International Standard

ISO 16610-21

Geometrical product specifications (GPS) — Filtration —

Part 21: **Linear profile filters: Gaussian filters**

Spécification géométrique des produits (GPS) — Filtrage —

Partie 21: Filtres de profil linéaires: Filtres gaussiens

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 290, *Dimensional and geometrical product specification and verification*, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

This second edition cancels and replaces the first edition (ISO 16610-21:2011), which has been technically revised.

The main changes are as follows:

— providing implementation details for open and closed profiles,

— providing the treatment of end effects.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences chain links C and E in the GPS matrix structure.

The ISO GPS matrix model given in ISO 14638 gives an overview of the ISO GPS system of which this document is a part. The fundamental rules of ISO GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this document, unless otherwise indicated.

For more information on the relationship of this document to the filtration matrix model, see Annex C.

For more detailed information on the relation of this document to other standards and the GPS matrix model, see Annex D.

This document develops the terminology and concepts of linear Gaussian filters for surface profiles. Linear Gaussian filters for surface profiles have a transmission of 50 % for sinusoidal surface profiles with wavelengths equal to the cut-off wavelength. It separates the large- and small-scale lateral components of surface profiles in such a way that the surface profiles can be reconstructed without altering.

Geometrical product specifications (GPS) — Filtration —

Part 21:
Linear profile filters: Gaussian filters

1 Scope

This document specifies linear Gaussian filters for the filtration of surface profiles. It defines, in particular, how to separate large- and small-scale lateral components of surface profiles.

The concept presented for closed profiles are applicable to the case of roundness filtration. Where appropriate, these concept can be extended to generalized closed profiles, especially for surface profiles with re-entrant features.

Implementation details are given in Annex A for open profiles and Annex B for closed profiles.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1, ISO 16610-20, ISO/IEC Guide 99 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
surface profile
line resulting from the intersection between a surface portion and an ideal plane

Note 1 to entry: The orientation of the ideal plane is usually perpendicular to the tangent plane of the surface portion.

Note 2 to entry: See ISO 17450-1:2011, 3.3 and 3.3.1, for the definition of an ideal plane.

[SOURCE: ISO 16610-1:2015, 3.1.2, modified — Note 2 to entry replaced.]

3.1.1
open profile
finite length *surface profile* (3.1) with two ends

Note 1 to entry: An open profile has a compact support, i.e. within a certain interval the height values of an open profile can be equal to any real number. Outside the interval, the height values of an open profile are set to zero.

[SOURCE: ISO 16610-1:2015, 3.7, modified — Note 1 to entry replaced.]

3.1.2
unbounded open profile
infinite length *surface profile* (3.1) without ends

Note 1 to entry: In this document, the term "unbounded" refers to the X-axis.

Note 2 to entry: The concept of the unbounded open profile is ideal and do not apply to real surface profiles.

3.1.3
closed profile
connected finite length *surface profile* (3.1) without ends

Note 1 to entry: A closed profile is a closed curve which is periodic with the finite period length *L*.

Note 2 to entry: A typical example of a closed profile is one from a roundness measurement.

[SOURCE: ISO 16610-1:2015, 3.8, modified — Note 1 to entry replaced and Note 2 to entry added.]

3.2
linear profile filter
profile filter which separates *surface profiles* (3.1) into large- and small-scale lateral components and is also a linear function

Note 1 to entry: If F is a function and X and Y are surface profiles, and if a and b are independent from X and Y, then F being a linear function implies $F(a\,X + b\,Y) = a\,F(X) + b\,F(Y)$.

[SOURCE: ISO 16610-20:2015, 3.1, modified — In definition "profiles" replaced by "surface profiles" and "long wave" and "short wave" replaced by "large-scale lateral" and "small-scale lateral"; Note 1 to entry replaced.]

3.3
weighting function
function to calculate large-scale lateral components by convolution of the surface profile heights with this function

Note 1 to entry: The convolution (see ISO 16610-20:2015, 4.1) performs a weighted moving average of the surface profile heights. The weighting function, reflected at the X-axis, defines the weighting coefficients for the averaging process.

3.4
transmission characteristic of a filter
characteristic that indicates the amount by which the amplitude of a sinusoidal surface profile is attenuated as a function of its wavelength

Note 1 to entry: The transmission characteristic is the Fourier transformation of the *weighting function* (3.3).

[SOURCE: ISO 16610-20:2015, 3.4, modified — "surface" added before "profile".]

3.5
cut-off wavelength
λ_c
wavelength of a sinusoidal surface profile of which 50 % of the amplitude is transmitted by the profile

Note 1 to entry: Linear profile filters are identified by the filter type and the cut-off wavelength value.

Note 2 to entry: The cut-off wavelength is the nesting index for linear profile filters.

[SOURCE: ISO 16610-20:2015, 3.5, modified — "surface" added before "profile", "profile filter" replaced by "profile" and in Note 2 to entry “recommended” deleted.]

3.6
undulations per revolution
UPR
integer number of sinusoidal undulations contained in a *closed profile* (3.1.3)

Note 1 to entry: In this document, UPR is a frequency and is denoted by f.

3.7
cut-off frequency in undulations per revolution
f_c
frequency in UPR of a sinusoidal *closed profile* (3.1.3) of which 50 % of the amplitude is transmitted by the profile filter

4 Characteristics of the Gaussian filter for unbounded open profiles

4.1 General

In this clause, the ideal filtration of unbounded open profiles is considered. For this purpose, the unbounded open profiles are convolved with the ideal Gaussian weighting function of infinite length. The treatment of open profiles is considered in Annex A.

4.2 Gaussian weighting function

The Gaussian weighting function with cut-off wavelength λ_c (see Figure 1) for unbounded open profiles is defined according to Formula (1):

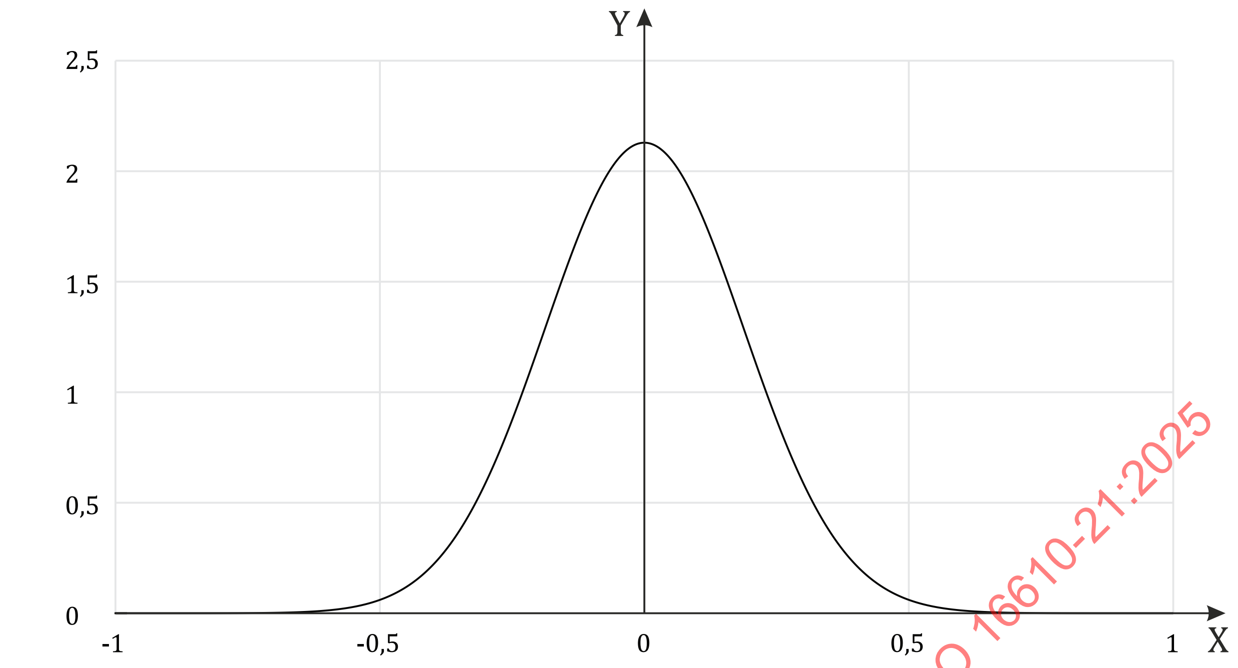
$$s(v) = \frac{1}{\alpha \lambda_c} e^{-\pi \left(\frac{v}{\alpha \lambda_c} \right)^2} \tag{1}$$

where

v	is the distance from the centre (maximum) of the Gaussian weighting function;
$s(v)$	is the Gaussian weighting function depending on v;
λ_c	is the cut-off wavelength;
α	is the constant to provide 50 % transmission characteristic at the cut-off wavelength λ_c.

The constant α is given by Formula (2):

$$\alpha = \sqrt{\frac{\ln 2}{\pi}} \approx 0{,}469\ 7 \approx \frac{318}{677} \approx \frac{31}{66} \tag{2}$$



Key

X v / λ_c

Y $s(v)\,\lambda_c$

Figure 1 — Weighting function of the Gaussian filter for unbounded open profiles

4.3 Filter equations

4.3.1 Determination of the large-scale lateral component

The large-scale lateral component of an unbounded open profile is determined by convolution of the heights of this unbounded open profile with the Gaussian weighting function according to Formula (3):

$$w(x) = \int_{-\infty}^{\infty} z(u)\, s(x-u)\, \mathrm{d}u \tag{3}$$

where

x	is the given x-coordinate;
u	is the integration variable along the X-axis of the unbounded open profile;
$z(u)$	is the unbounded open profile depending on u;
$s(x-u)$	is the Gaussian weighting function reflected at the ordinate axis at the given x-coordinate and depending on u;
$w(x)$	is the large-scale lateral component of the unbounded open profile depending on x.

4.3.2 Determination of the small-scale lateral component

The small-scale lateral component of an unbounded open profile is determined by subtracting the large-scale lateral component of this unbounded open profile, Formula (3), from this unbounded open profile according to Formula (4):

$$r(x) = z(x) - w(x) \tag{4}$$

where

x is the given x-coordinate;

$z(x)$ is the unbounded open profile depending on x;

$w(x)$ is the large-scale lateral component of the unbounded open profile depending on x;

$r(x)$ is the small-scale lateral component of the unbounded open profile depending on x.

4.4 Transmission characteristics

4.4.1 Transmission characteristic for the large-scale lateral component

The transmission characteristic for the large-scale lateral component of an unbounded open profile (see Figure 2) is determined from the Gaussian weighting function by means of the Fourier transformation and is given by Formula (5):

$$\frac{a_1}{a_0} = e^{-\pi\left(\frac{\alpha\,\lambda_c}{\lambda}\right)^2} = 2^{-\left(\frac{\lambda_c}{\lambda}\right)^2} \tag{5}$$

where

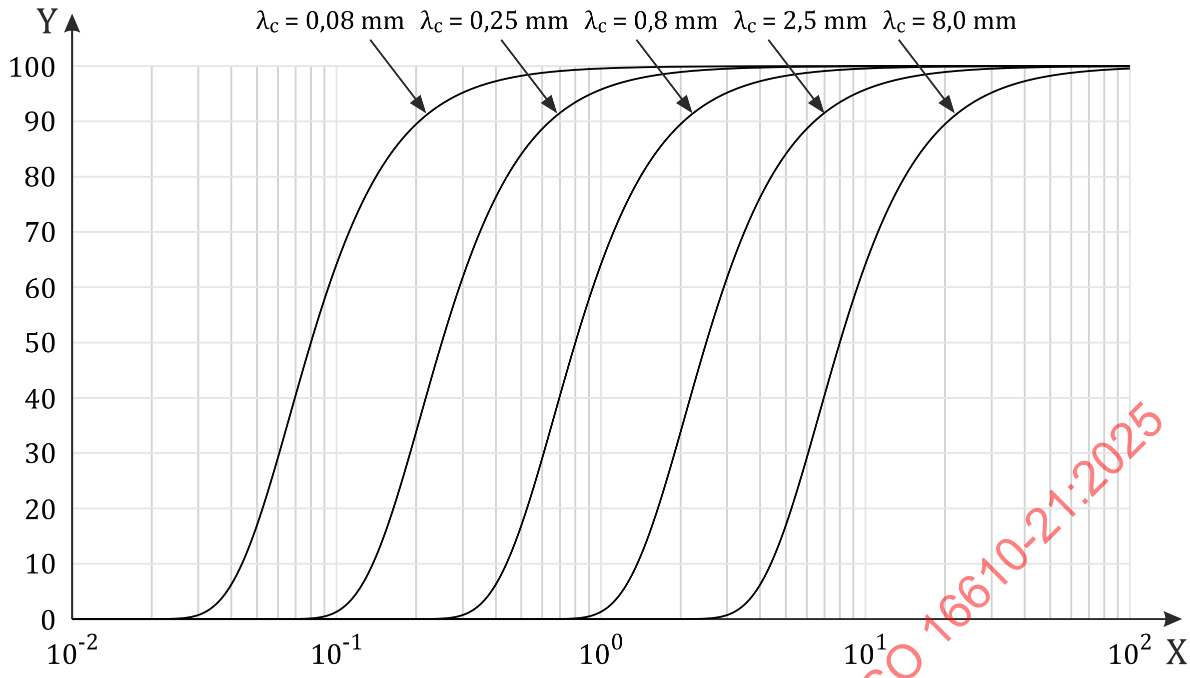
a_0 is the amplitude of a sinusoidal unbounded open profile before filtration;

a_1 is the amplitude of this sinusoidal unbounded open profile after filtration;

λ is the wavelength of this sinusoidal unbounded open profile;

λ_c is the cut-off wavelength;

α is the constant to provide 50 % transmission characteristic at the cut-off wavelength λ_c and is defined according to Formula (2).



Key

X wavelength λ in mm

Y amplitude transmission a_1 / a_0 in per cent

Figure 2 — Transmission characteristic for the large-scale lateral component of unbounded open profiles

4.4.2 Transmission characteristic for the small-scale lateral component

The transmission characteristic for the small-scale lateral component of an unbounded open profile (see Figure 3) is complementary to the transmission characteristic for the large-scale lateral component of this unbounded open profile, Formula (5), and is given by Formula (6):

$$\frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} = 1 - 2^{-\left(\frac{\lambda_c}{\lambda}\right)^2} \quad (6)$$

where

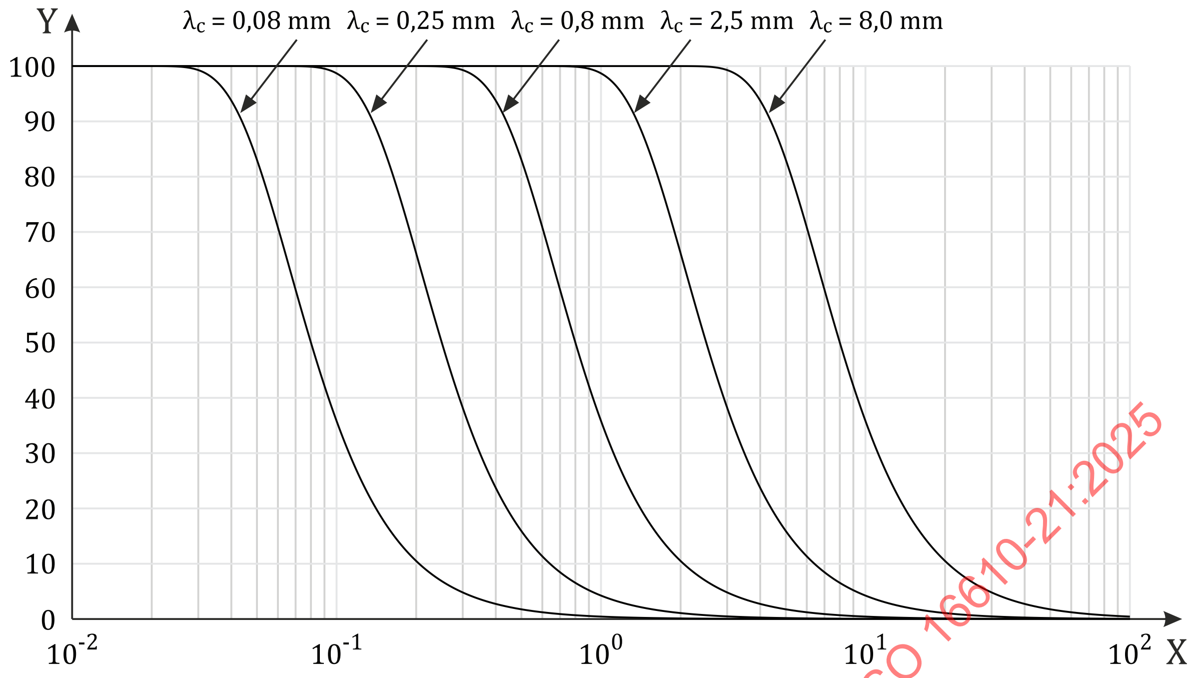
a_0 is the amplitude of a sinusoidal unbounded open profile before filtration;

a_1 is the amplitude of this sinusoidal unbounded open profile after filtration;

a_2 is the amplitude of the small-scale lateral component of this sinusoidal unbounded open profile;

λ is the wavelength of this sinusoidal unbounded open profile;

λ_c is the cut-off wavelength.



Key

X wavelength λ in mm

Y amplitude transmission a_2 / a_0 in per cent

Figure 3 — Transmission characteristic for the small-scale lateral component of unbounded open profiles

5 Characteristics of the Gaussian filter for closed profiles

5.1 General

In this clause, the ideal filtration of closed profiles applied to roundness profiles is considered. For this purpose, the closed profiles are convolved with the ideal Gaussian weighting function of infinite length. The treatment of a truncated Gaussian weighting function with finite length is considered in Annex B.

5.2 Gaussian weighting function

The Gaussian weighting function with cut-off frequency in UPR f_c (see Figure 4) for closed profiles is defined according to Formula (7):

$$\tilde{s}(v) = \frac{f_c}{\tilde{\alpha} L} e^{-\pi \left(\frac{v f_c}{\tilde{\alpha} L} \right)^2} \tag{7}$$

where

v is the distance from the centre (maximum) of the Gaussian weighting function;

$\tilde{s}(v)$ is the Gaussian weighting function depending on v;

f_c is the cut-off frequency in UPR;

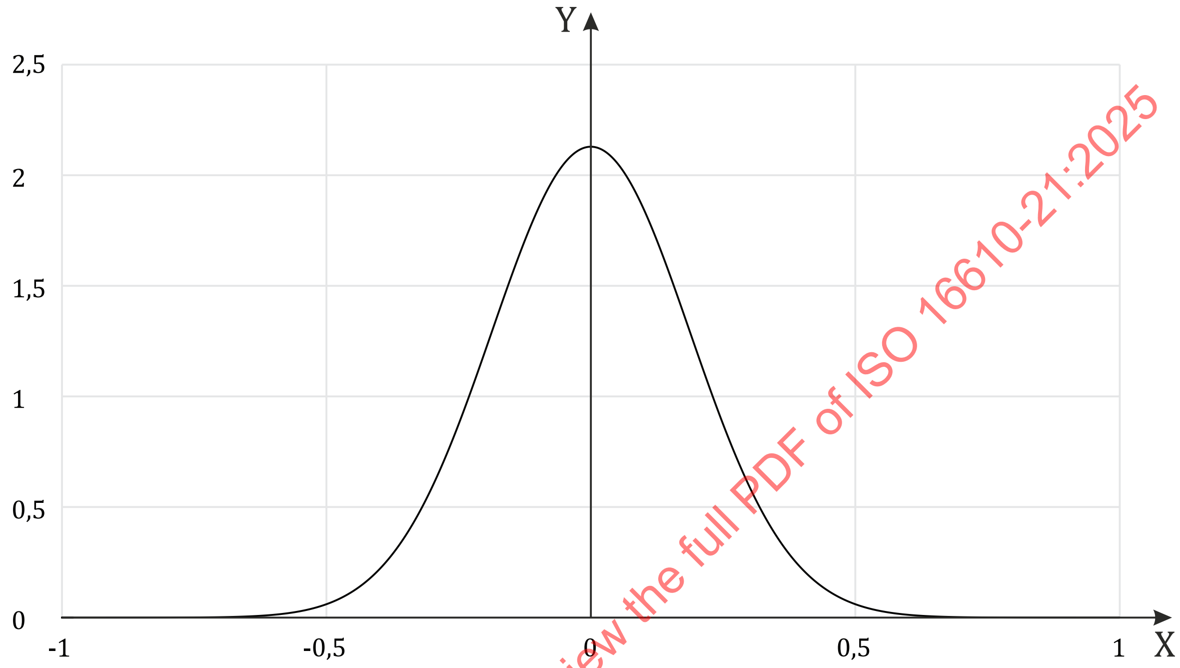
L is the period length of the closed profile;

$\tilde{\alpha}$ is the constant to provide 50 % transmission characteristic at the cut-off frequency in UPR f_c.

The constant $\tilde{\alpha}$ is given by Formula (8):

$$\tilde{\alpha} = \sqrt{\frac{\ln 2}{\pi}} \approx 0{,}469\ 7 \approx \frac{318}{677} \approx \frac{31}{66} \tag{8}$$

NOTE With the relationship between the finite period length of the closed profile L and the cut-off frequency in UPR f_c, the cut-off wavelength λ_c is given by $\lambda_c = L / f_c$. This relationship is applied for Formula (7).



Key

X $v \cdot f_c / L$

Y $\tilde{s}(v) \cdot L / f_c$

Figure 4 — Weighting function of the Gaussian filter for closed profiles

5.3 Filter equations

5.3.1 Determination of the large-scale lateral component

The large-scale lateral component of a closed profile is determined by convolution of the heights of this closed profile with the Gaussian weighting function according to Formula (9):

$$\tilde{w}(x) = \int_{-\infty}^{\infty} \tilde{z}(u)\,\tilde{s}(x-u)\,\mathrm{d}u \tag{9}$$

where

x is the given x-coordinate;

u is the integration variable along the X-axis of the closed profile;

$\tilde{z}(u)$ is the closed profile depending on u;

$\tilde{s}(x-u)$ is the Gaussian weighting function reflected at the ordinate axis at the given *x*-coordinate and depending on *u*;

$\tilde{w}(x)$ is the large-scale lateral component of the closed profile depending on *x*.

NOTE For a closed profile $\tilde{z}(x)$ applies $\tilde{z}(x)=\tilde{z}(x+L)$, where L is the finite period length of the closed profile, thus $\tilde{w}(x)=\tilde{w}(x+L)$.

5.3.2 Determination of the small-scale lateral component

The small-scale lateral component of a closed profile is determined by subtracting the large-scale lateral component of this closed profile, Formula (9), from this closed profile according to Formula (10):

$$\tilde{r}(x)=\tilde{z}(x)-\tilde{w}(x) \tag{10}$$

where

x is the given *x*-coordinate;

$\tilde{z}(x)$ is the closed profile depending on *x*;

$\tilde{w}(x)$ is the large-scale lateral component of the closed profile depending on *x*;

$\tilde{r}(x)$ is the small-scale lateral component of the closed profile depending on *x*.

NOTE $\tilde{z}(x)$ and $\tilde{w}(x)$ are periodic with the finite period length *L*, thus $\tilde{r}(x)=\tilde{r}(x+L)$.

5.4 Transmission characteristics

5.4.1 Transmission characteristic for the large-scale lateral component

The transmission characteristic for the large-scale lateral component of a closed profile (see Figure 5) is determined from the Gaussian weighting function by means of the Fourier transformation and is given by Formula (11):

$$\frac{\tilde{a}_1}{\tilde{a}_0}=\mathrm{e}^{-\pi\left(\frac{\tilde{\alpha} f}{f_\mathrm{c}}\right)^2}=2^{-\left(\frac{f}{f_\mathrm{c}}\right)^2} \tag{11}$$

where

$\tilde{a}_0$ is the amplitude of a sinusoidal closed profile before filtration;

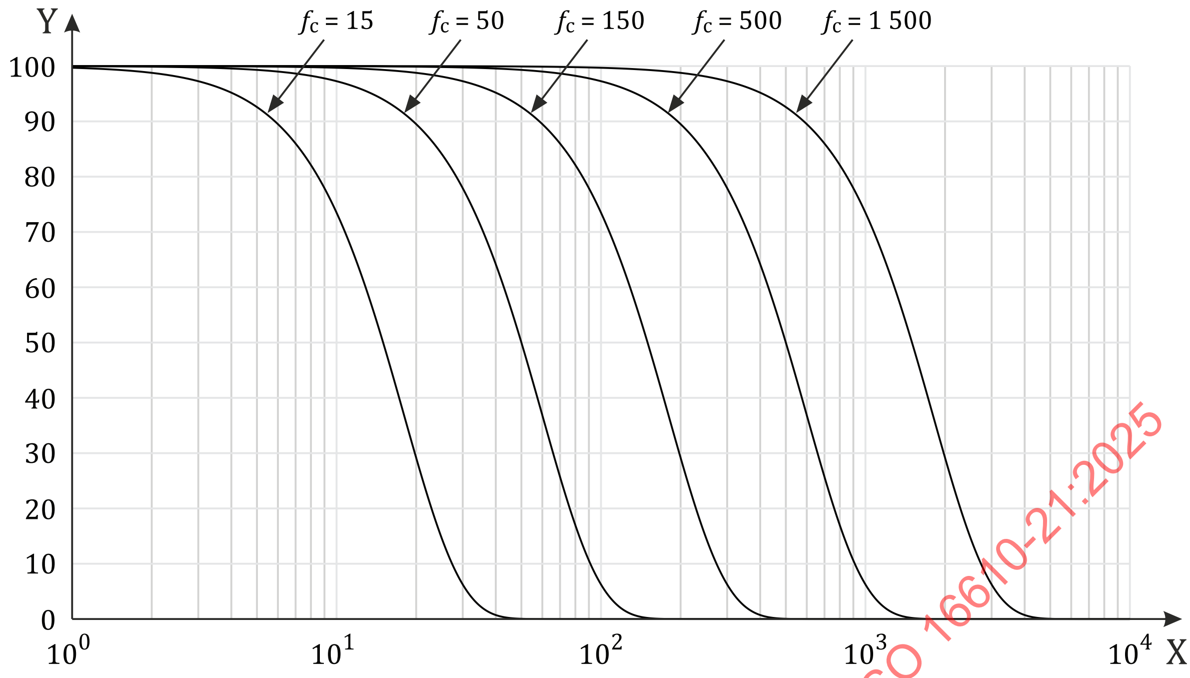
$\tilde{a}_1$ is the amplitude of this sinusoidal closed profile after filtration;

f is the frequency in UPR of this sinusoidal closed profile;

f_c is the cut-off frequency in UPR;

$\tilde{\alpha}$ is the constant to provide 50 % transmission characteristic at the cut-off frequency in UPR f_c and is defined according to Formula (8).

NOTE The relationship between the frequency in UPR *f* and the wavelength λ is given by $\lambda=L/f$, where *L* is the period length of the closed profile.



Key

X frequency f in UPR

Y amplitude transmission $\tilde{a}_1 / \tilde{a}_0$ in per cent

Figure 5 — Transmission characteristic for the large-scale lateral component of closed profiles

5.4.2 Transmission characteristic for the small-scale lateral component

The transmission characteristic for the small-scale lateral component of a closed profile (see Figure 6) is complementary to the transmission characteristic for the large-scale lateral component of this closed profile, Formula (11), and is given by Formula (12):

$$\frac{\tilde{a}_2}{\tilde{a}_0} = 1 - \frac{\tilde{a}_1}{\tilde{a}_0} = 1 - 2^{-\left(\frac{f}{f_c}\right)^2} \qquad (12)$$

where

$\tilde{a}_0$ is the amplitude of a sinusoidal closed profile before filtration;

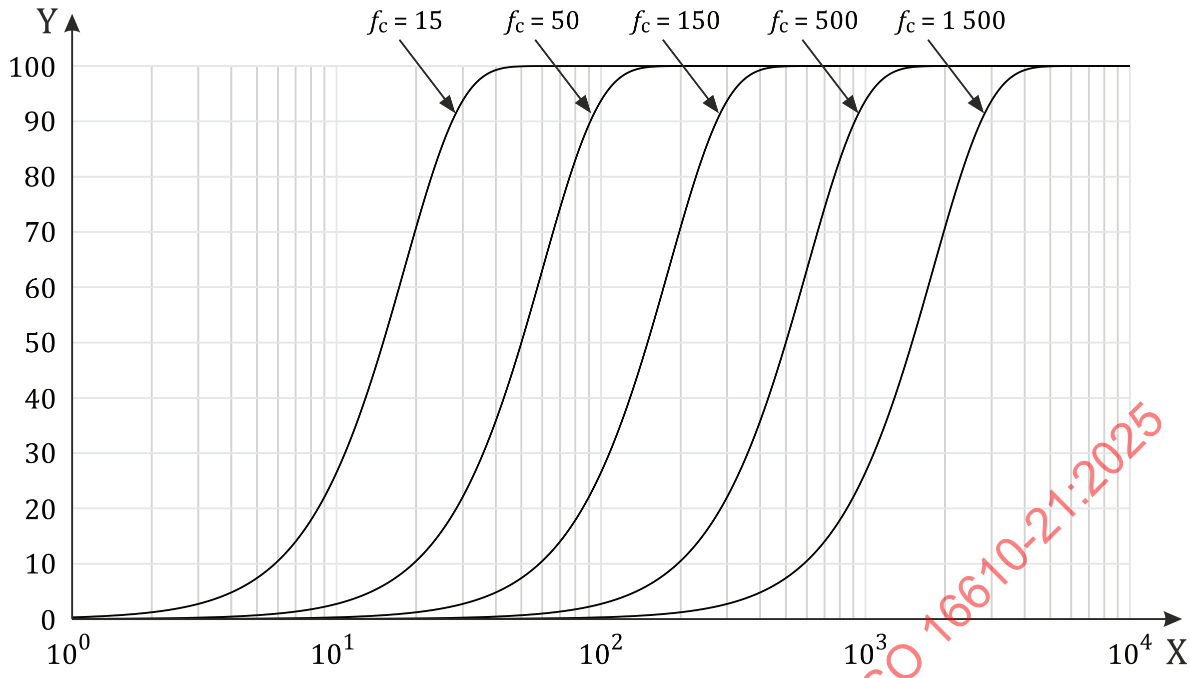
$\tilde{a}_1$ is the amplitude of this sinusoidal closed profile after filtration;

$\tilde{a}_2$ is the amplitude of the small-scale lateral component of this sinusoidal closed profile;

f is the frequency in UPR of this sinusoidal closed profile;

f_c is the cut-off frequency in UPR.

NOTE The relationship between the frequency in UPR f and the wavelength λ is given by $\lambda = L / f$, where L is the period length of the closed profile.



Key

X frequency f in UPR

Y amplitude transmission $\tilde{a}_2 / \tilde{a}_0$ in per cent

Figure 6 — Transmission characteristic for the small-scale lateral component of closed profiles

6 Series of nesting index values

For unbounded open profiles or open profiles and if not otherwise specified, the nesting index (cut-off wavelength λ_c) from the following series of values shall be used:

$\lambda_c = \ldots;\ 2{,}5\,\mu m;\ 8\,\mu m;\ 25\,\mu m;\ 80\,\mu m;\ 250\,\mu m;\ 0{,}8\,mm;\ 2{,}5\,mm;\ 8\,mm;\ 25\,mm;\ \ldots$

For roundness profiles exemplarily used as closed profiles in this document and if not otherwise specified, the nesting index (cut-off frequency in UPR f_c) from the following series of values shall be used:

$f_c = 5;\ 15;\ 50;\ 150;\ 500;\ 1\ 500;\ 5\ 000;\ \ldots$

7 Filter designation

Gaussian filters in accordance with this document and in accordance with ISO 16610-1:2015, Clause 5, shall be designated as:

FPLG

NOTE FPLG corresponds to filter for surface profiles of the linear Gauss type.

Annex A
(informative)

Implementation details of the Gaussian filter for open profiles

A.1 General

This annex describes the implementation of the Gaussian filter for open profiles. Clause A.2 describes the implementation by truncation of the Gaussian weighting function. This procedure also results in truncation of the evaluable surface profile. Clause A.3 describes the implementation by the method of moment retainment. This method does not truncate the evaluable surface profile. Examples are given for both implementation methods.

A.2 Treatment of the end effect regions by truncation

A.2.1 Truncated Gaussian weighting function

In theory, the domain of Gaussian weighting function is from minus infinity to plus infinity. For any practicable implementation the Gaussian weighting function is truncated. Taking into account that the Gaussian weighting function approaches zero in regions far enough from its centre (for $|v| > \lambda_c$ it follows that $s_c(v)\,\lambda_c < 1{,}394\ 2\times10^{-6}$), the truncated Gaussian weighting function can be expressed by Formula (A.1):

$$s_c(v) = \begin{cases} \dfrac{1}{\alpha\,\lambda_c}\, e^{-\pi\left(\frac{v}{\alpha\,\lambda_c}\right)^2} & -L_c\,\lambda_c \le v \le L_c\,\lambda_c \\ 0 & \text{otherwise} \end{cases} \qquad \text{(A.1)}$$

where

v	is the distance from the centre (maximum) of the truncated Gaussian weighting function;
$s_c(v)$	is the truncated Gaussian weighting function depending on v;
λ_c	is the cut-off wavelength;
L_c	is the truncation constant with $L_c > 0$;
α	is the constant defined according to Formula (2).

A.2.2 Filter equations

A.2.2.1 Determination of the large-scale lateral component

Using the truncated Gaussian weighting function, the large-scale lateral component of an open profile is determined according to Formula (A.2):

$$w(x) = \begin{cases} \dfrac{\displaystyle\int_{x-L_c\,\lambda_c}^{x+L_c\,\lambda_c} z(u)\, s_c(x-u)\,du}{b_0} & \{x \mid x-L_c\,\lambda_c \in \Omega \wedge x+L_c\,\lambda_c \in \Omega\} \\ 0 & \text{otherwise} \end{cases} \qquad \text{(A.2)}$$

where

x	is the given x-coordinate;
u	is the integration variable along the X-axis of the open profile;
$z(u)$	is the open profile depending on u;
$s_c(x-u)$	is the truncated Gaussian weighting function reflected at the ordinate axis at the given x-coordinate and depending on u;
λ_c	is the cut-off wavelength;
L_c	is the truncation constant with $L_c > 0$;
$w(x)$	is the large-scale lateral component of the open profile depending on x;
Ω	is the finite interval, expressed as a set, in which the open profile can be any real number;
b_0	is a value to provide 100 % transmission characteristic at the wavelength $\lambda \to \infty$.

The value b_0 is calculated according to Formula (A.3):

$$b_0 = \int_{-L_c \lambda_c}^{L_c \lambda_c} s_c(v)\,dv \tag{A.3}$$

A.2.2.2 Determination of the small-scale lateral component

The small-scale lateral component of an open profile is determined by subtracting the large-scale lateral component of this open profile, Formula (A.2), from this open profile according to Formula (A.4):

$$r(x) = z(x) - w(x) \tag{A.4}$$

where

x	is the given x-coordinate;
$z(x)$	is the open profile depending on x;
$w(x)$	is the large-scale lateral component of the open profile depending on x;
$r(x)$	is the small-scale lateral component of the open profile depending on x.

A.2.3 Transmission characteristics

A.2.3.1 Transmission characteristic for the large-scale lateral component

The transmission characteristic for the large-scale lateral component of an open profile is determined from the truncated Gaussian weighting function by means of the Fourier transformation and is given by Formula (A.5):

$$\frac{a_1}{a_0} = 2^{-\left(\frac{\lambda_c}{\lambda}\right)^2} \frac{\mathrm{Re}\left[\mathrm{erf}\left(\frac{\pi L_c}{\sqrt{\ln 2}} + \mathrm{i}\sqrt{\ln 2}\,\frac{\lambda_c}{\lambda}\right)\right]}{\mathrm{erf}\left(\frac{\pi L_c}{\sqrt{\ln 2}}\right)} \tag{A.5}$$

where

a_0	is the amplitude of a sinusoidal open profile before filtration;
a_1	is the amplitude of this sinusoidal open profile after filtration;
λ	is the wavelength of this sinusoidal open profile;
λ_c	is the cut-off wavelength;
L_c	is the truncation constant with $L_c > 0$;
$\mathrm{erf}(k)$	is the error function for complex numbers $k \in \mathbf{C}$;
i	is the imaginary unit ($\mathrm{i}^2 = -1$);
Re	is the real part of a complex number.

A.2.3.2 Transmission characteristic for the small-scale lateral component

The transmission characteristic for the small-scale lateral component of an open profile is complementary to the transmission characteristic for the large-scale lateral component for this open profile, Formula (A.5), and is given by Formula (A.6):

$$\frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} = 1 - 2^{-\left(\frac{\lambda_c}{\lambda}\right)^2} \frac{\mathrm{Re}\left[\mathrm{erf}\left(\frac{\pi L_c}{\sqrt{\ln 2}} + \mathrm{i}\sqrt{\ln 2}\,\frac{\lambda_c}{\lambda}\right)\right]}{\mathrm{erf}\left(\frac{\pi L_c}{\sqrt{\ln 2}}\right)} \quad \text{(A.6)}$$

where

a_0	is the amplitude of a sinusoidal open profile before filtration;
a_1	is the amplitude of this sinusoidal open profile after filtration;
a_2	is the amplitude of the small-scale lateral component of this sinusoidal open profile;
λ	is the wavelength of this sinusoidal open profile;
λ_c	is the cut-off wavelength;
L_c	is the truncation constant with $L_c > 0$;
$\mathrm{erf}(k)$	is the error function for complex numbers $k \in \mathbf{C}$;
i	is the imaginary unit ($\mathrm{i}^2 = -1$);
Re	is the real part of a complex number.

A.2.4 End effect regions

Due to the truncation of the Gaussian weighting function and the finite length of the open profile, the length of the filtered open profile is $2 L_c \lambda_c$ shorter than the length of the unfiltered open profile. The regions where the convolution cannot be applied is called the end effect regions of the filter.

A.2.5 Choice of truncation constant L_c

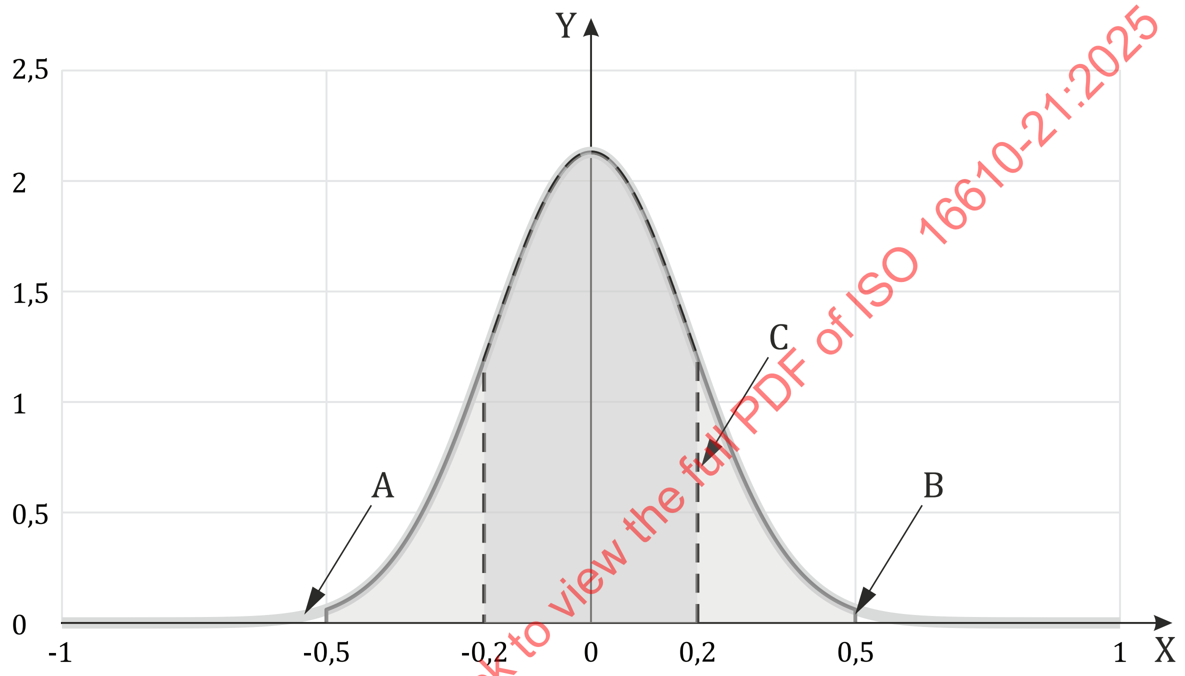
Different truncation values lead to different transmission errors, which in turn depend on the application. For applications where high precision is required, such as reference software, a truncation constant $L_c = 1$

is recommended. For general use, a truncation constant $L_c = 0{,}5$ can be used. The use of a truncation constant smaller than $L_c = 0{,}5$ is not recommended.

A.2.6 Examples

Figure A.1 shows truncated Gaussian weighting functions with different truncation constants L_c. For truncation constants L_c smaller than 0,5, the weighting function is significantly different from the ideal Gaussian weighting function, which also significantly changes the transmission characteristic of the filter.

To show the influence of the truncation constant L_c on the filter behaviour, the two truncation constants $L_c = 0{,}5$ and $L_c = 0{,}2$ are applied in comparison to the truncation constant $L_c = 1$.



Key

X v / λ_c

Y $s_c(v)\,\lambda_c$

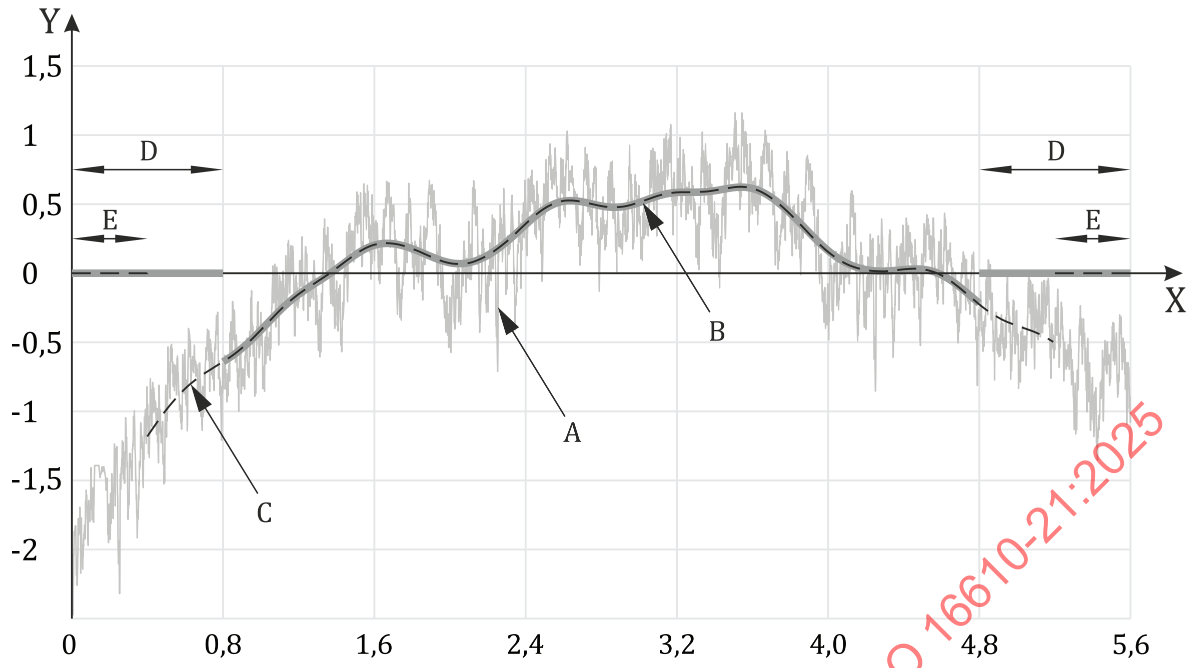
A truncation constant $L_c = 1$ (light grey solid line)

B truncation constant $L_c = 0{,}5$ (dark grey solid line)

C truncation constant $L_c = 0{,}2$ (black dashed line)

Figure A.1 — Truncated Gaussian weighting function for different truncation constants

Figure A.2 shows the filter behaviour on a ground surface profile in the presence of nominal shape. The selected cut-off wavelength is $\lambda_c = 0{,}8$ mm and the two different truncation constants are $L_c = 1$ (key item B) and $L_c = 0{,}5$ (key item C), respectively. The large-scale lateral components using $L_c = 1$ and $L_c = 0{,}5$ are almost identical.



Key

X length, in mm

Y height, in µm

A unfiltered open profile

B large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm, $L_c = 1$ (grey solid line)

C large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm, $L_c = 0{,}5$ (black dashed line)

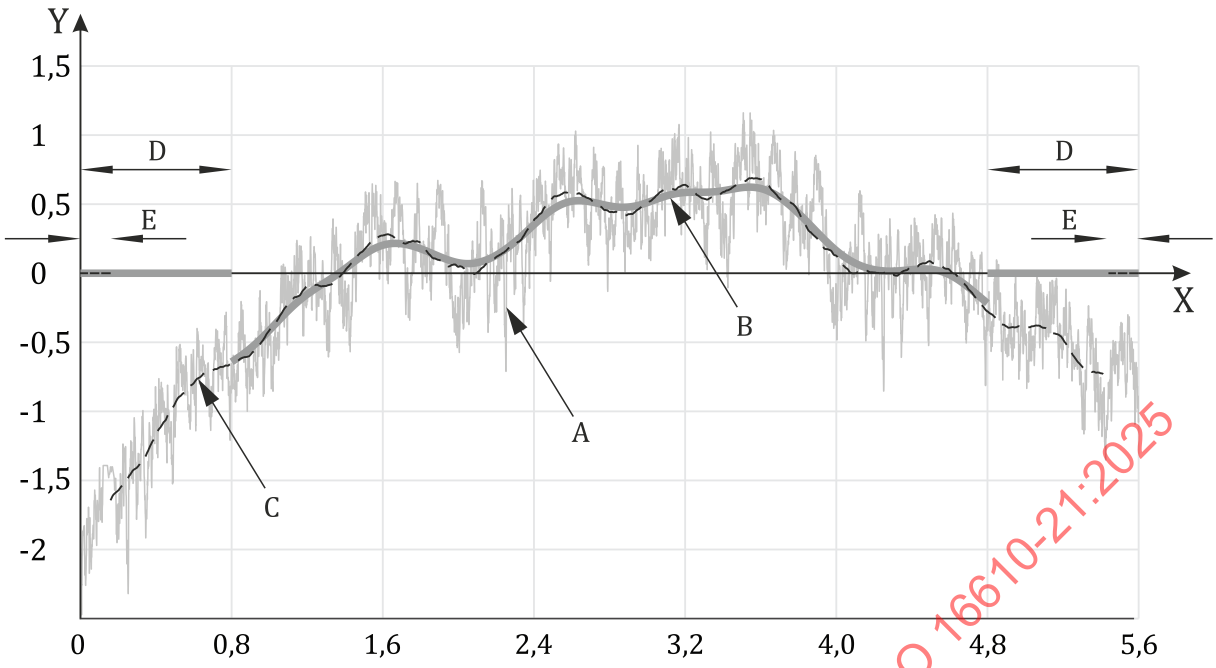
D end effect regions: $L_c = 1$ (grey solid line)

E end effect regions: $L_c = 0{,}5$ (black dashed line)

NOTE Shown are the large-scale lateral components of the surface profile for two different truncation constants $L_c = 1$ and $L_c = 0{,}5$ of the Gaussian weighting function.

Figure A.2 — Gaussian filtration applied to a ground surface profile in the presence of nominal shape

Figure A.3 shows the filter behaviour on the same ground surface profile (see Figure A.2) for the cut-off wavelength $\lambda_c = 0{,}8$ mm and the two different truncation constants $L_c = 1$ (key item B) and $L_c = 0{,}2$ (key item C). The large-scale lateral components of the surface profile using $L_c = 1$ and $L_c = 0{,}2$ respectively, differ. As can be seen in Figure A.1, the Gaussian weighting function was significantly changed by truncation with $L_c = 0{,}2$ and thus also its transmission characteristic.



Key

X length in mm

Y height in µm

A unfiltered open profile

B large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm, $L_c = 1$ (grey solid line)

C large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm, $L_c = 0{,}2$ (black dashed line)

D end effect regions: $L_c = 1$ (grey solid line)

E end effect regions: $L_c = 0{,}2$ (black dashed line)

NOTE Shown are the large-scale lateral components of the surface profile for two different truncation constants and of the Gaussian weighting function.

Figure A.3 — Gaussian filtration applied to a ground surface profile in the presence of nominal shape

A.3 Treatment of the end effect regions applying the method of moment retainment

A.3.1 General

This annex defines the moment retainment method (see Reference [6]) to treat the end effect regions of the Gaussian filter for open profiles. The moment retainment method performs an automatic boundary correction with defined properties (see A.3.3.1). By agreement with the customer, other methods described in ISO 16610-28 can be used to treat the end effect regions.

A.3.2 Filter equations

A.3.2.1 General

Depending on the selected cut-off wavelength λ_c and the truncation constant L_c, the length of a filtered open profile can be significantly smaller than the length of an unfiltered open profile due to the end effect regions. If the remaining evaluation part of the profile becomes too short after filtration (in order to retain valuable parameters), Formula (A.7) or Formula (A.9) should be applied to determine either the large-scale lateral component or the small-scale lateral component of the open profile.

A.3.2.2 Determination of the large-scale lateral component

Using the method of moment retainment, the large-scale lateral component of an open profile is determined according to Formula (A.7):

$$w(x)=\begin{cases}\dfrac{\displaystyle\int_{x-L_c\lambda_c}^{x+L_c\lambda_c} z(u)\left[b_2(x)-p\,b_1(x)(x-u)\right]s_c(x-u)\,\mathrm{d}u}{b_0(x)\,b_2(x)-p\,b_1(x)^2} & x\in\Omega\\ 0 & \text{otherwise}\end{cases} \tag{A.7}$$

where

x	is the given x-coordinate;
u	is the integration variable along the X-axis of the open profile;
$z(u)$	is the open profile depending on u;
$s_c(x-u)$	is the truncated Gaussian weighting function reflected at the ordinate axis at the given x-coordinate and depending on u;
λ_c	is the cut-off wavelength;
L_c	is the truncation constant with $L_c>0$;
$w(x)$	is the large-scale lateral component of the open profile depending on x;
Ω	is the finite interval, expressed as a set, in which the open profile can be any real number;
$b_j(x)$	are the correction terms ($j=0,1,2$) depending on x;
p	is the moment retainment at end effect regions ($p=0,1$).

The correction terms $b_j(x)$ are calculated according to Formula (A.8):

$$b_j(x)=\begin{cases}\displaystyle\int_{x-L_c\lambda_c}^{x+L_c\lambda_c} q(u)(x-u)^j\,s_c(x-u)\,\mathrm{d}u & x\in\Omega \text{ and } j=0,1,2\\ 0 & \text{otherwise}\end{cases} \tag{A.8}$$

where $q(u)=\begin{cases}1 & u\in\Omega\\ 0 & \text{otherwise}\end{cases}$

A.3.2.3 Determination of the small-scale lateral component

The small-scale lateral component of an open profile is determined by subtracting the large-scale lateral component of this open profile, Formula (A.7), from this open profile according to Formula (A.9):

$$r(x)=z(x)-w(x) \tag{A.9}$$

where

x	is the given x-coordinate;
$z(x)$	is the open profile depending on x;
$w(x)$	is the large-scale lateral component of the open profile depending on x;
$r(x)$	is the small-scale lateral component of the open profile depending on x;

A.3.3 Choice of parameter p and truncation constant L_c

A.3.3.1 General

The approach given by Formula (A.7) has the following properties:

— Within the inner region $\{x \mid x - L_c\ \lambda_c \in \Omega \wedge x + L_c\ \lambda_c \in \Omega\}$, the Gaussian weighting function for an open profile remains unchanged and the choice of parameter p has no effect on calculating the large-scale lateral component of the open profile. In this case, Formula (A.2) and Formula (A.7) lead to the same result for the large-scale lateral component $w(x)$.

— Within the inner region $\{x \mid x - L_c\ \lambda_c \in \Omega \wedge x + L_c\ \lambda_c \in \Omega\}$, the filtration result follows a 1st degree polynomial locally. This also applies to Formula (A.2).

— Within the end effect regions, the shape of the Gaussian weighting function changes depending on the distance to the profile ends, where parameter p controls the degree of following large-scale lateral components close to the open profile ends.

— Theoretically, any value $L_c > 0$ for the truncation constant can be used.

A.3.3.2 Choice of parameter p

Regarding the choice of parameter p, the following applies:

— $p = 0$ can be used for an open profile with an F-operation before applying the Gaussian filter. In this case, the Gaussian weighting function is additionally truncated at the ends of the open profile. For each position x within the end effect regions, the area of the Gaussian weighting function is normalized by the correction term $b_0(x)$, Formula (A.8), to one to provide 100 % transmission characteristic at wavelength $\lambda \to \infty$.

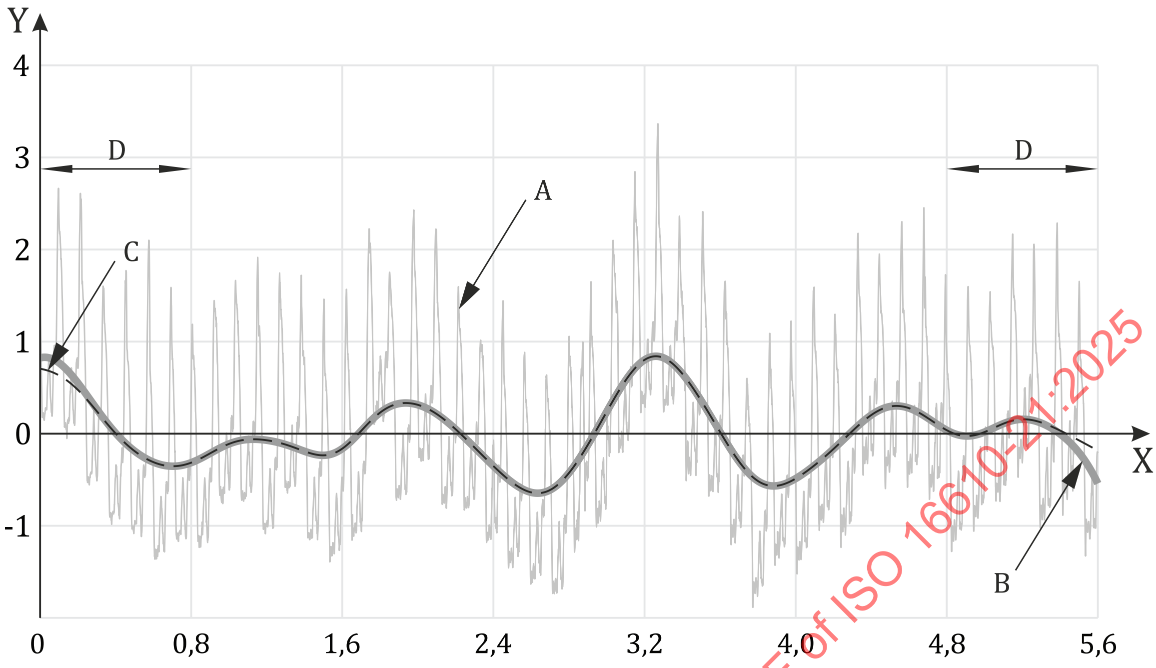
— $p = 1$ can be used for an open profile in the presence of a nominal shape. In this case, the Gaussian weighting function is additionally truncated at the ends of the open profile. For each position x within the end effect regions, the area of the Gaussian weighting function is normalized by the correction terms $b_j(x)$, Formula (A.8), to one to provide 100 % transmission characteristic at wavelength $\lambda \to \infty$. Furthermore, the filtration result follows a 1st degree polynomial within the end effect regions. Therefore, this applies within the entire open profile, and p = 1 is the recommended choice.

A.3.3.3 Choice of truncation constant L_c

Different truncation values lead to different transmission errors, which in turn depend on the application. For applications where high precision is required, such as reference software, a truncation constant $L_c = 1$ is recommended. For general use, a truncation constant $L_c = 0{,}5$ can be used. The use of a truncation constant smaller than $L_c = 0{,}5$ is not recommended.

A.3.4 Examples

Figure A.4 shows the filter behaviour applying the moment retainment method to a turned surface profile. The selected cut-off wavelength is $\lambda_c = 0{,}8$ mm and the truncation constant is $L_c = 1$. Shown is the large-scale lateral component of the surface profile for two different parameters $p=1$ (key item B) and $p=0$ (key item C).



Key

X length in mm

Y height in µm

A unfiltered open profile

B large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm, $p = 1$ (grey solid line)

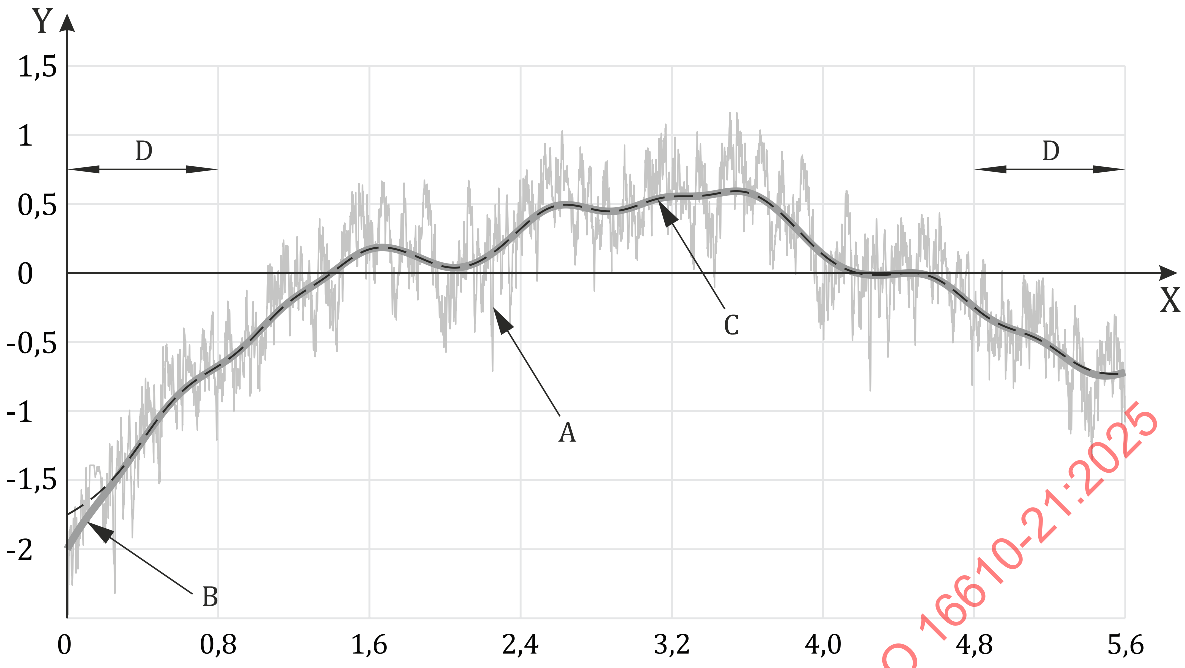
C large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm, $p = 0$ (black dashed line)

D end effect regions: $L_c = 1$

NOTE Shown are the large-scale lateral components of the surface profile for two different parameters $p=1$ and $p=0$.

Figure A.4 — Gaussian filtration with the moment retainment method for the treatment of the end-effect regions applied to a turned surface profile

Figure A.5 shows the filter behaviour applying the moment retainment method to a ground surface profile in the presence of nominal shape. The selected cut-off wavelength is $\lambda_c = 0{,}8$ mm and the truncation constant is $L_c = 1$. Shown is the large-scale lateral component of the surface profile for two different parameters $p=1$ (key item B) and $p=0$ (key item C).



Key

X length in mm

Y height in µm

A unfiltered open profile

B large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm , p = 1 (grey solid line)

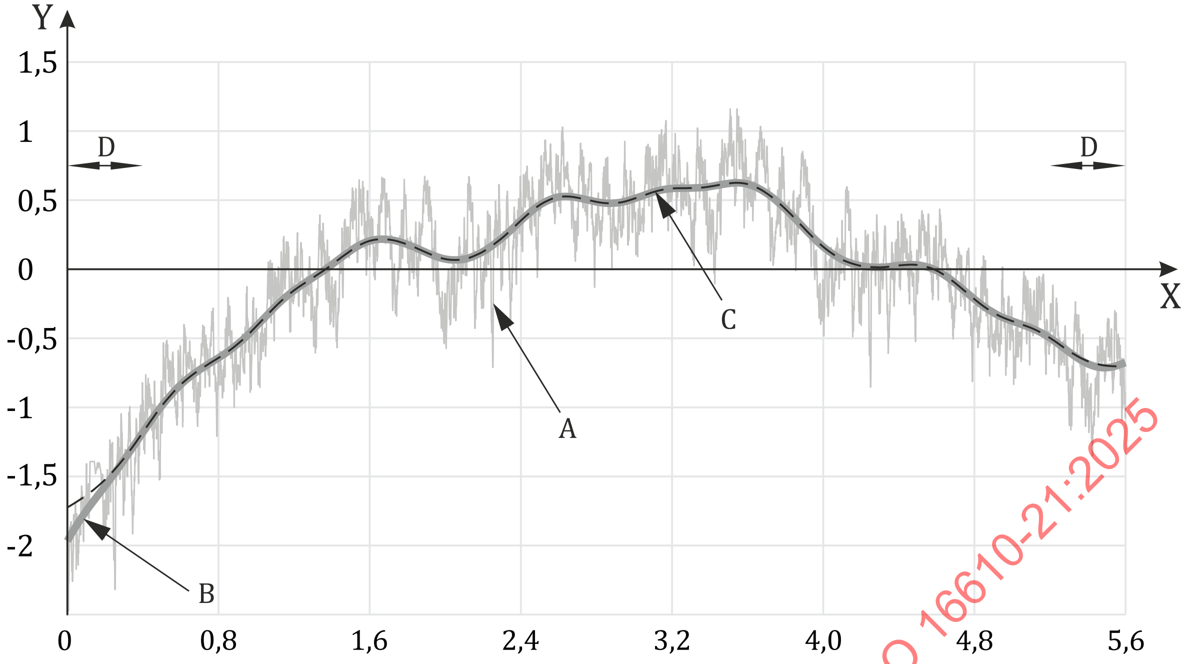
C large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm , p = 0 (black dashed line)

D end effect regions: $L_c = 1$

NOTE Shown are the large-scale lateral components of the surface profile for two different parameters $p = 1$ and $p = 0$. The truncation constant is $L_c = 1$.

Figure A.5 — Gaussian filtration with the moment retainment method for the treatment of the end-effect regions applied to a ground surface profile in presence of nominal shape

Figure A.6 shows the filter behaviour applying the moment retainment method for the same ground surface profile as in Figure A.5, but with a different truncation constant $L_c = 0{,}5$. The selected cut-off wavelength is $\lambda_c = 0{,}8$ mm. Shown is the large-scale lateral component of the surface profile for two different parameters $p = 1$ (key item B) and $p = 0$ (key item C). Comparing Figure A.5 and Figure A.6, the truncation constants $L_c = 1$ and $L_c = 0{,}5$ lead to almost the same filtration result.



Key

X length in mm

Y height in µm

A unfiltered open profile

B large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm , p = 1 (grey solid line)

C large-scale lateral component of the open profile after filtration: $\lambda_c = 0{,}8$ mm , p = 0 (black dashed line)

D end effect regions: $L_c = 0{,}5$

NOTE Shown are the large-scale lateral components of the surface profile for two different parameters $p = 1$ and $p = 0$. The truncation constant is $L_c = 0{,}5$.

Figure A.6 — Gaussian filtration with the moment retainment method for the treatment of the end-effect regions applied to a ground surface profile in presence of nominal shape