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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 6:
Rotating lasers

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 6: Lasers rotatifs

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This third edition cancels and replaces the second edition (ISO 17123-6:2012), which has been technically revised.

The main changes are as follows:

— more flexible configuration of the test line and updating of the mathematical model;

— harmonization of terminology and symbols;

— correction of errors.

A list of all parts in the ISO 17123 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document specifies field procedures for adoption when determining and evaluating the uncertainty of measurement results obtained by geodetic instruments and their ancillary equipment, when used in building and surveying measuring tasks. Primarily, these tests are intended to be field verifications of suitability of a particular instrument for the immediate task. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

The definition and concept of uncertainty as a quantitative attribute to the final result of measurement was developed mainly in the last two decades, even though error analysis has already long been a part of all measurement sciences. After several stages, the CIPM (Comité International des Poids et Mesures) referred the task of developing a detailed guide to ISO. Under the responsibility of the ISO Technical Advisory Group on Metrology (TAG 4), and in conjunction with six worldwide metrology organizations, a guidance document on the expression of measurement uncertainty was compiled with the objective of providing rules for use within standardization, calibration, laboratory, accreditation and metrology services. ISO/IEC Guide 98-3 was first published as the Guide to the Expression of Uncertainty in Measurement (GUM) in 1995.

With the introduction of uncertainty in measurement in ISO 17123 (all parts), it is intended to finally provide a uniform, quantitative expression of measurement uncertainty in geodetic metrology with the aim of meeting the requirements of customers.

ISO 17123 (all parts) provides not only a means of evaluating the precision (experimental standard deviation) of an instrument, but also a tool for defining an uncertainty budget, which allows for the summation of all uncertainty components, whether they are random or systematic, to a representative measure of accuracy, i.e. the combined standard uncertainty.

ISO 17123 (all parts) therefore provides, for each instrument investigated by the procedures, a proposal for additional, typical influence quantities, which can be expected during practical use. The customer can estimate, for a specific application, the relevant standard uncertainty components in order to derive and state the uncertainty of the measuring result.

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 6: Rotating lasers

1 Scope

This document specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of rotating lasers and their ancillary equipment when used in building and surveying measurements for levelling tasks. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

This document can be considered as one of the first steps in the process of evaluating the uncertainty of a measurement (more specifically a measurand). The uncertainty of a result of a measurement is dependent on a number of parameters. Therefore this document differentiates between different measures of accuracy and objectives in testing, like repeatability and reproducibility (between-day repeatability), and of course gives a thorough assessment of all possible error sources, as prescribed by ISO/IEC Guide 98-3 and ISO 17123-1.

These field procedures have been developed specifically for in situ applications without the need for special ancillary equipment and are purposefully designed to minimize atmospheric influences.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Buildings and civil engineering works — Procedures for setting out, measurement and surveying — Vocabulary*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

ISO 17123-2, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 2: Levels*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, ISO 17123-2, ISO/IEC Guide 98-3 and ISO/IEC Guide 99 apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

4 Symbols and abbreviated terms

4.1 Symbols

Symbol	Quantity	Unit
$\boldsymbol{A}$	design matrix	—
a	deflective deviation	m
b	deviation of the rotating axis	m
D	horizontal distance	m
$\bar{D}$	mean horizontal distance	m
$\tilde{d}$	height difference between target points	m
$\bar{\boldsymbol{d}}$	vector of mean height difference of target points	m
$\tilde{\boldsymbol{d}}$	vector of height differences of target points	m
h	height difference of levelling staff B and A	m
F	F (Fisher) distribution	—
f	number of target point	—
i	series of measurement	—
j	set of measurement	—
n	set of readings	—
$\boldsymbol{P}$	weight matrix of the observations	—
p	single weight factor	—
$\boldsymbol{Q}$	$\boldsymbol{Q}$ matrix is the inverse of the weight matrix $\boldsymbol{P}$	—
$\boldsymbol{r}$	residual vector of the height differences	m
r	residual	m
s, $\tilde{s}$	experimental standard deviation	m
t	t-distribution	—
u	standard uncertainty	m
x	measured reading at levelling staff	m
$\boldsymbol{x}$	observation vector of height differences	m
$\tilde{\boldsymbol{y}}$	vector of unknown parameters	m
$\bar{\boldsymbol{y}}$	mean vector of unknown parameters	m
ν	degrees of freedom	—
α	significance level	%
σ	theoretical standard deviation	m

Symbol	Quantity	Unit
χ^2	chi-squared distribution	—
Ω	sum of residual squares	m

4.2 Abbreviations

Abbreviation	Description
A	levelling point A
Ang	Angle
B	levelling point B
ISO-ROLAS	ISO specific for rotation lasers
ISO	International Organization for Standardization
S	instrument station
x, *y*, *z*	cartesian coordinate

5 General

5.1 Requirements

Before commencing surveying, it is important that the operator investigates that the precision in use of the measuring equipment is appropriate to the intended measuring task.

The rotating laser and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's handbook, and used with tripods and levelling staffs as recommended by the manufacturer.

The results of these tests are influenced by meteorological conditions, especially by the temperature gradient. An overcast sky and low wind speed guarantee the most favourable weather conditions. The particular conditions to be taken into account may vary depending on the location where the tasks are to be undertaken. Note should also be taken of the actual weather conditions at the time of measurements and the type of surface above which the measurements are performed. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

This document describes two different field procedures as given in Clauses 6 and 7. The operator shall choose the procedure which is most relevant to the project's particular requirements.

5.2 Procedure 1: simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given item of rotating-laser equipment is within the specified permitted deviation, according to ISO 4463-1.

This test procedure is normally intended for checking the precision (see ISO/IEC Guide 99:2007, 2.15) of a rotating laser to be used for area levelling applications, for tasks where measurements with unequal site lengths are common practice, e.g. building construction sites.

The simplified test procedure is based on a limited number of measurements. Therefore, a significant standard deviation and the standard uncertainty (Type A), respectively, cannot be obtained. If a more precise assessment of the rotating laser under field conditions is required, it is recommended to adopt the more rigorous full test procedure as given in Clause 7.

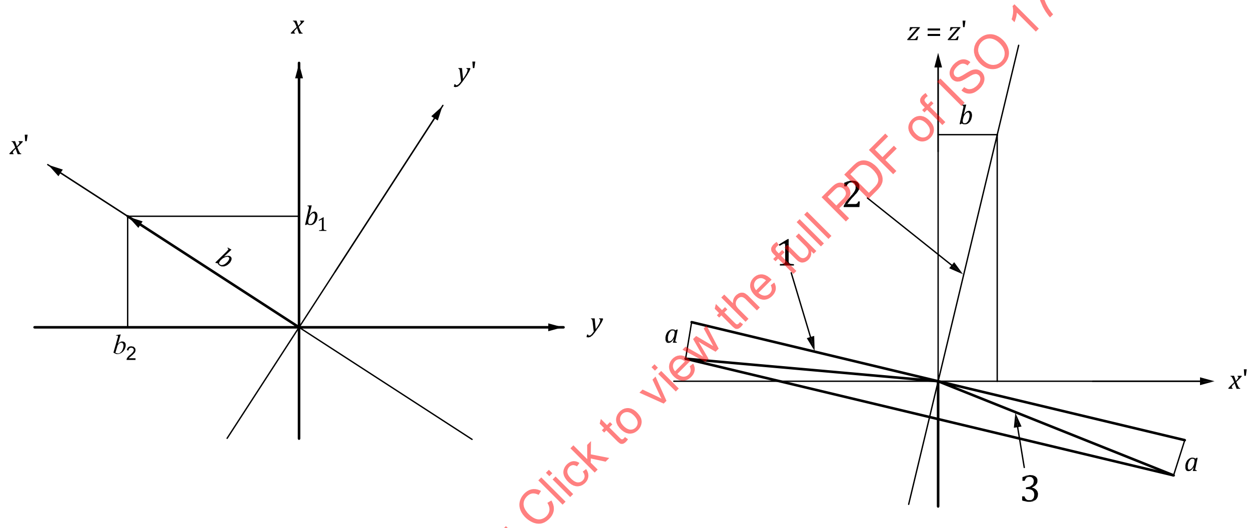
This test procedure relies on having a test field with height differences which are accepted as true values. If such a test field is not available, it is necessary to determine the unknown height differences (see Figures 1 and 2), using an optical level of accuracy (see ISO 17123-2) higher than the rotating laser required for the measuring task. If, however, a test field with known height differences cannot be established, it will be necessary to apply the full test procedure as given in Clause 7.

If no levelling instrument is available, the rotating laser to be tested can be used to determine the true values by measuring height differences between all points with central setups. At each setup, two height differences have to be observed by rotating the laser plane by 180°. The mean value of repeated readings in both positions will provide the height differences which are accepted as true.

5.3 Procedure 2: full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of a particular rotating laser and its ancillary equipment under field conditions, by a single survey team.

Further, this test procedure serves to determine the deflective deviation, a, and both components, b_1 and b_2, of the deviation of the rotating axis from the true vertical, $b=\sqrt{b_1{}^2+b_2{}^2}$ of the rotating laser (see Figure 1).



a) Horizontal plane (top view) **b) Vertical plane through x' (side view)**

Key

1 inclined plane

2 cone axis

3 inclined cone

a See Figure 5 also.

Figure 1 — Deflective deviations a and b

The recommended measuring distances within the test field (see Figure 3) are 40 m. Sight lengths greater than 40 m may be adopted for this precision-in-use test only, where the project specification may dictate, or where it is determining the range of the measure of precision of a rotating laser at respective distances.

The test procedure given in Clause 6 of this document is intended for determining the measure of precision in use of a particular rotating laser. This measure of precision in use is expressed in terms of the experimental standard deviation, s, of a height difference between the instrument level and a levelling staff (reading at the staff) at a certain distance. This experimental standard deviation corresponds to the standard uncertainty of Type A [see Formula (1)]:

$$s_{\text{ISO-ROLAS}} := u_{\text{ISO-ROLAS}} \tag{1}$$

Further, this procedure may be used to determine the standard uncertainty as a measure of precision in use of

— a single rotating laser and its ancillary equipment by a single survey team at a given time,

— a single rotating laser over time and differing environmental conditions, and

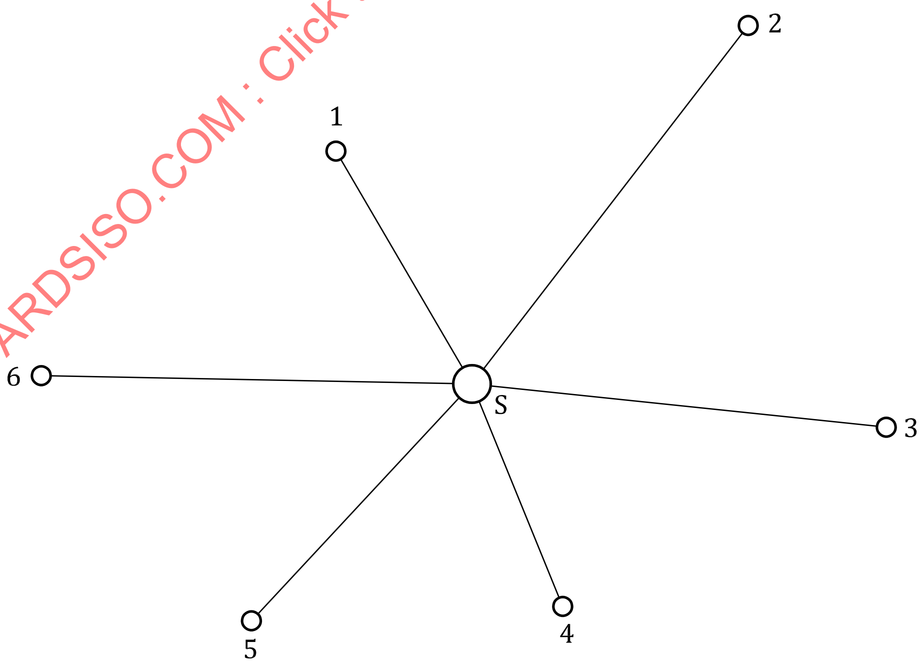
— several rotating lasers in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviation, *s*, obtained belongs to the population of the instrumentation's theoretical standard deviation, ⍰, whether two tested samples belong to the same population, whether the deflective deviation, *a*, is equal to zero, and whether the deviation, *b*, of the rotating axis from the true vertical of the rotating laser is equal to zero.

6 Simplified test procedure

6.1 Configuration of the test field

To keep the influence of refraction as small as possible, a reasonably horizontal test area shall be chosen. Six fixed target points, 1, 2, 3, 4, 5 and 6, shall be used to cover each horizontal quadrant at least with one target and shall be set up in approximately the same horizontal plane at different distances, between 10 m and 60 m apart from the instrument station S. The directions from the instrument to the six fixed points shall be spread over the horizon as equally as possible (see Figure 2).



Key

S instrument station

1, 2, 3, 4, 5, 6 fixed target points (f)

Figure 2 — Configuration of the test field for the simplified test procedure

To ensure reliable results, the target points shall be marked in a stable manner and reliably fixed during the test measurements, including repeat measurements.

The height differences between the six fixed points, 1 to 6, shall be determined using an optical level of known high accuracy as described in Clause 5.

The following five height differences between the 6 target points are known and calculated with Formula (2):

$$\tilde{\boldsymbol{d}} = \begin{pmatrix} \tilde{d}_{2,1} \\ \vdots \\ \tilde{d}_{f,f-1} \end{pmatrix} \quad f = 2,\ldots,6 \tag{2}$$

6.2 Measurements

The instrument shall be set up in a stable manner above point S. Before commencing the measurements, the laser beam shall become steady. To ensure that the laser plane of the instrument remains unchanged during the whole measuring cycle, a fixed target shall be observed before and after each set, j, of measurements, (j = 1,...,5).

Once the six target points are marked and reliably fixed, the six horizontal distances D_f between instrument station and target points shall be measured, e.g. by using a tape measure or laser distance meter.

Six separate readings, $x_{j,1}$ to $x_{j,6}$, on the scale of the levelling staff shall be carried out to each fixed target point, 1, 2, 3, 4, 5 and 6. Between two sets of readings the instrument shall be lifted, turned clockwise approximately 70°, placed in a slightly different position and relevelled. The time between any two sets of readings shall be at least 10 min.

Each reading shall be taken in a precise mode according to the recommendations of the manufacturer. Detection of height differences should be done by using a laser receiver that is typically part of the rotating laser set. This laser receiver should be set to the highest available sensitivity.

6.3 Calculation

The mean horizontal distance, $\bar{D}$, between instrument station and target points of the test configuration are calculated with Formula (3):

$$\bar{D}=\frac{1}{6}\sum_{f=1}^{6}D_f \tag{3}$$

$$f=1,\ldots,6$$

The evaluation of the readings, x_f, for each set, j, is based on the differences calculated with Formula (4):

$$\tilde{\boldsymbol{d}}_j=\begin{pmatrix}\tilde{d}_{j,2,1}\\ \vdots\\ \tilde{d}_{j,f,f-1}\end{pmatrix}=\begin{pmatrix}x_{j,2} & - & x_{j,1}\\ \vdots & - & \vdots\\ \tilde{d}_{j,f} & - & x_{j,f-1}\end{pmatrix}$$

$$j=1,\ldots,5 \tag{4}$$

$$f=2,\ldots,6$$

Calculating $\bar{\boldsymbol{d}}$, the mean of the differences, $\tilde{\boldsymbol{d}}_j$, the residual vector of the height differences in set, j, is obtained by Formula (5):

$$\boldsymbol{r}_j=\bar{\boldsymbol{d}}-\tilde{\boldsymbol{d}}_j \tag{5}$$

$$j=1,\ldots,5$$

The sum of the residual of the height differences in set j is defined as given in Formula (6):

$$\Omega_j=\boldsymbol{r}_j^T\boldsymbol{r}_j \tag{6}$$

Finally, the sum of the residual squares of all five sets yields is calculated with Formula (7):

$$\Omega=\sum_{j=1}^{5}\Omega_j \tag{7}$$

The experimental standard deviation, s, is calculated with Formula (8):

$$s=\sqrt{\frac{\Omega}{\nu}}=u_{\mathrm{ISO}} \tag{8}$$

and where ν is the corresponding number of degree of freedom as calculated according to Formula (9):

$$\nu=5\times(6-1)=25 \tag{9}$$

and u_{ISO} the standard uncertainty (Type A) of a single measured height difference, $\tilde{d}_{j,f,f-1}$, between two points of the test field. This represents in this document a measure of precision relative to the standard uncertainty of a Type A evaluation. This value includes systematic and random errors.

The experimental standard deviation s is expressed in the unit of length and refers to the specific size of the configured test field. The transformation in a more comparable angular unit yields to Formula (10):

$$s_{\mathrm{Ang}}=\tan^{-1}\left(\frac{s}{\bar{D}}\right) \tag{10}$$

A calculation example of the simplified test procedure is given in Annex A.

7 Full test procedure

7.1 Configuration of the test line

To keep the influence of refraction as small as possible, a reasonably horizontal test area shall be chosen. The ground shall be compact and the surface shall be uniform; roads covered with asphalt or concrete shall be avoided. If there is direct sunlight, the instrument and the levelling staffs shall be shaded, for example by an umbrella.

Two levelling points, A and B, shall be set up apart from each other in a distance which is typical for the working task and within the manufacturer's specification, e.g. 40 m. To ensure reliable results, the levelling staffs shall be set up in stable positions, reliably fixed during the test measurements, including any repeat measurements. The instrument shall be placed at the positions S1, S2 and S3. The distance from the instrument's position S2 and S3 to the nearest levelling point shall be between 1/4 and 1/2 of distance A-B. The position S1 shall be chosen equidistant between the levelling points A and B. See a configuration example of using 40 m as distance, D_{AB}, in Figure 3.

Distances in metres

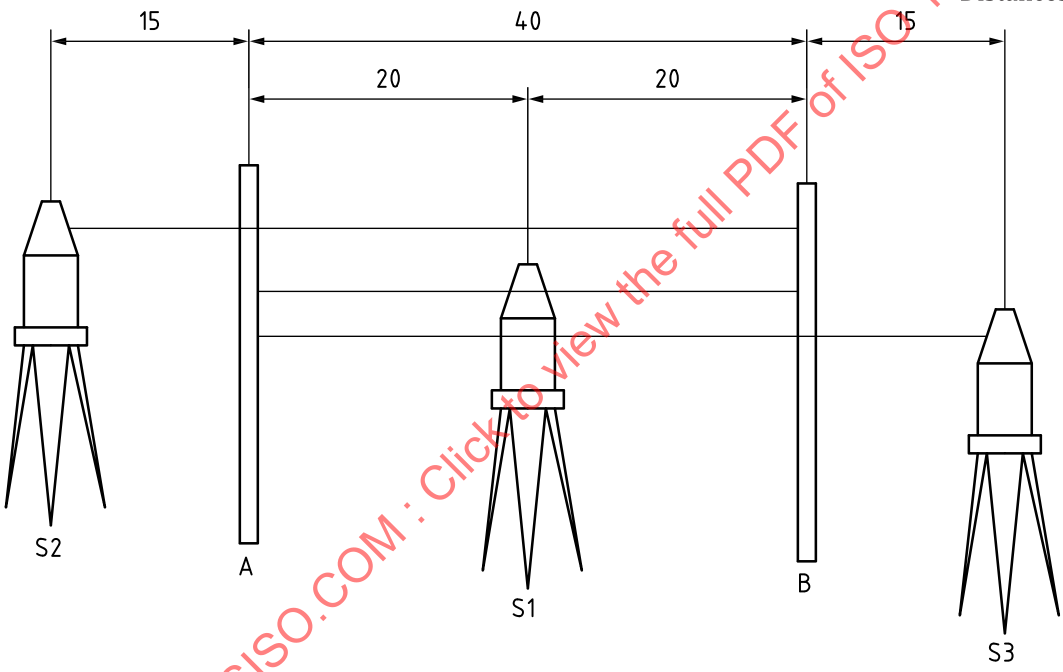


Figure 3 — Example of a configuration of the test line for the full test procedure

7.2 Measurements

Before starting measurements, the instrument shall be adjusted according to the manufacturer's specifications.

For the full test procedure, i = 4 series of measurements should be performed. In each series, three instrument setups S1, S2 and S3 are chosen, according to the configuration test line described before. At any setup n = 4 sets of readings are taken. Each set consists of two readings, x_{Aj} and x_{Bj}, namely to rod A and to rod B. After each set, the orientation of the instrument has to be changed clockwise about 90° (see Figure 4). Hence one series consists of $j = 3 \times 4 = 12$ readings for each rod. In order to ensure that the deviation b is aligned properly during the measurements, the instrument has to be oriented at the three positions S1, S2 and S3 in the same direction and the sense of rotation has to be maintained.

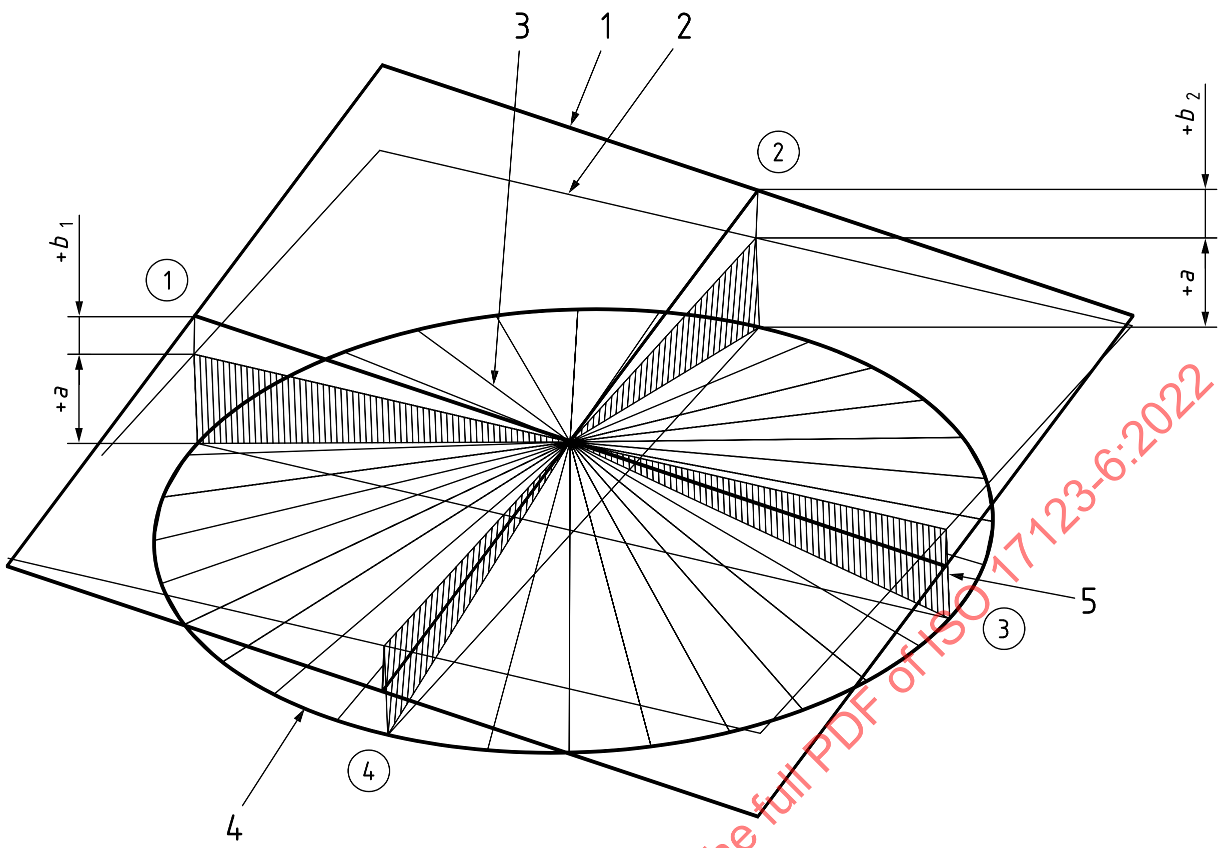
With each new setup of the chosen reference direction (reference marks on the tripod head), the instrument shall be relevelled carefully. If the instrument is provided with a compensator, care shall be taken that it functions properly. It is recommended to assign the four orientations of the instrument on the ground plate. The numbering of the 12 measurements can be represented for each measuring set as shown in Figure 4. All readings shall be taken in a precise mode according to the recommendations of the manufacturer. Detection of height differences should be done by using a laser receiver that is typically part of the rotating laser set. This laser receiver should be set to the highest available sensitivity.

		S1		S2		S3	
Set *n* orientation *n* = 1,...,4		Readings x_{Aj} and x_{Bj} *j* = 1,...,4		Readings x_{Aj} and x_{Bj} *j* = 5,...,8		Readings x_{Aj} and x_{Bj} *j* = 9,...,12	
Set 1		x_{A1}	x_{B1}	x_{A5}	x_{B5}	x_{A9}	x_{B9}
Set 2		x_{A2}	x_{B2}	x_{A6}	x_{B6}	x_{A10}	x_{B10}
Set 3		x_{A3}	x_{B3}	x_{A7}	x_{B7}	x_{A11}	x_{B11}
Set 4		x_{A4}	x_{B4}	x_{A8}	x_{B8}	x_{A12}	x_{B12}

Figure 4 — Arrangement of measurements

7.3 Calculation

The possible deviations of a rotating laser may be modelled as shown in Figure 5.



Key

1 horizontal plane

2 inclined plane, expressed by b_1 and b_2, (deviation of the rotating axis from the true vertical)

3 inclined cone

4 radius of cone = sighting distance

5 height of cone, *a*

①, ②, ③, ④ directions

Figure 5 — Example of a configuration of the test line for the full test procedure

In order to create a horizontal sighting in the described measuring configuration, the readings at the levelling staffs for selected sighting distances can be corrected in respect of the later to determine deviations *a* and *b* (see Table 1) by a correction factor with respect to the used distance.

From the observation formulae for the i^{th} series, the residuals, r_1 to r_{12}, are obtained (see Table 1).

Table 1 — Residuals for i^{th} series of measurement from observation formulae

S1	S2	S3
$r_1 = h\text{-}b_1\text{-}(x_{B1}\text{-}x_{A1})$	$r_5 = h\text{+a-}b_1\text{-}(x_{B5}\text{-}x_{A5})$	$r_9 = h\text{-}a\text{-}b_1\text{-}(x_{B9}\text{-}x_{A9})$
$r_2 = h\text{+}b_2\text{-}(x_{B2}\text{-}x_{A2})$	$r_6 = h\text{+a+}b_2\text{-}(x_{B6}\text{-}x_{A6})$	$r_{10} = h\text{-a+}b_2\text{-}(x_{B10}\text{-}x_{A10})$
$r_3 = h\text{+}b_1\text{-}(x_{B3}\text{-}x_{A3})$	$r_7 = h\text{+a+}b_1\text{-}(x_{B7}\text{-}x_{A7})$	$r_{11} = h\text{-a+}b_1\text{-}(x_{B11}\text{-}x_{A11})$
$r_4 = h\text{-}b_2\text{-}(x_{B4}\text{-}x_{A4})$	$r_8 = h\text{+a-}b_2\text{-}(x_{B8}\text{-}x_{A8})$	$r_{12} = h\text{-a-}b_2\text{-}(x_{B12}\text{-}x_{A12})$

With 12 observations and four unknown parameters, *a*, b_1, b_2 and *h* (height difference of levelling staff B and A), we have an over-determined system, which leads to a parametric adjustment. As the observation formulae are already linear, Table 1 can easily be transferred in matrix notation:

$$\boldsymbol{r} = \boldsymbol{A}\tilde{\boldsymbol{y}} - \boldsymbol{x} \quad (11)$$

where

$\boldsymbol{r}$ is the (12 × 1) residual vector of the r_j, j = 1,...12

$\boldsymbol{x} = x_B\text{-}x_A$ is the (12 × 1) quasi-observation vector of the height differences, with (12 × 1), reading vector x_{Aj}, j = 1,...,12 of the levelling staff A and (12 × 1), reading vector x_{Bj}, j = 1,...,12 of the levelling staff B;

$\tilde{\boldsymbol{y}} = [h\ a\ b_1\ b_2\ ]^T$ is the (4 × 1) vector of the unknown parameters, and

$\boldsymbol{A}$ is (12 × 4) the full-rank design matrix defined in Formula (12)

$$\boldsymbol{A}^{\boldsymbol{T}} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \end{pmatrix} \tag{12}$$

In order to derive the weight matrix of the observations, $\boldsymbol{P}$, we assume that error accumulates as a function of distance. The weighting factor is referred to the chosen sighting distance, D_{AB}, between the levelling points. We assume that all observations are uncorrelated. Therefore, the weight matrix will be a diagonal matrix with the corresponding weights:

$$p_i = \frac{D_{AB}}{\max(d_{AB}^j)} \tag{13}$$

$i = 1,\ldots,12$

Where D_{AB} is the configured distance between the two levelling points A and B, and $\max(d_{AB}^j)$ is the maximal distance between the instrument setup and its furthest levelling point (A or B) in every series, i = 1, 2, 3 (see Figure 4). Based on the configuration of the test line for the full test procedure given in Figure 3, the resulting weight matrix is calculated with Formula (14):

$$\boldsymbol{P} = \begin{pmatrix} p_1 & & 0 \\ & \ddots & \\ 0 & & p_{12} \end{pmatrix} \tag{14}$$

The optimal solution for the overdetermined system of equations, Formula (11), is obtained using the method of least squares. The estimated vector of the unknown parameters for each series, i, finally is calculated with Formula (15):

$$\tilde{\boldsymbol{y}}_{\boldsymbol{i}} = \begin{pmatrix} h_i \\ a_i \\ b_{1i} \\ b_{2i} \end{pmatrix} = \left(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{A}\right)^{-1} \boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{x}_{\boldsymbol{i}} = \boldsymbol{N}^{-1}\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{x}_{\boldsymbol{i}} \tag{15}$$

The experimental standard deviation for the full test procedure in each series is given by Formula (16):

$$s_i = \sqrt{\frac{\boldsymbol{r}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{r}}{\nu}} \tag{16}$$

with $\nu = 12 - 4 = 8$ and

from all series i = 1,...,4 of observations we can derive the mean values of the parameters with Formula (17):

$$\bar{\boldsymbol{y}} = \frac{1}{4}\sum_{i=1}^{4}\tilde{\boldsymbol{y}}_i = \frac{1}{4}\sum_{i=1}^{4}\begin{pmatrix} h_i \\ a_i \\ b_{1i} \\ b_{2i} \end{pmatrix} \tag{17}$$

Finally, we get the total deviation of the rotating axis from the true vertical of the rotating laser, referenced to the sighting distance, D_{AB}, according to Formula (18):

$$b = \sqrt{b_1^2 + b_2^2} \tag{18}$$

With Formula (16) the overall experimental standard deviation of all series i = 1,...,4 yields

$$s = \sqrt{\frac{\sum_{i=1}^{4} s_i^2}{4}} = \sqrt{\frac{\sum_{i=1}^{4} \boldsymbol{r}_i^T \boldsymbol{P} \boldsymbol{r}_i}{32}} \tag{19}$$

Herewith we can state the standard uncertainty (Type A) of a height difference between the instrument level and a levelling staff (reading at the levelling staff) referenced to a sighting distance, D_{AB}, as defined in Formula (20):

$$u_{ISO\text{-}ROLAS} := s \tag{20}$$

The experimental standard deviation for the parameters of all series can be calculated by Formula (21):

$$s_{\bar{y}} = s\sqrt{\frac{1}{4} diag \boldsymbol{Q}} \tag{21}$$

where

$$\boldsymbol{Q} = \left(\boldsymbol{A}^T \boldsymbol{P} \boldsymbol{A}\right)^{-1} \tag{22}$$

Thus, the standard deviations and the standard uncertainties (Type A), respectively, of the parameters a, b and h are given by Formulae (23) to (25):

$$s_h = u_h \tag{23}$$

$$s_a = u_a \tag{24}$$

$$s_{b1} = s_{b2} = s_{b12} \tag{25}$$

Applying the law of variance covariance propagation on Formula (18), the experimental standard deviation of the parameter *b* can be written as given in Formula (26):

$$s_b = \frac{1}{b}\sqrt{b_1^2 s_{b1}^2 + b_2^2 s_{b2}^2} \tag{26}$$

Using Formula (25) leads to Formula (27) and (28):

$$s_b = \frac{1}{b}\sqrt{b_1^2 s_{b1}^2 + b_2^2 s_{b12}^2} = s_{b12} \tag{27}$$

and

$$s_b = u_b \tag{28}$$

The standard deviations, s_a, s_{b1}, s_{b2} and s_b, are expressed in the unit of length and refer to the specific size of the configured test field. The transformation in a more comparable angular unit yields to Formulae (29) to (31):

$$s_{a,\mathrm{Ang}} = \tan^{-1}\left(\frac{s_a}{D_{\mathrm{AB}}}\right) \tag{29}$$

$$s_{b1,\mathrm{Ang}} = s_{b2,\mathrm{Ang}} = \tan^{-1}\left(\frac{s_{b12}}{D_{\mathrm{AB}}}\right) \tag{30}$$

$$s_{b,\mathrm{Ang}} = \tan^{-1}\left(\frac{s_b}{D_{\mathrm{AB}}}\right) \tag{31}$$

A calculation example of the full test procedure is given in Annex B.

7.4 Statistical test

7.4.1 General

Statistical tests are recommended for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using:

— the experimental standard deviation, *s*, of a height difference between the instrument level and a levelling staff (reading at the levelling staff) referenced to a sighting distance, D_{AB},

— the deflective deviation, a, referenced to a sighting distance, D_{AB}, and its standard deviation, s_a, and

— the total deviation, *b*, of the rotating axis from the true vertical of the rotating laser referenced to a sighting distance, D_{AB}, and its standard deviation, s_b;

in order to answer the following questions (see Table 2).

a) Is the calculated experimental standard deviation, *s*, for one reading at a levelling staff referenced to a sighting distance, D_{AB}, smaller than the value, σ, stated by the manufacturer or smaller than another predetermined value, σ?

   Usually the manufacturers state the precision by the deflective angle from the horizontal, which should be interpreted to the corresponding standard deviation, σ, at the sighting distance, D_{AB}.

b) Do two experimental standard deviations, s and $\tilde{s}$, as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same number of degrees of freedom, ν?

The experimental standard deviations, s and $\tilde{s}$, may be obtained from

— two samples of measurements by the same instrument at different times, or

— two samples of measurements by different instruments.

c) Is the deflective deviation, a, equal to zero?

d) Is the total deviation, b, of the rotating axis from the true vertical equal to zero?

For the following tests, a confidence level of $1 - \alpha = 0{,}95$ and, according to the design of the measurements, a number of degrees of freedom of $\nu = 32$ are assumed.

Table 2 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s \le \sigma$	$s > \sigma$
b)	$s = \tilde{s}$	$s \ne \tilde{s}$
c)	$a = 0$	$a \ne 0$
d)	$b = 0$	$b \ne 0$

7.4.2 Question a)

The null hypothesis stating that the experimental standard deviation, s, is smaller than or equal to a theoretical or a predetermined value, σ, is not rejected if the conditions of Formulae (32) to (36) are fulfilled:

$$s \le \sigma \cdot \sqrt{\frac{\chi^2_{1-\alpha}(\nu)}{\nu}} \tag{32}$$

$$s \le \sigma \cdot \sqrt{\frac{\chi^2_{0,95}(32)}{32}} \tag{33}$$

$$\chi^2_{0,95}(32) = 46{,}19 \tag{34}$$

$$s \le \sigma \cdot \sqrt{\frac{46{,}19}{32}} \tag{35}$$

$$s \le \sigma \cdot 1{,}20 \tag{36}$$

Otherwise, the null hypothesis is rejected.

7.4.3 Question b)

In the case of two different samples, a test indicates whether the experimental standard deviations, s and $\tilde{s}$, belong to the same population. The corresponding null hypothesis, $s = \tilde{s}$, is not rejected if the conditions of Formulae (37) to (40) are fulfilled:

$$\frac{1}{F_{1-\frac{\alpha}{2}}(\nu,\nu)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{1-\frac{\alpha}{2}}(\nu,\nu) \tag{37}$$

$$\frac{1}{F_{0,975}(32,32)} \leq \frac{s^2}{\tilde{s}^2} \leq F_{0,975}(32,32) \tag{38}$$

$$F_{0,975}(32,32) = 2,02 \tag{39}$$

$$0,50 \leq \frac{s^2}{\tilde{s}^2} \leq 2,02 \tag{40}$$

Otherwise, the null hypothesis is rejected.

7.4.4 Question c)

The null hypothesis, stating that the deflective deviation, a, of the rotating laser is equal to zero, is not rejected if conditions of Formulae (41) to (43) are fulfilled:

$$|a| \leq s_a \cdot t_{1-\frac{\alpha}{2}}(\nu) \tag{41}$$

$$|a| \leq s_a \cdot t_{0,975}(32) \tag{42}$$

$$t_{0,975}(32) = 2,04 \tag{43}$$

Otherwise, the null hypothesis is rejected.

7.4.5 Question d)

The null hypothesis stating that the total deviation, b, of the rotating axis from the true vertical of the rotating laser is equal to zero is not rejected if the conditions of Formulae (44) to (46) are fulfilled:

$$|b| \leq s_b \cdot t_{1-\frac{\alpha}{2}}(\nu) \tag{44}$$

$$|b| \leq s_b \cdot t_{0,975}(32) \tag{45}$$

$$t_{0,975}(32) = 2,04 \tag{46}$$

Otherwise, the null hypothesis is rejected.

A calculation example of the statistical tests is given in B.3.

NOTE In practice, the parameters a and b can significantly influence the height readings.

8 Influence quantities and combined standard uncertainty evaluation (Type A and Type B)

The standard uncertainty, $u_{ISO\text{-}ROLAS}$, calculated in 7.3, represents a measure of precision for the described test configuration under the prevailing test conditions. To achieve in practice a realistic estimation of a quantitative measure of accuracy for a special application, we have to assemble all possible influence quantities that can affect the measurement result. Table 3 lists a range of typical influence quantities which could be considered. Through this we can derive uncertainty components either by a Type A or Type B evaluation. Rules for evaluating the different standard uncertainties and their summation to the combined standard uncertainty are given in ISO 17123-1. Subsequent possible influence quantities using the rotating laser are compiled.

Table 3 — Typical influence quantities of the rotating laser

Sources of uncertainty (influence quantity)	Evaluation	Distribution
I. Relevant sources of the rotating laser		
Typical measure of precision for measuring distance D_{AB}	Type A $u_{ISO\text{-}ROLAS}$	normal
Expanded measurement range	Type A	normal
Non-orthogonality between rotating axis and laser path, parameter *a*	Type B	rectangular
Inclination of the rotating laser plane or standing (rotating) axis, parameter *b*	Type B	triangular
Instability of laser beam, variation of the parameters *a* and *b* by temperature	Type B	rectangular
II. Error sources of the staff		
Tilts of the staff, misadjustment of spot bubble	Type B	rectangular
Zero error	Type B	rectangular
Staff scale	Type B	rectangular
Junction error	Type B	rectangular
III. Error patterns of the electronic reading		
Round-off error	Type B	rectangular
Random error of electronic beam detection, e.g. vibration effects	Type A	normal
Dependence on light intensity	Type B	rectangular
IV. General influence quantities		
Dependence on atmospheric conditions	Type B	rectangular
Error due to subsidence of instrument	Type B	rectangular
Curvature of the earth	Type B	rectangular

To calculate the standard uncertainties of the individual influence quantities in consideration of their distributions, in many cases it is advised to determine or estimate upper and lower limits; additionally, it is necessary to state the probability that the value considerably lies in this interval. Detailed advice is additionally given in ISO/IEC Guide 98-3:2008, 4.3.

Sometimes it is useful, after the calculation of the combined standard uncertainty, to indicate the expanded uncertainty due to the higher level of confidence that will better meet the accuracy indication of a measuring tolerance.

An example for the calculation of a typical uncertainty budget of the rotating laser is given in Annex C.

Annex A
(informative)

Example of the simplified test procedure

A.1 Configuration of the test field

A level of known sufficient accuracy is used to determine the reference heights (relative heights) of the six target points of the test field.

The experimental standard deviation and the standard uncertainty, respectively, of a single height difference is determined according to the full test procedure as given in ISO 17123-2:2001, Clause 6.

$$s_{\bar{x}} = u_{\bar{x}} = 0{,}2\,\text{mm}$$

The relative heights of the six target points and the height differences were obtained as given in Table A.1.

Table A.1 — Relative heights and height differences

$\bar{x}_1 = 1{,}702\ 2$ m	
$\bar{x}_2 = 1{,}521\ 4$ m	$\bar{d}_{2,1} = -0{,}180\ 8$ m
$\bar{x}_3 = 1{,}637\ 6$ m	$\bar{d}_{3,2} = +0{,}116\ 2$ m
$\bar{x}_4 = 1{,}712\ 4$ m	$\bar{d}_{4,3} = +0{,}074\ 8$ m
$\bar{x}_5 = 1{,}561\ 0$ m	$\bar{d}_{5,4} = -0{,}151\ 4$ m
$\bar{x}_6 = 1{,}608\ 8$ m	$\bar{d}_{6,5} = +0{,}047\ 8$ m
	$\sum = -0{,}0934\,\text{m} = \tilde{x}_6 - \tilde{x}_1$

A.2 Measurements

Table A.2 contains the measured values, $x_{j,f}$ in columns 1 to 3, and the height differences $\bar{d}_{f,f-1}$ in column 5 as given in A.2.

Table A.2 — Measurements

1	2	3	4	5	6	7
j	f	x_f m	$\tilde{d}_{f,f\text{-}1}$ m	$\bar{d}_{f,f\text{-}1}$ m	$r_{f,f\text{-}1}$ mm	$r^2_{f,f\text{-}1}$ mm^2
1	1	2,215				
	2	2,033	– 0,182	– 0,180 8	+ 1,2	1,44
	3	2,150	+ 0,117	– 0,116 2	– 0,8	0,64
	4	2,225	+ 0,075	+ 0,074 8	– 0,2	0,04
	5	2,073	– 0,152	–0,151 4	+ 0,6	0,36
	6	2,122	+ 0,049	+ 0,047 8	– 1,2	1,44
2	1	1,915				
	2	1,736	– 0,179	– 0,180 8	– 1,8	3,24
	3	1,851	+ 0,115	– 0,116 2	+ 1,2	1,44
	4	1,926	+ 0,075	+ 0,074 8	– 0,2	0,04
	5	1,776	– 0,150	– 0,151 4	–1,4	1,96
	6	1,824	+ 0,048	+ 0,047 8	– 0,2	0,04
3	1	1,224				
	2	1,042	– 0,182	– 0,180 8	+ 1,2	1,44
	3	1,158	+ 0,116	– 0,116 2	+ 0,2	0,04
	4	1,232	+ 0,074	+ 0,074 8	+ 0,8	0,64
	5	1,081	– 0,151	– 0,151 4	– 0,4	0,16
	6	1,128	+ 0,047	+ 0,047 8	+ 0,8	0,64
4	1	1,585				
	2	1,404	– 0,181	– 0,180 8	+ 0,2	0,04
	3	1,521	+ 0,117	– 0,116 2	– 0,8	0,64
	4	1,595	+ 0,074	+ 0,074 8	+ 0,8	0,64
	5	1,443	– 0,152	– 0,151 4	+ 0,6	0,36
	6	1,489	+ 0,046	+ 0,047 8	+ 1,8	3,24
5	1	1,777				
	2	1,596	– 0,181	– 0,180 8	+ 0,2	0,04
	3	1,712	+ 0,116	– 0,116 2	+ 0,2	0,04
	4	1,788	+ 0,076	+ 0,074 8	– 1,2	1,44
	5	1,637	– 0,151	– 0,151 4	– 0,4	0,16
	6	1,684	+ 0,047	+ 0,047 8	+ 0,8	0,64
$\sum$			– 0,469	– 0,093 4	+ 2,0	20,80
$\sum_{j=1}^{5}(x_{j,6} - x_{j,1})$		– 0,469				

A.3 Calculation

First, the height differences, $\tilde{d}_{f,f-1}$, are calculated according to Formula (4) (see column 4 in Table A.2). Then the residuals, $r_{f,f-1}$, are obtained according to Formula (5) (see column 6 in Table A.2). The sum of squares of the residuals, $\sum r^2$ is equal to 20,80 mm^2 (see the last line of column 7 in Table A.2). Since the number of degrees of freedom, ν, is equal to 25, the standard deviation and the standard uncertainty, respectively, of a height difference, $\tilde{d}_{f,f-1}$, are calculated according to Formula (8):

$$s = \sqrt{\frac{20{,}80\,\text{mm}^2}{25}} = u_{\text{ISO}} = 0{,}9\,\text{mm}$$

There are two arithmetic checks in Table A.2:

— the value in the last line of column 3 shall be equal to the sum of column 4: −0,469 = −0,469;

— five times the sum of column 5, minus the sum of column 4 shall be the sum of column 6: 5 × (−0,093 4)−(−0,469) = 0,002.

Annex B
(informative)

Example of the full test procedure

B.1 Measurements

Table B.1 contains the measured values, x_{Aj} and x_{Bj} of the i^{th} series of measurement (the series of measurements numbers 2, 3 and 4 were not printed).

Observer:	S. Miller
Weather:	sunny, +10 °C
Instrument type and number:	NN xxx 630401
Date:	1999-04-15
Configuration of test line (distance D_{AB}):	40 m as an example, same configuration as in Figure 3

Table B.1 — Measurement and residuals of series No. 1

1	2	3	4	5
Instrument setup	j	x_{Aj} m	x_{Bj} m	x_j m
1	1	1,779	1,537	− 0,242
	2	1,780	1,536	− 0,244
	3	1,783	1,535	− 0,248
	4	1,783	1,536	− 0,247
2	5	1,596	1,352	− 0,244
	6	1,600	1,352	− 0,248
	7	1,604	1,353	− 0,251
	8	1,601	1,353	− 0,248
3	9	1,633	1,393	− 0,240
	10	1,634	1,390	− 0,244
	11	1,630	1,387	− 0,243
	12	1,631	1,390	− 0,241
$\sum$		20,054	17,114	− 2,940

B.2 Calculation

Based on the configuration of the test line by using 40 m and according to Formula (13) and (14), the weight matrix results in:

$$diag(\boldsymbol{P}) = \left(\frac{40}{20}\ \frac{40}{20}\ \frac{40}{20}\ \frac{40}{20}\ \frac{40}{55}\ \frac{40}{55}\ \frac{40}{55}\ \frac{40}{55}\ \frac{40}{55}\ \frac{40}{55}\ \frac{40}{55}\ \frac{40}{55} \right) = (2\ 2\ 2\ 2\ 0{,}7\ 0{,}7\ 0{,}7\ 0{,}7\ 0{,}7\ 0{,}7\ 0{,}7\ 0{,}7)$$

The four unknown parameters, h, a, b_1 and b_2 are calculated according to Formula (15):

$$\tilde{y}_1 = \begin{pmatrix} 0{,}147 & 0{,}147 & 0{,}147 & 0{,}147 & 0{,}051\,5 & 0{,}051\,5 & 0{,}051\,5 & 0{,}051\,5 & 0{,}051\,5 & 0{,}051\,5 & 0{,}051\,5 & 0{,}051\,5 \\ 0 & 0 & 0 & 0 & 0{,}125 & 0{,}125 & 0{,}125 & 0{,}125 & -0{,}125 & -0{,}125 & -0{,}125 & -0{,}125 \\ -0{,}294\,1 & 0 & 0{,}294\,1 & 0 & -0{,}102\,9 & 0 & 0{,}102\,9 & 0 & -0{,}102\,9 & 0 & 0{,}102\,9 & 0 \\ 0 & 0{,}294\,1 & 0 & -0{,}294\,1 & 0 & 0{,}102\,9 & 0 & -0{,}102\,9 & 0 & 0{,}102\,9 & 0 & -0{,}102\,9 \end{pmatrix} \begin{pmatrix} -0{,}242 \\ -0{,}244 \\ -0{,}248 \\ -0{,}247 \\ -0{,}244 \\ -0{,}248 \\ -0{,}251 \\ -0{,}248 \\ -0{,}240 \\ -0{,}244 \\ -0{,}243 \\ -0{,}241 \end{pmatrix} (\mathrm{m})$$

$$\tilde{y}_1 = \begin{pmatrix} -0{,}245\,1 \\ -0{,}002\,9 \\ -0{,}002\,8 \\ 0{,}000\,6 \end{pmatrix} (\mathrm{m})$$

Following Formula (11), the residuals can be calculated to:

$$r = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \\ 1 & -1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} -0{,}245\,1 \\ -0{,}002\,9 \\ -0{,}002\,8 \\ 0{,}000\,6 \end{pmatrix} (\mathrm{m}) - \begin{pmatrix} -0{,}242 \\ -0{,}244 \\ -0{,}248 \\ -0{,}247 \\ -0{,}244 \\ -0{,}248 \\ -0{,}251 \\ -0{,}248 \\ -0{,}240 \\ -0{,}244 \\ -0{,}243 \\ -0{,}241 \end{pmatrix} (\mathrm{m}) = \begin{pmatrix} -0{,}3 \\ -0{,}5 \\ 0{,}1 \\ 1{,}3 \\ -1{,}2 \\ 0{,}6 \\ 0{,}2 \\ -0{,}5 \\ 0{,}6 \\ 2{,}4 \\ -2{,}0 \\ -1{,}8 \end{pmatrix} (\mathrm{mm})$$

According to Formula (16), the standard deviation of the first series over a distance of 40 m (standard deviation of the weighting unit) yields:

$$s_1 = \sqrt{\frac{14{,}96\,\mathrm{mm}^2}{8}} = 1{,}4\,\mathrm{mm}$$

The corresponding results of the second, third and fourth series are for the parameters:

$$\tilde{y}_2 = \begin{pmatrix} -0{,}244\,8 \\ -0{,}003\,3 \\ -0{,}002\,5 \\ 0{,}001\,3 \end{pmatrix} (\mathrm{m})$$

$$\tilde{y}_3 = \begin{pmatrix} -0{,}245\ 4 \\ -0{,}003\ 6 \\ -0{,}002\ 1 \\ 0{,}000\ 8 \end{pmatrix} (\text{m})$$

$$\tilde{y}_4 = \begin{pmatrix} -0{,}245\ 4 \\ -0{,}003\ 6 \\ -0{,}002\ 7 \\ 0{,}000\ 7 \end{pmatrix} (\text{m})$$

and for the standard deviations:

$$s_2 = 0{,}9\ \text{mm}$$

$$s_3 = 1{,}1\ \text{mm}$$

$$s_4 = 0{,}9\ \text{mm}$$

For every series, the following holds:

$$\nu = \nu_1 = \nu_2 = \nu_3 = \nu_4 = 8$$

The standard uncertainty (Type A) $u_{\text{ISO-ROLAS}}$ and the parameters h, a, b_1, b_2 over all series are calculated according to Formulae (19), (20), (17) and (18):

$$\nu = 4 \times \nu_1 = 32$$

$$s = \sqrt{\frac{(1{,}2\ \text{mm})^2 + (0{,}9\ \text{mm})^2 + (1{,}1\ \text{mm})^2 + (0{,}9\ \text{mm})^2}{4}} = 1{,}0\ \text{mm}$$

$$u_{ISO\text{-ROLAS}} = 1{,}0\ \text{mm}$$

$$h = \frac{0{,}245\ 1 - 0{,}244\ 8 - 0{,}245\ 4 - 0{,}245\ 4}{4} = -0{,}245\ 2\ \text{m}$$

$$a = \frac{-0{,}0029 - 0{,}0033 - 0{,}0036 - 0{,}0036}{4} = -0{,}0034\ \text{m}$$

$$b_1 = \frac{-0{,}002\ 8 - 0{,}002\ 5 - 0{,}002\ 1 - 0{,}002\ 7}{4} = -0{,}002\ 5\ \text{m}$$

$$b_2 = \frac{0{,}006 + 0{,}001\ 3 + 0{,}000\ 8 + 0{,}007}{4} = +0{,}000\ 8\ \text{m}$$

