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Geometrical product specification (GPS) — Filtration —

Part 60:
Linear areal filters — Basic concepts

Spécification géométrique des produits (GPS) — Filtrage —

Partie 60: Filtres surfaciques linéaires — Concepts de base

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

— *Part 1: Overview and basic concepts*

— *Part 20: Linear profile filters: Basic concepts*

— *Part 21: Linear profile filters: Gaussian filters*

— *Part 22: Linear profile filters: Spline filters*

— *Part 28: Profile filters: End effects*

— *Part 29: Linear profile filters: Spline wavelets*

— *Part 30: Robust profile filters: Basic concepts*

— *Part 31: Robust profile filters: Gaussian regression filters*

— *Part 32: Robust profile filters: Spline filters*

— *Part 40: Morphological profile filters: Basic concepts*

— *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*

— *Part 49: Morphological profile filters: Scale space techniques*

— *Part 60: Linear areal filters: Basic concepts*

— *Part 61: Linear areal filters: Gaussian filters*

— *Part 71: Robust areal filters: Gaussian regression filters*

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— *Part 85: Morphological areal filters: Segmentation*

The following parts are planned:

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

— *Part 45: Morphological profile filters: Segmentation*

— *Part 62: Linear areal filters: Spline filters*

— *Part 69: Linear areal filters: Spline wavelets*

— *Part 70: Robust areal filters: Basic concepts*

— *Part 72: Robust areal filters: Spline filters*

— *Part 80: Morphological areal filters: Basic concepts*

— *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*

— *Part 89: Morphological areal filters: Scale space techniques*

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Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences chain links C and F of all chains of standards.

The ISO/GPS Matrix model given in ISO 14638 gives an overview of the ISO/GPS system of which this part of ISO 16610 is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this part of ISO 16610 and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this part of ISO 16610, unless otherwise indicated.

For more detailed information about the relation of this part of ISO 16610 to the GPS matrix model, see Annex C.

This part of ISO 16610 develops the basic concepts of linear areal filters, which include Gaussian filter, Spline filters, and Wavelet filters.

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Geometrical product specification (GPS) — Filtration —

Part 60:
Linear areal filters — Basic concepts

1 Scope

This part of ISO 16610 sets out the basic concepts of linear areal filters.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO 16610-21:2011, *Geometrical product specifications (GPS) — Filtration — Part 21: Linear profile filters: Gaussian filters*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1, ISO 16610-20, ISO 16610-21, ISO/IEC Guide 99, and the following apply.

3.1
linear areal filter
areal filter which separates surfaces into long wave and short wave components and is also a linear function

Note 1 to entry: If F is a function and X and Y are surfaces then F is a linear function implies F(aX + bY) = aF(X) + bF(Y).

Note 2 to entry: A linear areal filter is for surfaces in a specified coordinate system, for example, planar and cylindrical.

Note 3 to entry: Linear areal filter examples include Gaussian, spline, spline wavelet, and complex wavelet.

3.1.1
linear planar filter
linear areal filter (3.1) which separates planar surfaces into long wave and short wave components, which applies to nominal planar surfaces

Note 1 to entry: A planar surface is open in all directions.

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3.1.2
linear cylindrical filter
linear areal filter (3.1) which separates cylindrical surfaces into long wave and short wave components, which applies to nominal cylindrical surface

Note 1 to entry: A cylindrical surface is open in the axial direction and closed in the circumferential direction.

3.2
phase correct areal filter
linear areal filter (3.1) which does not cause phase shifts leading to asymmetrical surface distortions

Note 1 to entry: Phase correct filters are a particular kind of linear phase filters because any linear phase filter can be transformed (simply by shifting its weighting function) to a zero phase filter, which is a phase correct filter.

3.3
mean surface
long wave surface component determined from the surface by application of an areal filter

3.4
weighting function
function for calculating the mean surface, which indicates for each point the weight attached by the surface in the vicinity of that point

3.5
filter equation
equation for the mathematical description of the filter

Note 1 to entry: Filter equations do not necessarily specify an algorithm for the numerical realization of the filter.

[SOURCE: ISO 16610-1:2015, 3.10]

3.6
transmission characteristic of an areal filter
characteristic that indicates the amount by which the amplitude of a sinusoidal surface is attenuated as a function of its wavelengths

Note 1 to entry: The transmission characteristic is the Fourier transformation of the weighting function.

3.7
cut-off wavelength (nesting index)
wavelength of a sinusoidal surface of which 50 % of the amplitude is transmitted by the *linear areal filter* (3.1)

Note 1 to entry: Linear areal filters are identified by the filter type and the cut-off wavelength.

Note 2 to entry: The cut-off value for the linear areal filter is an example of a nesting index.

Note 3 to entry: The cut-off 50 % value is by convention.

3.8
filter bank
set of high-pass and low-pass filters, arranged in a specified structure

[SOURCE: ISO 16610-20:2015, 3.6]

3.9
multiresolution analysis
decomposition of a surface by a *filter bank* (3.8) into portions of different scales

Note 1 to entry: The portions at different scales are also referred to as resolutions.

[SOURCE: ISO 16610-20:2015, 3.7]

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4 Basic concepts

4.1 General

A filter claiming to comply with this part of ISO 16610 shall exhibit the characteristics described in 4.1, 4.2, 4.3, and 4.4.

NOTE A concept diagram for linear areal filters is given in Annex A. The relationship to the filtration matrix model is given in Annex B.

The most general linear areal filter is defined by Formula (1):

$$w(x,y)=\iint K(x,y;\mu,\upsilon)z(\mu,\upsilon)d\mu d\upsilon \tag{1}$$

where

$z\,(\mu,\upsilon)$	is the unfiltered surface;
$w\,(x,y)$	is the filtered surface;
$K\,(x,y{:}\,\mu,\upsilon)$	is the kernel of the filter, which is real, symmetric, and spatial invariant.

If $K(x,y;\mu,\upsilon)=K(x-\mu,y-\upsilon)$, the filtering is a convolution,

$$w(x,y)=\iint K(x-\mu,y-\upsilon)z(\mu,\upsilon)d\mu d\upsilon \tag{2}$$

and the kernel is also called the weighting function of the filter.

However, extracted data are always discrete. Therefore, the filters described here are also discrete. In cases that the weighting function is not discrete, the discrete nature of the extracted data shall be taken into account (see 4.3).

NOTE An alternative approach is to use a unique interpolation scheme on the discrete extracted data to create a continuous signal (with finite degrees of freedom) and use this as input to subsequent filtration operations.

4.2 Separable weighting functions

If the weighting function is separable, i.e. it can be written as a tensor product of profile filter weighting functions

$$K(x,y)=u(x)v(y) \tag{3}$$

the convolution is also a tensor product:

$$w(x,y)=\int u(x-\mu)\left[\int v(y-\upsilon)z(\mu,\upsilon)d\upsilon\right]d\mu \tag{4}$$

i.e. the convolution is separable, too. Thus, the convolution can be calculated in a two-step process, using profile filters instead of areal filters:

$$g(x,y)=\int v(y-\upsilon)z(x,\upsilon)d\upsilon \tag{5}$$

and

$$w(x,y)=\int u(x-\mu)g(\mu,y)d\mu \tag{6}$$

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4.3 Discrete representation of data

A sampled surface can be represented by an $n \times m$ matrix of heights, Z. The length n of the row vectors and the length m of the column vectors are equal to the number of data points in x and y directions respectively. The sampling is assumed to be uniform in x and y direction, i.e. the sampling interval Δ is constant in x and y direction. Thus the matrix element of the row i and the column j is given by $z_{ij} = z(x_i, y_j)$, with $x_i = i\Delta$ $(i = 1 \ldots n)$ and $y_j = j\Delta$ $(j = 1 \ldots m)$.

4.4 Discrete representation of the linear areal filter

A discrete linear areal filter is represented by an array *H*, which is the tensor product of two matrices *U* and *V*, provided the filter has a separable kernel, i.e.

$$H = U \otimes V \qquad h_{irjs} = u_{ir} v_{js} \tag{7}$$

is valid. The matrices *U* and *V* are both square matrices, with the dimension being equal to the number of data points to be filtered along the respective direction, i.e. if the input data are given by an $n \times m$ matrix, *U* is an $n \times n$ matrix and *V* is an $m \times m$ matrix.

Three types of areal filters are possible, depending on the periodicity of the filter in both directions. If the filter is periodic in both directions, the areal filter is called a periodic areal filter; if it is non-periodic in both directions, the surface filter is called a non-periodic areal filter; if it is periodic in only one direction, the areal filter is called a semi-periodic areal filter.

NOTE Non-periodic areal filters are used for open surfaces, for example, planar surfaces, periodic areal filters are used for closed surfaces, for nominal example, toroidal surfaces, and semi-periodic areal filters are used for semi-closed surfaces, for example, cylindrical surfaces.

If the filter is non-periodic along the respective direction, the corresponding matrix is a constant diagonal (Toeplitz) matrix

$$\begin{pmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & & & & \\ & c' & b' & a & b & c & & & \\ & & c' & b' & a & b & c & & \\ & & & c' & b' & a & b & c & \\ & & & & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \tag{8}$$

otherwise, if the filter is periodic along the respective direction, the corresponding matrix is a circulant matrix

$$\begin{pmatrix} a & b & c & \cdots & \cdots & c' & b' \\ b' & a & b & c & \cdots & \cdots & c' \\ c' & b' & a & b & c & \cdots & \cdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & \cdots & c' & b' & a & b & c \\ c & \cdots & \cdots & c' & b' & a & b \\ b & c & \cdots & \cdots & c' & b' & a \end{pmatrix} \tag{9}$$

If the filter is phase correct, the matrices representing the filter are both symmetrical, i.e. $b = b'$, $c = c'$,... (generally $a_{ij} = a_{ji}$) is valid.

The sum of the matrix elements a_{ij} of each row *i* is constant and for low-pass filters equals unity, i.e.

$$\sum_j a_{ij} = 1 \tag{10}$$

NOTE In case of a symmetrical matrix, the sum of the matrix elements a_{ij} of each column *j* is also constant and equals unity, i.e. $\sum_i a_{ij} = 1$ is also valid.

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4.5 Discrete representation of the weighting function

As each row of the matrices used in the tensor product representation of the filter is identical after being shifted accordingly, the matrix elements can be represented by only one row. Thus one has

$$u_{ir} = f_k \text{ with } k = i - r \tag{11}$$

and

$$v_{js} = g_l \text{ with } l = j - s \tag{12}$$

yielding the tensor product

$$h_{irjs} = u_{ir} v_{js} = f_k g_l = h_{kl} \text{ with } k = i - r \text{ and } l = j - s \tag{13}$$

The values h_{kl} form a matrix h of a dimension equal to the dimension of the input or output data matrix, respectively. This matrix is the discrete representation of the weighting function of the filter.

NOTE 1 Normally, the definition area of the weighting function is much smaller than the area of the data sets. Then *h* contains zeros outside the definition area of the weighting function.

EXAMPLE 1 The moving average areal filter is frequently used for easy smoothing of a data set (not necessarily an optimal method and used here as an illustration only). This is an example of a filter with a discrete weighting function. This weighting function (a length of 3 has been taken in each direction) is given by

$$\frac{1}{9}\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 1 & 1 & 1 & 0 & \cdots \\ \cdots & 0 & 1 & 1 & 1 & 0 & \cdots \\ \cdots & 0 & 1 & 1 & 1 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{14}$$

NOTE 2 The weighting function is often also called the impulse response function because it is the output data set of the filter if the input data set is only a single unity impulse.

$$\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 1 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & \cdots \\ & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{15}$$

If the weighting function is given as a continuous function, it should be sampled, in order to get a discrete data set. The sampling intervals used shall be equal to the sampling interval of the measured data. Subsequently, it is mandatory to re-normalize the sampled data of the weighting function in order to fulfil the condition that they shall sum to unity, thus avoiding bias effects (for details concerning bias effects, see Reference [3]).

EXAMPLE 2 The Gaussian filter according to ISO 16610-21 is an example of a continuous weighting function $s(x)$ defined by Formula (16):

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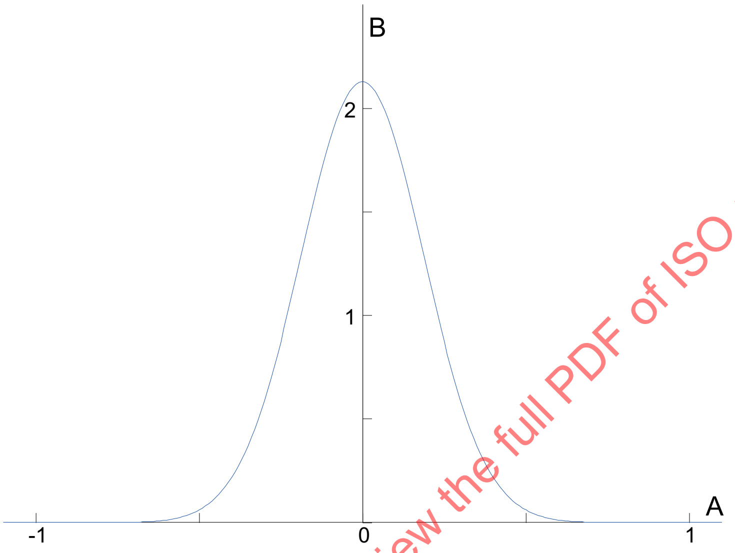
$$s(x) = \frac{1}{\alpha\lambda_c} \exp\left[-\pi \left(\frac{x}{\alpha\lambda_c} \right)^2 \right] \tag{16}$$

with x being the distance from the centre (maximum) of the weighting function, λ_c the cut-off wavelength, and a constant given by

$$\alpha = \sqrt{\frac{\ln 2}{\pi}} = 0,4697$$

The graph of this weighting function is shown in Figure 1.



Key

A $\frac{X}{\lambda_c}$

B $\lambda_c \cdot Z$

Figure 1 — Example of a continuous weighting function (Gaussian filter)

The sampled data s_k of the weighting function after a re-normalization are given by Formulae (17) and (18):

$$s_k = \frac{1}{C} \exp\left[-\pi \left(\frac{\Delta}{\alpha\lambda_c} \right)^2 k^2 \right] \tag{17}$$

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$$C=\sum_{k}\exp\left[-\pi\left(\frac{\Delta}{\alpha\lambda_{\mathrm{c}}}\right)^{2}k^{2}\right] \tag{18}$$

Using the discrete weighting function of the Gaussian profile filter, an areal Gaussian filter according to ISO 16610-61 can be constructed by the tensor product

$$h_{kl}=s_{k}s_{l} \tag{19}$$

yielding the weighting function

$$h_{kl}=\frac{1}{C^{2}}\exp\left[-\pi\left(\frac{\Delta}{\alpha\lambda_{\mathrm{c}}}\right)^{2}\left(k^{2}+l^{2}\right)\right] \tag{20}$$

of the discrete areal Gaussian filter.

This is a non-periodic filter and should not be applied to periodic and semi-periodic surfaces.

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5 Linear areal filters

5.1 Filter equations

If the filter is represented by the matrices *U* and *V*, the input data by the matrix *Z*, and the output data by the matrix *W*, then the filtering process is described by the linear operation

$$W=\left(U\otimes V\right)Z \tag{21}$$

This equation is called the filter equation. If $\left(U\otimes V\right)^{-1}$ is the inverse of the tensor product $\left(U\otimes V\right)$, then

$$Z=\left(U\otimes V\right)^{-1}W \tag{22}$$

is also a valid filter equation.

NOTE 1 The filter can be defined by the tensor product $\left(U\otimes V\right)$ or by the inverse $\left(U\otimes V\right)^{-1}$, whichever leads to a simpler definition. However, the weighting function is only given by the tensor product $\left(U\otimes V\right)$.

NOTE 2 The inverse of the tensor product might not always exist, in which case the filtering process is not invertible, i.e. data reconstruction is impossible. The invertibility of a filter can be seen from its transfer function (see 5.3). A filter which is not invertible has a transfer function $H(\omega_x,\omega_y)$, which is zero for at least one frequency pair (ω_x,ω_y).

EXAMPLE 3 The moving average filter (EXAMPLE 1) mentioned above is not invertible since the absolute value of the transfer function is zero for certain frequencies. If the filter is changed to the weighted moving average filter with the weighting function ($\alpha<1/2$),

$$\frac{1}{(1+2\alpha)^{2}}\begin{pmatrix}\ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots\\ \cdots & 0 & \alpha^{2} & \alpha & \alpha^{2} & 0 & \cdots\\ \cdots & 0 & \alpha & 1 & \alpha & 0 & \cdots\\ \cdots & 0 & \alpha^{2} & \alpha & \alpha^{2} & 0 & \cdots\\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots\\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix} \tag{23}$$

it becomes invertible.

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5.2 Discrete convolution

If the weighting function is separable and is a convolution filter, the filter equation can be written as

$$w_{ij} = \sum_r \sum_s h_{irjs} z_{rs} = \sum_r u_{i-r} \left(\sum_s v_{j-s} z_{rs} \right) \tag{24}$$

or

$$t_{rj} = \sum_s v_{j-s} z_{rs} \tag{25}$$

and

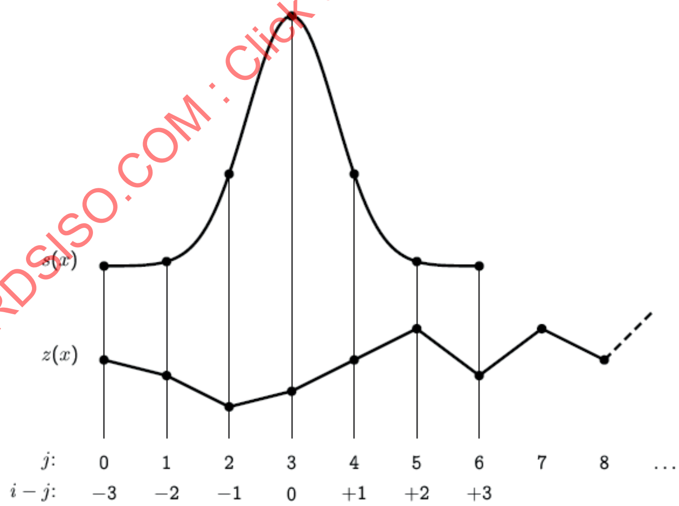
$$w_{ij} = \sum_r u_{i-r} t_{rj} \tag{26}$$

The letter expressions are known as discrete convolutions with the abbreviated notation $t = v * z$ and $w = u * t$, respectively. If the filter matrices U or V are circulant, the respective convolution is circular, i.e. the coefficients u_{i-r} v_{j-s}, respectively, shall be seen as being extended periodically at both ends (wrapped around).

NOTE The circular convolution can be calculated using the discrete Fourier transformation (DFT), which is often faster than the usual convolution. In the case of open profiles, take care of possible end effects.

EXAMPLE 4 An example of a discrete profile convolution is shown in Figure 2. Here, the filtered value w_i for $i = 3$ is calculated by multiplication of the data values at the points $j = 0 \ldots 6$ with the sampled values of the weighting function at the points $i - j$ and a subsequent summation.

$$w_i = \sum_{j=0}^{n} s_{i-j} z_j \tag{27}$$



NOTE Figure 2 is one direction only, similar in the other direction.

Figure 2 — Example of a discrete convolution

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5.3 Transfer function

Taking the discrete Fourier transformation of the discrete convolution yields

$$\Im(W) = \Im(H)\Im(Z) \tag{28}$$

with $\Im(Z)$ being the discrete Fourier transform of the input matrix Z, $\Im(W)$ being the discrete Fourier transform of the output matrix W, and $\Im(H)$ being the discrete Fourier transform of the discrete representation of the weighting function H. The function $\Im(H)$ is called the transfer function of the filter. It depends on the wavelengths $\left(\lambda_x, \lambda_y\right)$ or the angular frequencies $\omega_x = 2\pi/\lambda_x$ and $\omega_y = 2\pi/\lambda_y$, respectively, as the Fourier transformation transforms to the wavelength or frequency domain.

NOTE 1 The *discrete Fourier transform* (DFT) is a function of discrete frequencies. Here, continuous frequencies are used; consequently, the corresponding transform is mathematically the *discrete time Fourier transform* (DTFT). For simplicity, and to avoid confusion between the terms *time* and *wavelength*, the term *discrete Fourier transformation* (DFT) will be used instead of the correct term *discrete time Fourier transform* (DTFT) throughout this part of ISO 16610. For more information concerning the discrete time Fourier transform (DTFT) and the difference between *time*- and *wavelength*-based Fourier transforms, see Reference [2].

The Fourier transformation $\Im(H)\left(\omega_x, \omega_y\right)$ of the discrete representation of the weighting function given by the matrix H with components h_{kl} is calculated by

$$\Im(H)(\omega_x, \omega_y) = \sum\sum h_{kl} e^{-i(\omega_x k + \omega_y l)} \tag{29}$$

If the weighting function is separable, i.e. $h_{kl} = u_k v_l$ is valid, this can be simplified to

$$\Im(H)(\omega_x, \omega_y) = \left(\sum_k u_k e^{-i\omega_x k}\right)\left(\sum_l v_l e^{-i\omega_x l}\right) \tag{30}$$

Generally, the transfer function turns out to be complex valued. However, if the weighting function is symmetrical, i.e. $u_{-k} = u_k$ (for all k) and $v_{-l} = v_l$ (for all l) is valid, the formula is simplified to

$$\Im(H)(\omega_x, \omega_y) = \left(u_0 + 2\sum_{k>0} u_k \cos\omega_k k\right)\left(u_0 + 2\sum_{l>0} v_l \cos\omega_l l\right) \tag{31}$$

which is a real transfer function.

For a phase correct filter, the transfer function is always a real function, i.e. the imaginary part is zero. This is due to the fact that the imaginary part represents a phase shift, which is always zero for phase correct filters.

EXAMPLE 5 The transfer function for the moving average areal filter mentioned above is

$$\Im(H)(\omega_x, \omega_y) = \frac{(1 + 2\cos\omega_x)(1 + 2\cos\omega_y)}{9} \tag{32}$$

The graph of this transfer function is shown in Figure 3. This filter is not invertible because $\Im(H)(\omega_x, \omega_y) = 0$. Further, this average filter has quite bad suppression of high frequencies due to the side lobes between $|\omega| > 2\pi/3$ and $|\omega| < \pi$.

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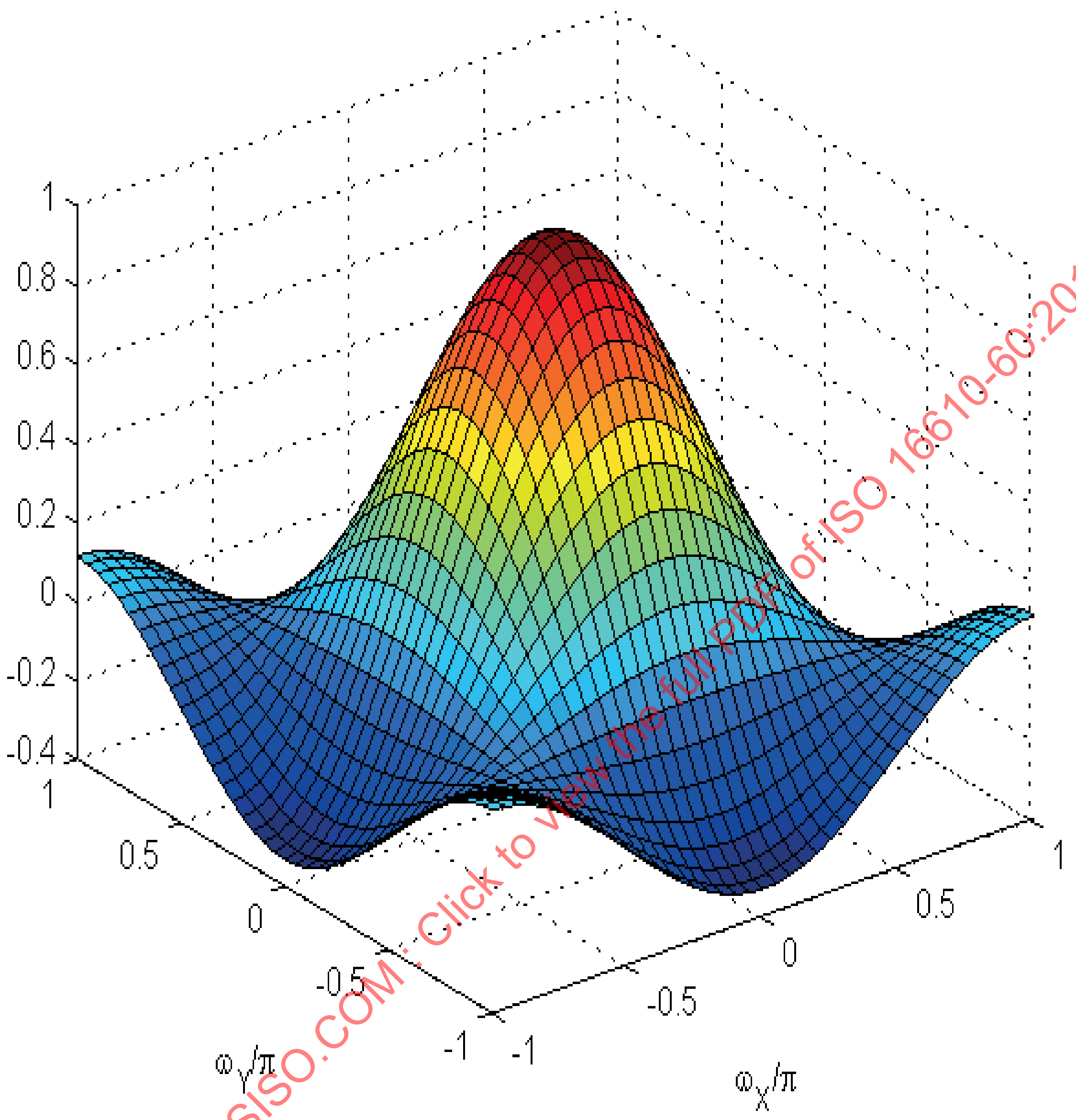

Figure 3 — Transfer function of the moving average filter of length 3 × 3

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The moving areal average filter shown in Figure 3 is a low-pass filter because $\Im(H)(\omega_x,\omega_y)$ has highest values around the frequencies $\omega_x = 0$ and $\omega_y = 0$. In contrary, for a high-pass filter $\Im(H)(\omega_x,\omega_y)$ would have highest values in the high-frequency region near $\omega_x = \pm\pi$ and $\omega_y = \pm\pi$. If a low-pass filter transfer function $\Im(H_0)(\omega_x,\omega_y)$ is given, the simplest way to get a high-pass filter transfer function $\Im(H_1)(\omega_x,\omega_y)$ is to calculate $\Im(H_1)(\omega_x,\omega_y)=[1-\Im(H_0)(\omega_x,0)][1-\Im(H_0)(0,\omega_y)]$.

However, this is not always the best possible choice.

EXAMPLE 6 The above-mentioned modified, stable areal moving average has the (low pass) transfer function

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$$\Im(H_0)(\omega_x,\omega_y)=\frac{(1+2\alpha\cos\omega_x)(1+2\alpha\cos\omega_y)}{(1+2\alpha)^2} \tag{33}$$

The high-pass filter has then the transfer function

$$\Im(H_1)(\omega_x,\omega_y)=\left(\frac{2\alpha}{1+2\alpha}\right)^2(1-\cos\omega_x)(1-\cos\omega_y) \tag{34}$$

The weighting function of the low-pass filter is

$$\frac{1}{(1+2\alpha)^2}\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & \alpha^2 & \alpha & \alpha^2 & 0 & \cdots \\ \cdots & 0 & \alpha & 1 & \alpha & 0 & \cdots \\ \cdots & 0 & \alpha^2 & \alpha & \alpha^2 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{35}$$

The weighting function of the high-pass filter can easily be shown to be

$$\frac{\alpha^2}{(1+2\alpha)^2}\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 1 & -2 & 1 & 0 & \cdots \\ \cdots & 0 & -2 & 4 & -2 & 0 & \cdots \\ \cdots & 0 & 1 & -2 & 1 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{36}$$

This filter is called an areal (weighted) moving difference filter.

5.4 Separable filter banks

A filter bank is a set of filters. In a profile two-channel filter bank, the two filters are normally a high-pass and a low-pass filter. In a separable filter bank, two profile filter banks are used: For example in a two profile two-channel filter bank, a profile two-channel filter bank is first implemented in the x direction, and subsequently, a profile two-channel filter bank is implemented in the y direction, leading to four outputs (see Figure 4). One of these outputs is the smoothed version of the original surface; the other three contain high-frequency details of the original surface.

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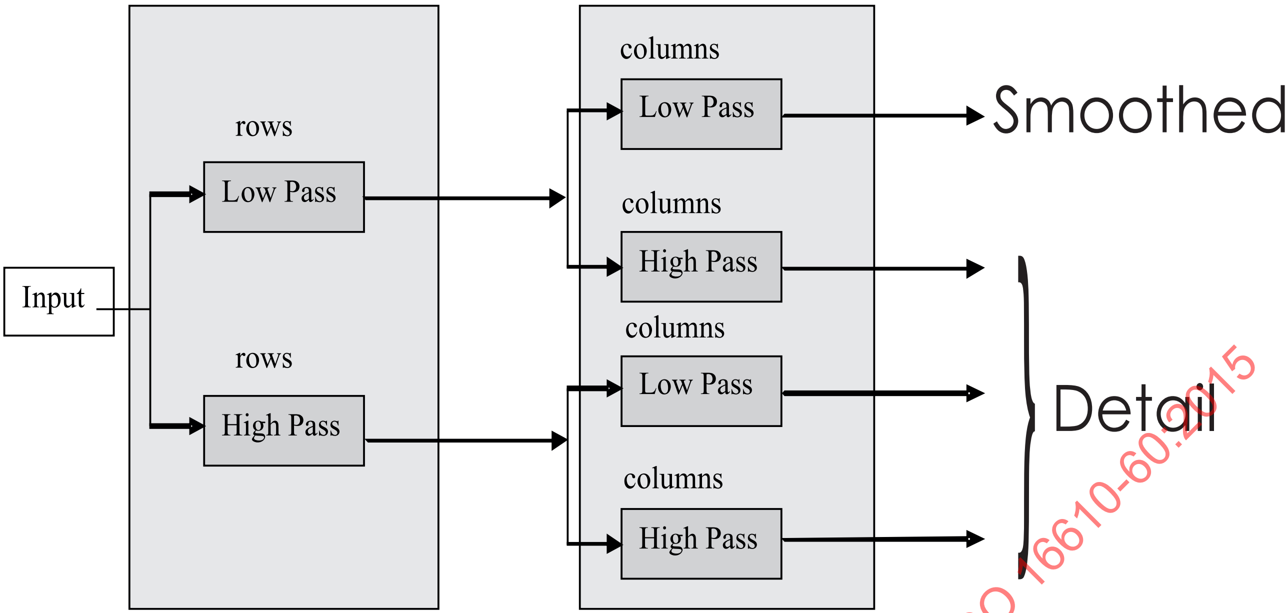

Figure 4 — Example of a separable filter bank

NOTE Cascading of filter banks leads to multiresolution analysis. Each filtering stage gives smoother details of the surface data. They appear at multiple scales. However, filter banks have to be specifically designed to achieve multiresolution.

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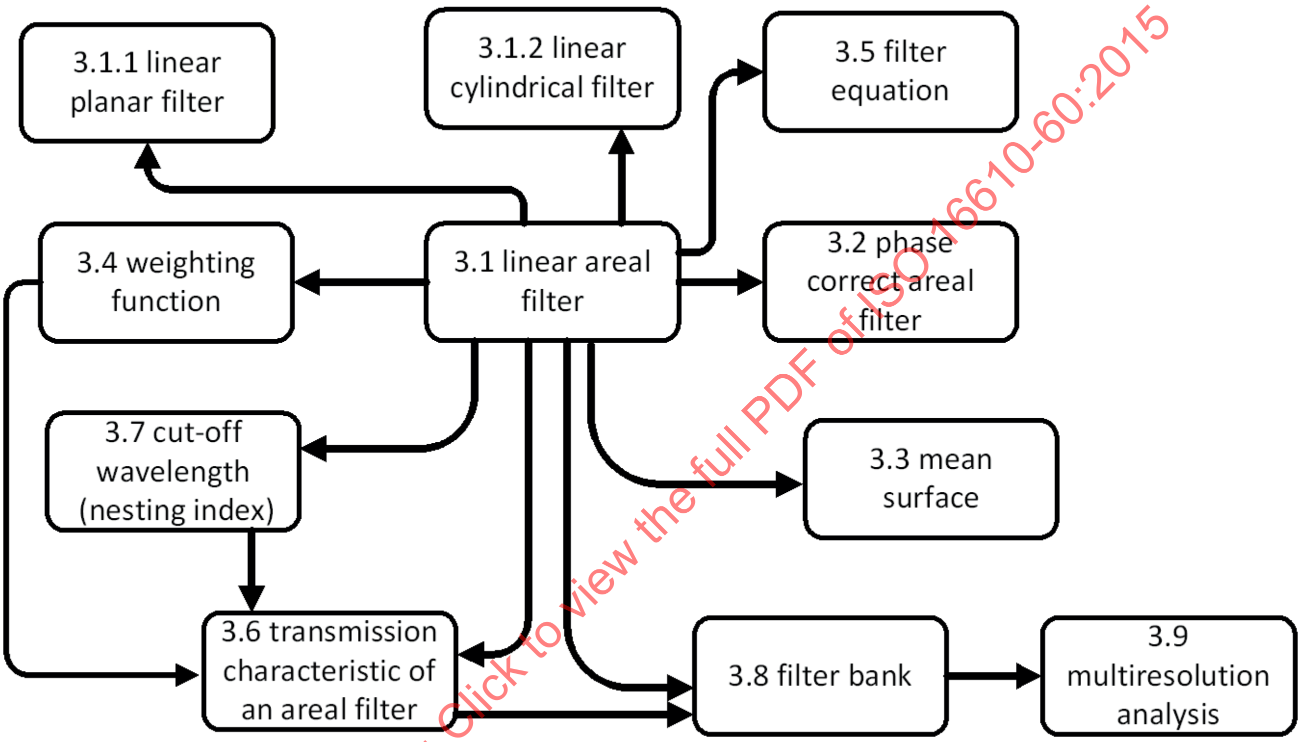
Annex A
(informative)

Concept diagram

The following is a concept diagram for this part of ISO 16610.



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