INTERNATIONAL STANDARD

# ISO 16610-20

First edition
2015-04-15

# Geometrical product specifications (GPS) — Filtration —

## Part 20:
## Linear profile filters: Basic concepts

*Spécification géométrique des produits (GPS) — Filtrage —*

*Partie 20: Filtres de profil linéaires: Concepts de base*

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Reference number
ISO 16610-20:2015(E)

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Published in Switzerland

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# Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition cancels and replaces ISO/TS 16610-20:2006 which has been technically revised.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

— *Part 1: Overview and basic concepts*

— *Part 20: Linear profile filters: Basic concepts*

— *Part 21: Linear profile filters: Gaussian filters*

— *Part 22: Linear profile filters: Spline filters*

— *Part 28: Profile filters: End effects*

— *Part 29: Linear profile filters: Spline wavelets*

— *Part 30: Robust profile filters: Basic concepts*

— *Part 31: Robust profile filters: Gaussian regression filters*

— *Part 32: Robust profile filters: Spline filters*

— *Part 40: Morphological profile filters: Basic concepts*

— *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*

— *Part 49: Morphological profile filters: Scale space techniques*

— *Part 60: Linear areal filters: Basic concepts*

— *Part 61: Linear areal filters: Gaussian filters*

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— *Part 71: Robust areal filters: Gaussian regression filters*

— *Part 85: Morphological areal filters: Segmentation*

The following parts are planned:

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

— *Part 45: Morphological profile filters: Segmentation*

— *Part 62: Linear areal filters: Spline filters*

— *Part 69: Linear areal filters: Spline wavelets*

— *Part 70: Robust areal filters: Basic concepts*

— *Part 72: Robust areal filters: Spline filters*

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— *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*

— *Part 89: Morphological areal filters: Scale space techniques*

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# Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO/TR 14638). It influences chain links 3 and 5 in the GPS matrix structure.

The ISO/GPS Masterplan given in ISO 14638 gives an overview of the ISO/GPS system of which this part of ISO 16610 is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this part of ISO 16610 and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this part of ISO 16610, unless otherwise indicated.

For more detailed information about the relation of this part of ISO 16610 to the GPS matrix model, see Annex C.

This part of ISO 16610 develops the basic concepts of linear filters, which include spline filters and spline wavelets, and the Gaussian filters.

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# Geometrical product specifications (GPS) — Filtration —

## Part 20:
## Linear profile filters: Basic concepts

## 1 Scope

This part of ISO 16610 describes the basic concepts of linear profile filters.

## 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99, ISO 16610-1, and the following apply.

**3.1**
**linear profile filter**
profile filter which separates profiles into long wave and short wave components and is also a linear function

Note 1 to entry: If F is a function and X and Y are profiles, then F being a linear function implies F(aX + bY) = aF(X) + bF(Y).

**3.2**
**phase correct profile filter**
**phase correct linear profile filter**
*linear profile filter* (3.1) which does not cause phase shifts leading to asymmetrical profile distortions

Note 1 to entry: Phase correct filters are a particular kind of the so called linear phase filters because any linear phase filter can be transformed (simply by shifting its weighting function) to a zero phase filter which is a phase correct filter.

**3.3**
**weighting function**
function for calculating the mean line which indicates, for each point, the weight attached by the profile in the vicinity of that point

Note 1 to entry: The transmission characteristic of the mean line is the Fourier transformation of the weighting function.

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**3.4**
**transmission characteristic of a filter**
characteristic that indicates the amount by which the amplitude of a sinusoidal profile is attenuated as a function of its wavelength

Note 1 to entry: The transmission characteristic is the Fourier transformation of the weighting function.

**3.5**
**cut-off wavelength**
wavelength of a sinusoidal profile of which 50 % of the amplitude is transmitted by the profile filter

Note 1 to entry: Linear profile filters are identified by the filter type and the cut-off wavelength value.

Note 2 to entry: The cut-off wavelength is the recommended nesting index for linear profile filters.

**3.6**
**filter bank**
set of high-pass and low-pass filters arranged in a specified structure

Note 1 to entry: See 5.4 for further details.

**3.7**
**multiresolution analysis**
decomposition of a profile by a *filter bank* (3.6) into portions of different scales

Note 1 to entry: The portions at different scales are also referred to as resolutions.

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# 4 Basic concepts

## 4.1 General

For a filter to conform with this part of ISO 16610, it shall exhibit the characteristics described in 5.1, 5.2, 5.3, and 5.4.

NOTE A concept diagram for linear profile filters is given in Annex A. The relationship to the filtration matrix model is given in Annex B.

The most general linear profile filter is defined by

$$y(x)=\int K(x,\xi)z(\xi)\mathrm{d}\xi \tag{1}$$

where

$z(\xi)$ is the unfiltered profile;

$y(x)$ is the filtered profile;

$K(x,\xi)$ is a real symmetric and spatial invariant kernel.

If $K(x,\xi)=K(x-\xi)$, the filtering is a convolution,

$$y(x)=\int K(x-\xi)z(\xi)\mathrm{d}\xi \tag{2}$$

and the kernel is called the weighting function of the filter.

However, extracted data are always discrete. Consequently, the filters described here are also discrete. If the weighting function is not discrete (see 4.4, Example 2), it shall be converted into a discrete representation.

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## 4.2 Discrete representation of data

An extracted profile can be represented by a vector. The length *n* of this vector is equal to the number of data points. The sampling is assumed to be uniform, i.e. the sampling interval is constant. The *i*th data point of the profile is therefore the *i*th component of the vector.

$$z=(a_1 a_2 ... a_i ... a_{n-1} a_n) \tag{3}$$

## 4.3 Discrete representation of the linear profile filter

A linear profile filter is represented by a square matrix. The dimension of this matrix is equal to the number of data points to be filtered. If the filter is non-periodic, the matrix is a constant diagonal (Toeplitz) matrix:

$$S=\begin{pmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & & & & \\ & a & b & c & b & c & & & \\ & & c' & b' & a & b & c & & \\ & & & c' & b' & c' & b' & a & \\ & & & & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \tag{4}$$

Otherwise, if the filter is periodic, the matrix is a circulant matrix:

$$S=\begin{pmatrix} a & b & c & \dots & \dots & c' & b' \\ b' & a & b & c & \dots & \dots & c' \\ c' & b' & a & b & c & \dots & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & \dots & c' & b' & a & b & c \\ c & \dots & \dots & c' & b' & a & b \\ b & c & \dots & \dots & c' & b' & a \end{pmatrix} \tag{5}$$

If the filter is phase correct, the matrix representing the filter is symmetrical, i.e. $b=b', c=c', \dots$ (generally $a_{ij}=a_{ji}$) is valid. The sum of the matrix elements $a_{ij}$ of each row $i$ is constant and for low-pass filters equals one, i.e.

$$\sum_j a_{ij}=1 \tag{6}$$

NOTE 1 In the case of a symmetrical matrix, the sum of the matrix elements $a_{ij}$ of each column $j$ is also constant and equals one, i.e. $\sum_i a_{ij}=1$ is also valid.

NOTE 2 The relationship between the matrix *S*, the input data by the vector z, and the output data by the vector w is given in Formula (13).

## 4.4 Discrete representation of the weighting function

Given that each row of the matrix representation of the filter is identical after being shifted accordingly, the matrix elements may be represented by one single row, i.e.

$$a_{ij}=s_k \text{ with } k=i-j \tag{7}$$

The values $s_k$ form a vector $s$ of a dimension equal to the length of the input or output data vector respectively. This vector is the discrete representation of the weighting function of the filter.

NOTE 1 The length of the weighting function is usually much smaller than the length of the data set. In this case, *s* contains zeros at each end.

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EXAMPLE 1 The moving average filter is frequently used for easy smoothing of a data set which is not necessarily an optimal method. In the following example of a filter with a discrete weighting function, where a length of 3 has been taken, the weighting function is given by

$$\left( ...0,0,\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0... \right) \tag{8}$$

NOTE 2 The weighting function is often also called the impulse response function because it is the output data set of the filter if the input data set is only a single unity impulse $(...0,0,0,1,0,0,0...)$.

If the weighting function is given as a continuous function, it shall be sampled in order to obtain a discrete data set. The sampling interval used shall be equal to the sampling interval of the extracted data. It is mandatory to renormalize the sampled data of the weighting function subsequently in order to fulfil the condition that they shall sum to unity, thus, avoiding bias effects (for details concerning bias effects, see Reference [3]).

EXAMPLE 2 The Gaussian filter, in accordance with ISO 16610-21, is an example of a continuous weighting function $s(x)$ defined by Formula (9):

$$s(x) = \frac{1}{\alpha \lambda_c} \exp\left[ -\pi \left( \frac{x}{\alpha \lambda_c} \right)^2 \right] \tag{9}$$

where

$x$ is the distance from the centre (maximum) of the weighting function;

$\lambda_c$ is the cut-off wavelength;

$\alpha$ is a constant given by the following equation:

$$\alpha = \sqrt{\frac{\ln 2}{\pi}} = 0,4697... \tag{10}$$

The graph of this weighting function is shown in Figure 1.

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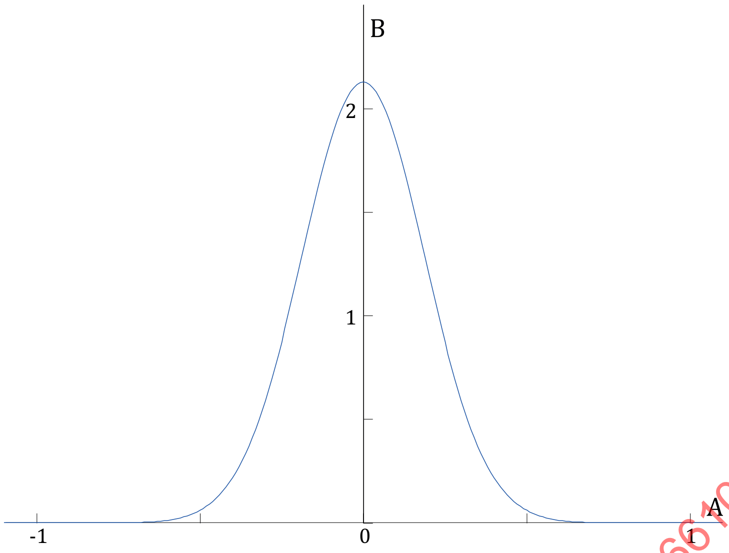


**Key**

A = $\dfrac{X}{\lambda_c}$

B = $\lambda_c Z$

**Figure 1 — Example of a continuous weighting function (Gaussian filter)**

The sample data $s_k$ of the weighting function after a renormalization are given by

$$s_k = \frac{1}{C}\exp\left[-\pi\left(\frac{\Delta x}{\alpha\lambda_c}\right)^2 k^2\right] \tag{11}$$

with the sampling interval $\Delta x$, and the normalization constant

$$C = \sum_k \exp\left[-\pi\left(\frac{\Delta x}{\alpha\lambda_c}\right)^2 k^2\right] \tag{12}$$

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# 5 Linear profile filters

## 5.1 Filter equations

If the filter is represented by the matrix $S$, the input data by the vector $z$, and the output data by the vector $w$, then the filtering process is described by the linear equation

$$w = Sz \tag{13}$$

This equation is the filter equation. If $S^{-1}$ is the inverse matrix of the filter matrix $S$, then

$$z = S^{-1}w \tag{14}$$

is also a valid filter equation.

NOTE 1 The filter can be defined by the matrix $S$ or by the inverse matrix $S^{-1}$, whichever leads to a simpler definition. However, the weighting function is only given by the rows of the matrix $S$.

NOTE 2 The inverse matrix sometimes does not exist. In which case, the filtering process is not invertible, i.e. data reconstruction is impossible. The vertibility of a filter can be seen from its transfer function (see 5.3). A filter which is not invertible has a transfer function $H(\omega)$, which is zero for at least one frequency $\omega$.

EXAMPLE The matrix of the moving average filter mentioned above

$$\frac{1}{3}\begin{pmatrix} \ddots & \ddots & \ddots & & & \\ & 1 & 1 & 1 & & \\ & & 1 & 1 & 1 & \\ & & & 1 & 1 & 1 \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \tag{15}$$

is not invertible. If the filter is changed to a moving average filter $(\alpha < 1/2)$

$$\frac{1}{1+2\alpha}\begin{pmatrix} \ddots & \ddots & \ddots & & & \\ & \alpha & 1 & \alpha & & \\ & & \alpha & 1 & \alpha & \\ & & & \alpha & 1 & \alpha \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \tag{16}$$

it becomes invertible.

The inverse matrix $S^{-1}$ is a constant diagonal matrix or a circulant matrix, if $S$ is, respectively, a constant diagonal matrix or a circulant matrix. The inverse matrix $S^{-1}$ is symmetrical if $S$ is symmetrical.

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## 5.2 Discrete convolution

The filter equation can be written as

$$w_i = \sum a_{ij} z_j = \sum s_{i-j} z_j \tag{17}$$

The latter expression is known as a discrete convolution with the abbreviated notation $w = s \times z$. If the filter matrix is circulant, the convolution is circular, i.e. the coefficients $s_{i-j}$ shall be seen as being extended periodically at both ends (wrapped around).

NOTE The circular convolution can be calculated by using the Fast Fourier Transform (FFT) which is often faster than the usual convolution.


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EXAMPLE An example of a discrete convolution is shown in Figure 2. Here, the filtered value $w_i$ for $i = 3$ is calculated by multiplication of the data values at the points $j = 0 \ldots 6$, with the sampled values of the weighting function at the points $i - j$, and a subsequent summation.

$$w_i = \sum_{j=0}^{n} s_{i-j} z_j \tag{18}$$

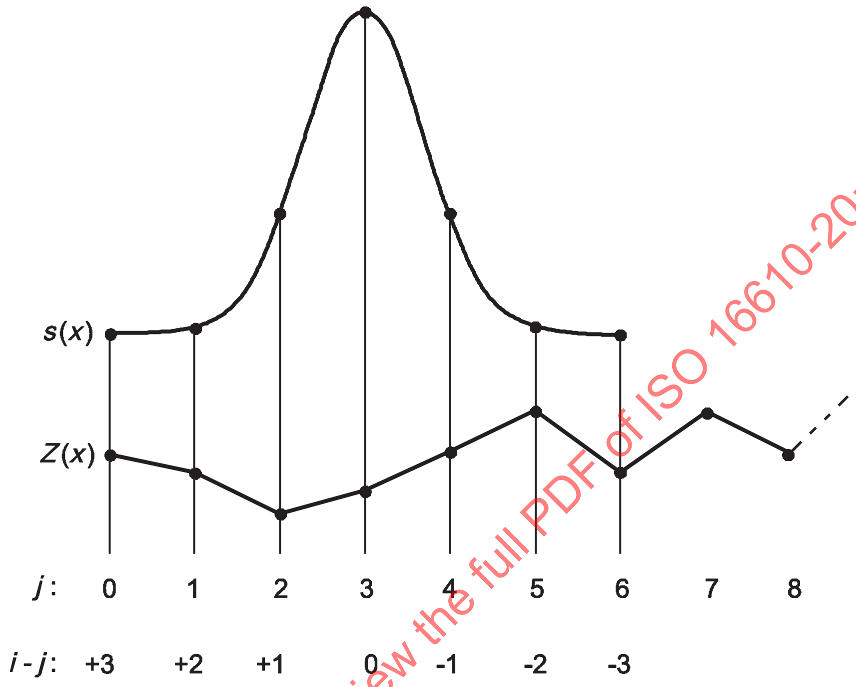


**Figure 2 — Example of a discrete convolution**

## 5.3 Transfer function

Taking the discrete Fourier transformation of the discrete convolution yields

$$W = HZ \tag{19}$$

where

- $W$ is the discrete Fourier Transform of the output vector $w$;
- $H$ is the discrete Fourier Transform of the discrete representation of the weighting function $s$;
- $Z$ is the discrete Fourier Transform of the input vector $z$.

NOTE 1 The *discrete Fourier transform* (DFT) is a function of discrete frequencies. Here, continuous frequencies are used. Consequently, the corresponding transform is mathematically the *discrete time Fourier transform* (DTFT). For simplicity and to avoid confusion between the terms *time* and *wavelength*, the term *discrete Fourier transformation* (DFT) will be used instead of the correct term *discrete time Fourier transform* (DTFT) throughout this part of ISO 16610. For more information concerning the DTFT and the difference between *time* and *wavelength* based Fourier transforms, see Reference [2].

The function $H$ is called the transfer function of the filter. It depends on the wavelength $\lambda$ or the angular frequency $\omega = 2\pi / \lambda$ as the Fourier transformation transforms to the wavelength or frequency domain.

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The Fourier transformation $H(\omega)$ of the discrete representation of the weighting function by the vector $s$ with components $s_k$ is calculated by

$$H(\omega)=\sum_k s_k e^{-i\omega k}=s_0+\sum_{k\neq 0} s_k\left(\cos\omega k+i\sin\omega k\right) \tag{20}$$

Generally speaking, the transfer function turns out to be complex valued. However, if the weighting function is symmetrical, i.e. $s_{-k}=s_k$ is valid, the formula is simplified to

$$H(\omega)=s_0+2\sum_{k>0} s_k\cos\omega k \tag{21}$$

leading to a real transfer function.

For a phase correct filter, the transfer function is always a real function, i.e. the imaginary part is zero. This is due to the fact that the imaginary part represents a phase shift which is not permitted for phase correct filters.

EXAMPLE 1 The transfer function for the moving average filter mentioned above is

$$H(\omega)=\frac{1+2\cos\omega}{3} \tag{22}$$

The graph of this transfer function is shown in Figure 3. This filter is not invertible because $H(\omega)=0$ if $\omega=\pm 2\pi/3$.

The moving average filter shown in Figure 3 is a low-pass filter because $H(\omega)$ has its highest values around the frequency $\omega=0$. By contrast, for a high-pass filter, $H(\omega)$ would have its highest values in the high-frequency region near $\omega=\pm\pi$. If a low-pass filter transfer function $H_0(\omega)$ is given, the simplest way to get a high-pass filter transfer function $H_1(\omega)$ is to calculate $H_1(\omega)=1-H_0(\omega)$. However, this is not always the best possible choice.

EXAMPLE 2 The high-pass filter is complimentary to a moving average filter and is therefore invertible. The (weighted) moving average filter has the (low-pass) transfer function

$$H_0(\omega)=\frac{1+2\alpha\cos\omega}{1+2\alpha} \tag{23}$$

The high-pass filter then has the transfer function

$$H_1(\omega)=1-H_0(\omega)=\frac{2\alpha}{1+2\alpha}\left(1-\cos\omega\right) \tag{24}$$

Both transfer functions are shown in Figure 4.

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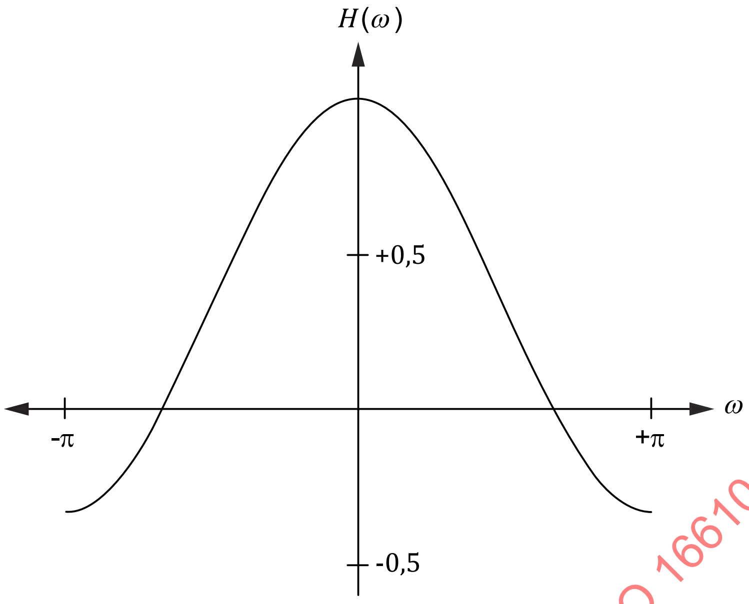
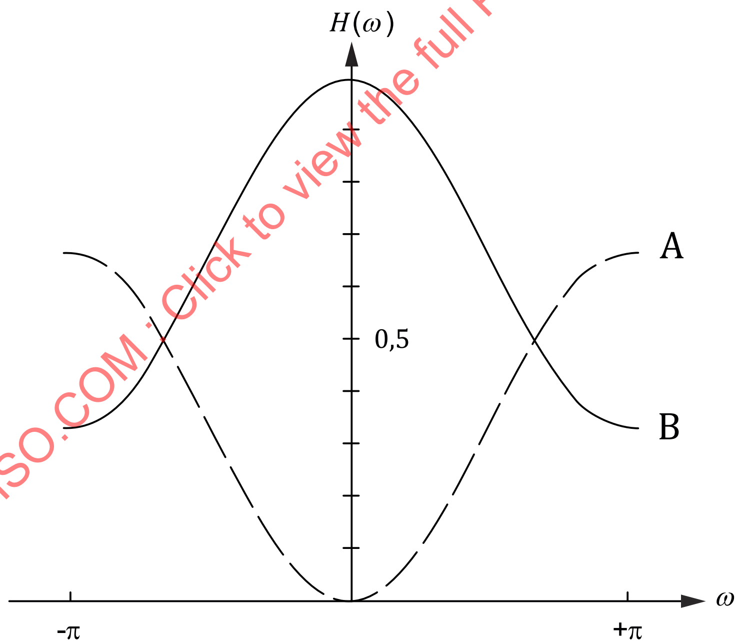



**Figure 3 — Transfer function of the moving average filter of length 3**




**Key**

A high-pass transfer function

B low-pass transfer function

**Figure 4 — Low-pass and high-pass transfer function with α = 0,25**

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The weighting function of the low-pass filter is

$$\left(...0,0,\frac{\alpha}{1+2\alpha},\frac{1}{1+2\alpha},\frac{\alpha}{1+2\alpha},0,0...\right) \tag{25}$$

The weighting function of the high-pass filter can easily be shown to be

$$\left(...0,0,-\frac{\alpha}{1+2\alpha},\frac{2\alpha}{1+2\alpha},-\frac{\alpha}{1+2\alpha},0,0...\right) \tag{26}$$

This filter is called a (weighted) moving difference filter.

## 5.4 Filter banks

In a two-channel filter bank, the two filters are normally a high-pass and a low-pass filter. These are indicated by their transfer functions $H_0(\omega)$ and $H_1(\omega)$. The object is to separate the input data into a low-frequency (long wavelength) component and a high-frequency (short wavelength) component.

EXAMPLE In terms of roughness, the profile $z(x)$ is separated into a waviness component $w(x)$ and a roughness component $r(x)$ by a low-pass filter $H_0(\omega)$ and a high-pass filter $H_1(\omega)$ (see Figure 5).

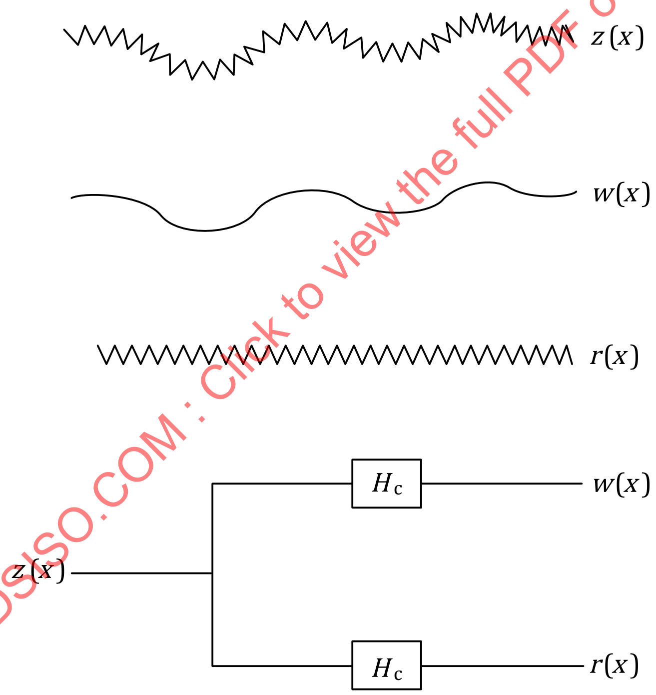


**Figure 5 — Separation of a measured profile into a waviness component $H_0(\omega)$ and a roughness component $H_1(\omega)$ by a low-pass filter and a high-pass filter**

Generally speaking, the transfer functions of the low-pass and high-pass filters overlap (with non-zero values at the same wavelength, as in Figure 4). This cannot be avoided in the practical implementation of the filter. The separation is not ideal because input data portions, whose frequencies fall within the overlap region, go partly to both channels. This results in aliasing in each channel. Any reconstruction of the input data from the filtered data needs to take this fact into account.

Cascading of filter banks leads to multiresolution analysis. Each filtering stage gives finer details of the profile data. They appear at multiple scales. However, filter banks shall be specifically designed to achieve multiresolution.

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# Annex A
(informative)

# Concept diagram

The following is a concept diagram for this part of ISO 16610.

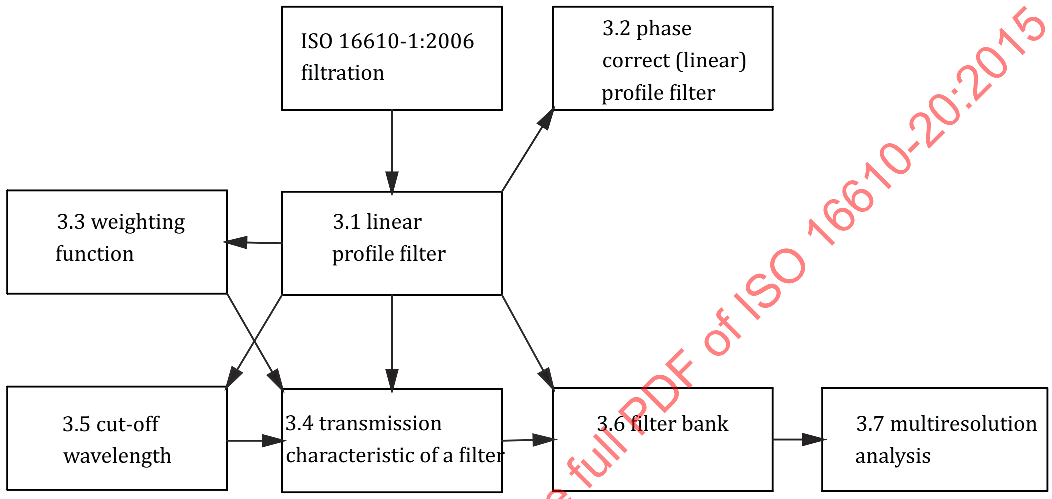


**Figure A.1 — Concept diagram**

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