INTERNATIONAL STANDARD

ISO 16610-28

First edition
2016-12-15

Geometrical product specifications (GPS) — Filtration —

Part 28:
Profile filters: End effects

Spécification géométrique des produits (GPS) — Filtrage —

Partie 28: Filtres de profil: Effets de bords

COPYRIGHT PROTECTED DOCUMENT

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition of ISO 16610-28 cancels and replaces ISO/TS 16610-28:2010, which has been technically revised.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences the chain link C of all chains of standards.

The ISO/GPS Matrix model given in ISO 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

For more detailed information of the relation of this document to the GPS matrix model, see Annex C.

This document develops the concept of handling end effects in the case of linear profile filters.

Geometrical product specifications (GPS) — Filtration —

Part 28:
Profile filters: End effects

1 Scope

This document provides methods for treating the end effects of linear profile filters where such effects occur.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO 16610-21, *Geometrical product specifications (GPS) — Filtration — Part 21: Linear profile filters: Gaussian filters*

ISO 16610-22, *Geometrical product specifications (GPS) — Filtration — Part 22: Linear profile filters: Spline filters*

ISO 16610-31, *Geometrical product specifications (GPS) — Filtration — Part 31: Robust profile filters: Gaussian regression filters*

ISO/TS 16610-32, *Geometrical product specifications (GPS) — Filtration — Part 32: Robust profile filters: Spline filters*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99, ISO 16610-1, ISO 16610-20, ISO 16610-21, ISO 16610-22, ISO 16610-31, ISO/TS 16610-32 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— IEC Electropedia: available at http://www.electropedia.org/

— ISO Online browsing platform: available at http://www.iso.org/obp

3.1
end effect
unintentional changes in the filtration response in the end portions of an open profile

3.2
end effect region
end portion of an open profile where end effects are significant

3.3
moment
nth moment, μ_n, of a real valued function $f(x)$, defined by

$$\mu_n = \int_{-\infty}^{\infty} x^n \times f(x) \times \mathrm{d}x$$

3.4
moment criterion
criterion applying to the shift invariant filter class of a linear profile filter where the weighting function of the filtration operation has vanishing moments up to the nth order, as expressed by

$$\int_{\Omega} x^p \times s(x) \times \mathrm{d}x = 0, \quad p = 1, ..., n$$

where

$s(x)$ is the weighting function of the filter and Ω the definition area of the weighting function

4 End effect correction methods

4.1 General

A linear shift invariant profile filter can be implemented as a weighted moving average with a constant weighting function, $s(x)$, e.g. the Gaussian bell curve according to ISO 16610-21. Because the measured profile, $z(x)$, is always finite, $s(x)$ shall have a local support, $-l_1 \leq x \leq l_2$, which is normally much smaller than the traversing length. Therefore, the filter formula for the low-pass filter based on the convolution is defined as

$$w(x) = \int_{-l_1}^{l_2} z(x-u) \times s(u) \times \mathrm{d}u = \int_{x-l_2}^{x+l_1} z(u) \times s(x-u) \times \mathrm{d}u, \quad l_2 \leq x \leq l_t - l_1 \tag{1}$$

where

$w(x)$ is the reference line;

$z(x)$ is the measured profile;

l_t is the measuring length.

In contrast to profile $z(x)$, reference line $w(x)$ is only valid for $l_2 \leq x \leq l_t - l_1$. The end effect regions are $B_2 = [0,l_2]$ and $B_1 = [l_t - l_1, l_t]$.

NOTE 1   For simplicity, only continuous weighting functions $s(x)$ are considered in this document. The methods are also valid for discrete weighting functions.

NOTE 2   The procedure can be applied directly to the profile or can modify the filtration operation.

EXAMPLE   In the case of the standardized Gaussian filter (see ISO 16610-21), the weighting function has a local support, e.g. $l_1 = l_2 = \lambda_c/2$. As shown in Figure 1, the filter formula cannot be applied over the whole traversing length. In the end effect region, either the left side or the right side of the Gaussian bell curve lies outside the profile.

Key
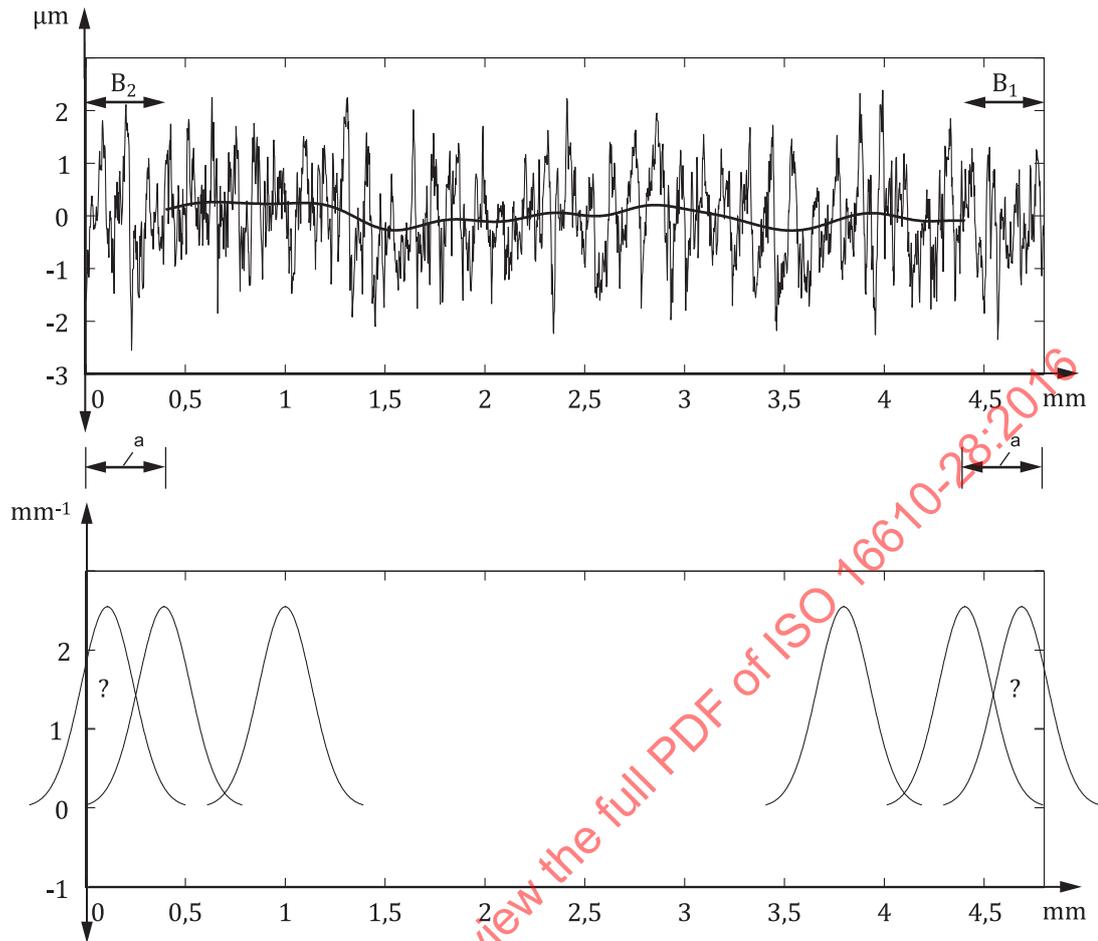
B1 l_2 right end effect region

B2 l_1 left end effect region

a   End effect region.

Figure 1 — End effects using standardized Gaussian filter

Due to their mathematical definition, the filters specified in ISO 16610-22, ISO 16610-29, ISO/TS 16610-32 (spline filter) and ISO 16610-31 (Gaussian regression filter) have an automatic end effect correction. Annex A presents the corresponding weighting function for different positions of the linear spline filter and the linear Gaussian regression filter.

## 4.2 Extrapolation of the profile — Methods

4.2.1 Zero padding

Zero padding is a simple method for retaining the traversing length after filtering the profile. Profile $z(x)$ is padded with zeros over length l_2 at the left side and over length l_1 at the right side of the profile:

$$\tilde{z}(x) = \begin{cases} 0 & \text{for} & -l_2 \leq x < 0 \\ z(x) & \text{for} & 0 \leq x \leq l_t \\ 0 & \text{for} & l_t < x \leq l_t + l_1 \end{cases} \qquad (2)$$

The formula filter in 3.4 can be rewritten as

$$w(x) = \int_{-l_1}^{l_2} \tilde{z}(x-u) \times s(u) \times du = \int_{x-l_2}^{x+l_1} \tilde{z}(u) \times s(x-u) \times du, \quad 0 \leq x \leq l_t \qquad (3)$$

EXAMPLE 1 Figure 2 shows zero padding using the Gaussian weighting function with $l_1 = l_2 = \lambda_c/2$ and a profile without a slope.
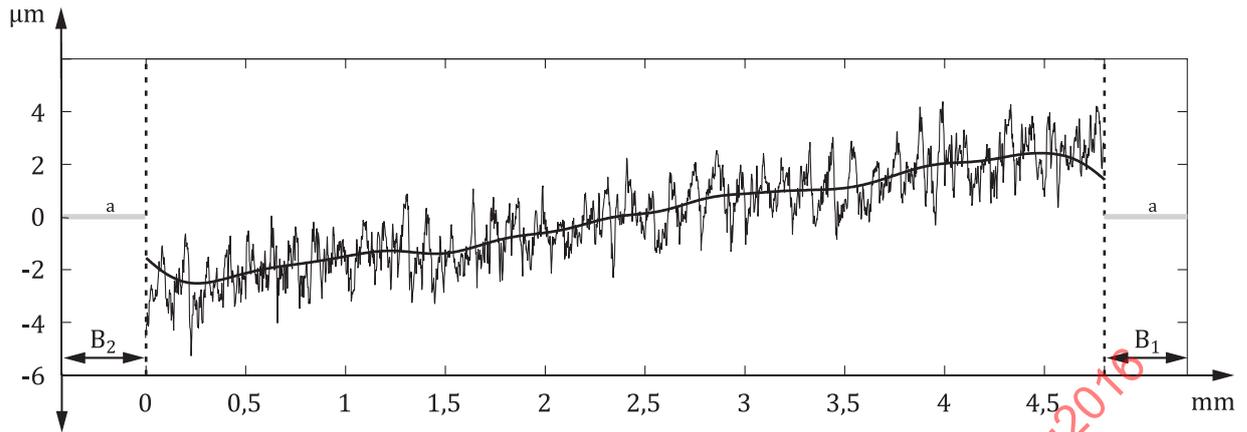

Key
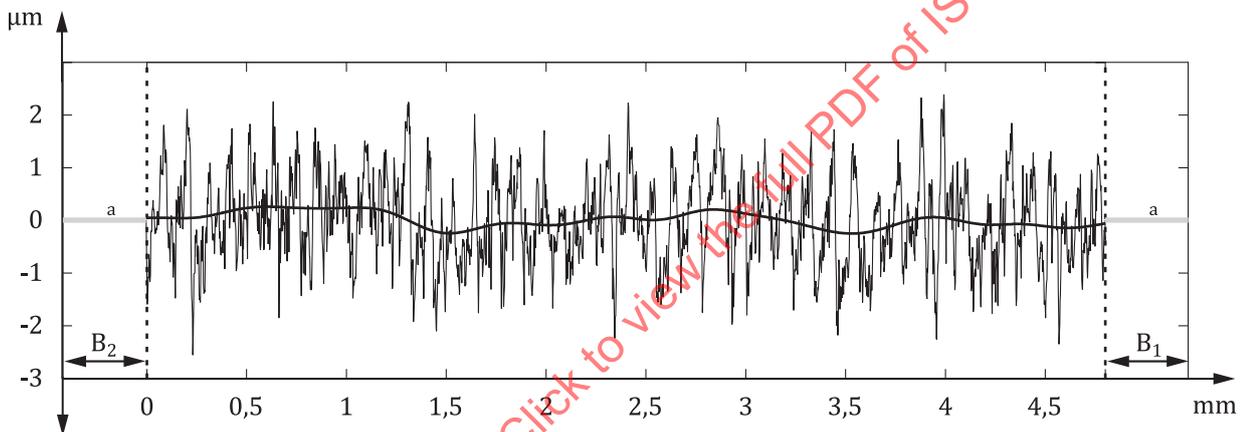
B1 l_2 right end effect region

B2 l_1 left end effect region

a Zero.

Figure 2 — Zero padding using standardized Gaussian filter and profile without slope

EXAMPLE 2 Figure 3 shows zero padding using the Gaussian weighting function with $l_1 = l_2 = \lambda_c/2$ and a profile with a slope.

Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a Zero.

NOTE   In Example 2, the end effects have not been eliminated.

Figure 3 — Zero padding using standardized Gaussian filter and profile with slope

4.2.2   Linear extrapolation

In the case of linear extrapolation, a least-squares line is fitted to the profile within the left and right end effect regions:
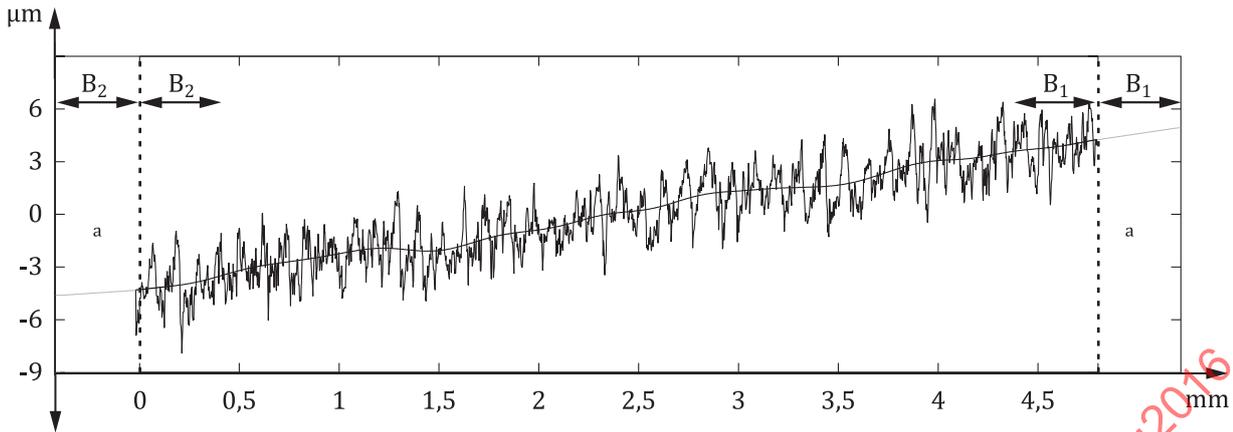
$$\int_0^{l_2} \left[z(x) - m_l \times x - t_l \right]^2 \times dx \quad \rightarrow \quad \underset{m_l, t_l}{\text{Min}} \quad \text{and} \quad \int_{l_t - l_1}^{l_t} \left[z(x) - m_r \times x - t_r \right]^2 \times dx \quad \rightarrow \quad \underset{m_r, t_r}{\text{Min}} \quad (4)$$

The profile is now extended to

$$\tilde{z}(x) = \begin{cases} m_l \times x + t_l & \text{for} \quad -l_2 \leq x < 0 \\ z(x) & \text{for} \quad 0 \leq x \leq l_t \\ m_r \times x + t_r & \text{for} \quad l_t < x \leq l_t + l_1 \end{cases} \quad (5)$$

Inserting $\tilde{z}(x)$ in Formula (3) yields the reference line.

EXAMPLE   Figure 4 shows the linear extrapolation method using the Gaussian weighting function with $l_1 = l_2 = \lambda_c/2$ and a profile with a slope.

Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a   Linear extrapolation.

NOTE   In cases where more information with regard to the shape of the profile is known, more sophisticated approximation methods can be used, e.g. higher-order polynomials.

Figure 4 — Linear extrapolation using standardized Gaussian filter and profile with slope

4.2.3   Symmetric extension

4.2.3.1   General

A measured profile is extended by symmetric extension on the left hand and right hand respectively.
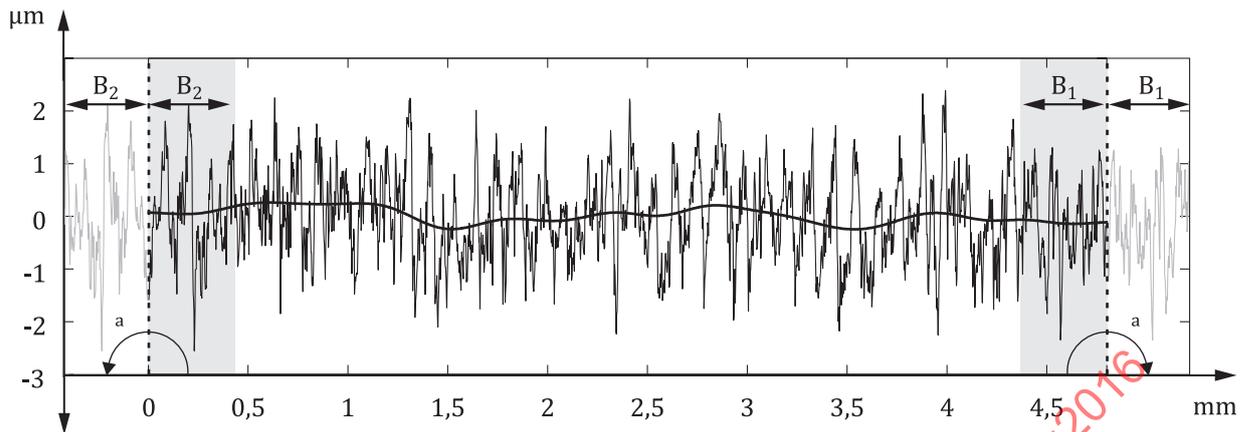
4.2.3.2   Line symmetrical reflection

A measured profile is extended by horizontal reflection on the left hand and right hand, respectively, and is defined by

$$\tilde{z}\left(x\right) = \begin{cases} z\left(-x\right) & \text{for} \quad -l_2 \leq x < 0 \\ z\left(x\right) & \text{for} \quad\;\; 0 \leq x \leq l_t \\ z\left(2 \times l_t - x\right) & \text{for} \quad l_t < x \leq l_t + l_1 \end{cases} \tag{6}$$

Inserting $\tilde{z}\left(x\right)$ in Formula (3) yields the reference line.

EXAMPLE 1   Figure 5 shows the line symmetrical reflection method using the Gaussian weighting function with $l_1 = l_2 = \lambda_c/2$.
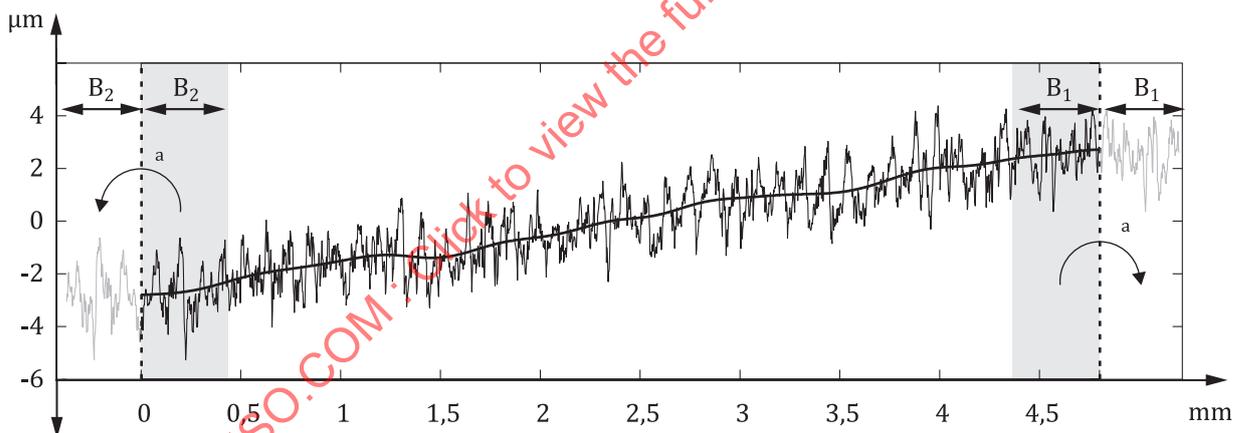
Key

B1 l_2 right end effect region
B2 l_1 left end effect region
a   Reflected.

Figure 5 — Line symmetrical profile reflection using standardized Gaussian filter

EXAMPLE 2   Figure 6 shows the line symmetrical reflection method using the Gaussian weighting function with $l_1 = l_2 = \lambda_c/2$ and a profile with a slope.

Key

B1 l_2 right end effect region
B2 l_1 left end effect region
a   Reflected.

Figure 6 — Line symmetrical profile reflection using standardized Gaussian filter and profile with slope

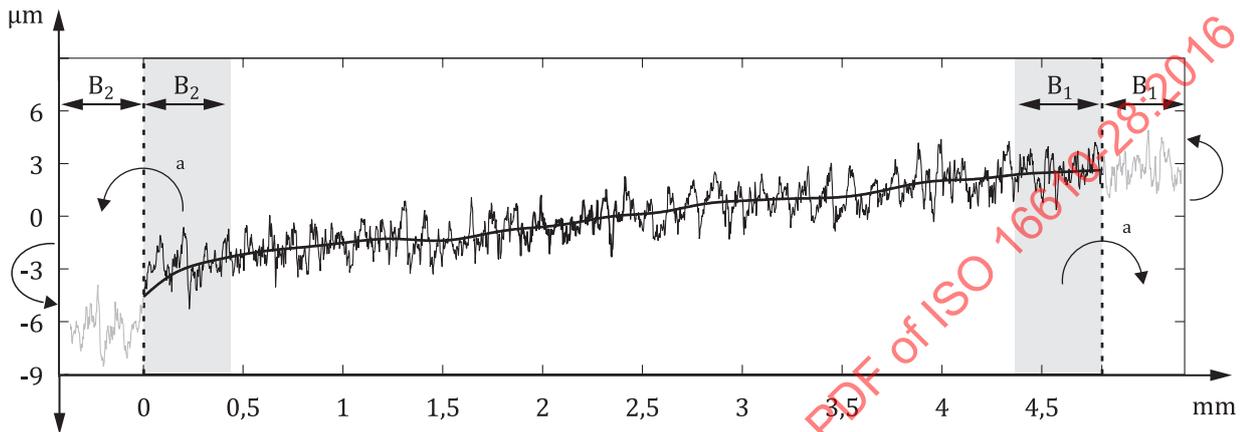#### 4.2.3.3   Point symmetrical reflection

A measured profile is extended by horizontal reflection in conjunction with vertical reflection on the left and right hand respectively. Both reflection lines shall intersect at the respective end point of the profile.

The point symmetrical reflection is defined by

$$\tilde{z}(x) = \begin{cases} 2 \times z(x=0) - z(-x) & \text{for} & -l_2 \leq x < 0 \\ z(x) & \text{for} & 0 \leq x \leq l_t \\ 2 \times z(x=l_t) - z(2 \times l_t - x) & \text{for} & l_t < x \leq l_t + l_1 \end{cases} \qquad (7)$$

Inserting $\tilde{z}(x)$ in Formula (3) yields the reference line.

EXAMPLE 1   Figure 7 shows the point symmetrical reflection method using the Gaussian weighting function with $l_1 = l_2 = \lambda_c/2$.

Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a   Reflected.

Figure 7 — Point symmetrical profile reflection using standardized Gaussian filter

4.3   Moment retainment criterion

In order to retain the moment criterion for a given weighting function, $s(x)$, at the end effect region, the filter formula of Formula (1) should be modified to

$$w(x) = \int_{\max(x-l_2,0)}^{\min(x+l_1,l_t)} z(\xi) \times \left(\sum_{p=0}^{n} b_p(x) \times (x-u)^p \times s(x-u) \right) \times \mathrm{d}u, \quad 0 \leq x \leq l_t \qquad (8)$$

where

n is the last vanishing moment of the weighting function, $s(x)$;

$b_p(x)$ is the shift variant correction function.

In the interior $-l_2 \le x \le l_t - l_1$, the filter formula is equal to [Formula (1)](#) and $b_p(x)$ results in

$$b_p\left(x\right) = \begin{cases} 1 & \text{for} \quad p = 0 \\ 0 & \text{for} \quad p > 0 \end{cases}, \quad l_2 \le x \le l_t - l_1 \tag{9}$$

At the end effect region, $0 \le x < l_2$ and $l_t - l_1 < x \le l_t$, $b_p(x)$ can be calculated by solving the matrix formula:

$$\begin{bmatrix} \mu_0\left(x\right) & \mu_1\left(x\right) & \cdots & \mu_n\left(x\right) \\ \mu_1\left(x\right) & \mu_2\left(x\right) & \cdots & \mu_{n+1}\left(x\right) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_n\left(x\right) & \mu_{n+1}\left(x\right) & \cdots & \mu_{2n}\left(x\right) \end{bmatrix} \cdot \begin{bmatrix} b_0\left(x\right) \\ b_1\left(x\right) \\ \vdots \\ b_n\left(x\right) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{10}$$

with

$$\mu_p\left(x\right) = \int\limits_{\max\left(x-l_2,0\right)}^{\min\left(x+l_1,l_t\right)} \left(x - u\right)^p \times s\left(x - u\right) \times \mathrm{d}u, \quad p = 0, .., 2n \tag{11}$$

EXAMPLE 1   Due to its symmetrical weighting function, the standardized Gaussian filter has one vanishing moment, $p = 1$. When solving [Formula (10)](#), correction function $b_p(x)$ at the end effect region results in
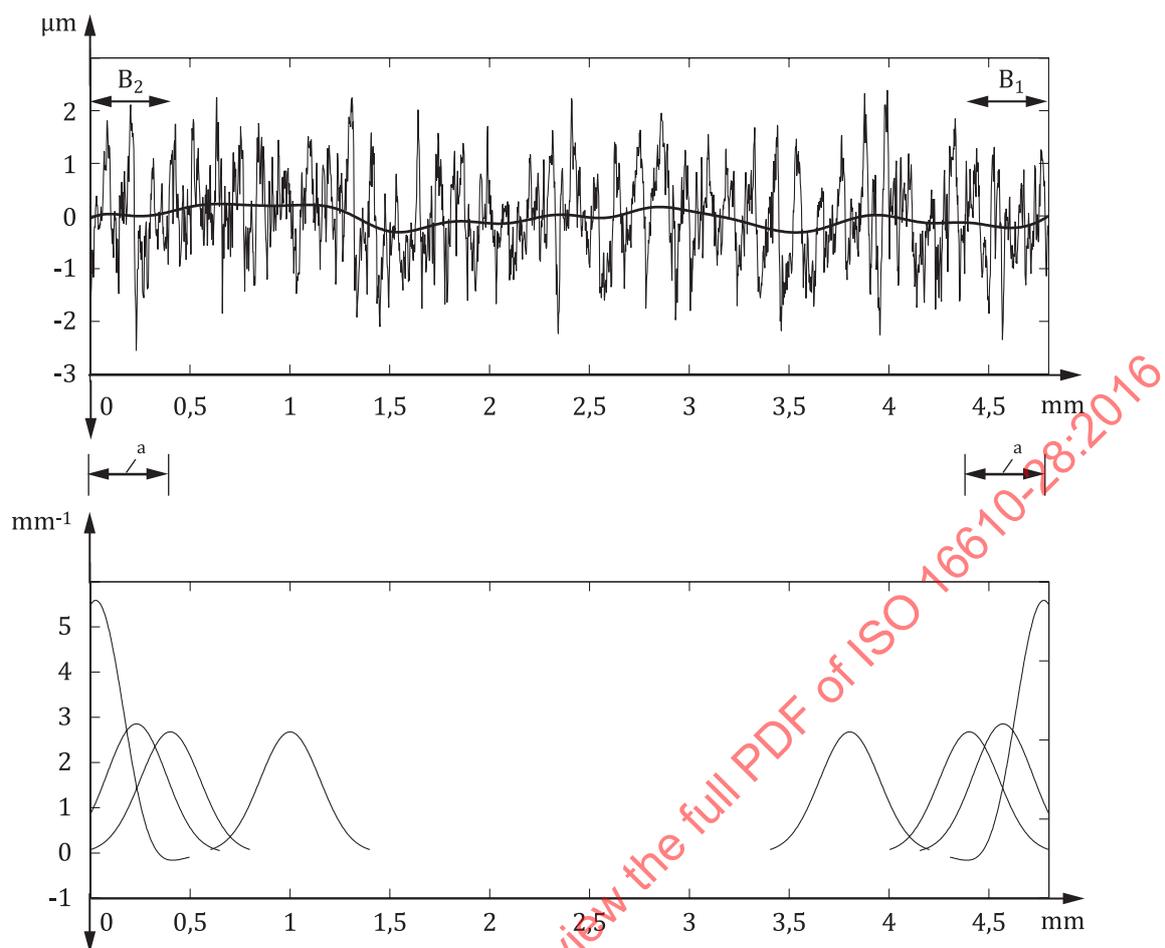
$$b_0\left(x\right) = \mu_2\left(x\right) / \det\left(x\right), \quad b_1\left(x\right) = -\mu_1\left(x\right) / \det\left(x\right) \tag{12}$$

with

$$\det\left(x\right) = \mu_2\left(x\right) \times \mu_0\left(x\right) - \mu_1\left(x\right)^2$$

The corresponding weighting function for $l_1 = l_2 = \lambda_c/2$ at different positions is shown in [Figure 8](#). For further information, see Reference [5].
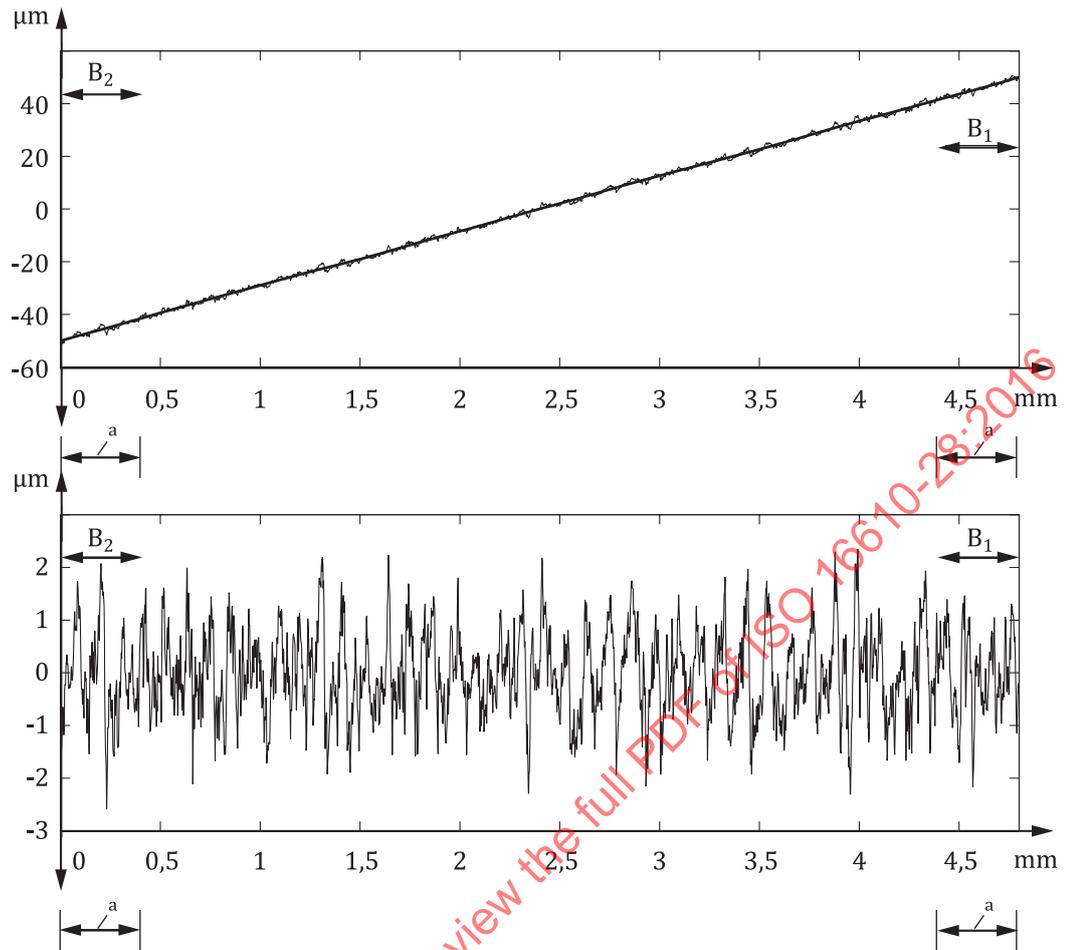
Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a End effect region.

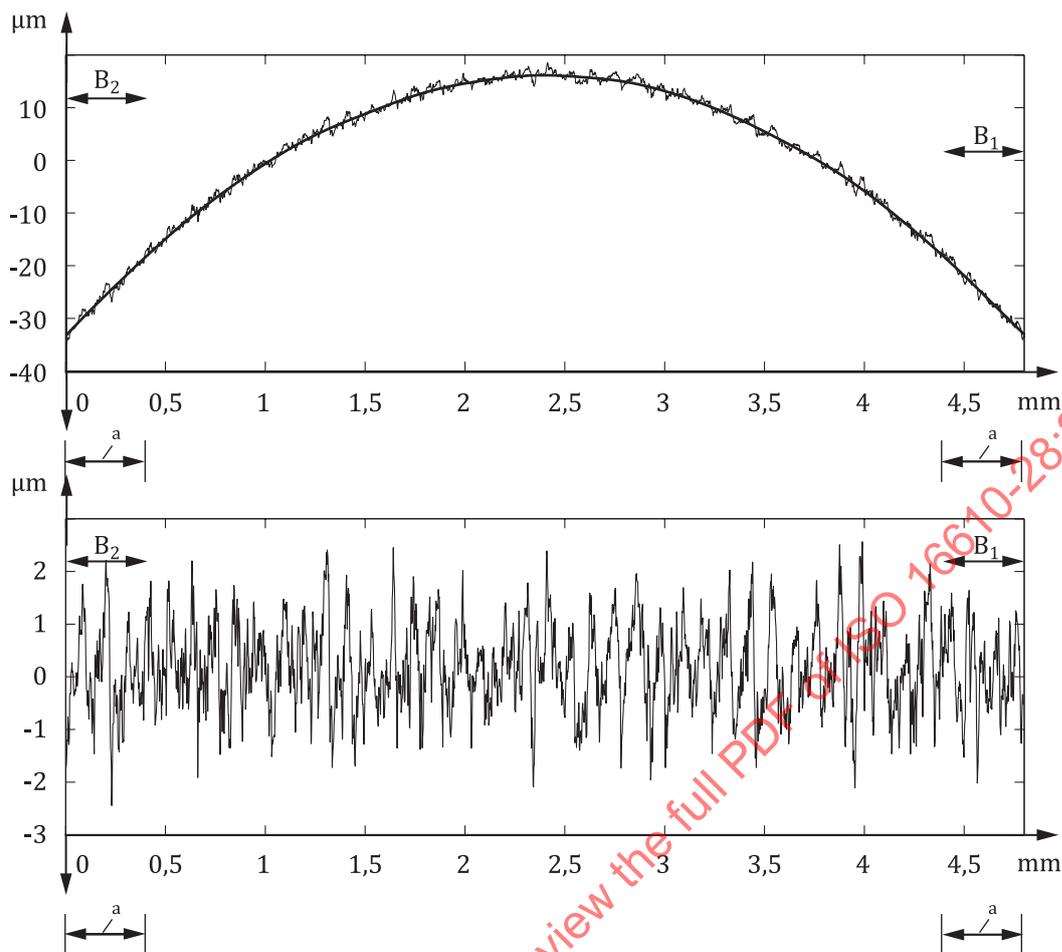**Figure 8 — Retained moment condition $n = 1$, using standardized Gaussian filter**

Due to the chosen moment condition, $n = 1$, the filter can exactly approximate a tilted profile as shown in Figure 9.

Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a End effect region.

Figure 9 — Retained moment condition *n* = 1, using standardized Gaussian filter and tilted profile

As shown in Figure 10, moment condition *n* = 1 leads also to a good approximation of a profile with an arc.
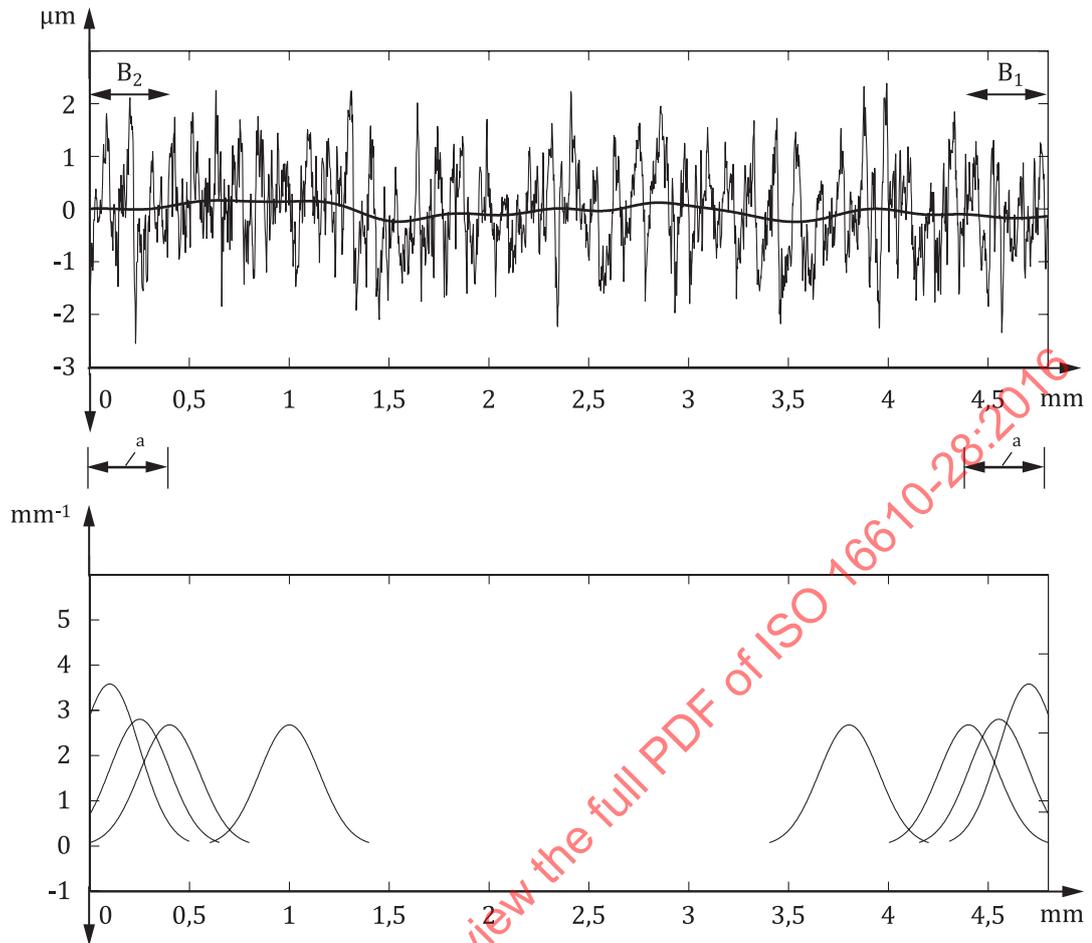
Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a   End effect region.

Figure 10 — Retained moment condition $n = 1$, using standardized Gaussian filter and profile with arc

EXAMPLE 2   Retaining only the 0th moment leads to a very simple expression for the correction function:

$$b_0\left(x\right) = 1 / \mu_0\left(x\right) \tag{13}$$
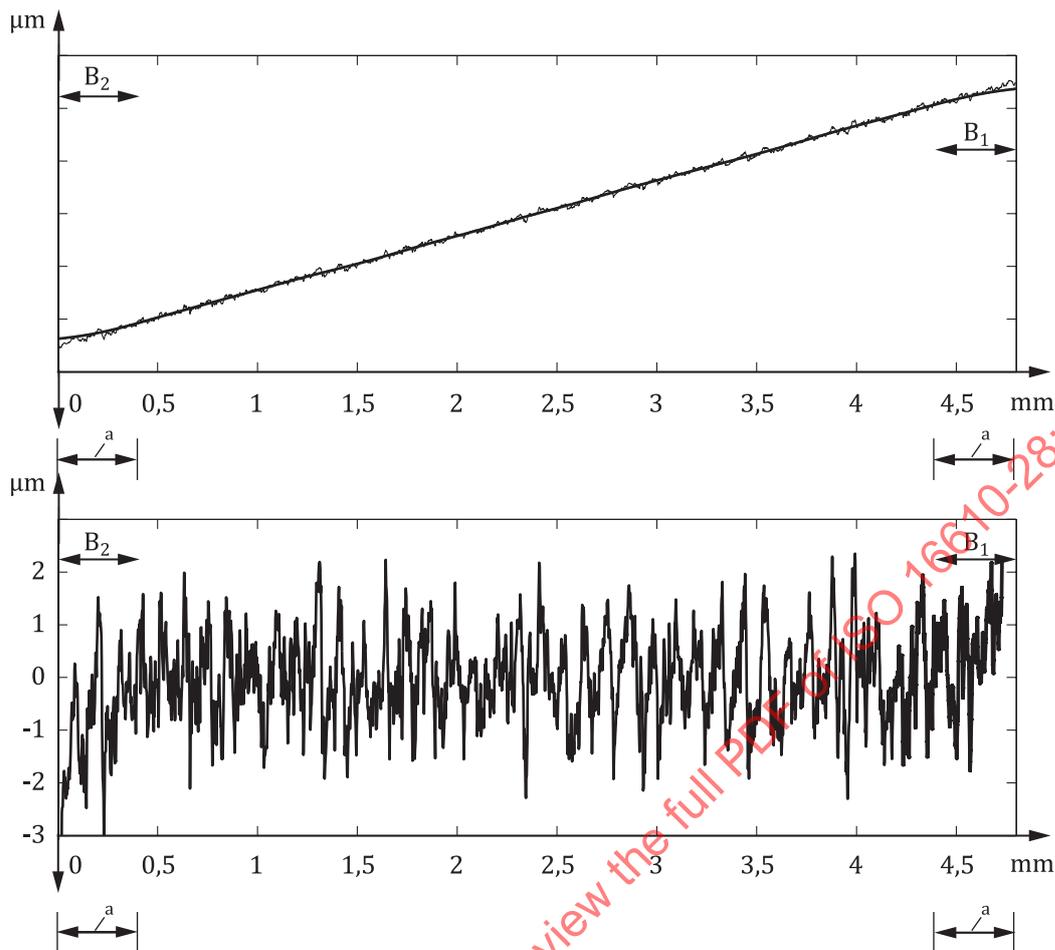
The result is shown in <u>Figure 11</u> for $l_1 = l_2 = \lambda_c/2$.

Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a   End effect region.

Figure 11 — Retained moment condition $n = 0$, using standardized Gaussian filter and $n = 0$

In contrast to Example 1, the filter has no vanishing moment and leads to a distorted roughness profile at the end effect region when filtering a highly tilted profile (see Figure 12).

Key

B1 l_2 right end effect region

B2 l_1 left end effect region

a   End effect region.

Figure 12 — Retained moment condition $n = 0$, using standardized Gaussian filter and tilted profile

5   Recommendations

5.1   Default end correction

If not otherwise specified, the default end correction shall be no end correction.

It is recommended that if an end correction is applied the moment retainment criterion be used (see 4.3).

5.2 End correction designations

End corrections according to this document are designated:

End Correction	Designation	Parameter(s)
Zero Padding	$E_C\ Z_P\ l_1\ l_2$	Profile is padded with zeros over length l_2 at the left side and over length l_1 at the right side of the profile
Linear Extrapolation	$E_C\ L_E\ l_1\ l_2$	Profile is extrapolated over length l_2 at the left side and over length l_1 at the right side of the profile
Line symmetrical reflection	$E_C\ L_{SR}\ l_1\ l_2$	Profile is extended over length l_2 at the left side and over length l_1 at the right side of the profile
Point symmetrical reflection	$E_C\ P_{SR}\ l_1\ l_2$	Profile is extended over length l_2 at the left side and over length l_1 at the right side of the profile
Moment retainment criterion	$E_C\ M_{RC}\ n\ l_1\ l_2$	n is the number of moments. Profile is extended over length l_2 at the left side and over length l_1 at the right side of the profile

Annex A
(normative)

Filters according to ISO 16610 with automatic correction of end effects

Figures A.1 and A.2 show the mean line and weighting function of filters according to ISO 16610-22 (spline filter) and ISO 16610-31 (Gaussian regression filter) with an automatic correction of end effects.
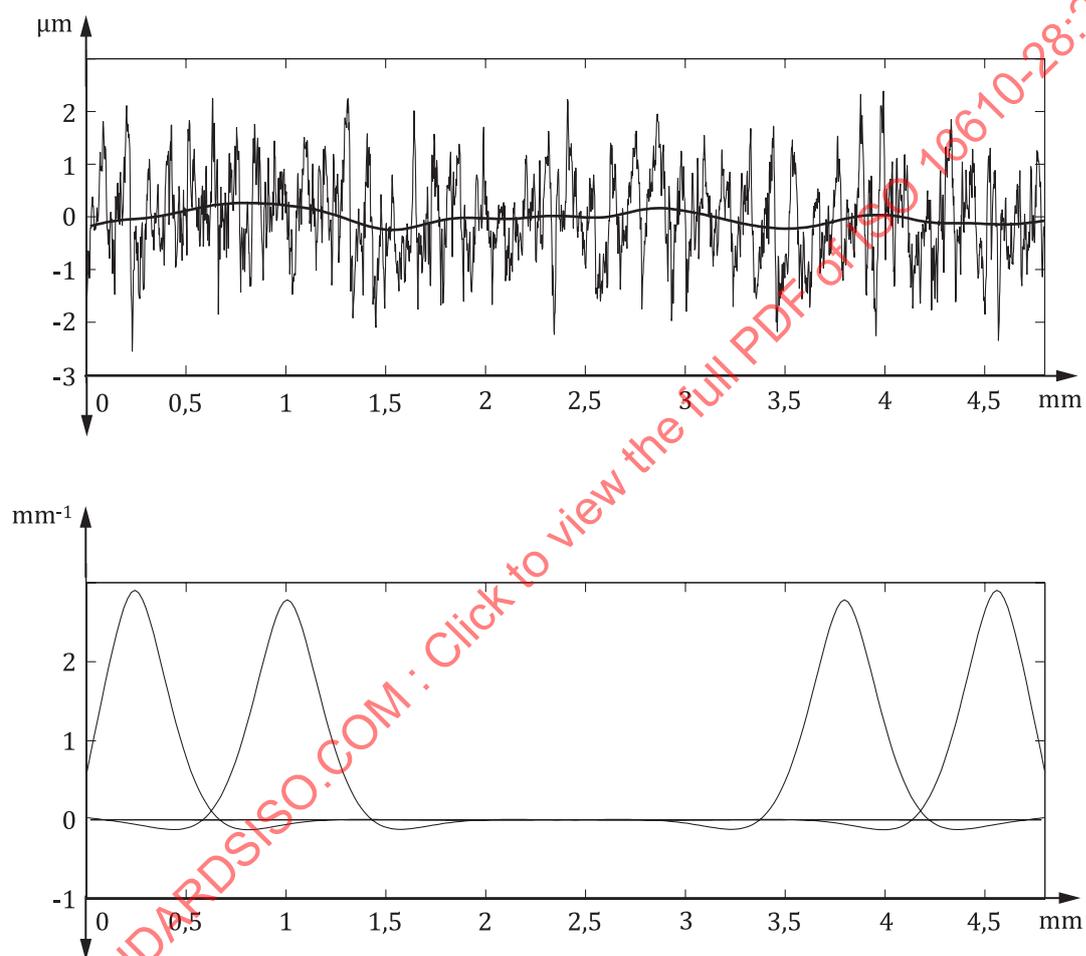

Figure A.1 — Automatic correction of end effects of spline filter