepsilon_{D\,i,j}$, $\varepsilon_{h\,i,j}$	are deviations of the horizontal distance and height difference, respectively.

If any deviation fails to satisfy either of the two conditions in Formula (2), the inclusion of an outlier (or outliers) in the corresponding measurements is suspected; repeat the test procedure.

$$\begin{aligned} \left|\varepsilon_{D\,i,j}\right| &\le 2,5 \times \sqrt{2} \times s_{xy} \\ \left|\varepsilon_{h\,i,j}\right| &\le 2,5 \times \sqrt{2} \times s_h \end{aligned} \tag{2}$$

where

s_{xy} and s_h	are either the predetermined standard deviation according to the full test procedure or the values specified by the manufacturer.

6 Full test procedure

6.1 Measurements

For the full test procedure, three series of measurements shall be taken. The sequence of the measurements in each series is identical to the case of the simplified test. The start times of consecutive series shall be separated by at least 90 min.

6.2 Calculation

6.2.1 General

Calculation is performed in two steps. In the first step, individual measurements are compared directly with the nominal values available in order to detect any measurement with gross error. The statistical values of interest are calculated in the second step. All procedures of the two steps are described in the adjacent clauses.

6.2.2 Preliminary measurement check

The same procedure described previously in the simplified procedure shall be applied to all measurements in all three series.

6.2.3 Calculation of statistical values

Firstly, applying the least squares adjustment on overall measurements in all series, the estimates of *x*, *y*, and *h* for each rover point *k* (= 1, 2) are calculated as

$$\begin{aligned} \bar{x}_k &= \frac{1}{15}\sum_{i=1}^{3}\sum_{j=1}^{5} x_{i,j,k} \\ \bar{y}_k &= \frac{1}{15}\sum_{i=1}^{3}\sum_{j=1}^{5} y_{i,j,k} \qquad k = 1,\ 2 \\ \bar{h}_k &= \frac{1}{15}\sum_{i=1}^{3}\sum_{j=1}^{5} y_{i,j,k} \end{aligned} \tag{3}$$

Then the residuals of *x*, *y*, and *h* for all measurements in three series are calculated as

$$\begin{aligned} r_{x\,i,j,k} &= \bar{x}_k - x_{i,j,k} \\ r_{y\,i,j,k} &= \bar{y}_k - y_{i,j,k} \qquad k = 1, 2, j = 1, \ldots, 5,\ i = 1, 2, 3 \\ r_{h\,i,j,k} &= \bar{h}_k - h_{i,j,k} \end{aligned} \tag{4}$$

The above residuals are all squared and summed including measurements for all point index *k* = 1 and *k* = 2 for *x*, *y*, and *h* separately as

$$\begin{aligned} \sum r_x^2 &= \sum_{i=1}^{3}\sum_{j=1}^{5}\sum_{k=1}^{2} r_{x\,i,j,k}^2 \\ \sum r_y^2 &= \sum_{i=1}^{3}\sum_{j=1}^{5}\sum_{k=1}^{2} r_{y\,i,j,k}^2 \\ \sum r_h^2 &= \sum_{i=1}^{3}\sum_{j=1}^{5}\sum_{k=1}^{2} r_{h\,i,j,k}^2 \end{aligned} \tag{5}$$

The degrees of freedom for *x*, *y*, and *h* are identical. These are calculated as

$$v_x = v_y = v_h = (m \cdot n - 1) \cdot p = (3 \times 5 - 1) \times 2 = 28 \tag{6}$$

where

- m is the number of series, = 3;
- n is the number of sets in a series, = 5;
- p is the number of rover points, = 2.

Finally, the experimental standard deviations of a single measurement of *x*, *y*, and *h* are calculated as

$$s_x = \sqrt{\frac{\sum r_x^2}{\nu_x}} = \sqrt{\frac{\sum r_x^2}{28}}$$
$$s_y = \sqrt{\frac{\sum r_y^2}{\nu_y}} = \sqrt{\frac{\sum r_y^2}{28}} \tag{7}$$
$$s_h = \sqrt{\frac{\sum r_h^2}{\nu_h}} = \sqrt{\frac{\sum r_h^2}{28}}$$

and therefore the experimental standard deviation of a single position (*x*, *y*) yields

$$s_{xy} = \sqrt{s_x^2 + s_y^2} \tag{8}$$

Herewith, we can state the standard uncertainties (Type A) of a single position (*x*, *y*)

$$u_{\text{ISO-GNSS RTK-}xy} = s_{xy} \tag{9}$$

and of a single height

$$u_{\text{ISO-GNSS RTK-}h} = s_h \tag{10}$$

6.3 Statistical tests

6.3.1 General

Statistical tests are practicable for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using the experimental standard deviations s_{xy} and s_h obtained from the measurements and their respective degrees of freedom in order to answer the following questions (see Table 2).

a) Is the calculated experimental standard deviation, s_{xy}, of a single position, *x*, *y*, smaller than or equal to a corresponding value, σ_{xy}, stated by the manufacturer or another predetermined value, σ_{xy}?

b) Is the calculated experimental standard deviation, s_h, of a single height, *h*, smaller than or equal to a corresponding value, σ_h, stated by the manufacturer or another predetermined value, σ_h?

c) Do two experimental standard deviations, s_{xy} and $\tilde{s}_{xy}$, of a single position (*x*, *y*) as determined from two different samples of measurement belong to the same population, assuming that both samples have the same number of degrees of freedom, $\nu_x + \nu_y$ and $\tilde{\nu}_x + \tilde{\nu}_y$, corresponding to s_{xy} and $\tilde{s}_{xy}$ respectively?

d) Do two experimental standard deviations, s_h and $\tilde{s}_h$, of a single height, *h*, as determined from two different samples of measurement belong to the same population, assuming that both samples have the same number of degrees of freedom, ν_h and $\tilde{\nu}_h$, corresponding to s_h and $\tilde{s}_h$ respectively?

The experimental standard deviations, s and $\tilde{s}$, may be obtained from the following:

— two samples of measurements by the same equipment;

— two samples of measurements by different equipment.

For the following tests, a confidence level of $1-\alpha=0,95$ and the degrees of freedom of $\nu_x+\nu_y=56$ or $\nu_h=28$ are assumed according to the design of measurements.

Table 2 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s_{xy}\leq\sigma_{xy}$	$s_{xy}>\sigma_{xy}$
b)	$s_h\leq\sigma_h$	$s_h>\sigma_h$
c)	$\sigma_{xy}=\tilde{\sigma}_{xy}$	$\sigma_{xy}\neq\tilde{\sigma}_{xy}$
d)	$\sigma_h=\tilde{\sigma}_h$	$\sigma_h\neq\tilde{\sigma}_h$

6.3.2 Question a)

The null hypothesis stating that the experimental standard deviation, s_{xy}, of a single position, x, y, is smaller than or equal to a theoretical or a predetermined value, σ_{xy}, is not rejected if the following condition is fulfilled:

$$s_{xy}\leq\sigma_{xy}\cdot\sqrt{\frac{\chi^2_{0,95}\cdot(\nu_x+\nu_y)}{\nu_x+\nu_y}} \tag{11}$$

$$s_{xy}\leq\sigma_{xy}\cdot\sqrt{\frac{\chi^2_{0,95}\cdot(56)}{56}} \tag{12}$$

$$\chi^2_{0,95}(56)=74,47 \tag{13}$$

$$s_{xy}\leq\sigma_{xy}\times\sqrt{\frac{74,74}{56}}=\sigma_{xy}\times1,15 \tag{14}$$

Otherwise, the null hypothesis is rejected.

6.3.3 Question b)

The null hypothesis stating that the experimental standard deviation, s_h, of a single height, *h*, is smaller than or equal to a theoretical or a predetermined value, σ_h, is not rejected if the following condition is fulfilled:

$$s_h\leq\sigma_h\cdot\sqrt{\frac{\chi^2_{0,95}(\nu_h)}{\nu_h}} \tag{15}$$

$$s_h\leq\sigma_h\cdot\sqrt{\frac{\chi^2_{0,95}(28)}{28}} \tag{16}$$

$$\chi^2_{0,95}(28)=41,34 \tag{17}$$

$$s_h \leq \sigma_h \times \sqrt{\frac{41,34}{28}} = \sigma_h \times 1,22 \tag{18}$$

Otherwise, the null hypothesis is rejected.

6.3.4 Question c)

In the case of two different samples, the test indicates whether the experimental standard deviations, s_{xy} and $\tilde{s}_{xy}$, of a single position (x, h) belong to the same population. The corresponding null hypothesis, $\sigma_{xy} = \tilde{\sigma}_{xy}$, is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\tilde{v}_x + \tilde{v}_y, v_x + v_y)} \leq \frac{s_{xy}^2}{\tilde{s}_{xy}^2} \leq F_{1-\alpha/2}(v_x + v_y, \tilde{v}_x + \tilde{v}_y) \tag{19}$$

$$\frac{1}{F_{0,975}(56,56)} \leq \frac{s_{xy}^2}{\tilde{s}_{xy}^2} \leq F_{0,975}(56,56) \tag{20}$$

$$F_{0,975}(56,56) = 1,70 \tag{21}$$

$$0,59 \leq \frac{s_{xy}^2}{\tilde{s}_{xy}^2} \leq 1,70 \tag{22}$$

Otherwise, the null hypothesis is rejected.

6.3.5 Question d)

The hypothesis that the two experimental standard deviations, s_h and $\tilde{s}_h$, of a single height, h, belong to the same population, is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\tilde{v}_h, v_h)} \leq \frac{s_h^2}{\tilde{s}_h^2} \leq F_{1-\alpha/2}(v_h, \tilde{v}_h) \tag{23}$$

$$\frac{1}{F_{0,975}(28,28)} \leq \frac{s_h^2}{\tilde{s}_h^2} \leq F_{0,975}(28,28) \tag{24}$$

$$F_{0,975}(28,28) = 2,13 \tag{25}$$

$$0,47 \leq \frac{s_h^2}{\tilde{s}_h^2} \leq 2,13 \tag{26}$$

Otherwise, the null hypothesis is rejected.

6.4 Combined standard uncertainty evaluation (Type A and Type B)

The sources of uncertainty (influence quantities) are described in Table 3 as an uncertainty budget.

Table 3 — Typical influence quantities of the GNSS (RTK)

Sources of uncertainty	Symbol	Evaluation	Distribution
I. Result of measurement			
Standard uncertainty of *xy*-coordinates	$u_{\text{ISO-GNSS-}xy}$	Type A	Normal
Standard uncertainty of *h*-coordinates	$u_{\text{ISO-GNSS-}h}$	Type A	Normal
II. Relevant sources of the GNSS Receiver			
Sensitivity of the tubular level	u_{bub}	Type B	Specified by the manufacturer
Display round-off error *x*, *y*	u_{disp}	Type B	Rectangular
III. Error pattern from the setting of the instruments			
Centring	u_{c}	Type B	Normal
Antenna height (see 4.2)	u_{ha}	Type B	Normal
Stability of a tripod height (ISO 12858-2:1999)	u_{hs}	Type B	Rectangular
Antenna phase centre off-set-parameter d*x* and d*y*	$u_{\text{d}x}$ $u_{\text{d}y}$	Type B	Normal
Antenna phase centre off-set-parameter d*h*	$u_{\text{d}h}$	Type B	Normal
multi path			Not considered here
Clock in the GNSS receiver or satellite			Not considered here
Orbit of the satellite			Not considered here
Ionospheric delay			Not considered here
Tropospheric delay			Not considered here
IV. Mathematical modelling			
Transformation	u_{tr}	Type A	Normal
Geoid undulation	u_{dH}	Type B	Rectangular

Combined uncertainty on the horizontal coordinate system is described as

$$u_{xy} = \sqrt{u_{\text{ISO-GNSS}-xy}^2 + \left[h_\text{a} \ \tan(u_{\text{bub}})\right]^2 + 2u_{\text{disp}}^2 + u_\text{c}^2 + u_{\text{d}x}^2 + u_{\text{d}y}^2 + u_{\text{tr}}{}^2} \qquad (27)$$

where

h_a is the antenna height.

Combined uncertainty on the vertical coordinate system is described as

$$u_h = \sqrt{u_{\text{ISO-GNSS}-h}^2 + u_{\text{disp}}^2 + u_{ha}^2 + u_{hs}^2 + u_{\text{d}h}^2 + u_{\text{dH}}{}^2} \tag{28}$$

Expanded uncertainty:

With coverage factor k = 2

$$U_{xy} = 2 \cdot u_{xy} \tag{29}$$

$$U_h = 2 \cdot u_h \tag{30}$$

Annex A
(informative)

Example of the simplified test procedure

A.1 Measurements

— Observer: Tokio

— Weather: Fine, +5 °C

— Instruments type and number: AAA 01234

— Antenna type and number or built in: BBB 05678

— Date: 2006-01-21

— Base line (nominal value) horizontal distance D^* :19,996 m, height difference $\varnothing h^*$: 0,038 m

— Predefined standard deviation s_{xy} = 15 mm; s_h = 25 mm

A.2 Calculation

According to Formula (1), the calculated data are shown in the Table A.1

Table A.1 — Measurements and deviations

Seq. No.	Set	Rover point	Measurements m			Horizontal distance m	Height difference m	Deviations mm	
	j	k	x	y	h	D_j	Δh_j	$\varepsilon_{D\,i,j}$	$\varepsilon_{h\,i,j}$
1	1	1	−67 637,433	−63 945,554	320,732	—	—	—	—
2	1	2	−67 654,082	−63 934,442	320,781	20,017	0,049	21	11
3	2	1	−67 637,448	−63 945,550	320,732	—	—	—	—
4	2	2	−67 654,084	−63 934,451	320,774	19,999	0,042	3	4
5	3	1	−67 637,450	−63 945,550	320,745	—	—	—	—
6	3	2	−67 654,083	−63 934,454	320,793	19,994	0,048	−2	10
7	4	1	−67 637,453	−63 945,541	320,731	—	—	—	—
8	4	2	−67 654,077	−63 934,447	320,783	19,986	0,052	−10	14
9	5	1	−67 637,450	−63 945,555	320,740	—	—	—	—
10	5	2	−67 654,083	−63 934,452	320,778	19,998	0,038	2	0
Limit of each deviation mm			—	—	—	—	—	±53	±88

All deviations satisfy the condition of Formula (2). No outlier is suspected.

Annex B
(informative)

Example of the full test procedure

B.1 Measurements

— Observer: Bonn

— Weather: Fine, +5 °C

— Instruments type and number: BBB 01234

— Antenna type and number or built in: CCC 05678

— Date: 2006-09-22

— Base line (nominal value) horizontal distance D^* :19,994 m, height difference Δh^* : 0,028 m

— Pre-defined standard deviation s_{xy} = 15 mm; s_h = 25 mm

B.2 Calculation

B.2.1 Preliminary check

According to the Formula (1), the calculated data are shown on the Table B.1

Table B.1 — Measurements and experimental standard deviations

Seq No.	Series	Set	Rover point	Measurement			Horizontal distance	Height difference	Deviations	
				m			m	m	mm	
	i	j	K	x	y	h	D_j	Δh_j	$\varepsilon_{D\,i,j}$	$\varepsilon_{h\,i,j}$
1	1	1	1	−67 635,470	−63 943,197	320,792	—	—	—	—
2	1	1	2	−67 652,389	−63 932,527	320,799	20,003	0,007	9	−21
3	1	2	1	−67 635,479	−63 943,188	320,788	—	—	—	—
4	1	2	2	−67 652,376	−63 932,525	320,824	19,980	0,036	−14	8
5	1	3	1	−67 635,480	−63 943,189	320,789	—	—	—	—
6	1	3	2	−67 652,387	−63 932,529	320,810	19,987	0,021	−7	−7
7	1	4	1	−67 635,476	−63 943,192	320,793	—	—	—	—
8	1	4	2	−67 652,393	−63 932,530	320,808	19,997	0,015	3	−13
9	1	5	1	−67 635,481	−63 943,192	320,794	—	—	—	—
10	1	5	2	−67 652,390	−63 932,522	320,803	19,994	0,009	0	−19
11	2	1	1	−67 635,478	−63 943,191	320,800	—	—	—	—
12	2	1	2	−67 652,399	−63 932,535	320,823	19,997	0,023	3	−5
13	2	2	1	−67 635,479	−63 943,193	320,798	—	—	—	—
14	2	2	2	−67 652,392	−63 932,528	320,828	19,995	0,030	1	2

Table B.1 *(continued)*

Seq No.	Series	Set	Rover point	Measurement m			Horizontal distance m	Height difference m	Deviations mm	
	i	j	K	x	y	h	D_j	Δh_j	$\varepsilon_{D\,i,j}$	$\varepsilon_{h\,i,j}$
15	2	3	1	−67 635,477	−63 943,194	320,780	—	—	—	—
16	2	3	2	−67 652,396	−63 932,530	320,797	19,999	0,017	5	−11
17	2	4	1	−67 635,475	−63 943,191	320,786	—	—	—	—
18	2	4	2	−67 652,395	−63 932,532	320,812	19,998	0,026	4	−2
19	2	5	1	−67 635,476	−63 943,191	320,784	—	—	—	—
20	2	5	2	−67 652,391	−63 932,534	320,812	19,992	0,028	−2	0
21	3	1	1	−67 635,479	−63 943,194	320,798	—	—	—	—
22	3	1	2	−67 652,391	−63 932,529	320,826	19,994	0,028	0	0
23	3	2	1	−67 635,478	−63 943,195	320,805	—	—	—	—
24	3	2	2	−67 652,398	−63 932,532	320,823	20,000	0,018	6	−10
25	3	3	1	−67 635,485	−63 943,199	320,799	—	—	—	—
26	3	3	2	−67 652,400	−63 932,534	320,813	19,996	0,014	2	−14
27	3	4	1	−67 635,474	−63 943,195	320,804	—	—	—	—
28	3	4	2	−67 652,394	−63 932,532	320,831	20,000	0,027	6	−1
29	3	5	1	−67 635,483	−63 943,200	320,793	—	—	—	—
30	3	5	2	−67 652,398	−63 932,537	320,833	19,995	0,040	1	12
Limit of each deviation mm				—	—	—	—	—	±53	±88

All deviations satisfy the conditions of Formula (2). No outlier is suspected.

B.2.2 Calculation of statistical values

Table B.2 — Measurements, residuals, and experimental standard deviation

Seq No.	Series	Set	Rover point	Measurement m			Residual mm			Squared residual mm^2		
	i	J	k	x	y	h	r_x	r_y	r_h	r_x^2	r_y^2	r_h^2
1	1	1	1	−67 635,470	−63 943,197	320,792	8	−4	−2	64	16	4
2	1	1	2	−67 652,389	−63 932,527	320,799	4	3	−17	16	9	289
3	1	2	1	−67 635,479	−63 943,188	320,788	−1	5	−6	1	25	36
4	1	2	2	−67 652,376	−63 932,525	320,824	17	5	8	289	25	64
5	1	3	1	−67 635,480	−63 943,189	320,789	−2	4	−5	4	16	25
6	1	3	2	−67 652,387	−63 932,529	320,810	6	1	−6	36	1	36
7	1	4	1	−67 635,476	−63 943,192	320,793	2	1	−1	4	1	1
8	1	4	2	−67 652,393	−63 932,530	320,808	0	0	−8	0	0	64
9	1	5	1	−67 635,481	−63 943,192	320,794	−3	1	0	9	1	0
10	1	5	2	−67 652,390	−63 932,522	320,803	3	8	−13	9	64	169
11	2	1	1	−67 635,478	−63 943,191	320,800	0	2	6	0	4	36
12	2	1	2	−67 652,399	−63 932,535	320,823	−6	−4	7	36	16	49

Table B.2 *(continued)*

Seq No.	Series	Set	Rover point	Measurement m			Residual mm			Squared residual mm²		
	i	*J*	*k*	*x*	*y*	*h*	r_x	r_y	r_h	r_x^2	r_y^2	r_h^2
13	2	2	1	−67 635,479	−63 943,193	320,798	−1	0	4	1	0	16
14	2	2	2	−67 652,392	−63 932,528	320,828	1	2	12	1	4	144
15	2	3	1	−67 635,477	−63 943,194	320,780	1	−1	−14	1	1	196
16	2	3	2	−67 652,396	−63 932,530	320,797	−3	0	−19	9	0	361
17	2	4	1	−67 635,475	−63 943,191	320,786	3	2	−8	9	4	64
18	2	4	2	−67 652,395	−63 932,532	320,812	−2	−2	−4	4	4	16
19	2	5	1	−67 635,476	−63 943,191	320,784	−2	−2	−10	4	4	100
20	2	5	2	−67 652,391	−63 932,534	320,812	2	−4	−4	4	16	16
21	3	1	1	−67 635,479	−63 943,194	320,798	−1	−1	4	1	1	16
22	3	1	2	−67 652,391	−63 932,529	320,826	2	1	10	4	1	100
23	3	2	1	−67 635,478	−63 943,195	320,805	0	−2	11	0	4	121
24	3	2	2	−67 652,398	−63 932,532	320,823	−5	−2	7	25	4	49
25	3	3	1	−67 635,485	−63 943,199	320,799	−7	−6	5	49	36	25
26	3	3	2	−67 652,400	−63 932,534	320,813	−7	−4	−3	49	16	9
27	3	4	1	−67 635,474	−63 943,195	320,804	4	−2	10	16	4	100
28	3	4	2	−67 652,394	−63 932,532	320,831	−1	−2	15	1	4	225
29	3	5	1	−67 635,483	−63 943,200	320,793	−5	−7	−1	25	49	1
30	3	5	2	−67 652,398	−63 932,537	320,833	−5	−7	17	25	49	289
Average over the series			1	−67 635,478	−63 943,193	320,794	—	—	—	—	—	—
			2	−67 652,393	−63 932,530	320,816						
Summation of the squared residual				—	—	—	—	—	—	696	379	2621
Experimental standard deviation, s				s_x = 4,99	s_y = 3,68	s_h = 9,68	—	—	—	—	—	—

The degrees of freedom for *x*, *y*, and *h* are identical and they are calculated according to Formula (6) as

$$v_x = v_y = v_h = (m \cdot n - 1) \cdot p = (3 \times 5 - 1) \times 2 = 28$$

According to Formula (7), the standard deviations of a single measurement of *x*, *y*, and *h* are calculated as

$$s_x = \sqrt{\frac{r_x^2}{v_x}} = \sqrt{\frac{r_x^2}{28}} = \sqrt{\frac{696}{28}} = 4,99\ \text{mm}$$

$$s_y = \sqrt{\frac{r_y^2}{v_y}} = \sqrt{\frac{r_y^2}{28}} = \sqrt{\frac{379}{28}} = 3,68\ \text{mm}$$

$$s_h = \sqrt{\frac{r_h^{\ 2}}{v_h}} = \sqrt{\frac{r_h^{\ 2}}{28}} = \sqrt{\frac{2621}{28}} = 9,68\ \text{mm}$$

According to Formulae (8) and (9), related experimental standard deviations are

$$s_{xy} = \sqrt{s_x^2 + s_y^2} = \sqrt{4,99^2 + 3,68^2} = 6,20\ \text{mm}$$

$$s_h = 9,68\text{mm}$$

B.3 Statistical tests

B.3.1 Statistical test according to question a)

$$s_{xy} = 6,20\ \text{mm}\ ;\ \sigma_{xy} = 15,00\ \text{mm}\ ;\ v = v_x + v_y = 56$$

$$6,20 \le 15,00 \times 1,15$$

$$6,20 \le 17,2$$

The test condition is fulfilled, the calculated experimental standard deviation, s_{xy}, of a single position (*x, y*) is smaller than or equal to a corresponding value, σ_{xy}, stated by the manufacturer or another predetermined value, σ_{xy}.

B.3.2 Statistical test according to question b)

$$s_h = 9,68\ \text{mm}\ ;\ \sigma_h = 25,00\ \text{mm}\ ,\ v = 28\ \text{mm}$$

$$9,68 \le 25,00 \times 1,22$$

$$9,68 \le 30,5$$

The test condition is fulfilled, the calculated experimental standard deviation, s_h, of a single height, *h*, is smaller than or equal to a corresponding value, σ_h, stated by the manufacturer or another predetermined value, σ_h.

B.3.3 Statistical test according to question c)

$$s_{xy} = 6,20\ \text{mm}\ ;\ \tilde{s}_{xy} = 6,00\ \text{mm}\ ;\ v = v_x + v_y = 56$$

$$0,59 \le \frac{38,44}{36,00} \le 1,70$$

$$0,59 \le 1,07 \le 1,70$$

The test condition is fulfilled, the null hypothesis stating that the experimental standard deviations, s_{xy} and $\tilde{s}_{xy}$, belong to the same population is not rejected at the confidence level of 95 %.