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Space environment (natural and artificial) — Geomagnetic reference models

Environnement spatial (naturel et artificiel) — Modèles de référence du champ magnétique terrestre

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Contents

Page

Foreword

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The committee responsible for this document is ISO/TC 20, *Aircraft and space vehicles*, Subcommittee SC 14, *Space systems and operations*.

Introduction

For centuries, geomagnetic reference models have been used to describe the vector field (for example, by its direction and strength) as a function of position and date. Such models are widely used in upper-atmospheric, ionospheric, and magnetospheric research, and in characterizing the near-Earth space environment. They further provide an essential reference for navigation, heading, and attitude determination and control subsystems of spacecraft and ground-based systems.

Earth's magnetic field is represented in such models as a spherical harmonic expansion of the equivalent scalar magnetic potential. This representation was proposed by Karl Friedrich Gauss (1777-1855) and has been used ever since to describe the geomagnetic field. The spherical harmonic coefficients of the geomagnetic field are commonly called Gauss coefficients. In 1969, the International Association for Geomagnetism and Aeronomy (IAGA) introduced the International Geomagnetic Reference Field (IGRF), which uses the Gauss representation to describe Earth's magnetic field. This International Standard closely mirrors the established specification of such models, including formulae and computational procedure.

There are several internal and external sources contributing to the observed magnetic field. All of these sources affect a scientific or navigational instrument, but only some of them are represented in geomagnetic reference models. The strongest contribution, by far, is the magnetic field produced by motions in the Earth's liquid-iron outer core, which is called the core field. This core field changes perceptibly from year to year. When extrapolating the temporal evolution of the field into the future, a linear extrapolation of the Gauss coefficients is used. Geomagnetic reference models specify the Gauss coefficients for a start date (epoch) and provide their linear change over time as a set of so-called secular variation (SV) coefficients. Due to unpredictable nonlinear changes in the core field, predictive geomagnetic reference models are valid only for a limited period, and users subsequently have to update to a newer version.

Other sources also contribute to the magnetic field. Magnetic minerals in the crust and upper mantle give rise to magnetic anomalies which can be significant locally. Electric currents induced by the flow of conducting sea water through the ambient magnetic field make a further, albeit weak, contribution to the observed magnetic field. Time-varying electric currents in the upper atmosphere and near-Earth space generate an external magnetic field. The external field does not average to zero over time. Its steady contribution can therefore be included in a geomagnetic reference model using external Gauss coefficients with linear secular variation. Time-varying external magnetic fields further induce electric currents in the Earth and oceans, producing secondary internal magnetic fields. Since there is no general consensus on how to separate these various internal and external sources, it is left to the producer of a geomagnetic reference model to specify which of these internal and external sources are included in their model, and any radial limitation to the validity of the external part of their model.

Space environment (natural and artificial) — Geomagnetic reference models

1 Scope

This International Standard defines reference models representing the geomagnetic field. It closely mirrors and clarifies specifications which have been in use for many decades.

The approach is to represent the corresponding scalar magnetic potential by a spherical harmonic expansion having specified numerical coefficients, called Gauss coefficients. This International Standard covers models in which, at any one time, changes of the magnetic field are modelled by a linear time-dependence of each Gauss coefficient. A model might consist of a succession of sub-models, in each of which the coefficients change linearly with time. For such a step-wise linear model, the coefficients are continuous in time. This International Standard provides the formulae and a step-by-step computational procedure to evaluate a geomagnetic reference model for any desired location and date.

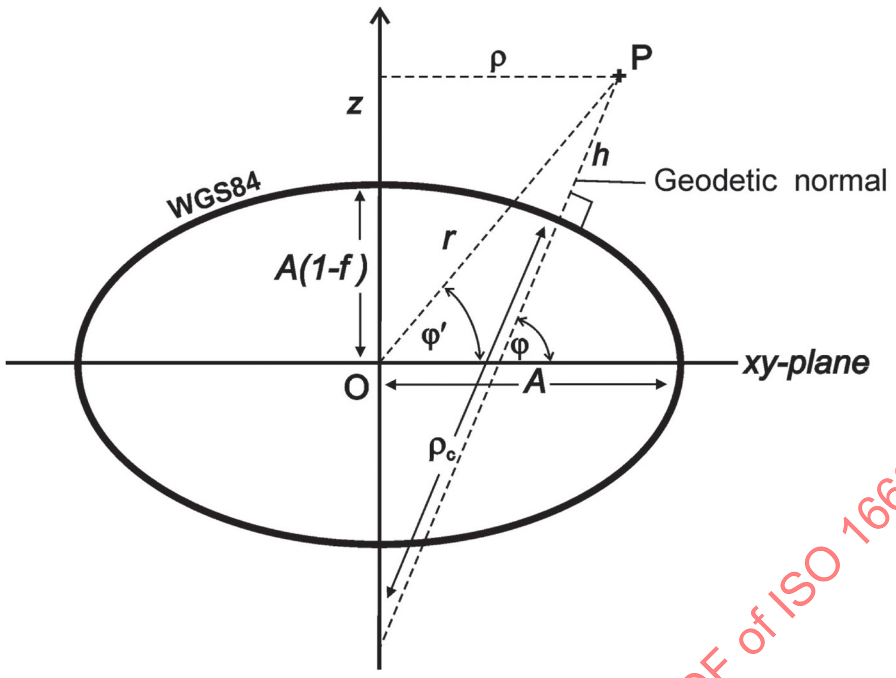
This International Standard does not specify the interpretation of the geophysical content of a geomagnetic reference model. It is left to the producer of a model to specify which internal and external magnetic sources are included in (or excluded from) their model.

2 Reference frames

2.1 General

For positions remote from the Earth, it is customary to use a geocentric reference frame and to resolve the magnetic field vector into components based on this geocentric frame. For positions on and near the Earth's surface, it is customary to use a geodetic reference frame, based on the standard World Geodetic System (WGS84) ellipsoid of rotation approximating the Earth's surface, and to resolve the magnetic field vector into components based on this geodetic frame.

The down axis in the geodetic frame is perpendicular to the surface of the WGS84 ellipsoid. Deflections of the vertical caused by local gravity anomalies have to be taken care of by the user and are not covered by this International Standard (see 5.1). Geocentric and geodetic coordinates are illustrated in Figure 1.



NOTE An axial cross-section through the point of interest P which is at longitude λ. This point P is specified in geocentric spherical polar coordinates by its distance r from the Earth centre O and the incidence angle of the OP line with the equatorial xy-plane. The same point P has geodetic coordinates given by its height h above the WGS84 reference ellipsoid and the incidence angle of its geodetic normal with the xy-plane.

Figure 1 — Geocentric and geodetic coordinates

2.2 Geocentric reference frame

A point location in geocentric Cartesian coordinates is given by the same (x, y, z), as used in geodetic Cartesian coordinates (see 3.1.2 of ISO 19111, but note that it uses capital X, Y, and Z). Equivalently, coordinates can be specified as geocentric spherical polar (λ, φ, r), where λ is the longitude, φ' is the geocentric latitude, and r is the distance from the Earth centre. The prime is used to distinguish geocentric from geodetic terms where necessary.

2.3 Geodetic reference frame

The geodetic reference frame is based on the WGS84 reference ellipsoid. This is an ellipsoid of rotation having a defined semi-major axis A (in the equatorial plane), and a flattening f. This leads to a semi-minor axis (essentially along the spin axis) of A(1–f). Specifically,

$$A = 6\,378\,137 \text{ m} \tag{1}$$

$$\frac{1}{f} = 298{,}257\,223\,563 \tag{2}$$

$$e^2 = f(2 - f) \tag{3}$$

where

e is the (first) eccentricity.

A point location in geodetic coordinates is given by (λ, φ, h), where λ is the longitude, φ is the geodetic latitude, and h is the height (distance normal to the ellipsoid) above the WGS84 reference ellipsoid. Geodetic and geocentric longitudes are identical. This nomenclature follows ISO 19111. A point location can also be specified by the Cartesian coordinates (x, y, z) in the WGS84 reference system, where the positive z and x axes point in the directions of the semi-minor axis and the prime meridian (λ = 0) in the equatorial plane, respectively; this is the same as the geocentric Cartesian coordinate system. For the WGS84 ellipsoid we then have for ρ_c, the radius of curvature of the normal section at the geodetic latitude φ,

$$\rho_c = \frac{A}{\sqrt{1 - e^2 \sin^2 \varphi}} \tag{4}$$

2.4 Geodetic to geocentric coordinate transform

The geodetic coordinates (λ, φ, h) are transformed into spherical geocentric coordinates (λ, φ', r) by recognizing that λ is the same in both coordinate systems, and that (φ', r) is computed from (φ, h) according to the formulae:

$$x = (\rho_c + h)\cos\varphi\cos\lambda \tag{5}$$

$$y = (\rho_c + h)\cos\varphi\sin\lambda \tag{6}$$

$$z = [\rho_c(1 - e^2) + h]\sin\varphi \tag{7}$$

$$\rho = \sqrt{(x^2 + y^2)} = (\rho_c + h)\cos\varphi \tag{8}$$

$$r = \sqrt{\rho^2 + z^2} \tag{9}$$

$$\varphi' = \arcsin\frac{z}{r} \tag{10}$$

where

ρ is the east–west (cylindrical) radius of curvature.

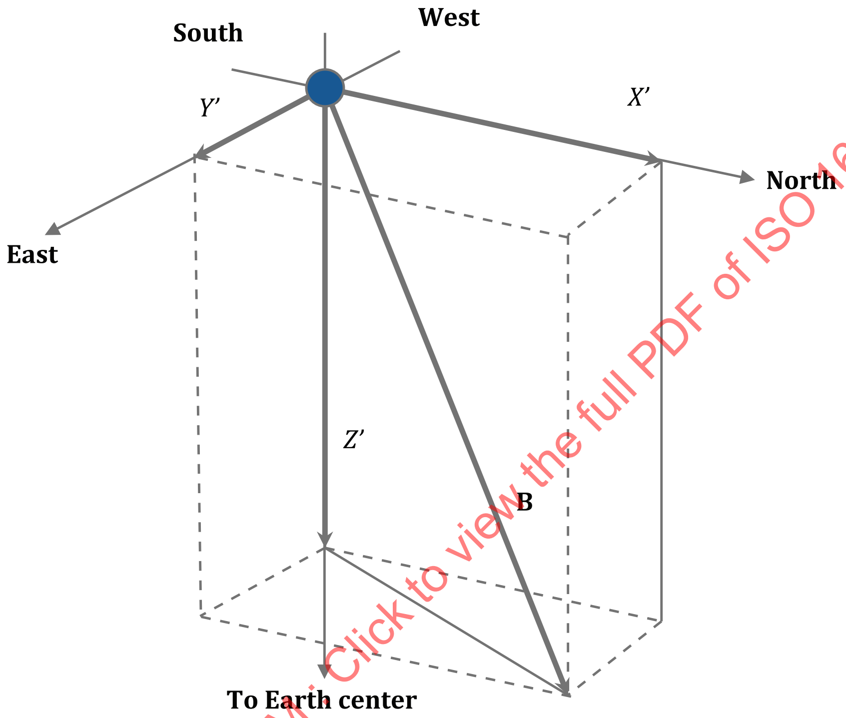
3 Specification of the geomagnetic field vector

3.1 General

When synthesizing the field from the Gauss coefficients, the output field is given in a geocentric frame. But the output field is often needed in the geodetic frame instead. This Clause therefore specifies and relates the magnetic field in the geocentric and geodetic frames.

3.2 Magnetic vector components in the geocentric frame

At the point of interest $P(\lambda, \varphi', r)$, the geomagnetic field vector, **B**, can be described by three orthogonal components in a local Cartesian coordinate system with origin at *P* and axes in the geocentric spherical-polar directions given by dλ, dφ', and d*r*. For historical reasons, however, the triplet (X', Y', Z') is used (see Figure 2), where X' is the northerly intensity, Y' the easterly intensity, and Z' the inward radial intensity, positive towards the Earth's centre. The unit vectors of the geomagnetic field components X', Y', and Z' thus point in the φ', λ, and negative-*r* directions, respectively. The quantities X', Y', and Z' are the sizes of perpendicular vectors that add vectorially to **B**. Note that the orientation of this coordinate system varies with angular position. The synthesis of the field from a spherical harmonic model initially produces these X', Y', and Z' components.

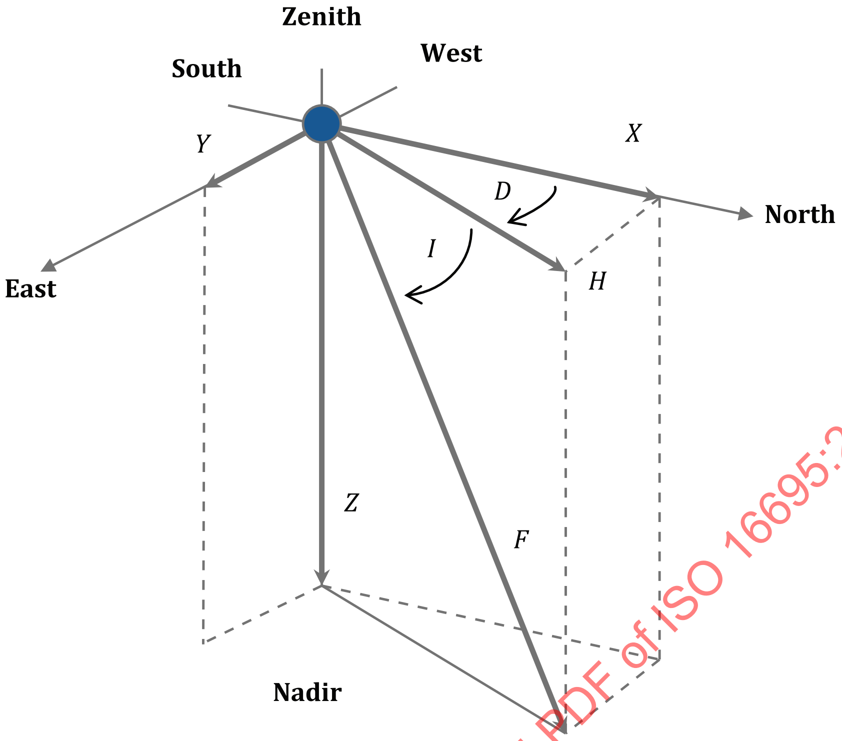


NOTE The components of the geomagnetic field vector **B** in the geocentric reference frame are the northerly intensity X', easterly intensity Y' and centerwardly intensity Z'.

Figure 2 — Components of the geomagnetic field vector in the geocentric reference frame

3.3 Magnetic elements in the geodetic reference frame

The geomagnetic field vector, **B**, is fully described by an appropriate set of three elements, but in practice, seven elements are variously used (see Figure 3). The main orthogonal set is the northerly intensity *X*, the easterly intensity *Y*, and the vertical intensity *Z* (positive downwards). Another orthogonal set is the total intensity *F*, the inclination angle *I* (also called the dip angle and measured from the horizontal plane to the field vector, positive downwards), and the declination angle *D* (also called the magnetic variation and measured clockwise from true north to the horizontal component of the field vector). The seventh element is the horizontal intensity *H*. In the descriptions of *X, Y, Z, F, H, I*, and *D* above, the vertical direction is perpendicular to the WGS84 ellipsoid model of the Earth's surface, the horizontal plane is perpendicular to the vertical direction, and the rotational directions clockwise and counter-clockwise are determined by a view from above.



NOTE The seven elements of the geomagnetic field vector, **B**, in the geodetic reference frame are: northerly intensity *X*, easterly intensity *Y*, vertical intensity *Z* (positive downwards), total intensity *F*, inclination angle *I* (also called the dip angle and measured from the horizontal plane to the field vector, positive downwards), declination angle *D* (also called the magnetic variation and measured clockwise from true north to horizontal component of the field vector), and horizontal intensity *H*.

Figure 3 — Elements of the geomagnetic field vector in the geodetic reference frame

The quantities *X, Y,* and *Z* are the sizes of perpendicular vectors that add vectorially to **B**; their directions form a local Cartesian coordinate system. The corresponding set of local spherical polar-coordinate components is denoted by (*F, I, D*). In both these local coordinate systems, the orientation varies with position on the surface.

The spherical harmonic representation initially produces the Cartesian (*X'*, *Y'*, *Z'*). These are then rotated into the geodetic (*X*, *Y*, *Z*), see 3.4. The magnetic elements *H, F, I*, and *D* are then computed from the orthogonal components (*X, Y, Z*):

$$H = \sqrt{X^2 + Y^2} \quad (11)$$

$$F = \sqrt{H^2 + Z^2} \quad (12)$$

$$I = \arctan(Z, H) \quad (13)$$

$$D = \arctan(Y, X) \quad (14)$$

where $\arctan(a,b)$ is $\tan^{-1}(a/b)$, taking into account the angular quadrant, avoiding a division by zero, and resulting in a declination in the range of −π to π and inclination of −π/2 to π/2. In formulae, and in computing sub-routines, angles are in radians, but typical user software allows input/output in degrees.

Conversely, *X*, *Y*, and *Z* can be determined from the quantities *F*, *I*, and *D*. The time variation (secular variation) of these elements is computed using Formulae (15) to (18).

$$\dot{H} = \frac{X \cdot \dot{X} + Y \cdot \dot{Y}}{H} \tag{15}$$

$$\dot{F} = \frac{X \cdot \dot{X} + Y \cdot \dot{Y} + Z \cdot \dot{Z}}{F} \tag{16}$$

$$\dot{I} = \frac{H \cdot \dot{Z} - Z \cdot \dot{H}}{F^2} \tag{17}$$

$$\dot{D} = \frac{X \cdot \dot{Y} - Y \cdot \dot{X}}{H^2} \tag{18}$$

where $\dot{I}$ and $\dot{D}$, are given in radians/year. These angular changes are typically converted by user software to arc-minutes/year.

3.4 Transform of magnetic vector components from geocentric to geodetic frame

The geocentric magnetic field vector components, X', Y', and Z', are rotated into the geodetic reference frame, using Formulae (19) to (21).

$$X = X'\cos(\varphi' - \varphi) - Z'\sin(\varphi' - \varphi) \tag{19}$$

$$Y = Y' \tag{20}$$

$$Z = X'\sin(\varphi' - \varphi) + Z'\cos(\varphi' - \varphi) \tag{21}$$

with corresponding formulae for the time derivatives of the vector components, $\dot{X}'$, $\dot{Y}'$, and $\dot{Z}'$.

$$\dot{X} = \dot{X}'\cos(\varphi' - \varphi) - \dot{Z}'\sin(\varphi' - \varphi) \tag{22}$$

$$\dot{Y} = \dot{Y}' \tag{23}$$

$$Z = X'\sin(\varphi' - \varphi) + Z'\cos(\varphi' - \varphi) \tag{24}$$

4 Specification of the geomagnetic reference model

4.1 Potential of the magnetic field

To the accuracy of present models, it is adequate to assume that $\mu = \mu_0$ everywhere. In the absence of local electric currents, the magnetic field, **B,** then behaves as a potential field and can be written as the negative spatial gradient of a scalar potential V, satisfying the Laplace formula $\nabla \cdot \nabla V = 0$. In geocentric spherical coordinates (λ, φ', r) this gives

$$\mathbf{B}(\lambda,\varphi',r,t) = -\nabla V(\lambda,\varphi',r,t) \tag{25}$$

The potential V is the sum of contributions by sources internal and external to the Earth. The internal contributions are due mostly to the geodynamo in the core and magnetization of the lithosphere. The external part accounts for a steady contribution of the magnetospheric ring current, which is present even during magnetically quiet periods. It changes gradually over the course of the 11-year solar cycle and is therefore well represented by the same piece-wise linear temporal representation as the internal field. The potential V is expanded in terms of spherical harmonics:

$$\begin{aligned} V(\lambda,\varphi',r,t) = {} & a\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left({}_i g_n^m(t)\cos m\lambda + {}_i h_n^m(t)\sin m\lambda\right)\breve{P}_n^m(\sin\varphi') \\ & + a\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n}\sum_{m=0}^{n}\left({}_e g_n^m(t)\cos m\lambda + {}_e h_n^m(t)\sin m\lambda\right)\breve{P}_n^m(\sin\varphi') \end{aligned} \tag{26}$$

where

N_i is the truncation degree of internal expansion;

N_e is the truncation degree of external expansion;

a is the geomagnetic reference radius (6 371 200 m);

λ is the longitude in a spherical geocentric reference frame;

φ' is the latitude in a spherical geocentric reference frame;

r is the radius in a spherical geocentric reference frame;

${}_i g_n^m(t)$ and ${}_i h_n^m(t)$ are the internal time-dependent Gauss coefficients of degree n;

${}_e g_n^m(t)$ and ${}_e h_n^m(t)$ are the external time-dependent Gauss coefficients of order m.

The Schmidt semi-normalized associated Legendre functions $\breve{P}_n^m(\mu)$, where $\mu = \sin\varphi'$ is a real number in the interval [-1;1], are defined as

$$\begin{aligned} \breve{P}_n^m(\mu) &= \sqrt{2\frac{(n-m)!}{(n+m)!}}P_{n,m}(\mu) \text{ if } m > 0 \\ \breve{P}_n^m(\mu) &= P_{n,m}(\mu) \text{ if } m > 0 \end{aligned} \tag{27}$$

This follows the definition of $P_{n,m}(\mu)$ commonly used in geodesy and geomagnetism (see Heiskenen and Moritz, 1967[5] and Langel, 1987[6]). Sample functions, for geocentric latitude, φ', are as follows.

$$P_{3,0}(\sin\varphi') = \frac{1}{2}(\sin\varphi')(5\sin^2\varphi' - 3) \tag{28}$$

$$P_{3,1}(\sin\varphi') = -\frac{3}{2}(\cos\varphi')(1 - 5\sin^2\varphi') \tag{29}$$

$$P_{3,2}(\sin\varphi') = 15(\sin\varphi')(1 - \sin^2\varphi') \tag{30}$$

$$P_{3,3}(\sin\varphi') = 15\cos^3\varphi' \tag{31}$$

These $P_{n,m}(\mu)$ are related to the $P_n^m(\mu)$ defined in Reference [7] (14.2, page 352) or in Chapter 8.7 of Reference [4] by $P_{n,m}(\mu) = (-1)^m P_n^m(\mu)$.

4.2 Geomagnetic reference radius

The geomagnetic reference radius used in the spherical harmonic expansion of the magnetic potential is fixed by convention at a = 6 371 200 m. The Gauss coefficients are referenced to this value. It is therefore important to use exactly this value in the computation of the field elements.

4.3 Epoch of a sub-model

The Gauss coefficients of a geomagnetic reference sub-model are specified in terms of a fixed base-date, t_0, the so-called epoch of the sub-model. The epoch is given in decimal year, for example, as *2010.0*.

4.4 Validity of a sub-model

Due to nonlinear changes of the Earth's magnetic field, linear geomagnetic reference models of the type described here are valid only for a limited period. The start and end dates of a model are given in decimal year. For example, a model validity of *2010.0* to *2015.0* means that the model is valid from *2010-Jan-01 00:00:00* to *2015-Jan-01 00:00:00*.

The 'external' coefficients represent fields from a source at some region outside $r = a$. For an observation point further out than the source, the corresponding field is now 'internal'. So there is a limit in radius beyond which the external coefficients are not appropriate. This radius will be specified by the model, along with the maximum degree of the external terms. Magnetic fields caused by electric currents in the magnetosphere are specifically covered by ISO 22009.

4.5 Time-dependence of Gauss coefficients

The Gauss coefficients ${}_i g_n^m(t)$, ${}_i h_n^m(t)$, ${}_e g_n^m(t)$, and ${}_e h_n^m(t)$ are determined for the desired time, t, from the model coefficients ${}_i g_n^m(t_0)$, ${}_i h_n^m(t_0)$, ${}_e g_n^m(t_0)$, and ${}_e h_n^m(t_0)$, and the linear secular variation model coefficients ${}_i \dot{g}_n^m(t_0)$, ${}_i \dot{h}_n^m(t_0)$, ${}_e \dot{g}_n^m(t_0)$, and ${}_e \dot{h}_n^m(t_0)$, at epoch t_0 as follows.

$$\begin{aligned}
{}_i g_n^m(t) &= {}_i g_n^m(t_0) + (t - t_0)\,{}_i \dot{g}_n^m(t_0) \\
{}_i h_n^m(t) &= {}_i h_n^m(t_0) + (t - t_0)\,{}_i \dot{h}_n^m(t_0) \\
{}_e g_n^m(t) &= {}_e g_n^m(t_0) + (t - t_0)\,{}_e \dot{g}_n^m(t_0) \\
{}_e h_n^m(t) &= {}_e h_n^m(t_0) + (t - t_0)\,{}_e \dot{h}_n^m(t_0)
\end{aligned} \tag{32}$$

Here, time is given in decimal years and t_0 is the epoch of the sub-model.

If for a sub-model extending from time t_1 to t_2 the secular variation coefficients are not explicitly provided, then they have to be computed by Formula (33).

$$\begin{aligned}
{}_i \dot{g}_n^m(t_1) &= [{}_i g_n^m(t_2) - {}_i g_n^m(t_1)]/(t_2 - t_1) \\
{}_i \dot{h}_n^m(t_1) &= [{}_i h_n^m(t_2) - {}_i h_n^m(t_1)]/(t_2 - t_1) \\
{}_e \dot{g}_n^m(t_1) &= [{}_e g_n^m(t_2) - {}_e g_n^m(t_1)]/(t_2 - t_1) \\
{}_e \dot{h}_n^m(t_1) &= [{}_e h_n^m(t_2) - {}_e h_n^m(t_1)]/(t_2 - t_1)
\end{aligned} \tag{33}$$

where

t_1 is the epoch of this particular sub-model.

In case the degree of the expansion differs between times t_1 and t_2, then the following applies: If the degree of the expansion is N_1 at t_1 and the degree is N_2 at t_2, then the secular variation coefficients are computed assuming that ${}_i g_n^m(t_1)$, ${}_i h_n^m(t_1)$, ${}_e g_n^m(t_1)$, and ${}_e h_n^m(t_1)$ are zero for $N_1 < n \leq N_2$. This avoids a discontinuity in the magnetic field at time t_2. For example, the International Geomagnetic Reference Field changes from degree 10 in 1995 to degree 13 in 2000. When computing the field during the period 1995 to 2000, the coefficients of degrees 11 to 13 are linearly interpolated from a value of zero in 1995 to their values in 2000.

4.6 Calculation of magnetic vector components in the geocentric reference frame

The magnetic field vector components X', Y', and Z' in the geocentric reference frame are computed as follows.

$$X'(\lambda,\varphi',r,t) = -\frac{1}{r}\frac{\partial V}{\partial \varphi'}$$

$$= -\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left({}_i g_n^m(t)\cos m\lambda + {}_i h_n^m(t)\sin m\lambda\right)\frac{\partial \breve{P}_n^m(\sin\varphi')}{\partial \varphi'} \quad (34)$$

$$-\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}\left({}_e g_n^m(t)\cos m\lambda + {}_e h_n^m(t)\sin m\lambda\right)\frac{\partial \breve{P}_n^m(\sin\varphi')}{\partial \varphi'}$$

$$Y'(\lambda,\varphi',r,t) = -\frac{1}{r\cos\varphi'}\frac{\partial V}{\partial \lambda}$$

$$= \frac{1}{\cos\varphi'}\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m\left({}_i g_n^m(t)\sin m\lambda - {}_i h_n^m(t)\cos m\lambda\right)\breve{P}_n^m(\sin\varphi') \quad (35)$$

$$+\frac{1}{\cos\varphi'}\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n} m\left({}_e g_n^m(t)\sin m\lambda - {}_e h_n^m(t)\cos m\lambda\right)\breve{P}_n^m(\sin\varphi')$$

$$Z'(\lambda,\varphi',r,t) = \frac{\partial V}{\partial r}$$

$$= -\sum_{n=1}^{N_i}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left({}_i g_n^m(t)\cos m\lambda + {}_i h_n^m(t)\sin m\lambda\right)\breve{P}_n^m(\sin\varphi') \quad (36)$$

$$+\sum_{n=1}^{N_e} n\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}\left({}_e g_n^m(t)\cos m\lambda + {}_e h_n^m(t)\sin m\lambda\right)\breve{P}_n^m(\sin\varphi')$$

Correspondingly, the secular variation of the magnetic field components are computed as follows.

$$\dot{X}'(\lambda,\varphi',r,t_0) = -\frac{1}{r}\frac{\partial \dot{V}}{\partial \varphi'}$$

$$= -\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left({}_i \dot{g}_n^m(t_0)\cos m\lambda + {}_i \dot{h}_n^m(t_0)\sin m\lambda\right)\frac{d\breve{P}_n^m(\sin\varphi')}{d\varphi'} \quad (37)$$

$$-\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}\left({}_e \dot{g}_n^m(t)\cos m\lambda + {}_e \dot{h}_n^m(t)\sin m\lambda\right)\frac{d\breve{P}_n^m(\sin\varphi')}{d\varphi'}$$

$$\dot{Y}'(\lambda,\varphi',r,t_0)=-\frac{1}{r\cos\varphi'}\frac{\partial\dot{V}}{\partial\lambda}$$

$$=\frac{1}{\cos\varphi'}\sum_{n=1}^{N_i}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}m({}_i\dot{g}_n^m(t_0)\sin m\lambda-{}_i\dot{h}_n^m(t_0)\cos m\lambda)\breve{P}_n^m(\sin\varphi') \quad (38)$$

$$+\frac{1}{\cos\varphi'}\sum_{n=1}^{N_e}\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}m({}_e\dot{g}_n^m(t)\sin m\lambda-{}_e\dot{h}_n^m(t)\cos m\lambda)\breve{P}_n^m(\sin\varphi')$$

$$\dot{Z}'(\lambda,\varphi',r,t_0)=\frac{\partial\dot{V}}{\partial r}$$

$$=-\sum_{n=1}^{N_i}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}({}_i\dot{g}_n^m(t_0)\cos m\lambda+{}_i\dot{h}_n^m(t_0)\sin m\lambda)\breve{P}_n^m(\sin\varphi') \quad (39)$$

$$+\sum_{n=1}^{N_e}n\left(\frac{r}{a}\right)^{n-1}\sum_{m=0}^{n}({}_e\dot{g}_n^m(t)\cos m\lambda+{}_e\dot{h}_n^m(t)\sin m\lambda)\breve{P}_n^m(\sin\varphi')$$

The derivative in Formulae (34) and (37) can be replaced by the following expression:

$$\frac{d\breve{P}_n^m(\sin\varphi')}{d\varphi'}=(n+1)(\tan\varphi')\breve{P}_n^m(\sin\varphi')-\sqrt{(n+1)^2-m^2}(\sec\varphi')\breve{P}_{n+1}^m(\sin\varphi') \quad (40)$$

Formulae (35) and (38) contain a factor of cos φ' in the denominator. This factor is zero at the poles, but the other functions are such that there is no infinity there. In practice, only the m = 1 terms give a horizontal field near the poles, and this is a nearly uniform field coming from an n = 1, m = 1, potential.

4.7 Spatial wavelength

A model of spherical harmonic degree N accounts for magnetic fields that have spatial wavelengths larger or equal to $360°/\sqrt{N(N+1)}$ in arc length (see Sec. 3.6.3 of Reference [3]).

4.8 Root-mean-square difference between two model fields

It is sometimes useful to assess the difference between two geomagnetic model fields. For example, these could be successive updates of the same model, or two entirely different models. Let $\mathbf{B}_1(\lambda, \varphi', a, t)$ and $\mathbf{B}_2(\lambda, \varphi', a, t)$ be the fields predicted by models 1 and 2, for a particular time t and up to degree N, over the sphere of reference radius a. The mean-square of the difference-vector ($\mathbf{B}_1$-$\mathbf{B}_2$) between the two model fields, is computed following Sec. 4.4 of Reference [3]:

$$R(t)=\frac{1}{4\pi}\int_{-\pi/2}^{\pi/2}\int_{0}^{2\pi}[\mathbf{B}_1(\lambda,\varphi',a,t)-\mathbf{B}_2(\lambda,\varphi',a,t)]^2\cos\varphi'\,d\lambda\,d\varphi'=\sum_{1}^{N}R_n(t) \quad (41)$$

$$R_n(t)=(n+1)\sum_{m=0}^{n}[({}_{1,i}g_n^m(t)-{}_{2,i}g_n^m(t))^2+({}_{1,i}h_n^m(t)-{}_{2,i}h_n^m(t))^2)]$$

$$+n\sum_{m=0}^{n}[({}_{1,e}g_n^m(t)-{}_{2,e}g_n^m(t))^2+({}_{1,e}h_n^m(t)-{}_{2,e}h_n^m(t))^2)] \quad (42)$$

where $R(t)$ is the mean-square of the difference-vector between the two model fields; the double integral provides the average over the geomagnetic reference sphere, and $R_n(t)$ is the mean-square of the magnitude of the difference-vector of degree n. Here, the Gauss coefficients ${}_{1,i}g_n^m(t)$, ${}_{1,i}h_n^m(t)$, ${}_{1,e}g_n^m(t)$, and ${}_{1,e}h_n^m(t)$ of the first model and the coefficients ${}_{2,i}g_n^m(t)$, ${}_{2,i}h_n^m(t)$, ${}_{2,e}g_n^m(t)$, and ${}_{2,e}h_n^m(t)$ of the second model are assumed zero for n larger than the respective degree of the expansion. The root-mean-square (RMS) difference at time t between the two model fields is then defined as

$$d_{\mathrm{RMS}}(t)=\sqrt{R(t)} \tag{43}$$

5 Examples of the use of geomagnetic reference models

5.1 Compute reference magnetic elements near the Earth's surface

Near the Earth's surface it is conventional to express the magnetic field vector in the geodetic frame. Computing such a reference magnetic field for a given location and date typically requires the following steps.

1. Choose the desired date t in decimal years and the longitude, geodetic latitude, and height in metres above the WGS84 ellipsoid. Heights above mean sea level (approximately following the equipotential surface known as the geoid) need to be converted to height above the ellipsoid. This can be achieved, for example, by using the online geoid calculator for the Earth Gravity Model (EGM, http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm96/intpt.html) at http://geographiclib.sourceforge.net/cgi-bin/GeoidEval, which gives the height of the geoid above the WGS84 ellipsoid.
2. Convert geodetic to geocentric coordinates (2.4).
3. Compute the Gauss coefficients ${}_i g_n^m(t)$, ${}_i h_n^m(t)$, ${}_e g_n^m(t)$, and ${}_e h_n^m(t)$ for all degrees n and orders m for the desired date t (4.5).
4. Compute magnetic field vector components X', Y', and Z' in geocentric reference frame (4.6).
5. Since the inclinations of the vertical and northward directions differ between the geocentric and geodetic reference frames, the magnetic field vector components have to be rotated from the geocentric (X', Y', Z') to the geodetic (X, Y, Z) (3.4).
6. Compute the magnetic declination and any other desired magnetic elements (3.3) in the geodetic reference frame.

The computed magnetic field elements in the geodetic reference frame are with respect to a local coordinate system where the down axis is perpendicular to the surface of the WGS84 ellipsoid. Deflections of the vertical caused by local gravity anomalies have to be taken care of by the user and are not covered by this International Standard.

5.2 Compute reference magnetic vector in near-Earth space

In Low Earth Orbit (LEO), it is conventional to express the magnetic field vector in the geocentric frame. Computing such a reference magnetic field vector, e.g. to determine the attitude of a spacecraft, typically requires the following steps.

1. Determine date t in decimal years and position of the spacecraft given in longitude, geocentric latitude, and radius.
2. Compute the Gauss coefficients ${}_i g_n^m(t)$, ${}_i h_n^m(t)$, ${}_e g_n^m(t)$, and ${}_e h_n^m(t)$ for all degrees n and orders m for the desired date t (4.5).
3. Compute model magnetic field vector components X', Y', and Z' in geocentric reference frame (4.6), e.g. to compare with measured magnetic field vector to determine attitude of the spacecraft.

5.3 Compute reference magnetic vector in magnetosphere

In the magnetosphere it is conventional to express the magnetic field vector in the geocentric frame. Computing such a reference magnetic field vector, e.g. to determine the attitude of a spacecraft, typically requires the following steps.

1. Determine date t in decimal years and position of the spacecraft given in longitude, geocentric latitude, and radius.

2. Compute the Gauss coefficients ${}_i g_n^m(t)$ and ${}_i h_n^m(t)$ for all degrees n and orders m for the desired date t (4.5).

3. The external Gauss coefficients ${}_e g_n^m(t)$ and ${}_e h_n^m(t)$ are not applicable in the magnetosphere because the sources of this magnetic field contribution are not external to the spacecraft location. Instead, a model of the magnetospheric magnetic field should be used, as specified in ISO 22009.

4. Compute model magnetic field vector components X', Y', and Z' in geocentric reference frame (4.6), e.g. to compare with measured magnetic field vector to determine attitude of the spacecraft.

Annex A
(informative)

Available geomagnetic reference models

A.1 International Geomagnetic Reference Field

The International Geomagnetic Reference Field (IGRF) was introduced by the International Association of Geomagnetism and Aeronomy (IAGA) in 1968 in response to the demand for a standard spherical harmonic representation of the Earth's main field. The model extends to degree and order 13, and it is updated at five-year intervals, produced and released by IAGA Working Group V-MOD.

A.1.1 Model website

http://www.ngdc.noaa.gov/IAGA/vmod/

A.1.2 Stand-alone software

The following public domain software for the IGRF is available for download at the IGRF website.

— **Geomag.c**: Program in C-language maintained by NOAA's National Geophysical Data Center, distributed both as source code and as precompiled versions for Windows®[1)] and Linux®[1)]. It computes the magnetic field vector and elements for given locations (geocentric or geodetic) and dates (or ranges of dates). The program further has command line and spreadsheet options to facilitate repeated processing.

— **Igrf.f**: A FORTRAN program maintained by the British Geological Survey. The IGRF coefficients are embedded into the source code. Options include values at different locations at different times (spot), values at same location at one year intervals (time series), grid of values at one time (grid); geodetic or geocentric coordinates, latitude and longitude entered as decimal degrees or degrees and minutes (not in grid), and choice of main field or secular variation or both (grid only).

A.1.3 Online calculators

A.1.3.1 Declination, single point

http://www.ngdc.noaa.gov/geomag-web/#declination

http://www-app3.gfz-potsdam.de/Declinationcalc/declinationcalc.html

A.1.3.2 Magnetic vector and elements, single point

http://www.ngdc.noaa.gov/geomag-web/#igrfwmm

http://wdc.kugi.kyoto-u.ac.jp/igrf/point/index.html

http://www.geomag.bgs.ac.uk/gifs/igrf_form.shtml

A.1.3.3 Magnetic vector and elements, grids and profiles

http://www.ngdc.noaa.gov/geomag-web/#igrfgrid

1) This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of this product.

http://omniweb.gsfc.nasa.gov/vitmo/igrf_vitmo.html

A.1.4 Charts of the magnetic elements

http://www.ngdc.noaa.gov/wist/magfield.jsp

http://wdc.kugi.kyoto-u.ac.jp/igrf/index.html

A.2 World Magnetic Model

The World Magnetic Model (WMM) is the standard model used by the US Department of Defence (DoD), the UK Ministry of Defence, and the North Atlantic Treaty Organization (NATO) for navigation, attitude, and heading referencing systems using the geomagnetic field. It is also used widely in civilian navigation and heading systems. Sponsored by the US National Geospatial-Intelligence Agency (NGA) and the UK Defence Geographic Centre (DGC), the WMM is currently produced by the US National Oceanographic and Atmospheric Administration's National Geophysical Data Center (NOAA/NGDC) and the British Geological Survey (BGS). The UK and US have been collaborating in the production of the World Magnetic Charts, World Chart Models, and World Magnetic Models since 1965.

The WMM is a purely predictive model extending to degree and order 12, produced in five-year intervals. The updated model is released in December before the new epoch (Dec-2014, Dec-2019, etc.).

A.2.1 Model website

http://www.ngdc.noaa.gov/geomag/WMM/

http://www.geomag.bgs.ac.uk/navigation.html

A.2.2 Stand-alone software

— Website: http://www.ngdc.noaa.gov/geomag/WMM/soft.shtml. All programs are maintained by NGDC and are distributed both as source code and pre-compiled versions for Windows and Linux.

— **wmm_point.c**: Computes the magnetic field vector and elements for a given location (geodetic), altitude (above mean sea level or WGS84 ellpsoid), and date.

— **wmm_file.c**: Computes the magnetic field vector for locations listed in the input file and outputs the results in a file suitable for importing to spreadsheet.

— **wmm_grid.c**: Computes the magnetic field vector for a grid or profile of input points specified by the user. The program can print the result either to the screen or to a file.

— **WMM GUI**: Windows-based graphical user interface. It computes the magnetic field elements and their annual change for a given location (geodetic), altitude (above mean sea level or WGS84 ellpsoid), and date. The location can be specified either in UTM coordinates or in geodetic coordinates.

A.2.3 Online calculators

This magnetic field calculator, maintained by NGDC, calculates the elements of the geomagnetic field and their annual change with the option to produce profiles and grids.

http://www.ngdc.noaa.gov/geomag/WMM/calculators.shtml

This magnetic field calculator, maintained by BGS, calculates the elements of the magnetic field and plots the result and location on a Google map.

http://www.geomag.bgs.ac.uk/data_service/models_compass/wmm_calc.html

