
**Geometrical product specifications
(GPS) — Filtration —**

Part 29:

Linear profile filters: Spline wavelets

*Spécification géométrique des produits (GPS) — Filtrage —
Partie 29: Filtres de profil linéaires: Ondelettes splines*

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: [Foreword - Supplementary information](#)

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition cancels and replaces ISO/TS 16610-29:2006 which has been technically revised.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

- Part 1: Overview and basic concepts
- Part 20: Linear profile filters: Basic concepts
- Part 21: Linear profile filters: Gaussian filters
- Part 22: Linear profile filters: Spline filters
- Part 28: Profile filters: End effects
- Part 29: Linear profile filters: Spline wavelets
- Part 30: Robust profile filters: Basic concepts
- Part 31: Robust profile filters: Gaussian regression filters
- Part 32: Robust profile filters: Spline filters
- Part 40: Morphological profile filters: Basic concepts
- Part 41: Morphological profile filters: Disk and horizontal line-segment filters
- Part 49: Morphological profile filters: Scale space techniques
- Part 60: Linear areal filters: Basic concepts
- Part 61: Linear areal filters: Gaussian filters

- Part 71: Robust areal filters: Gaussian regression filters
- Part 85: Morphological areal filters: Segmentation

The following parts are planned:

- Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets
- Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets
- Part 45: Morphological profile filters: Segmentation
- Part 62: Linear areal filters: Spline filters
- Part 69: Linear areal filters: Spline wavelets
- Part 70: Robust areal filters: Basic concepts
- Part 72: Robust areal filters: Spline filters
- Part 80: Morphological areal filters: Basic concepts
- Part 81: Morphological areal filters: Sphere and horizontal planar segment filters
- Part 89: Morphological areal filters: Scale space techniques

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Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO/TR 14638). It influences chain links 3 and 5 in the GPS matrix structure.

The ISO/GPS Masterplan given in ISO 14638 gives an overview of the ISO/GPS system of which this part of ISO 16610 is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this part of ISO 16610 and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this part of ISO 16610, unless otherwise indicated.

For more detailed information of the relation of this part of ISO 16610 to the GPS matrix model, see [Annex E](#).

This part of ISO 16610 develops the terminology and concepts for spline wavelets.

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Geometrical product specifications (GPS) — Filtration —

Part 29:

Linear profile filters: Spline wavelets

1 Scope

This part of ISO 16610 specifies spline wavelets for profiles and contains the relevant concepts. It gives the basic terminology for spline wavelets of compact support, together with their usage.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic terminology*

ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO 16610-22, *Geometrical product specifications (GPS) — Filtration — Part 22: Linear profile filters: Spline filters*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99, ISO 16610-1, ISO 16610-20, ISO 16610-22, and the following apply.

3.1

mother wavelet

function of one or more variables which forms the basic building block for wavelet analysis, related to a scalar function

Note 1 to entry: A mother wavelet, which usually integrates to zero, is localized in space and has a finite bandwidth. [Figure 1](#) provides an example of a real-valued mother wavelet.

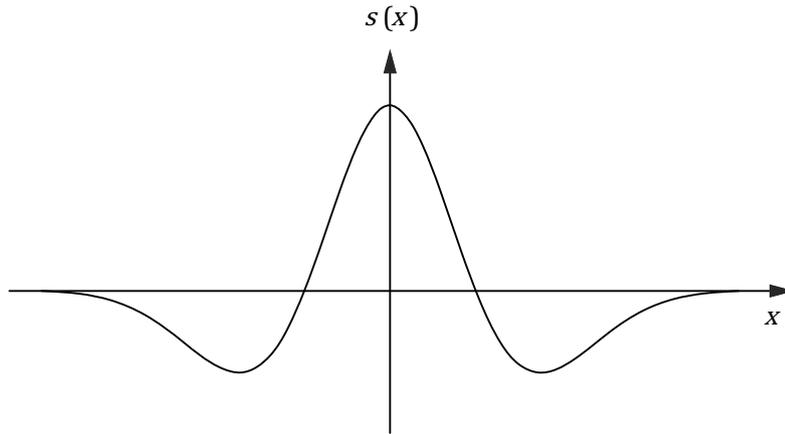


Figure 1 — Example of a real-valued mother wavelet

3.2 wavelet family

$g_{a,b}$
family of functions generated from the *mother wavelet* (3.1) by dilation and translation

Note 1 to entry: If $g(x)$ is the mother wavelet, then the wavelet family $g_{a,b}(x)$ is generated as follows:

$$g_{a,b}(x) = a^{-0,5} \times g\left(\frac{x-b}{a}\right) \tag{1}$$

where

- a is the dilation parameter;
- b is the translation parameter.

3.2.1 dilation

(wavelet) transformation which scales the spatial variable x by a factor a

Note 1 to entry: This transformation takes the function $g(x)$ to $a^{-0,5}g(x/a)$ for an arbitrary positive real number a .

Note 2 to entry: The factor $a^{-0,5}$ keeps the area under the function constant.

3.2.2 translation

transformation which shifts the spatial position of a function by a real number b

Note 1 to entry: This transformation takes the function $g(x)$ to $g(x - b)$ for an arbitrary real number b .

3.3 discrete wavelet transform

unique decomposition of a profile into a linear combination of a *wavelet family* (3.2) where the *translation* (3.2.2) parameters are integers and the *dilation* (3.2.1) parameters are powers of a fixed positive integer greater than 1

Note 1 to entry: The dilation parameters are usually powers of 2.

Note 2 to entry: Throughout the rest of this part of ISO 16610, the discrete wavelet transform is referred to as the wavelet transform.

3.4

multiresolution analysis

decomposition of a profile by a filter bank into portions of different scales

Note 1 to entry: The portions at different scales are also referred to as resolutions (see ISO 16610-20).

Note 2 to entry: See [Figure 2](#).

Note 3 to entry: Since there is, by definition, no loss of information, it is possible to reconstruct the original profile from the multiresolution ladder structure.

3.4.1

low-pass component

component obtained after convolution with a smoothing filter (low pass) and a decimation

3.4.2

high-pass component

component obtained after convolution with a difference filter (high pass) and a decimation

Note 1 to entry: The weighting function of the difference filter is defined by the wavelet from a particular family of wavelets, with a particular dilation parameter and no translation.

3.4.3

multiresolution ladder structure

structure consisting of all the orders of the difference components and the highest order smooth component

3.4.4

scalar function

function which defines the weighting function of the smoothing filter used to obtain the smooth component

Note 1 to entry: In order to avoid loss of information on the multiresolution ladder structure, the wavelet and scaling function are matched.

3.4.5

decimation

(wavelet) action which samples every k th point in a sampled profile, where k is a positive integer

Note 1 to entry: Typically, k is equal to 2.

3.5

spline wavelet

wavelet family ([3.2](#)) whose corresponding reconstructing *scalar functions* ([3.4.4](#)) are splines

4 General wavelet description

4.1 General

A spline wavelet claiming to comply with this part of ISO 16610 shall satisfy the equations given in [Annex A](#).

Note Examples of the application of cubic of interpolating spline wavelets are given in [Annex B](#). A concept diagram for the concepts for spline wavelet filters is given in [Annex C](#), and the relationship to the filtration matrix model is given in [Annex D](#).

4.2 Basic usage of wavelets

Wavelet analysis consists of decomposing a profile into a linear combination of wavelets $g_{a,b}(x)$, all generated from a single mother wavelet.^[5] This is similar to Fourier analysis, which decomposes a profile into a linear combination of sinewaves, but unlike Fourier analysis, wavelets can identify the location, as well as the scale of a feature in a profile. As a result, they can decompose profiles where the small-scale structure in one portion of the profile is unrelated to the structure in a different portion,

such as localized changes (i.e. scratches). Wavelets are also ideally suited for non-stationary profiles. Basically, wavelets decompose a profile into building blocks of constant shape, but of different scales.

4.3 Wavelet transform

The discretisation of the wavelet transform of a profile $s(t)$ given at fixed intervals $x_i = i\Delta x$, (where Δx is the sampling interval and $i = \dots, -2, -1, 0, 1, 2, \dots$) with the mother wavelet $g(x)$ is given by

$$S(i\Delta x, a) = \Delta x \sum_j s[(i-j)\Delta x] g_{a, j\Delta x}(j\Delta x) \tag{2}$$

The dilation parameter a is also restricted to discrete values. Consecutive values of a usually have a fixed ratio, i.e. $a_i/a_{i+1} = \text{constant}$

Note This constant is usually 2.

If the wavelet $g(x)$ has a finite spatial extent, the number of sampling points of $t(x)$ at the scale a grows linearly with a , such that calculations of S with an algorithm based on the above equation are generally impractical. Efficient wavelet algorithms depend on various properties of the mother wavelet. The algorithm considered here is that for multiresolution (see ISO 16610-20), which is valid for “biorthogonal” wavelets, to which spline wavelets belong.

The multiresolution form of the wavelet transform consists of constructing a ladder of smooth approximations to the profile (see Figure 2). The first rung is the original profile. Each rung in the ladder consists of a filter bank (see ISO 16610-20), where the profile S^i is split into two components:

- a smoother version of the profile S^{i+1} , which becomes the next rung;
- a component that is the “difference” between the two rungs d^{i+1} .

The action of the low-pass (smoothing) filter H_0 and the high-pass filter H_1 in the filter bank reduces the number of profile points by half.

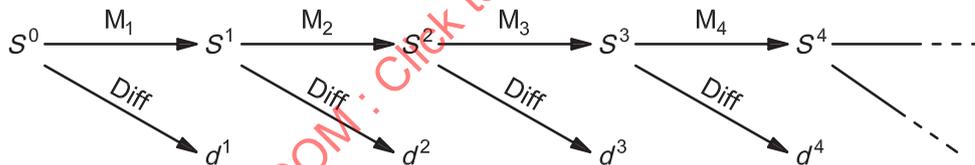


Figure 2 — Example of a multiresolution separation using a wavelet transform

The original profile can be reconstructed from $(d^1, d^2, d^3, \dots, d^n, S^n)$ by reversing the ladder structure and using a second pair of filters H'_0 and H'_1 . Wavelets do not consist of a single method (like the Fourier Transform), but of a multitude of transforms dependent on a mother wavelet, which determines the four filters, $H_0, H_1, H^*_0,$ and H^*_1 . Examples of wavelet transforms of profiles using multiresolution can be found in Annex B.

NOTE There are other forms of splitting the profile using filter banks. The above is just one example.

4.4 Spline wavelets

Symmetrical wavelets include spline wavelets. Spline wavelets are families of wavelets whose corresponding scalar functions are splines.

NOTE 1 The second-generation wavelet algorithm is an efficient method for computing wavelet transformations.

NOTE 2 All wavelets with a finite number of filter coefficients can be represented as second-generation wavelets.

4.5 Nested mathematical models

The multiresolution ladder structure lends itself naturally to a set of nested mathematical models of the profile, with the i th model, m^i , reconstructed from $(d^i, d^{i+1}, \dots, d^n, S^n)$, as illustrated in [Figure 2](#). The order of the model is equivalent to a cut-off value: the higher the order of the model, the smoother the representation. Thus, m^{i+1} is a smoother version of the profile than m^i .

A quantity similar to the “transmission bandwidth” can be constructed using the nested mathematical models, by calculating the height difference between two specified profiles, e.g.

$$m^{i,j} = m^i - m^j \quad (3)$$

where $i < j$.

Thus, in this particular example, order i is equivalent to cut-off value λ_s and order j is equivalent to cut-off value λ_c . The exact relationship between the model order and the cut-off value is dependent on the particular mother wavelet chosen.

5 Recommendations

5.1 Spline wavelet

If not otherwise specified, an interpolating cubic spline wavelet shall be used (see [Annex A](#)).

6 Filter designation

Spline filters in conformance with this part of ISO 16610 are designated

FPLW

See also ISO 16610-1:2015, Clause 5.

Annex A (normative)

Family of interpolating spline wavelets

A.1 General

The lifting scheme is used to define the family of interpolating spline wavelets. [3][4][9] Starting with the original sampled profile, each rung in the multiresolution ladder is calculated from the previous rung in three stages. These stages are called

- splitting,
- prediction, and
- updating.

The interpolating spline wavelet method described in this annex was introduced in Reference [3] for image processing purposes. Reference [9] has applied the method to surface characterisation.

A.2 Splitting stage

The lifting algorithm of the wavelet transform first of all divides the smoothed profile from the j th rung $A_{j,k}$ into “even” and “odd” subsets, in which each sequence contains half as many samples as $A_{j,k}$. The operator is given by

$$\begin{cases} a_{j+1,k} = A_{j,2k} \\ d_{j+1,k} = A_{j,2k+1} \end{cases} \quad (\text{A.1})$$

where

$A_{0,k} = Z_k$ the original sampled profile.

A.3 Prediction stage

The even and odd subsets are interspersed. If the profile has local correlation structure, the even and odd subsets will be highly correlated. It should thus be possible to predict the odd subset from the even subset with reasonable accuracy.

The prediction stage of the wavelet algorithm consists of predicting the odd subset from the even subset and then removing the predicted value from the odd subset value. The operator is given by

$$d_{j+1,k} = d_{j+1,k} - \rho(a_{j+1,k}) \tag{A.2}$$

For the family of interpolating spline wavelets, linear polynomials are used for the prediction. $\rho(a_{j+1,k})$ is a weighted prediction of a wavelet coefficient point given by

$$\rho(a_{j+1,k}) = \sum_{i=1}^N f_i(a_{j+1,k}) \tag{A.3}$$

The value of $\rho(a_{j+1,k})$ is based on the even set, where N denotes how many data points will attend the weighted prediction.

f_i are a set of filtering factors (weighting function) of one wavelet coefficient point and can be found by employing a “Neville’s polynomial interpolation” [6][7][8] with a degree $(N-1)$, with the following recursion:

$$f_i = f_{1,2,\dots,N}(x) = \frac{(x-x_1)f_{2,\dots,N}(x) - (x-x_N)f_{1,2,\dots,N-1}}{(x_N-x_1)} \tag{A.4}$$

Initial coefficients, f_1, f_2, \dots, f_N , are a set of Bezier coefficients of a spline interpolation, with degree $(N-1)$.

For example, if cubic polynomial interpolation is employed to create a weighting function, four neighbouring values will attend a weighted prediction. Five cases should be taken into account:

- a) two neighbouring points on either side of an interval;
- b) one sample point on the left and three on the right at the left boundary of an interval;
- c) vice versa at the right boundary;
- d) four sample points on the left;
- e) four sample points on the right.

These cases are considered in order to guarantee boundary “naturalness”, without including any artefacts (all filtering factors are indicated in [Table A.1](#)). The result of this is that running-in and running-out lengths of normal filtering techniques are not needed.

Table A.1 — Filter coefficients for cubic polynomial interpolation

Number of samples on left	Number of samples on right	$k - 7$	$k - 5$	$k - 3$	$k - 1$	$k + 1$	$k + 3$	$k + 5$	$k + 7$
0	4					2,187 5	-2,187 5	1,312 5	-0,312 5
1	3				0,312 5	0,937 5	-0,312 5	0,062 5	
2	2			-0,063	0,562 5	0,562 5	-0,063		
3	1		0,062 5	-0,312 5	0,937 5	0,312 5			
4	0	-0,312 5	1,312 5	-2,187 5	2,187 5				

For example, when there are two samples on the left and two samples on the right, the lifting factors are

$$f = \left(-\frac{1}{16}, \frac{9}{16}, \frac{9}{16}, -\frac{1}{16} \right) \tag{A.5}$$

and the wavelet coefficients can be updated to

$$d_{j+1,k} = d_{j+1,k} - \frac{1}{16} \left(-a_{j+1,k-2} + 9a_{j+1,k-1} + 9a_{j+1,k} - a_{j+1,k+1} \right) \tag{A.6}$$

A.4 Update stage

For every level of the multiresolution ladder, the resulting smoother profiles should preserve some of the properties of the original profile, e.g. the same average value and other higher moments. This is achieved in the updating stage.

The updating stage of the wavelet algorithm consists of updating the even subset from the odd subset, in order to preserve as many profile moments as possible. The operator is given by

$$A_{j+1,k} = a_{j+1,k} + \mu(d_{j+1,k}) \tag{A.7}$$

$\mu(d_{j+1,k})$ is a weighting update given by

$$\mu(d_{j+1,k}) = \sum_{i=1}^{\tilde{N}} l_i(d_{j+1,k}) \tag{A.8}$$

$\mu(d_{j+1,k})$ is based on the real wavelet coefficients, where \tilde{N} indicates how many wavelet coefficient points will attend the weighting update. The larger \tilde{N} is, the more profile moments are preserved. The l_i are referred to as lifting factors

The lifting factors can be calculated by the following algorithm. Firstly, an initial moment matrix is defined for all coefficients at the first level of the multiresolution ladder. The moment matrix M is defined by the number of points in the profile s and the value of \tilde{N} .

$$M[p,q] = \begin{bmatrix} m_{1,1} & \dots & m_{1,\tilde{N}} \\ \vdots & m_{p,q} & \vdots \\ m_{s,1} & \dots & m_{s,\tilde{N}} \end{bmatrix} = \begin{bmatrix} 1 & 1^2 & \dots & 1^{\tilde{N}} \\ 2 & 2^2 & \dots & 2^{\tilde{N}} \\ \vdots & \vdots & \ddots & \vdots \\ s & s^2 & \dots & s^{\tilde{N}} \end{bmatrix} \begin{matrix} 1 \text{ u p u s} \\ 1 \text{ u q u } \tilde{N} \end{matrix} \tag{A.9}$$

Updating the moment matrix requires an indication of how many filtering factors of corresponding wavelet coefficients will contribute to the update. When neighbouring point numbers on each side are the same, the moments can be expressed by

$$m_{2p,q} = m_{2p,q} + \sum_{t,j} f_i m_{t,q} \tag{A.10}$$

where

$$t = 2p - N + 1, 2p - N + 3, \dots, 2p + N - 1$$

$$i = 1, \dots, N$$

The lifting factors are the solution of the following linear system

$$\begin{bmatrix} m_{2^{p-\tilde{N}+2},1} & \cdots & m_{2^{p+\tilde{N}},1} \\ \vdots & m_{2^{p,q}} & \vdots \\ m_{2^{p-\tilde{N}+2},\tilde{N}} & \cdots & m_{2^{p+\tilde{N}},\tilde{N}} \end{bmatrix}_{\tilde{N},\tilde{N}} \begin{bmatrix} l_1 \\ \vdots \\ l_q \\ \vdots \\ l_{\tilde{N}} \end{bmatrix} = \begin{bmatrix} m_{2^{p+1},1} \\ \vdots \\ m_{2^{p+1},q} \\ \vdots \\ m_{2^{p+1},\tilde{N}} \end{bmatrix} \quad (\text{A.11})$$

For example, when a weighting update of a scalar coefficient is considered to be a cubic spline interpolation, the update can be calculated by using four neighbour wavelet coefficients. In this case, the lifting factors are $l = \left(-\frac{1}{32}, \frac{9}{32}, \frac{9}{32}, -\frac{1}{32}\right)$ and the scalar coefficients can be updated to

$$A_{j+1,k} = a_{j+1,k} + \frac{1}{32}(-d_{j+1,k-2} + 9d_{j+1,k-1} + 9d_{j+1,k} - d_{j+1,k+1}) \quad (\text{A.12})$$

A.5 Forward and inverse transforms

To summarize, the forward transform is given as

$$\text{Split} \begin{cases} a_{j+1,k} = A_{j,2k} \\ d_{j+1,k} = A_{j,2k+1} \end{cases} \quad (\text{A.13})$$

Predict $d_{j+1,k} = d_{j+1,k} - \rho(a_{j+1,k})$

Update $A_{j+1,k} = a_{j+1,k} + \mu(d_{j+1,k})$

One important property of the lifting scheme is that once the forward transform is defined, the inverse transform can immediately be obtained. The operations are just reversed and the + and - toggled. This leads to the following algorithm for the inverse transform.

$$\text{Update } a_{j+1,k} = A_{j+1,k} - \mu(d_{j+1,k}) \quad (\text{A.14})$$

Predict $d_{j+1,k} = d_{j+1,k} + \rho(a_{j+1,k})$

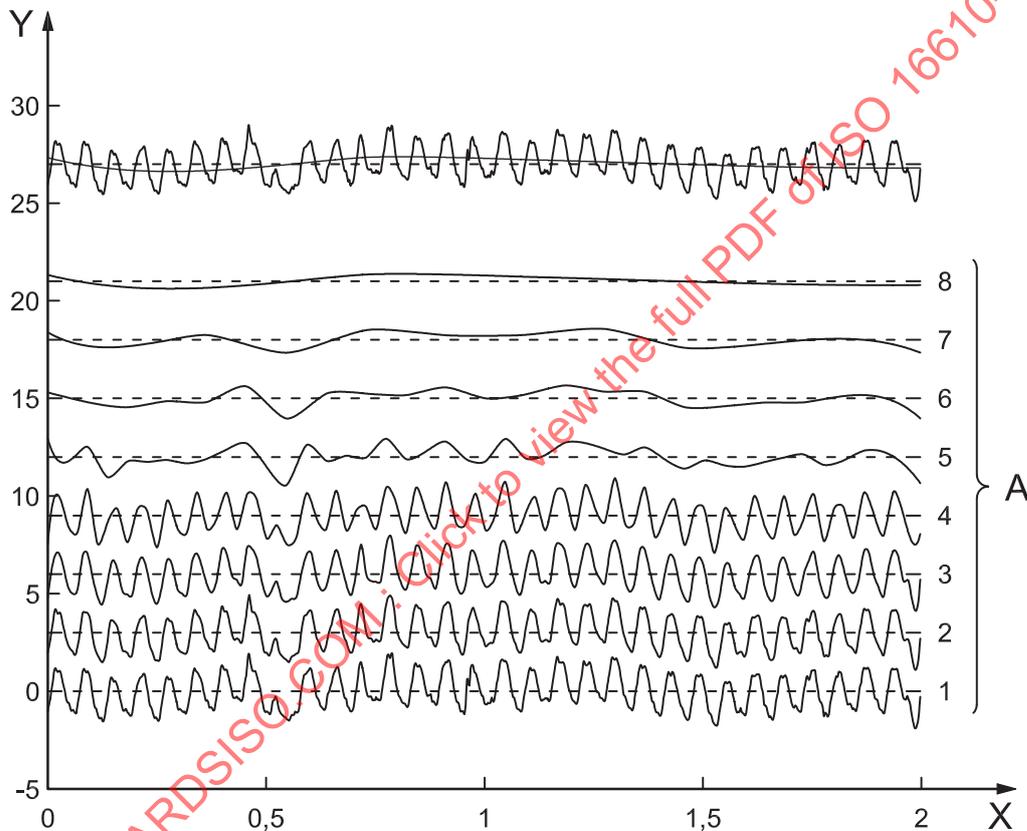
Combine = $\begin{cases} A_{j,2k} = a_{j+1,k} \\ A_{j,2k+1} = d_{j+1,k} \end{cases}$

Annex B (informative)

Examples of the application of cubic of interpolating spline wavelets

B.1 Profile from a milled surface

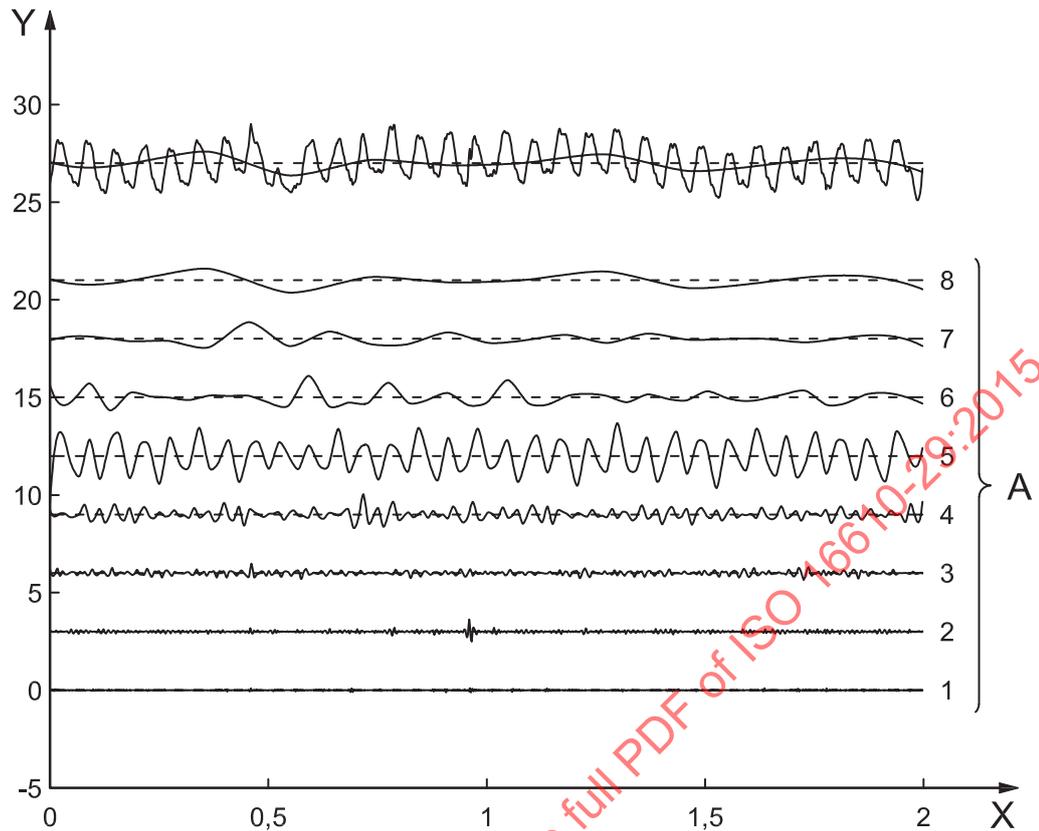
The profile is from a milled surface and is measured with a 5 μm tip stylus. [Figure B.1](#) shows the successively “smoothed” profiles, together with the original profile at the top, with the smoothest profile superimposed on top.



Key
 X distance, mm
 Y height, μm
 A levels

Figure B.1 — Successively smoothed profiles of a milled surface using cubic interpolating spline wavelet

[Figure B.2](#) shows the differences (details) between successive smoothings. The milling marks are easily identifiable at level 5.

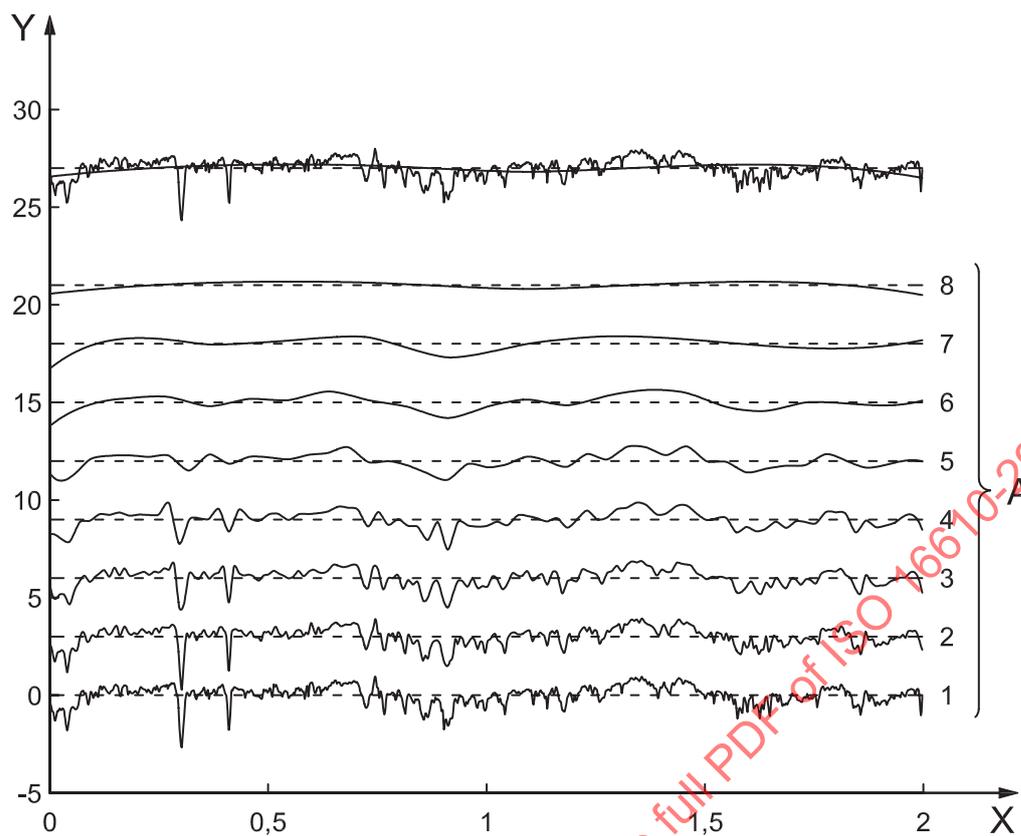


Key
 X distance, mm
 Y height, μm
 A levels

Figure B.2 — Differences on a milled surface using cubic interpolating spline wavelet

B.2 Profile from a ceramic surface

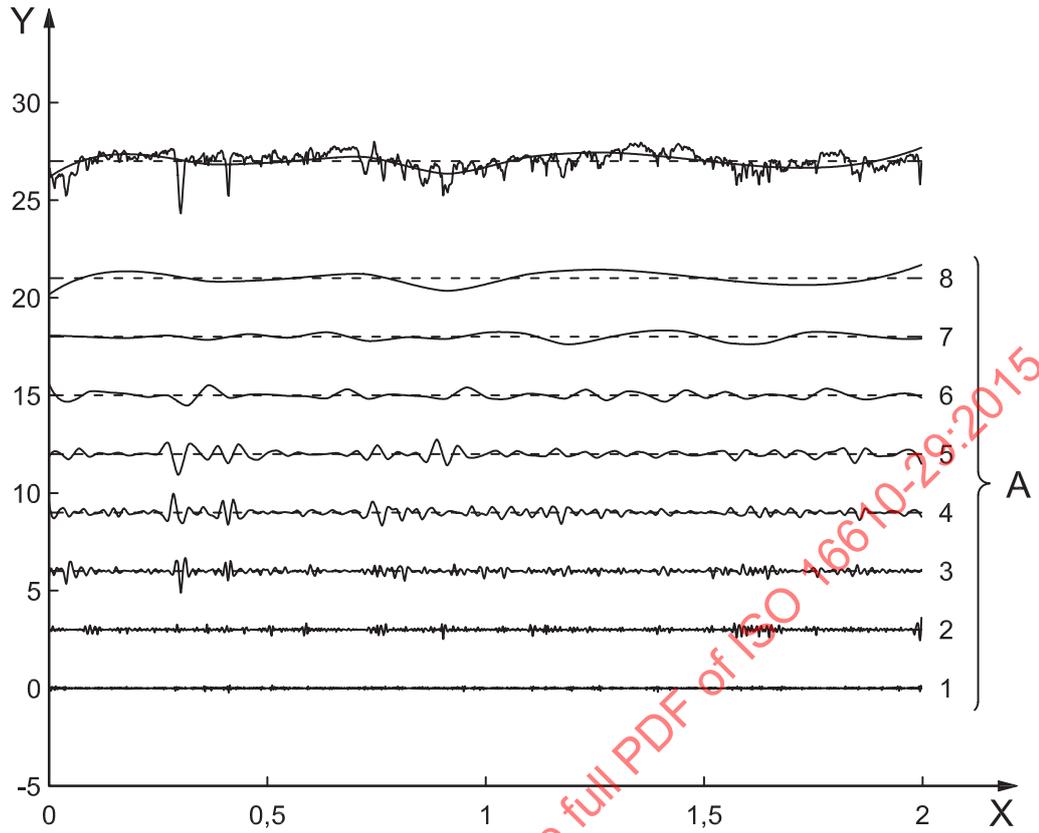
The profile is from a rough ceramic surface and is measured with a 5 mm tip stylus. [Figure B.3](#) shows the successively “smoothed” profiles, together with the original profile at the top.



Key
 X distance, mm
 Y height, μm
 A levels

Figure B.3 — Successively smoothed profiles of a ceramic surface using cubic interpolating spline wavelets

Figure B.4 shows the differences (details) between successive smoothings. The deep valleys are easily identifiable at levels 3 and 4, and various asperities can be seen at levels 1 and 2.



Key
 X distance, mm
 Y height, μm
 A levels

Figure B.4 — Differences on a ceramic surface using a cubic interpolating spline wavelet

Annex C (informative)

Concept diagram

The following is a concept diagram for this part of ISO 16610.

