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**Geometrical product specification  
(GPS) — Filtration —**

Part 61:

**Linear areal filters — Gaussian filters**

*Spécification géométrique des produits (GPS) — Filtrage —  
Partie 61: Filtres surfaciques linéaires : Filtres Gaussiens*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: [Foreword - Supplementary information](#)

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

- Part 1: Overview and basic concepts
- Part 20: Linear profile filters: Basic concepts
- Part 21: Linear profile filters: Gaussian filters
- Part 22: Linear profile filters: Spline filters
- Part 28: Profile filters: End effects
- Part 29: Linear profile filters: Spline wavelets
- Part 30: Robust profile filters: Basic concepts
- Part 31: Robust profile filters: Gaussian regression filters
- Part 32: Robust profile filters: Spline filters
- Part 40: Morphological profile filters: Basic concepts
- Part 41: Morphological profile filters: Disk and horizontal line-segment filters
- Part 49: Morphological profile filters: Scale space techniques
- Part 60: Linear areal filters: Basic concepts
- Part 61: Linear areal filters: Gaussian filters
- Part 71: Robust areal filters: Gaussian regression filters

— *Part 85: Morphological areal filters: Segmentation*

The following parts are planned:

- *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*
- *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*
- *Part 45: Morphological profile filters: Segmentation*
- *Part 62: Linear areal filters: Spline filters*
- *Part 69: Linear areal filters: Spline wavelets*
- *Part 70: Robust areal filters: Basic concepts*
- *Part 72: Robust areal filters: Spline filters*
- *Part 80: Morphological areal filters: Basic concepts*
- *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*
- *Part 89: Morphological areal filters: Scale space techniques*

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## Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO/TR 14638). It influences chain links 3 and 5 in the GPS matrix structure..

The ISO/GPS Masterplan given in ISO/TR 14638 gives an overview of the ISO/GPS system of which this part of ISO 16610 is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this part of ISO 16610 and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this part of ISO 16610, unless otherwise indicated.

For more detailed information about the relation of this part of ISO 16610 to the GPS matrix model, see [Annex D](#).

This part of ISO 16610 specifies the metrological characteristics of linear areal Gaussian filters for the rotationally symmetric filtration of nominal planar surfaces and the filtration of nominal cylindrical surfaces. It specifies, in particular, how to separate long and short wave components of a surface.

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# Geometrical product specification (GPS) — Filtration —

## Part 61:

### Linear areal filters — Gaussian filters

#### 1 Scope

This part of ISO 16610 specifies linear areal Gaussian filters for the rotationally symmetric filtration of nominal planar surfaces and the filtration of nominal cylindrical surfaces. It specifies, in particular, how to separate long and short wave components of a surface.

#### 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specification (GPS) — Filtration — Part 1: Overview and basic terminology*

ISO 16610-20, *Geometrical product specification (GPS) — Filtration — Part 20: Linear profile filters: basic concepts*

ISO 16610-21:2011, *Geometrical product specifications (GPS) — Filtration — Part 21: Linear profile filters: Gaussian filters*

ISO 16610-60, *Geometrical product specification (GPS) — Filtration — Part 60: Linear areal filters: Basic concepts*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM)*

#### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1, ISO 16610-20, ISO 16610-21, ISO 16610-60, ISO/IEC Guide 98-3:2008, ISO/IEC Guide 99, and the following apply.

##### 3.1

##### **linear areal filter**

areal filter which separates surfaces into long wave and short wave components and is also a linear function

[SOURCE: ISO 16610-60, 3.1]

##### 3.1.1

##### **linear planar filter**

*linear areal filter* (3.1) that separate surfaces into long wave and short wave components, which applies to nominal planar surfaces

[SOURCE: ISO 16610-60, 3.1.1]

### 3.1.2

#### linear cylindrical filter

*linear areal filter* (3.1) that separate surfaces into long wave and short wave components, which applies to nominal cylindrical surfaces

[SOURCE: ISO 16610-60, 3.1.2]

### 3.2

#### cut-off wavelength (nesting index)

wavelength of a sinusoidal surface of which 50% of the amplitude is transmitted by the *linear areal filter* (3.1)

Note 1 to entry: Linear areal filters are identified by the filter type and the cut-off wavelength.

[SOURCE: ISO 16610-60, 3.7]

Note 2 to entry: The cut-off value for the Gaussian filter is an example of a nesting index.

### 3.3

#### undulations per revolution

##### UPR

number of sinusoidal undulations contained in the roundness profile

#### 3.3.1

##### undulation cut-off (nesting index)

*cut-off wavelength* (3.2) of the filter applied to the extracted circumferential line

Note 1 to entry: These are usually defined in terms of undulations per revolution (UPR).

## 4 Characteristics of linear planar Gaussian filters

### 4.1 General

Linear planar Gaussian filters conforming to this part of ISO 16610 shall conform to 4.2 to 4.4.

### 4.2 Weighting function of linear planar filters

The weighting function of an areal filter (see Figure 1) has the formula of a rotationally symmetric Gaussian function with a cut-off wavelength,  $\lambda_c$ , given by Formula (1):

$$s(x, y) = \frac{1}{\alpha^2 \lambda_c^2} \exp \left[ -\frac{\pi}{\alpha^2} \left( \frac{x^2 + y^2}{\lambda_c^2} \right) \right] \quad (1)$$

where

$x$  is the distance from the centre (maximum) of the weighting function in X direction;

$y$  is the distance from the centre (maximum) of the weighting function in Y direction;

$\lambda_c$  is the cut-off wavelength;

$\alpha$  is the constant, to provide 50% transmission characteristic at the cut-off  $\lambda_c$ .

For a practical application, the weighting function of a filter (see Figure 1) is expressed within  $-L_c \lambda_c \leq \sqrt{x^2 + y^2} \leq L_c \lambda_c$  where  $L_c$  is the truncation indices of the Gaussian filter.

NOTE See ISO 16610-21, Annex A for recommended values of  $L_c$ .

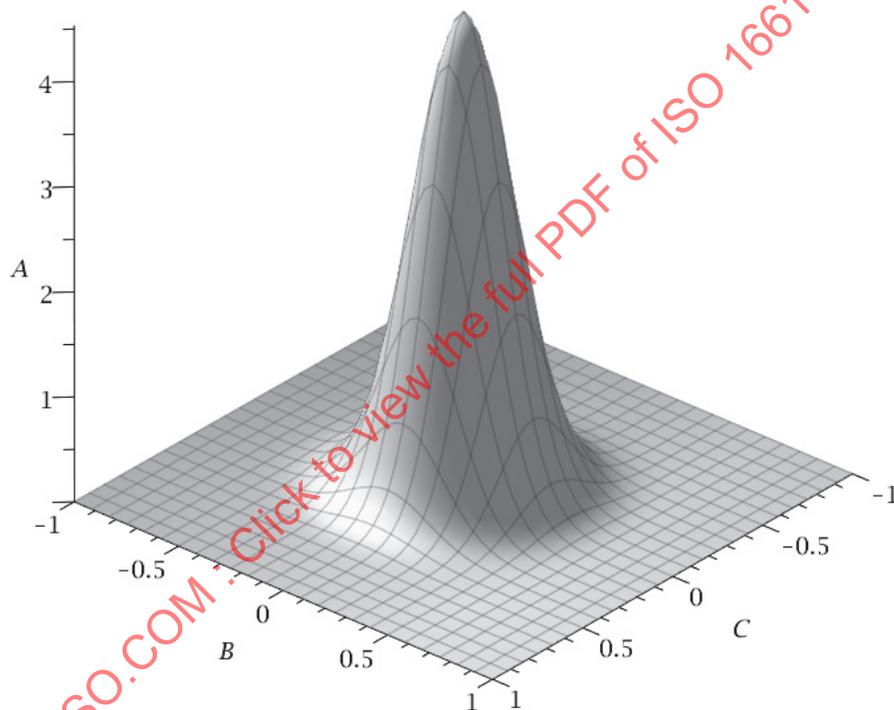
If smaller index values of  $L_c$  are used, then the uncertainty contributed by systematic error can become unacceptable,<sup>[3]</sup> as given in Formula (2):

$$s(x, y) = \begin{cases} \frac{1}{\alpha^2 \lambda_c^2} \exp \left[ -\frac{\pi}{\alpha^2} \left( \frac{x^2 + y^2}{\lambda_c^2} \right) \right], & -L_c \lambda_c \leq \sqrt{x^2 + y^2} \leq L_c \lambda_c \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where  $\alpha$  is given by Formula (3):

$$\alpha = \sqrt{\frac{\ln 2}{\pi}} \approx \frac{318}{677} \approx 0,4697 \approx \frac{31}{66} \quad (3)$$

The graph of the weighting function is shown in [Figure 1](#).



#### Key

- A weight modified to make it a unit number:  $\lambda_c^2 s(x, y)$
- B length modified to make it a unit number:  $\frac{y}{\lambda_c}$
- C length modified to make it a unit number:  $\frac{x}{\lambda_c}$

**Figure 1 — Weighting function of a Gaussian areal filter**

### 4.3 Transmission characteristics of linear planar Gaussian filters

#### 4.3.1 Transmission characteristic of the long wave component

The transmission characteristic is determined from the weighting function by means of the Fourier transformation. The transmission characteristic of the long wave component (mean) is given by Formula (4):

$$\frac{a_1}{a_0} = H(\lambda | \lambda_c) = \exp \left[ -\pi \left( \alpha \frac{\lambda_c}{\lambda} \right)^2 \right] \quad (4)$$

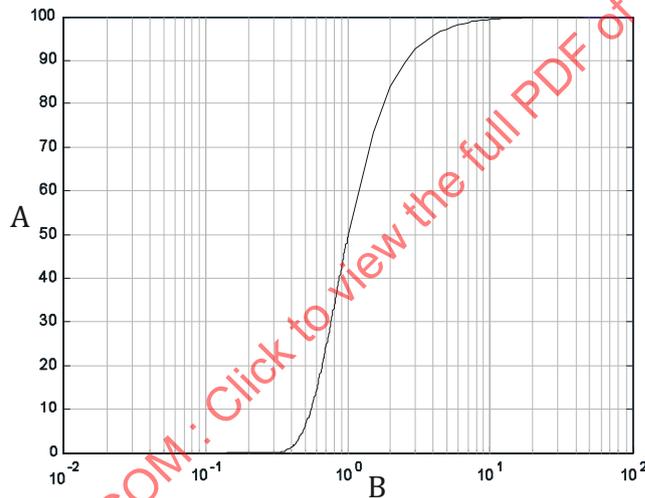
where

$a_0$  is the amplitude of a sine wave surface before filtering;

$a_1$  is the amplitude of the long wave component of a sine wave surface;

$\lambda$  is the wavelength of a sine surface in any direction.

The transmission characteristic of the long wave component with  $\lambda_c$  for a sine wave in any direction with wavelength  $\lambda$  is shown in [Figure 2](#).



#### Key

A amplitude transmission  $\frac{a_1}{a_0}$  in %

B  $\frac{\lambda}{\lambda_c}$

Figure 2 — Long wave transmission function of the areal Gaussian filter for planar surfaces with  $\lambda_c$

#### 4.3.2 Transmission characteristic of the short wave component

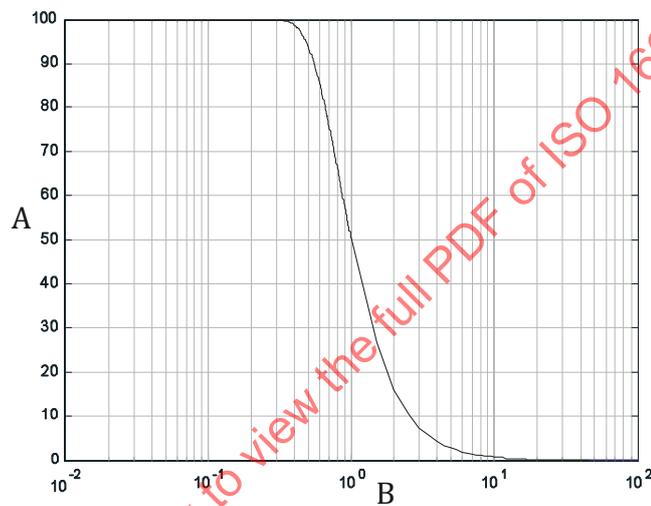
The transmission characteristic is determined from the weighting function by means of the Fourier transformation and is complementary to the transmission characteristic of the long wave profile component. The transmission characteristic of the short wave component is given by Formula (5):

$$\frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} = 1 - H(\lambda | \lambda_c) = 1 - \exp \left[ -\pi \left( \alpha \frac{\lambda_c}{\lambda} \right)^2 \right] \quad (5)$$

where

$a_2$  is the amplitude of the short wave component of a sine wave surface.

The transmission characteristic of the short wave component with  $\lambda_c$  for a sine wave in any direction with wavelength  $\lambda$  is shown in [Figure 3](#).



#### Key

A amplitude transmission  $\frac{a_2}{a_0}$  in %

B  $\frac{\lambda}{\lambda_c}$

**Figure 3** — Short wave transmission function of the areal Gaussian filter for planar surfaces with  $\lambda_c$

#### 4.4 Separable weighting functions

The linear planar Gaussian weighting function is separable. It can be written as a product of two linear open profile Gaussian weighting functions, as shown in Formula (6):

$$s(x, y | \lambda_c, \lambda_c) = s(x | \lambda_c) s(y | \lambda_c) \quad (6)$$

where

$$s(x | \lambda_c) = \frac{1}{\alpha \lambda_c} \exp \left[ -\pi \left( \frac{x}{\alpha \lambda_c} \right)^2 \right] \quad \text{is the weighting function in the x-direction;} \quad (7)$$

$$s(y | \lambda_c) = \frac{1}{\alpha \lambda_c} \exp \left[ -\pi \left( \frac{y}{\alpha \lambda_c} \right)^2 \right] \quad \text{is the weighting function in the y-direction.} \quad (8)$$

The filtered surface is given by Formula (9):

$$w(x, y) = \int s(x - \mu | \lambda_c) \left[ \int s(y - \nu | \lambda_c) z(\mu, \nu) d\nu \right] d\mu \quad (9)$$

where

$z(x, y)$  is the unfiltered surface;

$w(x, y)$  is the filtered surface.

i.e. the convolution is separable, too. Thus, the convolution can be calculated in a two-step process, using profile filters instead of areal filters, as given in Formula (10):

$$g(x, y) = \int s(y - \nu | \lambda_c) z(x, \nu) d\nu \quad (10)$$

and Formula (11)

$$w(x, y) = \int s(x - \mu | \lambda_c) g(\mu, y) d\mu \quad (11)$$

## 5 Characteristics of linear cylindrical Gaussian filters

### 5.1 General

Linear cylindrical Gaussian filters conforming to this part of ISO 16610 shall conform to [5.2](#) to [5.3](#).

### 5.2 Weighting function of linear cylindrical Gaussian filters

The linear cylindrical Gaussian weighting function is separable. It can be written as a product of two linear profile Gaussian weighting functions, which is given by Formula (12):

$$s(t, z | f_c, \lambda_{cz}) = s(t | f_c) s(z | \lambda_{cz}) \quad (12)$$

In the T direction (circumferential direction), use the linear closed profile Gaussian filter. The weighting function in the T direction (circumferential direction) has the equation of the Gaussian density function

wrapped around the cylindrical surface along the circumferential closed profile of length,  $L$ . With the cut-off frequency  $f_c = L/\lambda_c$ , use Formula (13):

$$s(t|f_c) = \begin{cases} \frac{f_c}{\alpha L} \exp \left[ -\pi \left( \frac{t f_c}{\alpha L} \right)^2 \right] & -\frac{L_{ct} L}{f_c} \leq t \leq \frac{L_{ct} L}{f_c} \\ 0 & \text{Otherwise} \end{cases} \quad (13)$$

where

$t$  is the distance from the centre (maximum) of the weighting function in T direction;

$f_c$  is the cut-off frequency in undulations per revolution;

$L$  is the length of the closed profile, for example, for a circle  $L = 2\pi R$ ;

$L_{ct}$  is the truncation index of Gaussian filter (see ISO 16610-21 for recommended values);

$\alpha$  is the constant, given by  $\alpha = \sqrt{\frac{\ln 2}{\pi}} \approx \frac{318}{677} \approx 0,4697 \approx \frac{31}{66}$ .

In Z direction (axial direction), the linear open profile Gaussian filter is used. The formula is given by Formula (14):

$$s(z|\lambda_{cz}) = \frac{1}{\alpha \lambda_{cz}} \exp \left[ -\pi \left( \frac{z}{\alpha \lambda_{cz}} \right)^2 \right] \quad (14)$$

where

$z$  is the distance from the centre (maximum) of the weighting function in Z direction;

$\lambda_{cz}$  is the cut-off wavelength in Z direction;

$L_{cz}$  is the truncation index of Gaussian filter (see ISO 16610-21 for recommended values).

### 5.3 Transmission characteristics of a linear cylindrical profile

#### 5.3.1 Transmission characteristic of the long wave component

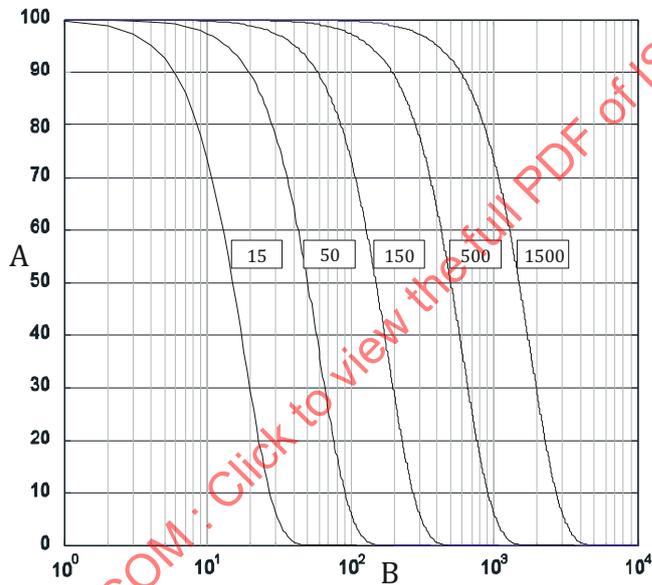
The transmission characteristic is determined from the weighting function by means of the Fourier transformation. The transmission characteristic is separable.

The filter characteristic for the mean line when  $\lambda_c \ll L$  in the circumferential direction of the long wave component (mean) can be approximated by Formula (15) (see [Figure 4](#)):

$$\frac{a_1}{a_0} = \exp \left[ -\pi \left( \frac{\alpha f}{f_c} \right)^2 \right] \tag{15}$$

where

- $a_0$  is the amplitude of a sine wave profile in the circumferential direction before profile filtering;
- $a_1$  is the amplitude of the long wave component of a sine wave profile in the circumferential direction;
- $f$  is the frequency of the sine wave profile in the circumferential direction in undulations per revolution.



**Key**

- A amplitude transmission  $\frac{a_1}{a_0}$  in %
- B undulations per revolution, UPR

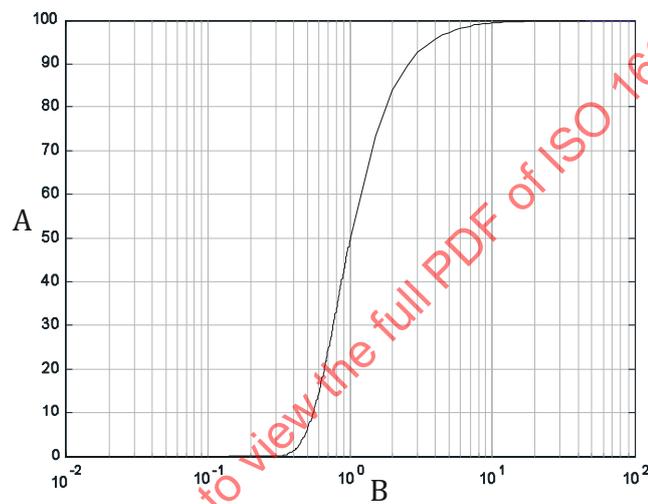
**Figure 4 — Long wave transmission function in T direction for a selected number of UPR**

The filter characteristic for the mean line in the axial direction of the long wave component (mean) can be described by Formula (16) (see [Figure 5](#)):

$$\frac{a_1}{a_0} = \exp \left[ -\pi \left( \frac{\alpha \lambda_{cz}}{\lambda} \right)^2 \right] \quad (16)$$

where

- $a_0$  is the amplitude of a sine wave profile in the axial direction before profile filtering;
- $a_1$  is the amplitude of the long wave component of a sine wave profile in the axial direction;
- $\lambda$  is the wavelength of the sine wave profile in the axial direction.



#### Key

A amplitude transmission  $\frac{a_1}{a_0}$  in %

B  $\frac{\lambda}{\lambda_{cz}}$

**Figure 5 — Long wave transmission function in Z direction**

#### 5.3.2 Transmission characteristic of the short wave component

The transmission characteristic of the short wave surface component is complementary to the transmission characteristic of the long wave surface component. The short wave surface component is the difference between the unfiltered surface and the long wave surface component.

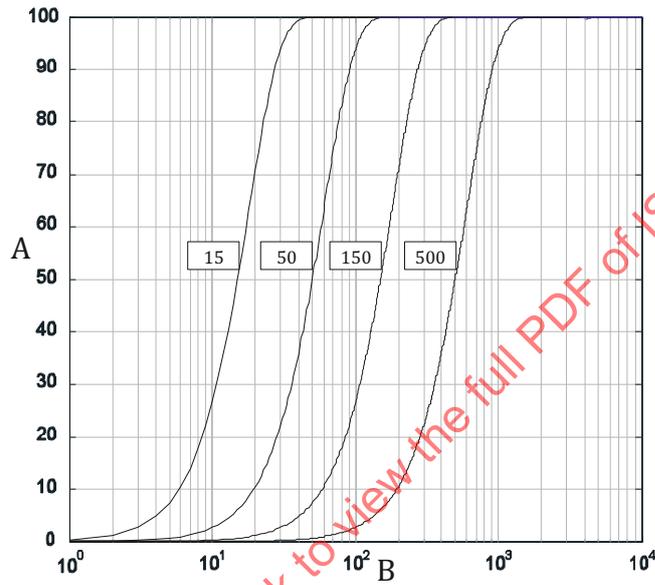
The transmission characteristic is determined from the weighting function by means of the Fourier transformation. The transmission characteristic is separable.

The filter characteristic for the mean line when  $\lambda_c \ll L$  in T direction (circumferential direction) of the short wave component (mean) can be approximated by Formula (17):

$$\frac{a_2}{a_0} = 1 - \exp \left[ -\pi \left( \frac{\alpha f}{f_c} \right)^2 \right] \tag{17}$$

where

$a_2$  is the amplitude of the short wave component of a sine wave profile in the circumferential direction.



**Key**

- A amplitude transmission  $\frac{a_2}{a_0}$  in %
- B undulations per revolution UPR

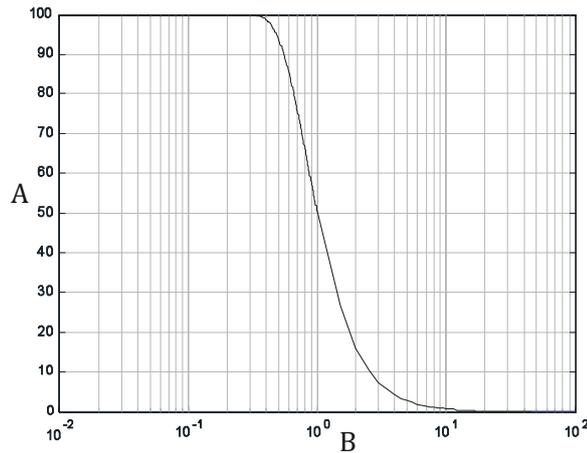
**Figure 6 — Short wave transmission function in T direction for a selected number of UPR**

The filter characteristic for the mean line in the Z direction (axial direction) of the long wave component (mean) is same as the open profile filter (see ISO 16610-21), as given in Formula (18):

$$\frac{a_2}{a_0} = 1 - \exp \left[ -\pi \left( \frac{\alpha \lambda_{cz}}{\lambda} \right)^2 \right] \tag{18}$$

where

$a_2$  is the amplitude of the short wave component of a sine wave profile in the axial direction.

**Key**

A amplitude transmission  $\frac{a_2}{a_0}$  in %

B  $\frac{\lambda}{\lambda_{cz}}$

**Figure 7 — Short wave transmission function in Z direction**

## 6 Other Information

### 6.1 General

Examples of linear areal Gaussian filters are given in [Annex A](#). The concept diagram is given in [Annex B](#). The relationship to the filtration matrix model is given in [Annex C](#). The relationship of this part of ISO 16610 to the GPS matrix is given in [Annex D](#).

### 6.2 Filter Designations

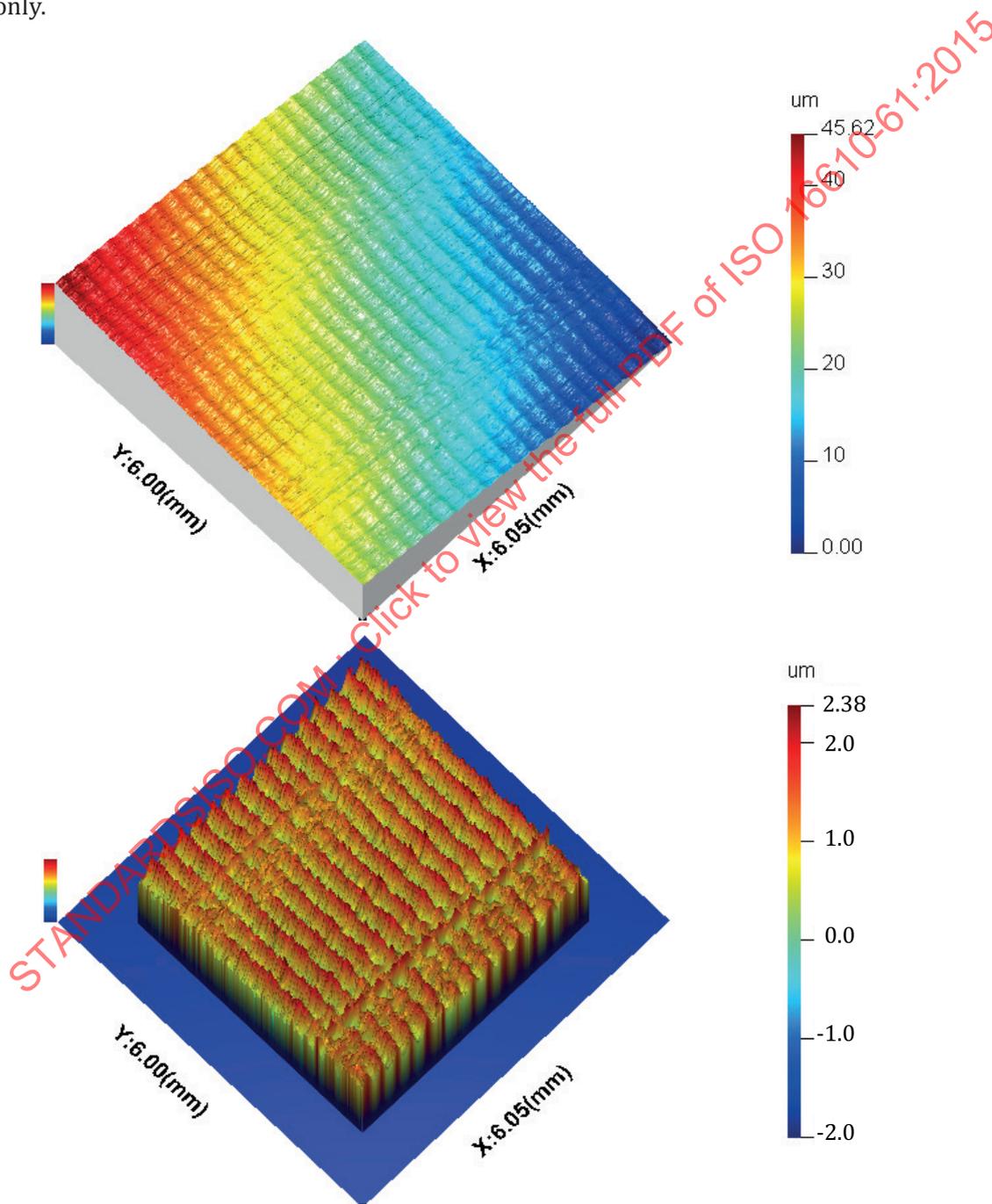
Linear areal filters according to this part of ISO 16610 are designated as follows:

	Filter designation
Linear planar filters	<b>FALGP</b>
Linear cylindrical filters	<b>FALGC</b>

## Annex A (informative)

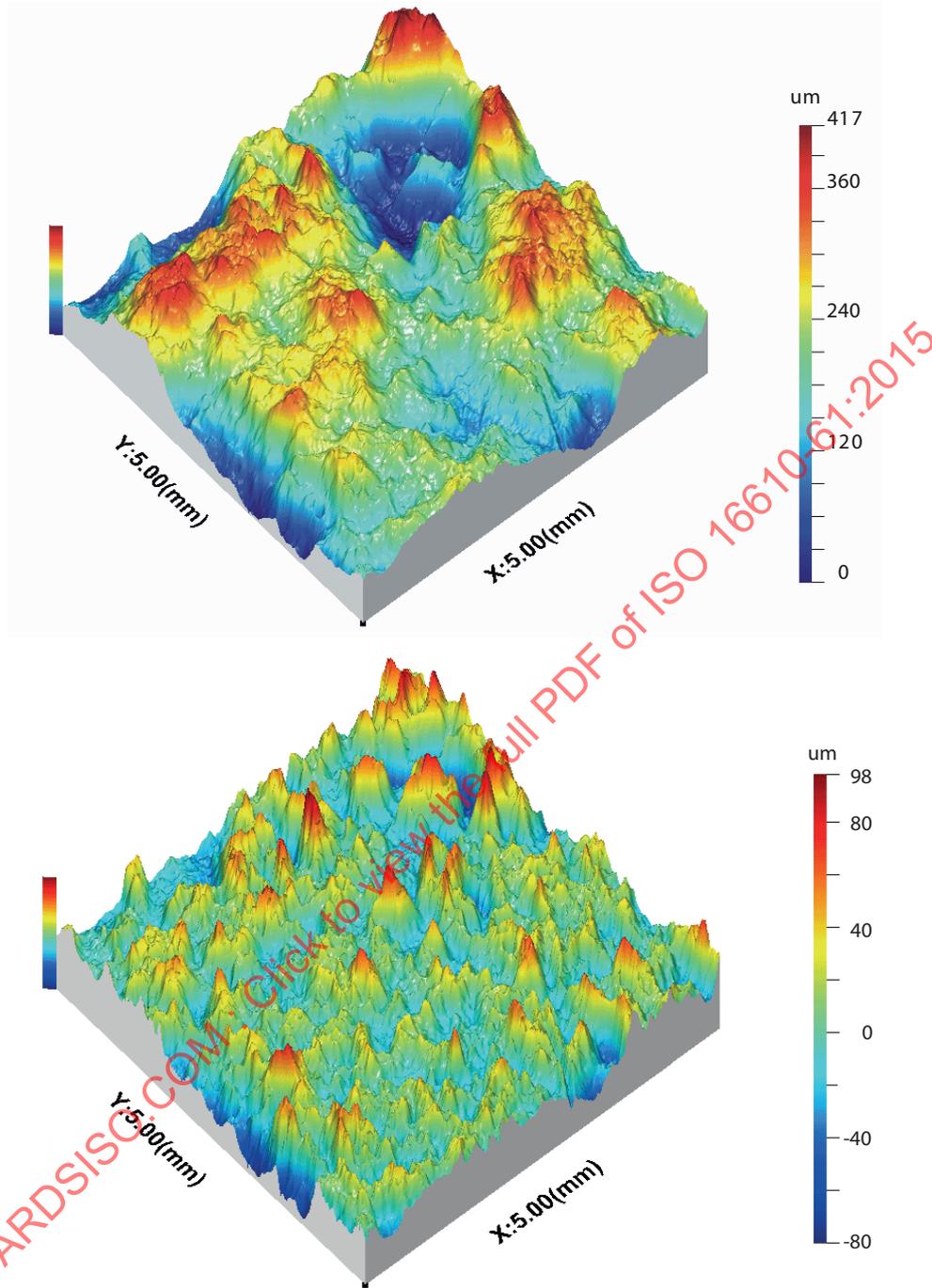
### Examples

Examples of the application of the areal Gaussian filter for a planar surface is given for information purposes only.



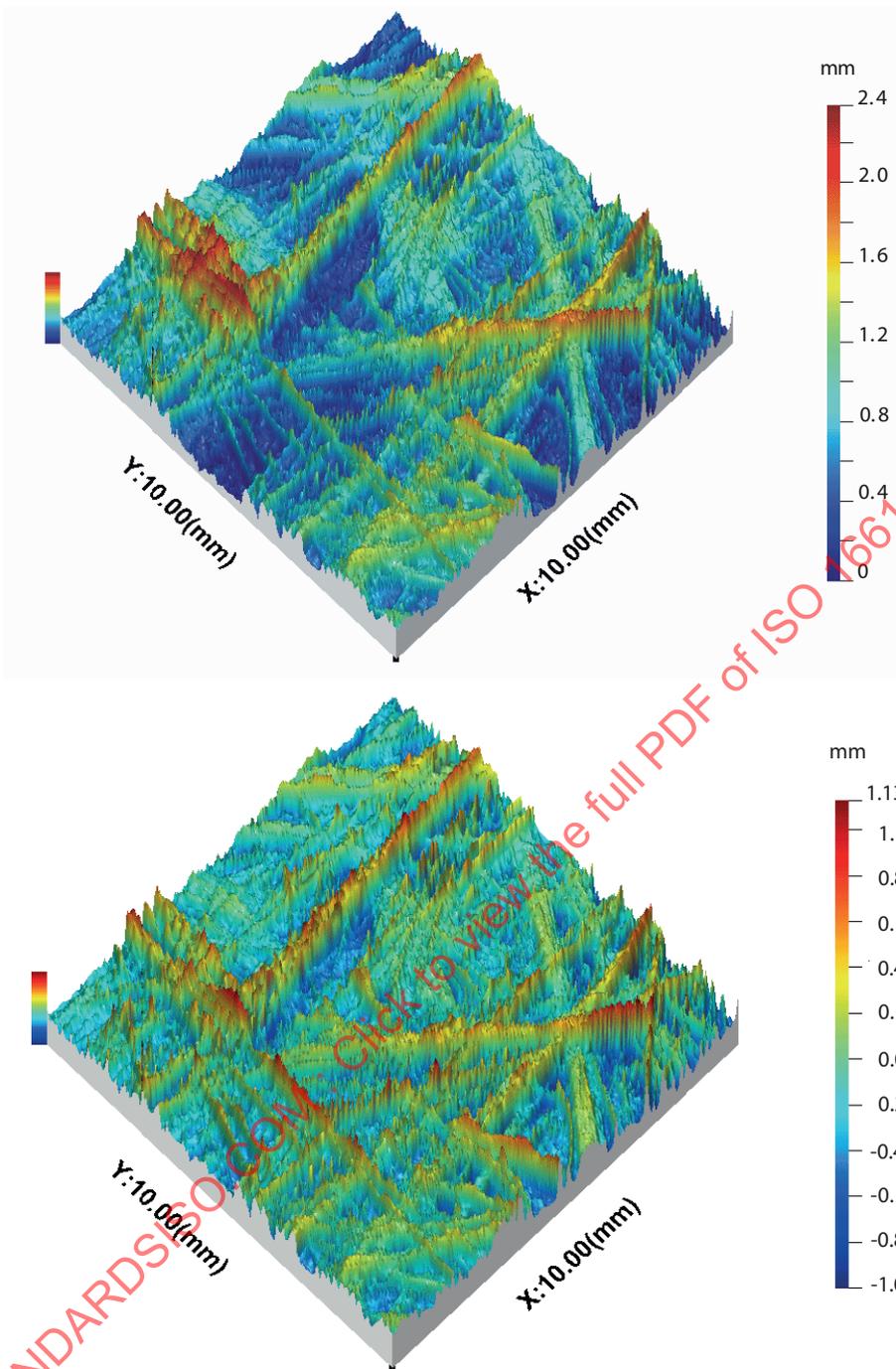
NOTE Top is the original surface, bottom is the filtered surface.

**Figure A.1 — Linear areal Gaussian filter with  $\lambda_c = 0,8$  mm for a milled surface**



NOTE. Top is the original surface, bottom is the filtered surface.

**Figure A.2 — Linear areal Gaussian filter with  $\lambda_c = 0,8$  mm for a stone surface**



NOTE Top is the original surface, bottom is the filtered surface.

Figure A.3 — Linear areal Gaussian filter with  $\lambda_c = 2,5$  mm for a carbon fibre surface