International Standard

ISO 16610-45

First edition
2025-02

Geometrical product specifications (GPS) — Filtration —

Part 45: Morphological profile filters: Segmentation

Spécification géométrique des produits (GPS) — Filtrage —

Partie 45: Filtres de profil morphologiques : Segmentation

Reference number
ISO 16610-45:2025(en)

ISO copyright office
CP 401 • Ch. de Blandonnet 8
CH-1214 Vernier, Geneva
Phone: +41 22 749 01 11
Email: copyright@iso.org
Website: www.iso.org

Published in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

ISO draws attention to the possibility that the implementation of this document may involve the use of (a) patent(s). ISO takes no position concerning the evidence, validity or applicability of any claimed patent rights in respect thereof. As of the date of publication of this document, ISO had not received notice of (a) patent(s) which may be required to implement this document. However, implementers are cautioned that this may not represent the latest information, which may be obtained from the patent database available at www.iso.org/patents. ISO shall not be held responsible for identifying any or all such patent rights.

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 290, *Dimensional and geometrical product specification and verification,* in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

A list of all parts in the ISO 16610 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences chain links C and E in the GPS matrix structure.

The ISO GPS matrix model given in ISO 14638 gives an overview of the ISO GPS system of which this document is a part. The fundamental rules of ISO GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

For more information on the relationship of this document to filtration matrix model, see Annex C.

For more detailed information on the relationship of this document to other standards and the GPS matrix model, see Annex E.

This document develops the terminology and concepts for profile segmentation.

This document will replace part of ISO 21920-2 as the source document for profile segmentation. ISO 21920-2 is under revision and its definitions will be aligned with this document.

Geometrical product specifications (GPS) — Filtration —

Part 45:
Morphological profile filters: Segmentation

1 Scope

This document defines the terminology and develops concepts for profile morphological segmentation. In particular it specifies the watershed segmentation method, the Wolf pruning method and the crossing-the-line method. This document assumes a continuous surface.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1 Terms related to geometrical features

3.1.1
topographic feature
line or point feature on a profile

3.1.1.1
line feature
hill or dale

3.1.1.2
point feature
peak or pit

3.1.2
peak
<watershed segmentation> point on the profile which is higher than all other points within the neighbourhood of that point

Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the peak is the middle single point on the plateau. Alternatively, techniques given in Annex B can be used.

3.1.3
peak
<reference line> highest point of a *hill* (3.1.7)

Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the peak is the middle single point on the plateau. Alternatively, techniques given in Annex B can be used.

3.1.4
pit
<watershed segmentation> point on the profile which is lower than all other points within the neighbourhood of that point

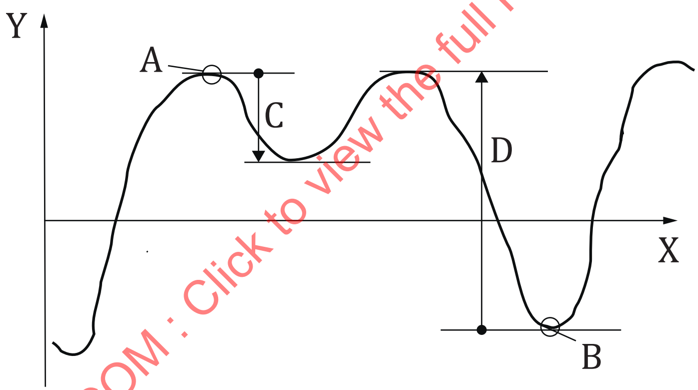
Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the pit is the middle single point on the plateau. Alternatively, techniques given in Annex B can be used.

3.1.5
pit
<reference line> lowest point of a *dale* (3.1.9)

Note 1 to entry: There is a theoretical possibility of a plateau. In this case, the pit is the middle single point on the plateau. Alternatively, techniques given in Annex B can be used.

3.1.6
hill
<watershed segmentation> region around a peak such that all maximal upward paths end at the peak

Note 1 to entry: See Figure 1.



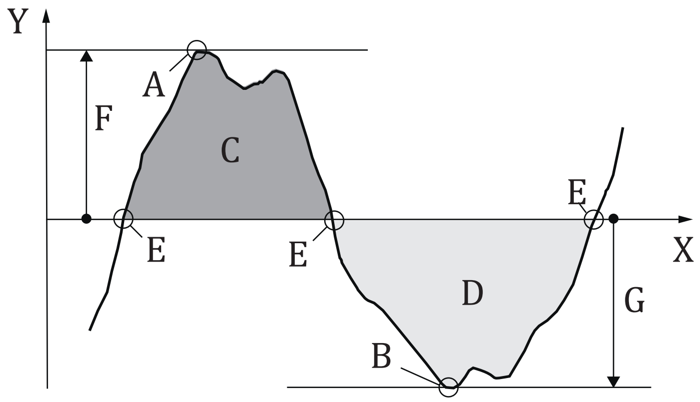
Key

X X-axis (reference line)
Y height
A peak
B pit
C hill local height
D dale local depth

Figure 1 — Hill local height and dale local depth (watershed segmentation)

3.1.7
hill
<reference line> outwardly directed (from material to surrounding medium) contiguous portion of the profile above the reference line bounded by the two adjacent points where the ordinate values change their sign

Note 1 to entry: See Figure 2.



Key

X X-axis (reference line)
Y height
A peak
B pit
C hill
D dale
E change of sign of the ordinate values
F peak height
G pit depth

Figure 2 — Peak height and pit depth (reference line)

3.1.8
dale
<watershed segmentation> region around a pit such that all maximal downward paths end at the pit

Note 1 to entry: See Figure 1.

3.1.9
dale
<reference line> inwardly directed (from surrounding medium to material) contiguous portion of the profile below the reference line bounded by the two adjacent points where the ordinate values change their sign

Note 1 to entry: See Figure 2.

3.2 Terms related to segmentation

3.2.1
segmentation
<profile> method which partitions a profile into distinct topographic features

Note 1 to entry: In this document, there are two types of segmentation:

a) watershed segmentation (see 4.3);

b) crossing-the-line segmentation (see 4.4).

3.2.1.1
watershed segmentation
filtration operation that spatially decomposes a profile into mutually exclusive portions of that profile

3.2.1.2
crossing-the-line segmentation
operation based on crossings of the reference line by a profile in conjunction with a combination algorithm, to leave a set of significant segments

Note 1 to entry: The crossing-the-line segmentation requires *height discrimination* (see 3.3.6.2).

3.2.1.3
event
<profile> mutually exclusive profile portions whose union covers the profile

EXAMPLE Ordinate values, motifs.

3.2.2
segmentation function
<profile> function which splits a set of events into two distinct sets, called the “significant events” and the “insignificant events”, and which satisfies the three segmentation properties

Note 1 to entry: A full mathematical description of the segmentation function and the three segmentation properties can be found in Reference [8].

3.2.3
first segmentation property
<profile> property where each event is allocated to the set of significant events or the set of insignificant events but not both

Note 1 to entry: This property can also be given as a mathematical equation as follows:

P1 $$\forall A \subseteq E,\ \Psi(A) \cup \Phi(A) = A \text{ and } \Psi(A) \cap \Phi(A) = \varnothing$$

where

- P1 is a first segmentation property;
- E is the set of all events;
- A is a subset of E;
- $\Psi(.)$ maps events onto the set of significant events;
- $\Phi(.)$ maps events onto the set of insignificant events (see Figure 3).

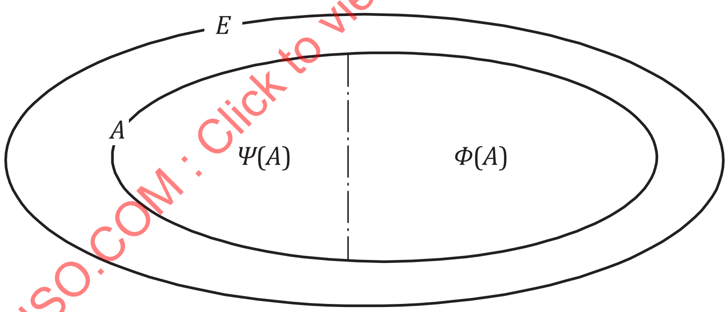


Figure 3 — Venn diagram of first segmentation property

Note 2 to entry: (.), and throughout this document, the dot is a placeholder for any set of events.

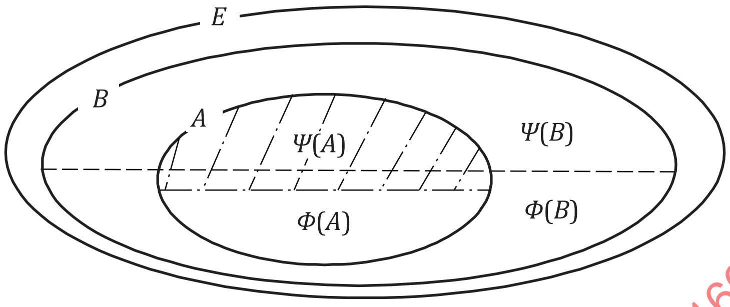
3.2.4
second segmentation property
<profile> property where a significant event is removed from the set of events, then the remaining significant events are contained in the new set of significant events

Note 1 to entry: This property can also be given as a mathematical equation as follows:

P2 $$\forall A \subseteq B \subseteq E, \quad \Phi(A) \subseteq \Phi(B)$$

where

P2	is a second segmentation property;
E	is the set of all events;
A and B	are subsets of E;
$\Phi(.)$	maps events onto the set of insignificant events (see Figure 4).



NOTE The shaded areas are the significant events for subset A.

Figure 4 — Venn diagram of second segmentation property

3.2.5
third segmentation property

<profile> property where an insignificant event is removed from the set of events, then the same set of significant events is obtained

Note 1 to entry: This property can also be given as a mathematical equation as follows:

P3 $$\forall A \subseteq B \subseteq E, \quad \Psi(B) \subseteq A \Rightarrow \Psi(A) = \Psi(B)$$

where

P3	is a third segmentation property;
E	is the set of all events;
A and B	are subsets of E;
$\Psi(.)$	maps events onto the set of significant events (see Figure 5).

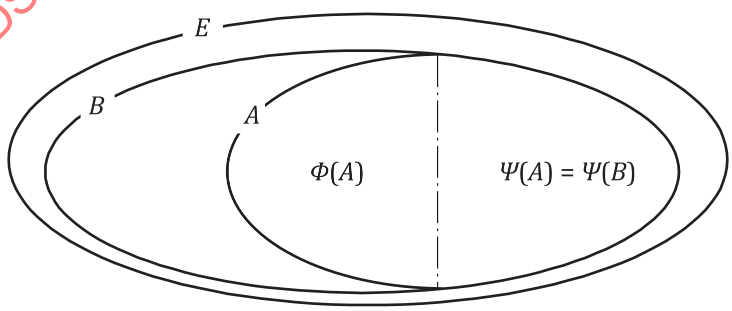


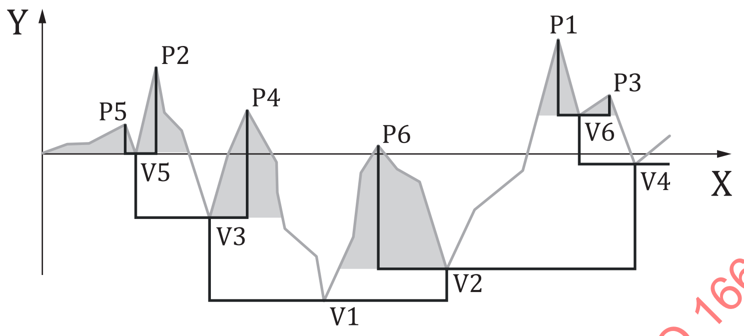
Figure 5 — Venn diagram of third segmentation property

3.3 Terms related to pruning

3.3.1
hill change tree
graph which describes the relationships between peaks and pits sorted by their *hill local heights* (3.3.5.1)

Note 1 to entry: Peaks are represented on a hill change tree by the ends of lines. Pits are represented on a hill change tree by joining lines.

Note 2 to entry: See Figure 6.



Key
X X-axis (reference line)
Y height
P peak
V pit

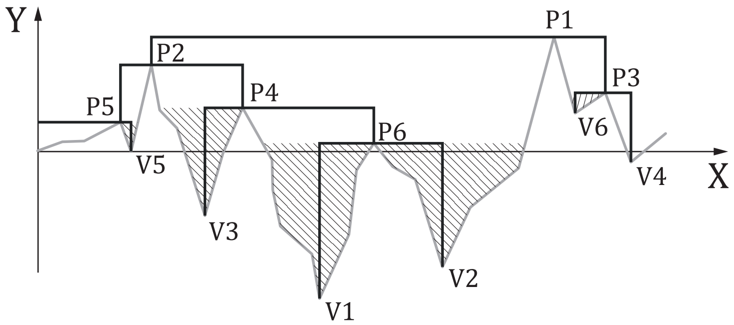
NOTE The filled areas indicate hills for watershed segmentation.

Figure 6 — Hill change tree

3.3.2
dale change tree
graph which describes the relationships between pits and peaks sorted by their *dale local depths* (3.3.5.4)

Note 1 to entry: Pits are represented on a dale change tree by the ends of lines. Peaks are represented on a dale change tree by joining lines.

Note 2 to entry: See Figure 7.



Key

X X-axis (reference line)

Y height

P peak

V pit

NOTE The filled areas indicate dales for watershed segmentation.

Figure 7 — Dale change tree

3.3.3
change tree
hill change tree or dale change tree

3.3.4
profile pruning
method to simplify a change tree in which lines from peaks (or pits) to their connected pits (or peaks) are removed

3.3.5
height
signed normal distance from the reference profile to the profile

Note 1 to entry: The distance is defined normal to the reference profile.

Note 2 to entry: The distance is negative if, from the reference profile, the profile lies in the direction of the material.

Note 3 to entry: Throughout this document, the term "height" is used either for a distance or for an absolute coordinate.

3.3.5.1
hill local height
height difference between a peak and the highest pit connected to that peak

Note 1 to entry: See Figure 1.

3.3.5.2
peak height
height difference between a peak and the reference line

Note 1 to entry: See Figure 2.

3.3.5.3
depth
height multiplied by minus one

3.3.5.4
dale local depth
height difference between a pit and the lowest peak connected to that pit

Note 1 to entry: See Figure 1.

3.3.5.5
pit depth
depth difference between a pit and the reference line

Note 1 to entry: See Figure 2.

3.3.6
height discrimination
profile height used as a threshold during segmentation

Note 1 to entry: Height discrimination is a nesting index.

3.3.6.1
height discrimination
<watershed segmentation> minimum hill local height or dale local depth of the profile

3.3.6.2
height discrimination
<reference line> minimum peak height or pit depth of the profile

3.3.6.3
peak height discrimination
minimum peak height used as a threshold during segmentation

3.3.6.4
pit depth discrimination
minimum pit depth used as a threshold during segmentation

3.3.6.5
Wolf pruning height
W
height discrimination used during watershed segmentation using *Wolf pruning* (3.3.8)

3.3.6.6
profile peak height discrimination
H_u
height discrimination used during the profile crossing-the-line segmentation method

3.3.7
depth discrimination
profile depth used as a threshold during segmentation

Note 1 to entry: Depth discrimination is a nesting index.

3.3.7.1
profile pit depth discrimination
H_l
depth discrimination used during the profile crossing-the-line segmentation method

3.3.8
Wolf pruning
pruning where lines are removed in order from a peak (or pit) with the smallest hill local height (or dale local depth) up to the peak (or pit) with a specified hill local height (or dale local depth)

Note 1 to entry: The peak local heights and pit local depths will change during Wolf pruning as removing lines from a change tree will also remove the associated pits and peaks.

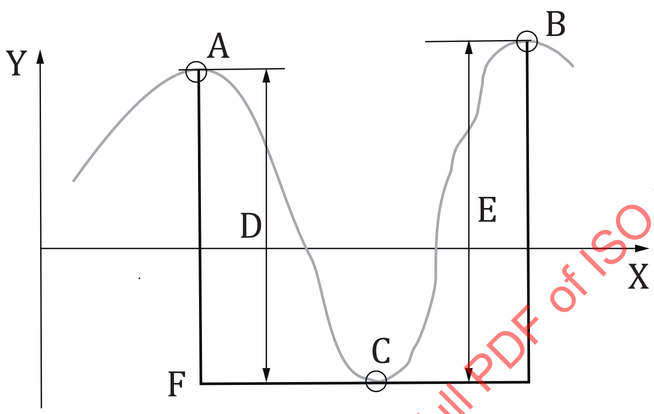
3.4 Terms related to the motif method

3.4.1
motif
portion of the primary profile between the highest points of two local peaks of the profile, which are not necessarily adjacent

3.4.2
S characteristic
height between the smallest of the end peaks to pit within a motif

Note 1 to entry: "Valley" instead of "pit" has been used in ISO 12085:1996.

Note 2 to entry: See an example of a motif in Figure 8.



Key

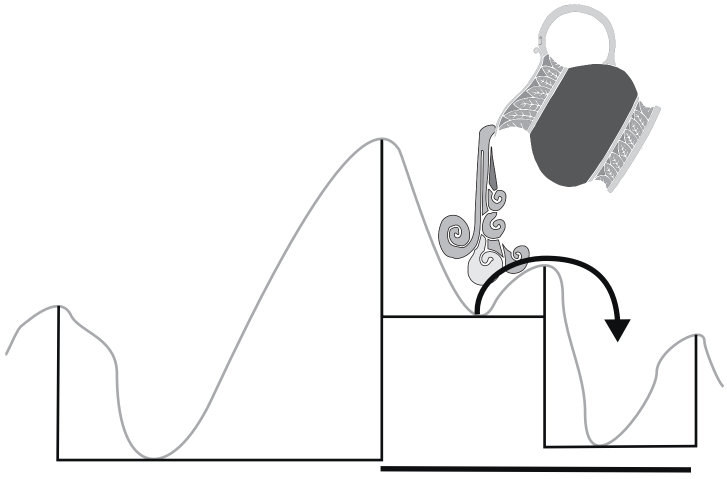
X X-axis (reference line)
Y height
A left peak of motif
B right peak of motif
C pit of motif
D height difference between A and C
E height difference between B and C
F motif defined by peaks A and B and pit C

NOTE Since D < E, the S characteristic's value is D.

Figure 8 — Example of a motif and S characteristic

3.4.3
profile watershed criteria
criteria which uses the concept of filling a motif with water and the adjacent motif, which the water first overflows into, being chosen for possible combination

Note 1 to entry: See Figure 9.



Key

1 combination

NOTE The middle motif overflows to the right and so the right motif is chosen for possible combination.

Figure 9 — Profile watershed criteria

4 Segmentation details

4.1 General

A profile watershed segmentation using Wolf pruning claiming to conform to this document shall conform to 4.3.

A profile crossing-the-line segmentation method claiming to conform to this document shall conform to 4.4.

4.2 Basic segmentation

Originally based upon a graphical segmentation of the profile into motifs that are then quantified in height and width. First approaches[7] were developed by a consortium involving PSA Peugeot Citroen and Renault, during the 1980s and 1990s.

In this document segmentation is a filtration operation (as defined in ISO 16610-1) that spatially decomposes a profile into mutually exclusive portions of that profile (see Figure 3).

Segmentation filtration requires:

— the objects being filtered, i.e. the segments;

— the rule for segment combination through profile pruning (e.g. the watershed rule);

— the rule which states which segments are significant (e.g. Wolf pruning).

From these mutually exclusive portions, the desired topological features are determined.

Associated with each segmentation method is a nesting index such that large values of the nesting index correspond to large surface portions and smaller values of the nesting index correspond to smaller surface portions.

4.3 Watershed segmentation using Wolf pruning

4.3.1 General

Watershed segmentation using Wolf pruning is based on motifs with a profile. The profile is initially segmented based on motifs between adjacent peaks and a procedure is then applied to prune out the least-significant motifs through a combination algorithm, defined in 4.3.2, to leave a set of significant motifs, which is the resulting watershed segmentation using Wolf pruning.[8][9]

The nesting index for watershed segmentation using Wolf pruning is defined as a Wolf height, with a Wolf pruning height, *W*, being set as a predefined height.

Watershed segmentation using Wolf pruning is a stable alternative to the motif combination algorithm, as contained in ISO 12085, and the subsequent calculation of the R and W parameters.

4.3.2 Calculation method for watershed segmentation using Wolf pruning

For watershed segmentation using Wolf pruning that uses height discrimination, *W*:

a) identify all motifs between internal adjacent peaks.

NOTE The ends of a profile do not count as peaks, resulting in the loss of a portion of the profile at both ends (see Figure 10).

b) Calculate the S characteristic for all the motifs.

c) Find the motif with the smallest S characteristic.

d) If the smallest S characteristic is lower than the Wolf discrimination height, *W*, combine it with an adjacent neighbouring motif using the profile watershed criteria so they become one motif and recalculate the S characteristic for the new combined motif.

e) If there are two or more motifs with the smallest S characteristic, then it does not matter in what order they are processed.

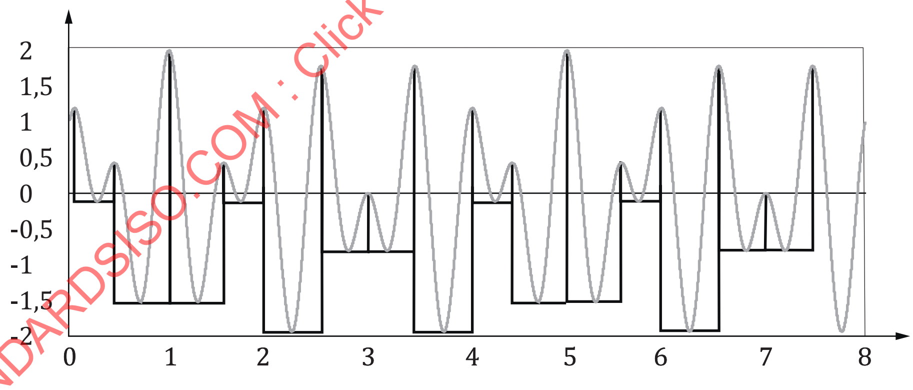
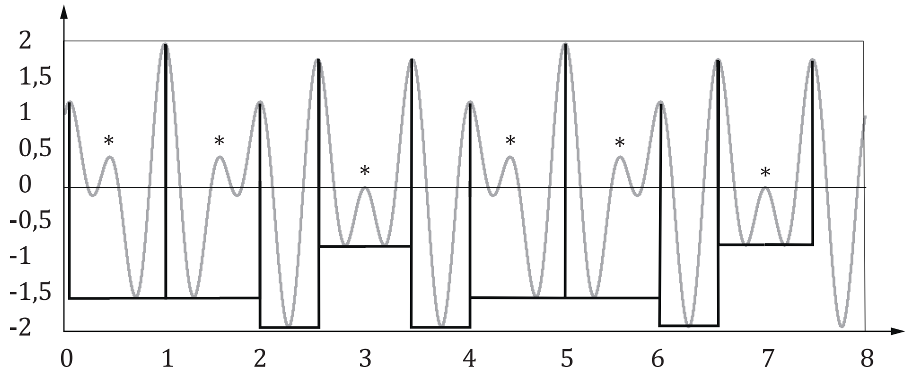


Figure 10 — Initial set of motifs with loss of profile at both ends

Continue until all motifs have S characteristics that are above the Wolf discrimination height, *W*. The remaining motifs are the required motifs for the watershed segmentation using Wolf pruning (see Figure 11).



Key

* eliminated peak

Figure 11 — Combined motifs

4.4 Crossing-the-line segmentation

4.4.1 General

Crossing-the-line segmentation is based on crossings of the reference line by a profile. The profile is initially segmented based on crossings of the reference line and a procedure is then applied to prune out the least significant through a combination algorithm, defined in 4.4.2, to leave a set of significant segments, which is the resulting crossing-the-line segmentation.[10]

NOTE 1 An alternative procedure is given in Annex A.

The nesting index for crossing-the-line segmentation is specified as the vector (H_u, H_l), where H_u is the height discrimination for the peaks and H_l is the depth discrimination for the pits; both values shall be predefined.

NOTE 2 H_u and H_l can take the same positive value or different positive values. One or both can also take the value zero.

4.4.2 Calculation method for crossing-the-line segmentation

Use the following method:

a) Step 1: Mark profile portions as follows:

1) If the profile contains positive, negative or zero values, mark all portions of the profile over or below the reference line. Keep zero values as part of the profile (see Figure 12 a)).

2) If the profile contains positive and zero values, mark all portions of the profile over and matching the reference line.

3) If the profile contains zero and negative values, mark all portions of the profile matching and below the reference line.

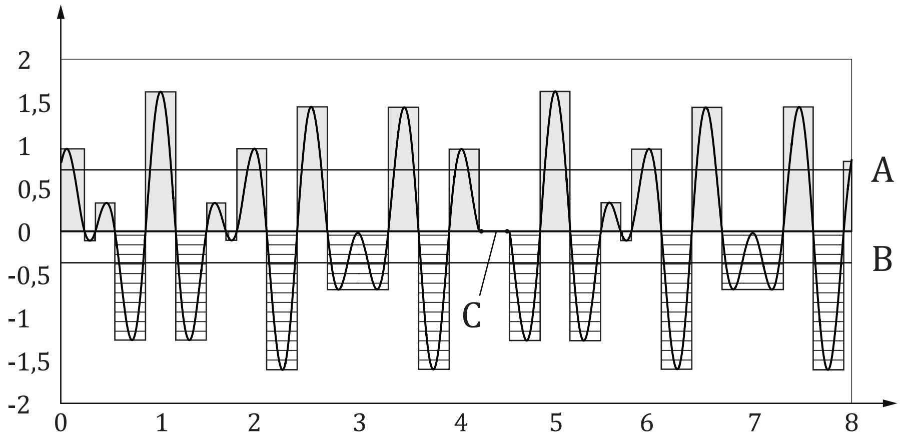
b) Step 2: Set profile values to zero which meet the following criteria (see Figure 12 b)):

1) the first and the last portion;

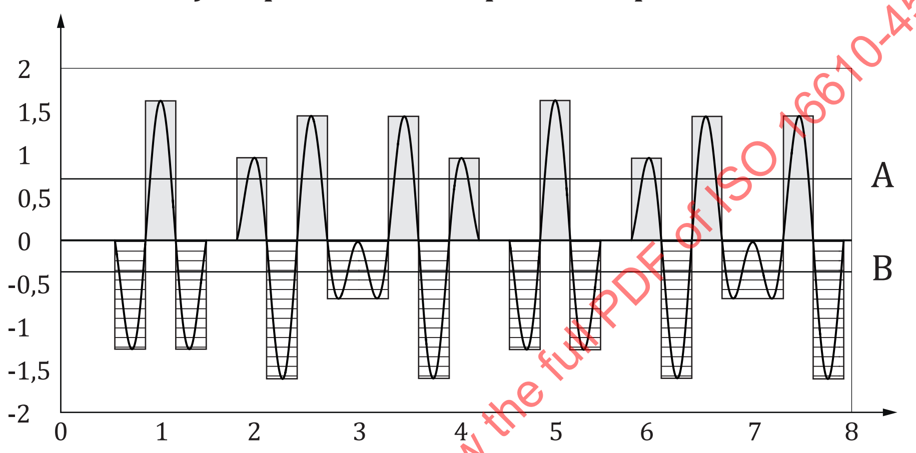
2) portions lower than their associated discrimination levels (H_u for peaks and H_l for pits).

c) Step 3: Combine as profile elements pairwise adjacent peaks/pits or pits/peaks (including zero values that can be in between) from left to right (see Figure 13 a)) and vice versa (see Figure 13 b)).

Zero values shall be expressed by the concept of hyperreal values.



a) Keep zero values as part of the profile



b) Set profile elements to zero which meet the discrimination criteria

Key

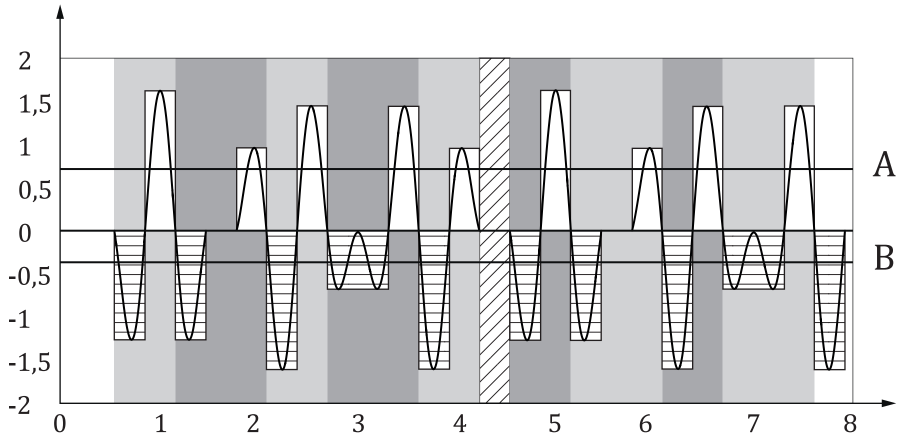
A H_U, profile peak height discrimination

B $-H_L$ minus profile pit depth discrimination

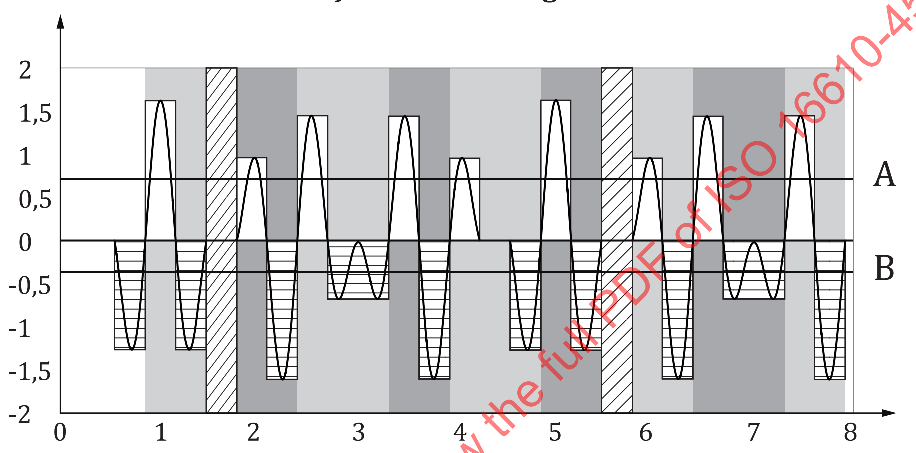
C zero values

NOTE Portions above the reference line are light-grey filled. Portions below the reference line are filled with bricks.

Figure 12 — Crossing-the-line segmentation calculation method (steps 1 and 2)



a) From left to right



b) From right to left

Key

A H_U, profile peak height discrimination

B $-H_L$ minus profile pit depth discrimination

NOTE All detected profile elements are coloured by alternating grey-filled rectangles. Mosaic-filled rectangles mark excluded profile sections.

Figure 13 — Combine as profile elements pairwise adjacent peaks/pits or pits/peaks (step 3)

5 General information

Other segmentation information is given in Annex B. The filtration matrix model is given in Annex C. The concept diagrams for this document are given in Annex D. The relationship between this document and the GPS matrix model is given in Annex E.

Segmentation filters according to this document are designated:

— Watershed segmentation using Wolf pruning: FPMSW

— Crossing-the-line segmentation: FPMSC

Annex A
(informative)

Crossing-the-line segmentation to determine profile elements

A.1 General

This annex specifies the determination of profile elements for feature parameters using crossing-the-line segmentation. Crossing-the-line segmentation is separated into four steps:

— Step 1: Determination of hills and dales within the evaluation length;

— Step 2: Determination of significant profile hills and profile dales;

— Step 3: Merging of adjacent significant profile hills or adjacent significant profile dales;

— Step 4: Determination of profile elements.

NOTE The following variables are used:

— n number of profile values;

— x_k position on the X-axis of profile value z_k with k = 1, 2, ..., n;

— z_k profile value with k = 1, 2, ..., n;

— n_{HD} total number of profile hills and profile dales;

— HD_k profile hill or profile dale with k = 1, 2, ..., n_{HD}; HD_k has four members:

  — $\mathrm{HD}.t \in -1,0,1$ to indicate a dale, a zero element or a hill;

  — $\mathrm{HD}.h$ peak height or pit depth;

  — $\mathrm{HD}.i_l$ index of the left boundary $x_{\mathrm{HD}.i_l}$ of a profile hill or a profile dale;

  — $\mathrm{HD}.i_r$ index of the right boundary $x_{\mathrm{HD}.i_r}$ of a profile hill or a profile dale;

— x_{left}, x_{right} intersection points with the reference line;

— H_{u} peak height discrimination $\in \mathrm{R}_0^+$;

— H_{l} pit depth discrimination $\in \mathrm{R}_0^+$;

— O_{H} threshold to suppress numerical noise $\in \mathrm{R}_0^+$ (outwardly directed);

— O_{D} threshold to suppress numerical noise $\in \mathrm{R}_0^+$ (inwardly directed);

— n_{pe} total number of profile elements;

— $X_{\mathrm{s},m}$ spacing of a profile element with m = 1, 2, ..., n_{pe};

— $Z_{\mathrm{t},m}$ height of a profile element with m = 1, 2, ..., n_{pe}.

A.2 Modified signum function

Function to determine the sign of a real number z depending on the positive real numbers u and l, see Formula (A.1):

$$\mathrm{sgm}(z,l,u)=\begin{cases}1 & \text{if} \quad z\geq u\\ -1 & \text{if} \quad z\leq -l\\ 0 & \text{otherwise}\end{cases} \tag{A.1}$$

A.3 Root function

Function to determine the intersection of the assessed profile with the reference line by linear interpolation, see Formula (A.2):

$$\mathrm{root}(x_a,z_a,x_b,z_b)=\begin{cases}\dfrac{(x_a+x_b)}{2} & \text{if} \quad z_a=z_b\\ \min\left(\max\left(\dfrac{(x_a z_b-x_b z_a)}{(z_b-z_a)},x_a\right),x_b\right) & \text{otherwise}\end{cases} \tag{A.2}$$

where $x_a, z_a \in \mathrm{R}$ and $x_b, z_b \in \mathrm{R}$ are the coordinates of the profile whose linear connection intersects the reference line.

NOTE If the intersection point lies outside the interval $[x_a, x_b]$, then the associated interval limit is used instead of the intersection point.

A.4 Step 1: Determination of hills and dales within the evaluation length

This clause defines an algorithm in order to detect hills and dales with arbitrary height and depth within the evaluation length. The result is a sequence HD of hills and dales. See Figure A.1.

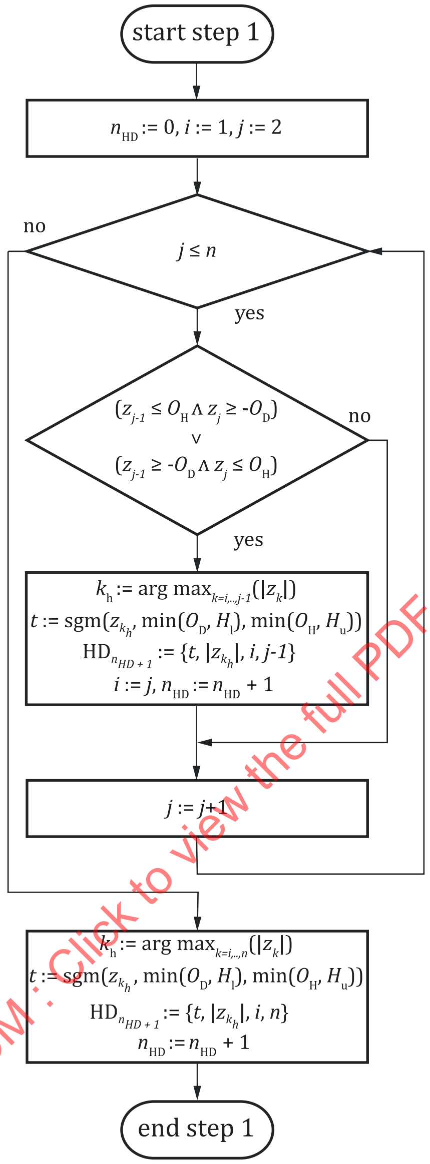


Figure A.1 — Step 1: Determination of hills and dales within the evaluation length

A.5 Step 2: Determination of significant profile hills and profile dales

This clause defines an algorithm in order to delete insignificant hills and dales applying the peak height discrimination and the pit depth discrimination. See Figure A.2.

NOTE The maximum operator between H_u and O_H as well as H_l and O_D is needed if O_H and O_D are constants.

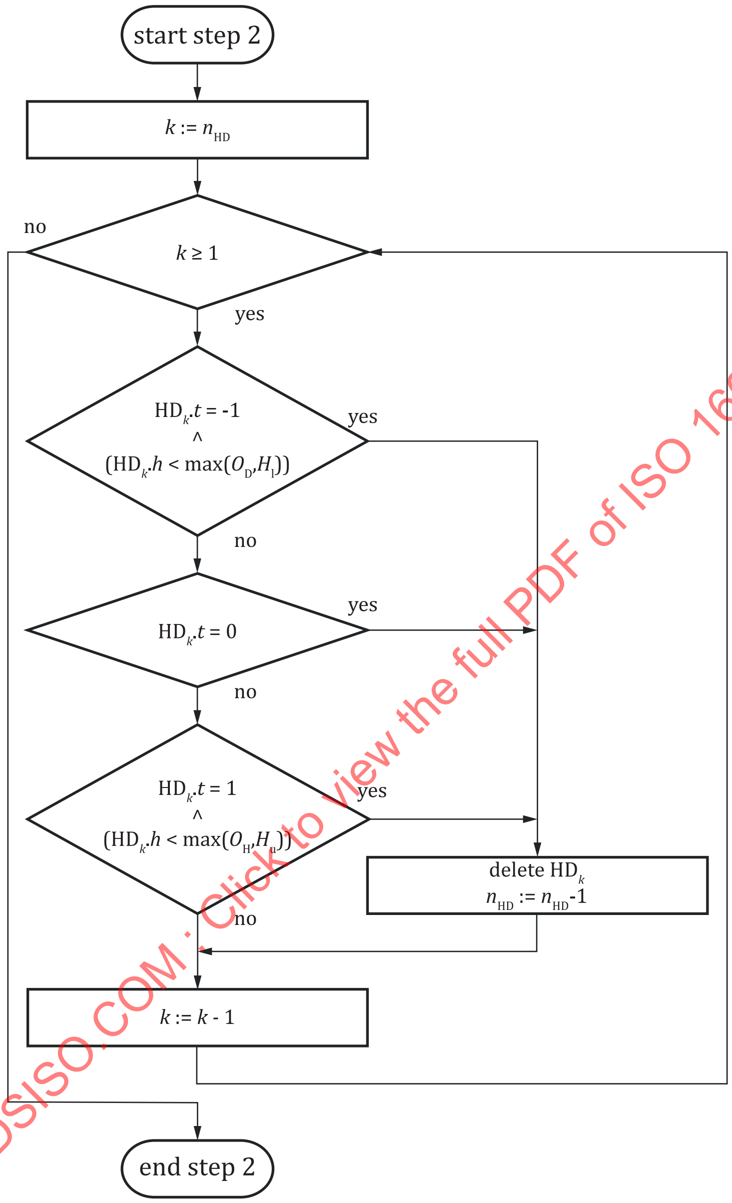


Figure A.2 — Step 2: Determination of significant profile hills and profile dales

A.6 Step 3: Merging of adjacent significant profile hills or adjacent significant profile dales

This clause defines an algorithm to merge adjacent hills or adjacent dales after applying the peak height discrimination for profile hills and the pit depth discrimination for profile dales. The result is a sequence HD of adjacent hills and dales or adjacent dales and hills. See Figure A3.

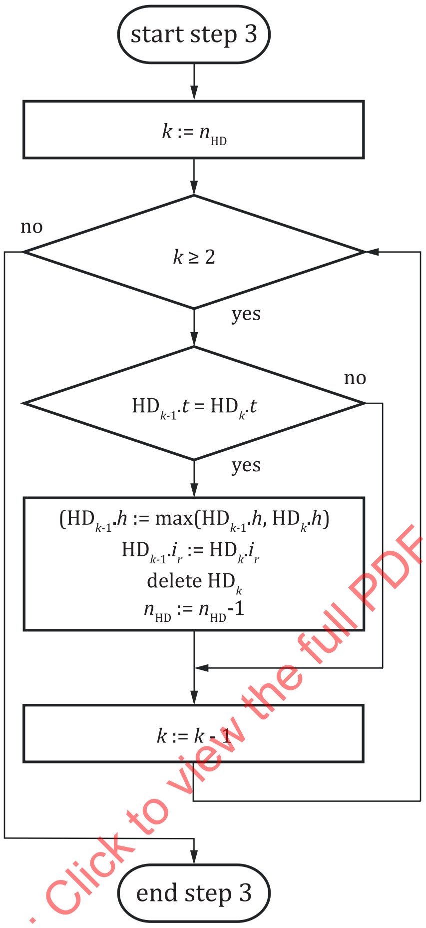


Figure A.3 — Step 3: Merging of adjacent hills or dales

A.7 Step 4: Determination of profile elements

This clause determines the individual spacing $X_{s,i}$ and height $Z_{t,i}$ of profile elements. Profile elements are determined from the beginning to the end of the evaluation length and vice versa. See Figure A.4.

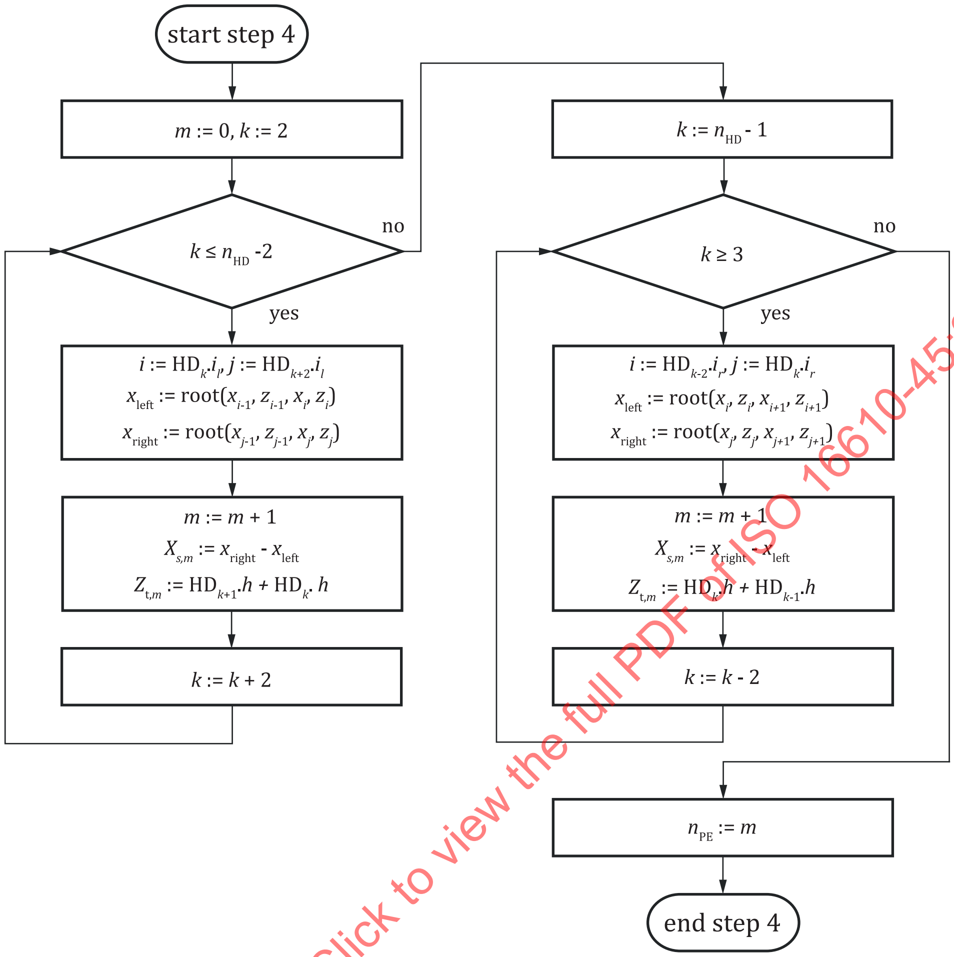


Figure A.4 — Step 4: Determination of profile elements

Annex B
(informative)

Other segmentation information — Hyperreal numbers

B.1 General

The hyperreals, or non-standard reals, *R, are an extension of the real numbers R that contain numbers greater than any finite positive integer. Such numbers are infinite and their reciprocals are infinitesimals.

The idea of the hyperreal system is to extend the real numbers R to form a system *R that includes infinitesimal and infinite numbers, but without changing any of the elementary axioms of algebra. Any statement of the form "any number z..." that is true for the reals is also true for the hyperreals (for more details, see References [11] and [12]).

B.2 Usage

In this document, only hyperreal numbers with reals and infinitesimals are used. Therefore, a hyperreal number, H_{rz}, will consist of two parts of the form:

$$H_{rz} = z + \varepsilon$$

where

z is a real number;

ε is a positive infinitesimal.

Non-standard reals are used to make sure that profile heights are all different, i.e. no two heights are equal. This greatly simplifies the segmentation algorithms. This is achieved by adding an infinitesimal tilt to the original profile height profile, as follows:

— Let (x, z) be the profile co-ordinates and let δ be a given infinitesimal.

— Define the hyperreal height as $H_{rz} = z + x \times \delta$.

Then it is trivial to show that all such hyperreal heights take different values with the following comparison rule:

$$H_{rz1} > H_{rz2}$$

If and only if $z_1 > z_2$ or if $z_1 = z_2$, then $x_1 > x_2$, where $H_{rz1} = z_1 + x_1 \times \delta$ and $H_{rz2} = z_2 + x_2 \times \delta$.

It is trivial to convert a hyperreal height $H_{rz} = z + x \times \delta$ back to a real height as follows:

$\mathrm{Real}(z + x \times \delta) = z$ i.e. just ignore the infinitesimal part.

B.3 Computer implementation

One possible computer implementation is as follows:

— Assume that the profile consists of N heights stored as double precision floats z_i with $i = 1, ..., N$.

NOTE It is assumed that double precision floats consist of a sign bit, an 11-bit excess − 1023 binary exponent and a 52-bit mantissa, plus the implied high-order 1 bit.

— Also assume that the resolution of the height values can be represented as a small number of significant bits, for example n, and let the given infinitesimal be the least significant bit δ (leaving approximately $m = 52 - n$ bits for the infinitesimal part of the hyperreal numbers). Then the real part of the height is the upper n bits of the mantissa of the double precision float and the infinitesimal part is the lower m bits.

— The hyperreal number of the i^{th} profile point becomes $H_{\text{rz}i} = z_i + i \times \delta$ with $i = 1, \ldots, N$ and the algebraic operations of this artificial hyperreal number (including comparison) are then just the standard algebraic operations of double precision floats, greatly simplifying the segmentation algorithms.