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Application of statistical and related methods to new technology and product development process — Robust tolerance design (RTD)

Application des méthodes statistiques et des méthodes liées aux nouvelles technologies et de développement de produit — Plans d'expériences robustes

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Foreword

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 8, *Application of statistical and related methodology for new technology and product development*.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The designer of a product typically decides the specifications of the product and passes them on to the manufacturing section for use in manufacturing the product. The specifications include the designed nominal values and tolerances for the parts and/or elements of the product. The optimum nominal values of the design parameters are determined by robust parameter design (RPD), and the optimum tolerances are determined by robust tolerance design (RTD).

RPD, as described in ISO 16336, is applied to the product prior to RTD. In RPD, the major noise factors are used to evaluate robustness as measured by the signal-to-noise ratio, which represents the variability of product output. It is a measure for comparing robustness between levels of control factors. RPD identifies the combination of the values of the design parameters as an optimum RPD condition for minimizing the variability, that is, maximizing the robustness.

RTD, as described in this document, is a method for selecting the degree of errors of the parts or elements of the product from the viewpoint of variability under the optimum RPD condition, that is, the combination of optimum nominal values of the design parameters. If a manufactured product has errors from the designed nominal values, the product output will deviate from the designed value. The error in a design parameter should be smaller than the designed error limit to keep the product output within the designed variability. This is why the design parameters need a tolerance.

The design of a product can be finalized by setting the optimum error limits of the design parameters by using RTD. The expected variance in output of a product manufactured with errored parts or elements can be estimated using RTD. After RPD is used to identify a set of optimum values for the design parameters, RTD is used to check whether the estimated variance is smaller than the target variance under the optimum RPD condition.

RPD can be used to set the optimum nominal values of the design parameters without increasing manufacturing cost while RTD is closely related to the manufacturing cost. Smaller tolerances, meaning higher-grade parts or elements, result in higher costs, while larger tolerances, meaning lower-grade parts or elements, result in lower costs. To finalize the product design, the cost of manufacturing the product is considered. The loss function in the Taguchi methods is used to transform the benefits of an improvement in quality into a monetary amount, the same as a cost.

The cost of the improvement and the benefits of the improvement in quality should be balanced in deciding the tolerances. RPD and RTD together provide a cost-effective way of optimizing product design.

If RPD cannot achieve the product variability smaller than the target variability, the tolerances of the design parameters are reduced to improve the variability, but smaller tolerances result in higher costs.

On the other hand, if RPD can achieve the product variability much smaller than the target variability, the tolerances of the design parameters are increased to reduce manufacturing cost, so larger tolerances result in lower costs.

Products manufactured with optimum nominal values and tolerances of design parameters are robust to noise situations under usage conditions after shipment. Robust products minimize users' quality losses due to defects, failures, and quality problems.

Application of statistical and related methods to new technology and product development process — Robust tolerance design (RTD)

1 Scope

This document specifies guidelines for applying the robust tolerance design (RTD) provided by the Taguchi methods to a product in order to finalize the design of the product.

NOTE 1 RTD is applied to the target product to set the optimum tolerances of the design parameters around the nominal values. RTD identifies the effects of errors in the controllable design parameters on product output and estimates the total variance of the product output if the tolerances are changed. Hence, RTD achieves the target variance of the output from the viewpoints of robustness, performance, and cost.

NOTE 2 The tolerance expresses a maximum allowable error in the value of a design parameter in the manufacturing process. In a perfect world, the parts or elements of every product have the designed nominal values of the design parameters. However, actual manufacturing does not reproduce the exact designed nominal values of the design parameters for all products. The actual products have errors in the values of their parts or elements. These errors are supposed to be within the designed tolerances.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16336, *Applications of statistical and related methods to new technology and product development process — Robust parameter design (RPD)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16336 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1
tolerance
difference between the upper specification limits and lower specification limits

3.2
robust tolerance design
RTD
method of setting optimum tolerances from the viewpoints of robustness, performance, and cost

4 Robust tolerance design

4.1 General

A company's product design section normally gives the specifications of a product, that is, the nominal values and tolerances of the design parameters, to the manufacturing section. The manufacturing section uses the designed specifications in manufacturing the product. When specifications specify the limits of a design parameter as $m \pm \Delta$, the parameter value x in the manufacturing process should satisfy the following restriction:

$$m - \Delta \leq x \leq m + \Delta, \quad (1)$$

where m and Δ denote a nominal value and its permissible difference, respectively. Only the symmetric ($\pm\Delta$) case is discussed in this document. In the symmetric case, the tolerance is 2Δ, and the permissible difference Δ is half the tolerance.

If the absolute error of a design parameter is larger than the specified permissible difference Δ, the variability in the product output cannot meet the designed performance and specifications.

RTD is used by the design section to set the optimum tolerance for each design parameter to achieve the designed performance, which is evaluated based on the total variance of the product output. The permissible difference of a design parameter is the maximum allowable error around the nominal value in the manufacturing process, and it is closely related to the cost of manufacturing.

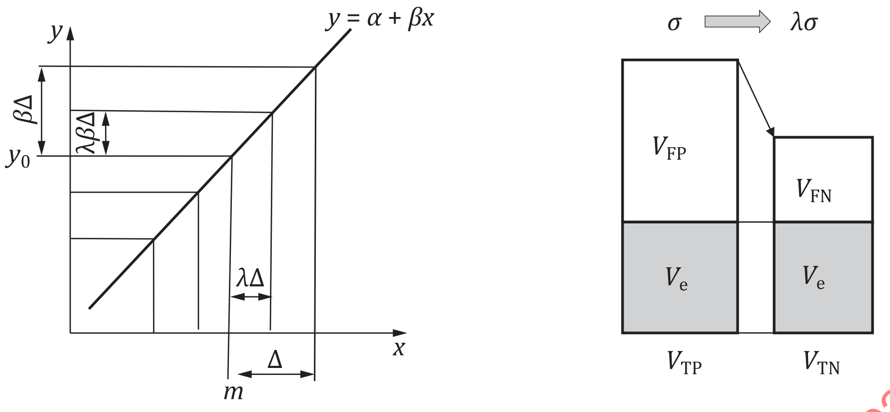
The optimum nominal values of the design parameters can be identified by robust parameter design (RPD) through robustness measure, signal-to-noise ratio[1]. The selection of a robust product by setting the nominal values as the optimum values using RPD prior to RTD is highly recommended. RPD can optimize the target product by choosing the optimum combination of design parameter nominal values from the viewpoint of the variability of the product output without increasing the cost[2].

If RPD cannot achieve a target variability, RTD is used to identify possible tolerances for achieving the target variability even at a higher cost. Smaller tolerances result in smaller variability, but this requires upgrading the parts or elements of the product, which leads to higher manufacturing cost. RTD is used to investigate the balance between product quality and improvement cost.

Even if RPD achieves the target variance, RTD is used, in some cases, to identify larger tolerances than those considered in RPD. Larger tolerances mean larger variability, but if the increased variability satisfies the target variability, the larger tolerances are applicable as they lead to reduced cost of manufacturing the designed product.

The purpose of RTD is to achieve the target variability by setting optimum tolerances from the viewpoints of robustness, performance, and cost. For this purpose, RTD estimates the total variance of the output of the designed product if the tolerance of a design parameter is changed. The total variance can be estimated based on the results of analysis of variance (ANOVA).

Assume that a value x of design parameter F has a linear effect on output y of the product, as shown in Figure 1 a). If the present permissible difference of x in F is $\Delta_P = \Delta$, the error distribution of F affects output y with a magnitude of $\beta\Delta$. If the permissible difference Δ of F is reduced to new permissible difference $\Delta_N = \lambda\Delta$ [λ<1 in Figure 1 a)], the effect of changing Δ in F on the output is reduced to $\lambda\beta\Delta$, and the variance in y due to changing Δ in F is reduced from the present variance V_{FP} to new variance $V_{FN} = \lambda^2 V_{FP}$. As a result, the total output variance is reduced from V_{TP} to V_{TN} [Figure 1 b)].



a) Linear dependence on x in F

b) Change in total variance

Figure 1 — Effect of changing Δ in design parameter F on total variance

The new total variance V_{TN} can be estimated as

$$V_{TN} = V_{FN} + V_e = \lambda^2 V_{FP} + V_e , \tag{2}$$

where $\lambda = \Delta_N / \Delta_P$ is assumed.

If the tolerance of a design parameter is reduced, that is, $\lambda<1$, the magnitude of error of the design parameter becomes smaller, and the total output variance is reduced. A smaller tolerance means that an upgraded part or element is used, so the cost of producing the new design can be higher than that of the present design.

If the tolerance of a design parameter is enlarged, that is, $\lambda>1$, the magnitude of error of the design parameter becomes larger, and the total output variance is enlarged. A larger tolerance means that a down-graded part or element is used, so the cost of producing the new design can be smaller than that of the present design.

RTD comprises two steps, as follows.

1) RTD experimentation: Collect data on the designed product, and analyse the data to determine the dependence of the product output on the design parameters.

2) Tolerance determination: Estimate the total variance if a tolerance is changed, and compare the effects in quality with the cost of the change to identify the optimum tolerance.

RTD experiments collect the output data of the designed product in which there are errors in the product design parameters, and estimate the total variance and its dependence on the design parameters. The experimental design plan is used to collect the data under the combination of design parameter errors. The ANOVA results show the effects of errors in the design parameters on the product output. The product output has a target variance from the viewpoints of robustness and performance.

In RTD experiments, the design parameters are taken as noise factors. A noise factor is an experimental factor which is taken into experiment for the purpose of estimating its variability. The variance in the linear effect of errors in the design parameters is estimated.

In RPD, on the other hand, the design parameters are taken as control factors. A control factor is an experimental factor which is taken into experiment for the purpose of selecting the optimum level of the factor. Designers can fix the nominal values of design parameters to the optimum RPD values. However, in actual manufacturing, the parts or elements of the product invariably have errors, so the designer cannot specify the error of a design parameter. The designer can set only the permissible difference Δ as an error limit.

Design parameter errors cause variability in product output. If the error of a design parameter has a linear effect on the product output, the output variance can be changed by resetting the tolerance of the design parameter. RTD experimentation is used to determine the contributions of the effects of errors in design parameters to the product output.

In the tolerance determination step of RTD, the change in the output variance due to resetting a tolerance is estimated, and the designer selects optimum tolerance for achieving the target output variance. The optimum tolerance can be determined by balancing the effect in quality due to a tolerance change against the cost of the tolerance change[3].

4.2 RTD experimentation

4.2.1 Data generation

RTD experimentation is used to determine the design parameters' linear effects for the designed product. The relationship between the output by the product and the errors in the design parameters is investigated. The output data can be generated in three ways:

1) by using a theoretical formula,

2) by experimentation with an actual product;

3) by simulation experimentation.

When the theoretical relationship between the product output and the design parameters is known, the output data can be directly calculated for various combination of the design parameter values. RTD offers multi-factor design as an experimental design for generating the output data in various combinations of the level of experimental factors, as shown in case study (1) in Clause 5. ANOVA is used for analysing the dependence of the product output on the factors.

Mathematical analysis can be applied in this case. Mathematical analysis consists of using variance estimates for a system by, for example, propagating an input variance through the system via Taylor series expansions of moment generating functions[4].

If an actual product can be constructed, it can be used for experimentation, and the data output can be collected using the actual experiment. However, in many cases, it is difficult to set the intended levels of the errors of design parameters in an actual product because the noise levels cannot be controlled within the error distribution of the design parameters. Simulation experimentation can be used in such cases. This is why simulation experiments are often used in RTD. A simulation program can provide the product output data, as shown in case study (2) in Clause 6.

4.2.2 Experimental design for data collection

RTD experimentation is used for collecting output data for the designed product under the combinations of design parameter errors. There are many design parameters, and a multi-factor experimental design is used to generate various such combinations. The purpose of RTD experimentation is to determine the main effects of experimental factors. An orthogonal array plan is recommended as a multi-factor experimental plan for collecting the data as it is an efficient way to collect data for an RTD experiment.

An orthogonal array plan can reduce the number of experimental runs compared with a full-factorial plan for the same number of factors and can assign the maximum number of factors in a plan for the same number of experimental runs. The main effects of factors can be estimated under the condition of a balanced combination of the other factors' levels. The choice of the orthogonal array depends on the numbers of factors and their levels[3].

An example orthogonal array (L_{18}) is shown in Table 1. Seven experimental factors with three levels (B-H) and one factor with two levels (A) can be assigned to the columns in the array. Rows represent the experimental run. The number in each cell represents the level of the factor assigned to the column. The experimental run of low No. 1 should be performed under the combination of factor's levels A1B1C1D1E1F1G1H1.

For RTD, the design parameters are assigned to the columns as noise factors. For the purpose of estimating the linear and non-linear effects of a factor, each factor has at least three levels. However, if the proportional property is obvious for a factor, a two-level setting is sufficient. Two-level factor is assigned to the first column. The last column in Table 1 shows the output data y_i calculated for the combination of factors' levels shown in the cells in the same low

Table 1 — Example of orthogonal array L_{18} and output data

Column	1	2	3	4	5	6	7	8	Data output
No.	A	B	C	D	E	F	G	H	
1	1	1	1	1	1	1	1	1	y_1
2	1	1	2	2	2	2	2	2	y_2
3	1	1	3	3	3	3	3	3	y_3
4	1	2	1	1	2	2	3	3	y_4
5	1	2	2	2	3	3	1	1	y_5
6	1	2	3	3	1	1	2	2	y_6
7	1	3	1	2	1	3	2	3	y_7
8	1	3	2	3	2	1	3	1	y_8
9	1	3	3	1	3	2	1	2	y_9
10	2	1	1	3	3	2	2	1	y_{10}
11	2	1	2	1	1	3	3	2	y_{11}
12	2	1	3	2	2	1	1	3	y_{12}
13	2	2	1	2	3	1	3	2	y_{13}
14	2	2	2	3	1	2	1	3	y_{14}
15	2	2	3	1	2	3	2	1	y_{15}
16	2	3	1	3	2	3	1	2	y_{16}
17	2	3	2	1	3	1	2	3	y_{17}
18	2	3	3	2	1	2	3	1	y_{18}

Table 2 shows an example of level setting of factors for RTD. The upper and lower permissible differences are assumed to be the same for simplicity. The levels of the factors are set around nominal value *m* with level width *d*. Nominal value *m* is set to an optimum value by RPD from the viewpoint of robustness. Level width *d* is set from the actual standard deviation of the design parameter if it is known.

Table 2 — Example of level settings of factors for RTD

Factor	1	2	3
A	$m_A - d_A$	$m_A + d_A$	—
B	$m_B - d_B$	m_B	$m_B + d_B$
C	$m_C - d_C$	m_C	$m_C + d_C$
D	$m_D - d_D$	m_D	$m_D + d_D$
E	$m_E - d_E$	m_E	$m_E + d_E$
F	$m_F - d_F$	m_F	$m_F + d_F$
G	$m_G - d_G$	m_G	$m_G + d_G$
H	$m_H - d_H$	m_H	$m_H + d_H$

When the actual standard deviation σ_x of the error in the design parameter is not exactly known, the assumption $\sigma_x = \Delta/2$ or $\sigma_x = \Delta/3$ can be applied.

When the actual standard deviation σ_x of the error in the design parameter is known, the level width *d* and the levels of the factors are set as follows.

For a two-level factor, $d = \sigma_x$:

X1: First level $x_1 = m - \sigma_x$, (3)

X2: Second level $x_2 = m + \sigma_x$. (4)

For a three-level factor, $d = \sqrt{\frac{3}{2}}\sigma_x$:

X1: First level $x_1 = m - d = m - \sqrt{\frac{3}{2}}\sigma_x$, (5)

X2: Second level $x_2 = m$, (6)

X3: Third level $x_3 = m + d = m + \sqrt{\frac{3}{2}}\sigma_x$. (7)

Setting the level of the factors in this way makes the estimated variance $\sigma_{y\ell}^2$ of output *y* caused by the linear effect of the error in the factor $\beta^2\sigma_x^2$, where *β* represents the linear coefficient of the relationship $y = \beta x$ between output *y* and input *x*.

If $y_{ij}\ (i = 1, \cdots, n,\ j = 1, \cdots, r)$ represents the output from *j*-th run in *r* repeated runs on *i*-th level x_i in *n* level factor, the linear coefficient *β* and the sum of squares of linear effect S_β are calculated as

$$\beta = \frac{\sum_{i=1}^{n}\sum_{j=1}^{r}(x_i - \bar{x})(y_{ij} - \bar{y})}{r\sum_{i=1}^{n}(x_i - \bar{x})^2}, \tag{8}$$

$$S_\beta = \frac{\left[\sum_{i=1}^{n}\sum_{j=1}^{r}(x_i - \bar{x})(y_{ij} - \bar{y})\right]^2}{r\sum_{i=1}^{n}(x_i - \bar{x})^2} = r\sum_{i=1}^{n}(x_i - \bar{x})^2 \cdot \beta^2 . \tag{9}$$

For a two-level factor A with levels $x_1 = \bar{x} - d$ and $x_2 = \bar{x} + d$, the sum of squares of linear effect S_β is calculated as $S_\beta = r \cdot 2d^2\beta^2$. If the linear effect of factor A is significant, S_β approximately represents $2r\sigma_{y\ell}^2$, where 2*r* denotes the number of data items and $\sigma_{y\ell}^2$ denotes the variance of each. If level width *d* is set to σ_x, $S_\beta = 2rd^2\beta^2 = 2r\sigma_x^2\beta^2 \cong 2r\sigma_{y\ell}^2$. Then variance $\sigma_{y\ell}^2$ in output *y* caused by the linear effect of the error in the factor becomes $\sigma_{y\ell}^2 = \beta^2\sigma_x^2$.

For a three-level factor B with levels $x_1 = \bar{x} - d, x_2 = \bar{x}$, and $x_3 = \bar{x} + d$, the sum of squares of linear effect S_β is calculated as $S_\beta = r \cdot 2d^2\beta^2$. If the linear effect of the factor is significant, S_β approximately represents $3r\sigma_{y\ell}^2$, where 3*r* denotes the number of data items. If the level width *d* is set to $\sqrt{\frac{3}{2}}\sigma_x$, $S_\beta = 2rd^2\beta^2 = 2r \cdot \frac{3}{2}\sigma_x^2 \cdot \beta^2 = 3r\sigma_x^2\beta^2 \cong 3r\sigma_{y\ell}^2$. Then variance $\sigma_{y\ell}^2$ in output *y* caused by the linear effect of the error in the noise factor becomes $\sigma_{y\ell}^2 = \beta^2\sigma_x^2$.

4.2.3 Analysis of variance

ANOVA is used to identify the linear effects of the factors and the ratios of their contributions to the total variance.

The ANOVA calculations for orthogonal array L_{18} are as follows.

Total sum of squares:

$$S_T = \sum_{i=1}^{18}(y_i - \bar{y})^2 = \sum_{i=1}^{18} y_i^2 - \frac{(\sum_{i=1}^{18} y_i)^2}{18}. \quad (10)$$

The total sum of squares is decomposed into sum of squares $S_{\bullet\ell}$ of the linear effect of each factor and sum of squares S_e of the error as follows:

$$S_T = S_A + S_{Bl} + S_{Cl} + S_{Dl} + S_{El} + S_{Fl} + S_{Gl} + S_{Hl} + S_e. \quad (11)$$

For calculating the factor effects, the sum of the data for each factor level is calculated:

$$\begin{aligned}
Y_{A1} &= y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 \\
Y_{A2} &= y_{10} + y_{11} + y_{12} + y_{13} + y_{14} + y_{15} + y_{16} + y_{17} + y_{18} \\
Y_{B1} &= y_1 + y_2 + y_3 + y_{10} + y_{11} + y_{12} \\
Y_{B2} &= y_4 + y_5 + y_6 + y_{13} + y_{14} + y_{15} \\
Y_{B3} &= y_7 + y_8 + y_9 + y_{16} + y_{17} + y_{18} \\
&\ldots \\
Y_{H3} &= y_3 + y_4 + y_7 + y_{12} + y_{14} + y_{17}
\end{aligned} \quad (12)$$

Table 3 summarizes the calculated sums of data.

Table 3 — Sums of data for each factor level

Factor	Sum of data		
	Level 1	**Level 2**	**Level 3**
A	Y_{A1}	Y_{A2}	-
B	Y_{B1}	Y_{B2}	Y_{B3}
C	Y_{C1}	Y_{C2}	Y_{C3}
D	Y_{D1}	Y_{D2}	Y_{D3}
E	Y_{E1}	Y_{E2}	Y_{E3}
F	Y_{F1}	Y_{F2}	Y_{F3}
G	Y_{G1}	Y_{G2}	Y_{G3}
H	Y_{H1}	Y_{H2}	Y_{H3}

For a two-level factor (A):

Main effect of factor A:

$$S_A = \frac{Y_{A1}^2 + Y_{A2}^2}{9} - \frac{(Y_{A1} + Y_{A2})^2}{18} = \frac{(Y_{A1} - Y_{A2})^2}{18}. \quad (13)$$

For a three-level factor (B, for example):

The effect of each three-level factor is separated into two parts, a linear term and a quadratic term.

Total effect of factor B:

$$S_B = \frac{Y_{B1}^2 + Y_{B2}^2 + Y_{B3}^2}{6} - \frac{\left(Y_{B1} + Y_{B2} + Y_{B3}\right)^2}{18}. \tag{14}$$

Linear term S_{Bl} and quadratic term S_{Bq}:

$$S_{Bl} = \frac{\left[(-1)\times Y_{B1} + 0\times Y_{B2} + 1\times Y_{B3}\right]^2}{6\times\left[(-1)^2 + 0^2 + 1^2\right]} = \frac{\left[(-1)\times Y_{B1} + 0\times Y_{B2} + 1\times Y_{B3}\right]^2}{6\times 2}.$$

$$S_{Bq} = \frac{\left[1\times Y_{B1} + (-2)\times Y_{B2} + 1\times Y_{B3}\right]^2}{6\times\left[1^2 + (-2)^2 + 1^2\right]} = \frac{\left[1\times Y_{B1} + (-2)\times Y_{B2} + 1\times Y_{B3}\right]^2}{6\times 6}. \tag{15}$$

The following relationship is useful for checking the calculations.

$$S_B = S_{Bl} + S_{Bq}. \tag{16}$$

The effects of the factors are also calculated for all other 3-level factors, factors C to H.

The results of the ANOVA calculations are shown in Table 4.

Table 4 — Results of ANOVA calculations

Source	*f*	*SS*	*V*
A	1	S_A	V_A
B*l*	1	S_{Bl}	V_{Bl}
B*q*	1	S_{Bq}	V_{Bq}
C*l*	1	S_{Cl}	V_{Cl}
C*q*	1	S_{Cq}	V_{Cq}
D*l*	1	S_{Dl}	V_{Dl}
D*q*	1	S_{Dq}	V_{Dq}
E*l*	1	S_{El}	V_{El}
E*q*	1	S_{Eq}	V_{Eq}
F*l*	1	S_{Fl}	V_{Fl}
F*q*	1	S_{Fq}	V_{Fq}
G*l*	1	S_{Gl}	V_{Gl}
G*q*	1	S_{Gq}	V_{Gq}
H*l*	1	S_{Hl}	V_{Hl}
H*q*	1	S_{Hq}	V_{Hq}
e	2	S_e	V_e
T	17	S_T	

If the linear term of a factor is smaller than the error term, it is pooled into the error effect.

If the quadratic term of a factor is small compared with the linear term and is comparable to the error term, it is pooled into the error effect.

If the quadratic term of a factor is large compared with the linear term, it is not pooled into the error effect because it can have higher order dependence than the linear effect. The tolerance of this factor cannot be changed without more investigation.

If the linear terms of all factors are larger than the error term, and all quadratic terms are pooled into the error effect, the sum of squares of error S_e is estimated as

$$S_e = S_T - (S_A + S_{Bl} + S_{Cl} + S_{Dl} + S_{El} + S_{Fl} + S_{Gl} + S_{Hl}). \tag{17}$$

Table 5 — Pooled ANOVA: Linear effects of factors and their contribution ratios

Source	f	SS	V	S'	ρ
A	1	S_{Al}	V_{Al}	S'_{Al}	ρ_{Al}
Bl	1	S_{Bl}	V_{Bl}	S'_{Bl}	ρ_{Bl}
Cl	1	S_{Cl}	V_{Cl}	S'_{Cl}	ρ_{Cl}
Dl	1	S_{Dl}	V_{Dl}	S'_{Dl}	ρ_{Dl}
El	1	S_{El}	V_{El}	S'_{El}	ρ_{El}
Fl	1	S_{Fl}	V_{Fl}	S'_{Fl}	ρ_{Fl}
Gl	1	S_{Gl}	V_{Gl}	S'_{Gl}	ρ_{Gl}
Hl	1	S_{Hl}	V_{Hl}	S'_{Hl}	ρ_{Hl}
e	9	S_e	V_e	S'_e	ρ_e
T	17	S_T	V_T		ρ_T

The resultant pooled ANOVA is shown in Table 5. It shows the linear effects of all factors, their pure sums of squares, and their contribution ratios. The contribution ratio ρ is defined as the portion of the pure sum of squares S' of a factor's effect to the total sum of squares S_T as:

$$\rho = \frac{S'}{S_T} \times 100 \quad (\%). \tag{18}$$

Consider the case in which the product has design parameter B and the total sum of squares S_T of the output. The pure sum of squares of linear term $S'_{B\ell}$ of B with degree of freedom $f_{B\ell} = 1$ and the pure sum of squares of error S'_e can be estimated as follows.

Pure sum of squares of linear term S'_{Bl}:

$$\begin{aligned} S'_{Bl} &= S_{Bl} - f_{Bl} \times V_e = S_{Bl} - V_e \quad && \textit{if } S_{Bl} > V_e, \\ S'_{Bl} &= 0 && \textit{if } S_{Bl} \leq V_e. \end{aligned} \tag{19}$$

If $S_{Bl} \leq V_e$, the linear term S_{Bl} is pooled into the error term S_e. If the linear terms of the factors are pooled into the error term, the degree of freedom of the error term f_e is changed, and the error variance is recalculated after all linear terms of the factors are checked. Table 5 shows the case in which the linear terms of all factors are not pooled.

Contribution ratio ρ_{Bl} of linear term S'_{Bl} of factor B:

$$\rho_{Bl} = \frac{S'_{Bl}}{S_T} \times 100 \quad (\%). \tag{20}$$

The error part $f_{Bl} \times V_e$ of sum of squares S_{Bl} is pooled to the pure sum of squares of error S'_e.

Pure sum of squares of error S'_{e}:

$$S'_{\mathrm{e}} = S_{\mathrm{e}} + f_{\mathrm{B}l} \times V_{\mathrm{e}} = (f_{\mathrm{e}} + f_{\mathrm{B}l}) \times V_{\mathrm{e}}. \quad (21)$$

If these calculations are carried out to the other factors C to H, S'_{e} is finally calculated as

$$S'_{\mathrm{e}} = S_{\mathrm{e}} + \left(\sum_{i=\mathrm{A},\cdots,\mathrm{H}} f_{il} \right) \times V_{\mathrm{e}} = f_{\mathrm{T}} \times V_{\mathrm{e}} = 17 \times V_{\mathrm{e}}. \quad (22)$$

The following formulas are used in these calculations:

$$\begin{aligned} & E[V_{\mathrm{B}l}] = \sigma_{\mathrm{e}}^2 + 2rd^2\beta_l^2 = V_{\mathrm{e}} + V'_{\mathrm{B}l} \quad \text{(for two-level factor)} \\ & E[V_{\mathrm{B}l}] = \sigma_{\mathrm{e}}^2 + 6rd^2\beta_l^2 = V_{\mathrm{e}} + V'_{\mathrm{B}l} \quad \text{(for three-level factor)} \\ & E[S_{\mathrm{B}l}] = f_{\mathrm{B}l}(V_{\mathrm{e}} + V'_{\mathrm{B}l}) = f_{\mathrm{B}l}V_{\mathrm{e}} + S'_{\mathrm{B}l} \\ & E[S'_{\mathrm{e}}] = f_T \times \sigma_{\mathrm{e}}^2 = f_T \times V_{\mathrm{e}} \\ & \sum_i \rho_i = 100 \quad (\%) \\ & \sum_i S'_i = S_{\mathrm{T}} \end{aligned} \quad (23)$$

where the linear term of factor B is estimated from level width *d*, and summation with subscript *i* denotes summation of all the terms.

In summary, the contribution ratio of the linear term of factor B is estimated as

$$\rho_{\mathrm{B}l} = \frac{S'_{\mathrm{B}l}}{S_{\mathrm{T}}} \times 100 = \frac{S_{\mathrm{B}l} - V_{\mathrm{e}}}{S_{\mathrm{T}}} \times 100 \quad (\%). \quad (24)$$

The contribution ratios of the other factors' linear terms are also estimated.

The contribution ratio of the final error term is estimated as

$$\rho_{\mathrm{e}} = \frac{S'_{\mathrm{e}}}{S_{\mathrm{T}}} \times 100 = \frac{f_{\mathrm{T}} \times V_{\mathrm{e}}}{S_{\mathrm{T}}} \times 100 \quad (\%). \quad (25)$$

The results of these estimations are shown in the pooled ANOVA in Table 5, which is used in the next step of RTD.

4.3 Tolerance determination

4.3.1 Estimating total variance if tolerance is changed

If the tolerance of a design parameter is changed, the new total variance of product output can be estimated using the pooled ANOVA in Table 5.

If a design parameter has a large linear effect on the output, the permissible difference of the design parameter can be changed to match the total variance to the target variance. If the contribution ratio of the linear effect of a factor is large, the effect of changing its tolerance on the output is large, so the new total variance of output is greatly changed. In this way, factors with large contribution ratios can be used to adjust the total variance.

If a design parameter has no significant effect on the output, its permissible difference can be increased. Doing this will lead to a larger tolerance and a lower cost part or element. If the estimated new total variance does not exceed the target variance, the tolerance of one or more factors can be enlarged. In this way, factors with small contribution ratios can be used to adjust the cost. Final decision on the tolerance is made in the next step.

There can be cases in which the estimation of the total variance is not reliable, for example, a case in which a tolerance is substantially enlarged and a case in which a factor has a large higher order effect on the output. In such cases, conducting a confirmation experiment is recommended. If the tolerance of factor B is enlarged by a factor of λ (λ>1), the linear effect of B is enlarged by a factor of λ^2, but the higher order effect of B can be enlarged even more. The quadratic effect of B, for example, is increased by a factor of λ^4. A confirmation experiment can be conducted by RTD experimentation. In the tolerance reduction case (λ<1), the higher order effects of B can be reduced, so the error variance does not change much.

If the present permissible difference Δ_P of the design parameter B is changed to new permissible difference Δ_N as $\Delta_N = \lambda\Delta_P$, a new total variance V_{TN} can be estimated from the present total variance V_{TP}:

$$V_{TN} = \left(\frac{\Delta_N}{\Delta_P}\right)^2 V_{B\ell} + V_e = \lambda^2 V_{Bl} + V_e = \frac{\lambda^2 \rho_{Bl} + \rho_e}{100} V_{TP} = \left[1 + \left(\lambda^2 - 1\right)\frac{\rho_{Bl}}{100}\right] \times V_{TP}, \tag{26}$$

where $\lambda = {\Delta_N}/{\Delta_P} = {d_N}/{d_P}$ is assumed.

When three factors (D, E, and F, for example) are used to adjust the total variance and their permissible differences are changed to $\lambda_D\Delta_D$, $\lambda_E\Delta_E$, and $\lambda_F\Delta_F$, respectively, the new total variance V_{TN} is estimated as

$$V_{TN} = \left[1 + \left(\lambda_D^2 - 1\right)\frac{\rho_{Dl}}{100} + \left(\lambda_E^2 - 1\right)\frac{\rho_{El}}{100} + \left(\lambda_F^2 - 1\right)\frac{\rho_{Fl}}{100}\right] \times V_{TP}, \tag{27}$$

where the independence of the factors' effects is assumed. If there is a high correlation between the factors, a more intensive study for estimating a new total variance is needed.

4.3.2 Deciding tolerance

In the final step of RTD, the improvement in quality is compared with the cost of improvement.

For estimating the improvement in quality, the benefits of an improvement in quality are transformed into a monetary amount. Taguchi's loss function is used to estimate the quality level. Taguchi's loss function expresses the level of the designed quality of a product on a monetary scale. Taguchi's loss function is used to transfer the total variance V_T into the equivalent monetary loss L.

The quality loss L per product can be estimated using Taguchi's loss function,

$$L = k\sigma^2 = kV_T = \frac{A}{\Delta^2} V_T, \tag{28}$$

where Δ denotes the permissible difference in the design parameter, and A denotes the loss per product when the error in the value of the design parameter exceeds the permissible difference. The quality loss L per product represents the quality level of the product in a monetary unit the same as for cost. The quality loss represents the loss in the market by the product which has variability in output. When quality loss L is large, the quality of the designed product is low.

If maintaining the quality level of the product has a cost C per product, the total loss L_T per product is estimated as

$$L_T = L + C. \tag{29}$$

Cost C includes, for example, the cost of purchasing parts or elements and the cost of maintaining the manufacturing process within the tolerance. If a part or element is upgraded through a tolerance change in the manufacturing process, for example, the cost of the upgraded part or element per product is included in cost C.

Total improvement gain G can be estimated as:

$$G = L_{TP} - L_{TN} = (L_P + C_P) - (L_N + C_N) = (L_P - L_N) + (C_P - C_N) = \Delta L + \Delta C, \quad (30)$$

where L with subscripts P and N denote the total loss under the present condition and that under the new condition, respectively, and ΔL and ΔC denote the improvement in quality and the improvement in cost, respectively.

If the cost of improvement ΔC is zero, the improvement in quality can be simply estimated as the difference ΔL between present loss L_{TP} and new loss L_{TN}:

$$G = \Delta L = L_{TP} - L_{TN} = kV_{TP} - kV_{TN} = k(V_{TP} - V_{TN}). \quad (31)$$

If the improvement in quality exceeds the improvement cost, that is, $G>0$, the new tolerance can be applied because the tolerance change is cost beneficial.

If the improvement cost exceeds the improvement in quality, that is, $G \le 0$, the new tolerance cannot be applied, and other tolerances should be tested.

These are the decisions made using RTD. If there are other restrictions, relevant decisions can be made after due consideration.

5 RTD case study (1) — Stabilizing a circuit by using theoretical formula

5.1 Experimentation

5.1.1 Objective

The target product is the constant voltage circuit shown in Figure 2. Output V_{out} is the voltage of resistor R_2; it should be constant even if the circuit elements have errors. The objective is stabilizing the circuit by setting optimum specifications for minimizing the variability of V_{out}.

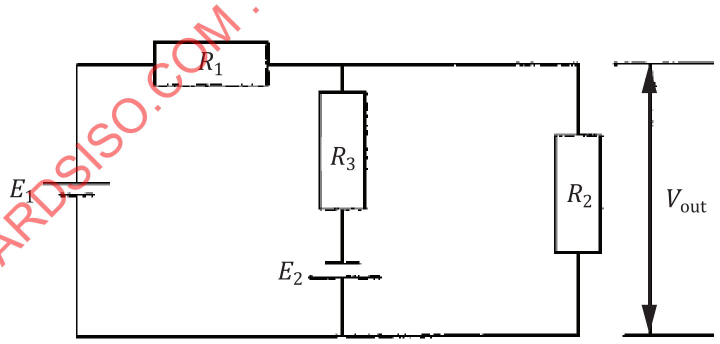


Figure 2 — Configuration of constant voltage circuit

The theoretical formula for the relationship between the output of the circuit and the values of the elements, such as resistors and batteries, is

$$V_{out} = R_2 I_2 = R_2 \frac{\left(1 - \frac{R_1 + R_3}{R_1}\right) E_1 + E_2}{\frac{R_2 (R_1 + R_3)}{R_1} + R_3}. \quad (32)$$

This case study is an example of RTD using a theoretical formula.

5.1.2 Experimental design for data collection and analysis of variance

RPD is applied prior to RTD to improve robustness. Table 6 shows the values of the design parameters before RPD is applied ("current values") and the optimum RPD values. The optimum RPD values are used to be the nominal values of the elements in the circuit.

Table 6 — Results of RPD application

Design parameters	Current value	Optimum value
B:R1 (Ω)	150	350
C:R2 (Ω)	70	15
D:R3 (Ω)	210	160
E:E1 (V)	5	3
F:E2 (V)	15	19

The experimental factors for RTD are taken from the design parameters as same as those for RPD. The levels of the factors for RTD are set around the nominal values as shown in Table 7. The second level is set to nominal value m in accordance with the optimum RPD condition. The first and third levels are set to the values of the errored design parameters, i.e. the deviated values from the nominal values. It is assumed that the standard deviation of the error is set as $\sigma = m/30$. Level width d of a factor is set as $d=\sqrt{\frac{3}{2}}\sigma$. The values shown in Table 7 are the calculated results rounded to five digits, but the calculations here and hereafter are done using all digits.

Table 7 — Levels of factors for RTD

Level	B:R1 (Ω)	C:R2 (Ω)	D:R3 (Ω)	E:E1 (V)	F:E2 (V)
1	335,71	14,388	153,47	2,877 5	18,224
2	350,00	15,000	160,00	3,000 0	19,000
3	364,29	15,612	166,53	3,122 5	19,776
NOTE Values are rounded to five digits.					

All the factors are three level factors, and then they are assigned to columns 2 to 6 of orthogonal array L_{18}, as shown in Table 8. They can be assigned to another orthogonal array, but for simplicity, orthogonal array L_{18} is used. Columns without assigned factors are error columns, then the experimental error can be reliably estimated with more degrees of freedom.

The data are calculated for the designated combination of the factor levels. Table 8 shows the data for the optimum RPD condition calculated using the theoretical Formula (32). Table 8 also shows the data for the current condition using the current values for comparison with the optimum RPD condition.

Table 8 — Assignment of factors to L_{18} and data for the circuit

Column	1	2	3	4	5	6	7	8	Data (V)	
No.	e	B	C	D	E	F	e	e	Current	Optimum
1	1	1	1	1	1	1	1	1	1,421	1,395
2	1	1	2	2	2	2	2	2	1,411	1,447
3	1	1	3	3	3	3	3	3	1,396	1,499
4	1	2	1	1	2	2	3	3	1,551	1,461
5	1	2	2	2	3	3	1	1	1,542	1,513
6	1	2	3	3	1	1	2	2	1,356	1,388
NOTE Data values are rounded to four digits.										

Table 8 *(continued)*

Column	1	2	3	4	5	6	7	8	Data (V)	
No.	e	B	C	D	E	F	e	e	Current	Optimum
7	1	3	1	2	1	3	2	3	1,674	1,474
8	1	3	2	3	2	1	3	1	1,338	1,342
9	1	3	3	1	3	2	1	2	1,639	1,572
10	2	1	1	3	3	2	2	1	1,228	1,335
11	2	1	2	1	1	3	3	2	1,686	1,579
12	2	1	3	2	2	1	1	3	1,327	1,432
13	2	2	1	2	3	1	3	2	1,285	1,335
14	2	2	2	3	1	2	1	3	1,436	1,402
15	2	2	3	1	2	3	2	1	1,742	1,638
16	2	3	1	3	2	3	1	2	1,523	1,412
17	2	3	2	1	3	1	2	3	1,485	1,451
18	2	3	3	2	1	2	3	1	1,635	1,518

NOTE Data values are rounded to four digits.

ANOVA calculation is carried out to the data for the optimum RPD condition. Table 9 shows the effects of all the columns. It indicates that the effects of the error columns 1,7, and 8 are comparable to that of the error term and that orthogonal array assignment is successfully applied.

Table 9 — ANOVA for optimum RPD condition

Source	*f*	*SS*	*V*
(Column 1)	1	0,000 009	0,000 009
B*l*	1	0,000 552	0,000 552
B*q*	1	0,000 011	0,000 011
C*l*	1	0,033 531	0,033 531
C*q*	1	0,000 003	0,000 003
D*l*	1	0,043 011	0,043 011
D*q*	1	0,000 033	0,000 033
E*l*	1	0,000 207	0,000 207
E*q*	1	0,000 001	0,000 001
F*l*	1	0,049 683	0,049 683
F*q*	1	0,000 002	0,000 002
(Column 7) *l*	1	0,000 005	0,000 005
(Column 7) *q*	1	0,000 001	0,000 001
(Column 8) *l*	1	0,000 041	0,000 041
(Column 8) *q*	1	0,000 002	0,000 002
e	2	0,000 034	0,000 017
T	17	0,127 126	—

NOTE Values are rounded to appropriate number of decimal places.

If the variance of an effect is smaller than or comparable to the error variance, the effect is pooled into the error term. All the variances of linear effects of factors B to F are larger than the error variance in this case, so they are not pooled into the error term. The quadratic terms of all factors are small compared with the corresponding linear terms and comparable to the error term. They are thus pooled into the error term. The effects of error columns 1, 7, and 8 are also pooled into the error term.

The resultant pooled ANOVA is shown in Table 10. The pure sum of squares *S'* and the contribution ratio ρ of each effect are shown in the table.

Table 10 — Pooled ANOVA for the optimum RPD condition

Source	*f*	*SS*	*V*	*S'*	*ρ*
B*l*	1	0,000 552	0,000 552	0,000 540	0,42
C*l*	1	0,033 531	0,033 531	0,033 520	26,37
D*l*	1	0,043 011	0,043 011	0,042 999	33,82
E*l*	1	0,000 207	0,000 207	0,000 195	0,15
F*l*	1	0,049 683	0,049 683	0,049 671	39,07
e	12	0,000 142	0,000 012	0,000 201	0,16
T	17	0,127 126	0,007 478	—	100,00
NOTE Values are rounded to appropriate number of decimal places.					

The pure sum of squares S'_{Bl} and the contribution ratio $\rho_{B\ell}$, of linear effect of factor B, for example, is calculated as follows:

$$\begin{aligned}\rho_{B\ell} &= \frac{S'_{Bl}}{S_T}\times 100 = \frac{S_{Bl} - f_{Bl}\times V_e}{S_T}\times 100 \quad (\%)\\ &= \frac{0{,}000552 - 1\times 0{,}000012}{0{,}127126}\times 100\\ &= \frac{0{,}000540}{0{,}127126}\times 100\\ &= 0{,}42 \quad (\%)\end{aligned} \tag{33}$$

The contribution ratios of the other factors, C to F, are calculated in the same way.

The contribution ratio of the error term is calculated as follows:

$$\begin{aligned}\rho_e &= \frac{S'_e}{S_T}\times 100 = \frac{f_T\times V_e}{S_T}\times 100 = \frac{17\times 0{,}000012}{0{,}127126}\times 100 \quad (\%)\\ &= \frac{0{,}000201}{0{,}127126}\times 100\\ &= 0{,}16 \quad (\%)\end{aligned} \tag{34}$$

ANOVA calculation is also carried out to the data for the current condition for comparison. Table 11 shows the resultant pooled ANOVA.

Table 11 — Pooled ANOVA for the current condition

Source	*f*	*SS*	*V*	*S'*	*ρ*
B*l*	1	0,056 630	0,056 630	0,056 595	14,64
C*l*	1	0,014 296	0,014 296	0,014 261	3,69
D*l*	1	0,129 569	0,129 569	0,129 534	33,51
E*l*	1	0,033 357	0,033 357	0,033 322	8,62
F*l*	1	0,152 300	0,152 300	0,152 266	39,39
e	12	0,000 416	0,000 035	0,000 589	0,15
T	17	0,386 567	0,022 739	—	100,00
NOTE Values are rounded to appropriate number of decimal places.					

The total variance of the optimum RPD condition of 0,007 478 (Table 10) is much smaller than that of the current condition of 0,022 739 (Table 11). It means that the standard deviations of the output are 0,086 V for the optimum RPD condition and 0,151 V for the current condition. RPD thus reduced the standard deviation of the output by more than 40 % compared with that for the current condition. This demonstrates the advantage of using RPD prior to RTD. RPD achieve small total variability by selecting nominal values without cost-up.

As shown in Table 10, the factors with large contribution ratios are factors C, D, and F. The factors with small contribution ratios are factors B and E. The effect of a tolerance change is considered in the next step.

5.2 Tolerance determination

The pooled ANOVA for the optimum RPD condition are shown in Table 10. If the results of the optimum RPD condition are not satisfactory, RTD is used to achieve the target variability. For example, if the target standard deviation is 0,050 V, the optimum RPD condition is not satisfactory, and RTD is used to reduce the tolerances in order to achieve the target standard deviation.

If the results of the optimum RPD condition are satisfactory in terms of the total output variance, RTD is used to examine the possibility of reducing manufacturing cost. For example, if the target standard deviation is 0,090 V, the optimum RPD condition is satisfactory, and RTD is used to widen the tolerances in order to reduce manufacturing cost.

Three cases are examined for tolerance determination to improve the quality and/or reduce the cost of a circuit.

Case 1: To improve quality, the permissible differences of factors C, D, and F are reduced by half; that is, $\lambda = 1/2$.

The new total variance is calculated as follows:

$$\begin{aligned} V_{\mathrm{TN}} &= \left[1+\left(\lambda_{\mathrm{C}}^2-1\right)\frac{\rho_{\mathrm{C}l}}{100}+\left(\lambda_{\mathrm{D}}^2-1\right)\frac{\rho_{\mathrm{D}l}}{100}+\left(\lambda_{\mathrm{F}}^2-1\right)\frac{\rho_{\mathrm{F}l}}{100}\right]\times V_{\mathrm{TP}} \\ &= \rho_{\mathrm{T}}\times V_{\mathrm{TP}} \\ &= \left[1+\left(\left(\tfrac{1}{2}\right)^2-1\right)\frac{26{,}37}{100}+\left(\left(\tfrac{1}{2}\right)^2-1\right)\frac{33{,}82}{100}+\left(\left(\tfrac{1}{2}\right)^2-1\right)\frac{39{,}07}{100}\right]\times 0{,}007\,478 \\ &= \frac{25{,}55}{100}\times 0{,}007\,478 \\ &= 0{,}001\,911 \\ &= (0{,}044)^2 \end{aligned} \quad (35)$$

Case 2: To reduce cost, the permissible differences of factors B and E are doubled; that is, λ = 2.

The new total variance is calculated as follows:

$$\begin{aligned} V_{TN} &= \left[1+\left(\lambda_B^2-1\right)\frac{\rho_{Bl}}{100}+\left(\lambda_E^2-1\right)\frac{\rho_{El}}{100} \right]\times V_{TP} \\ &= \rho_T \times V_{TP} \\ &= \left[1+\left(2^2-1\right)\frac{0{,}42}{100}+\left(2^2-1\right)\frac{0{,}15}{100} \right]\times 0{,}007\,478 \\ &= \frac{101{,}74}{100}\times 0{,}007\,478 \\ &= 0{,}007\,608 \\ &= (0{,}087)^2 \end{aligned} \tag{36}$$

Case 3: To improve quality and reduce cost, the changes made in cases 1 and 2 are applied simultaneously.

The new total variance is calculated as follows:

$$\begin{aligned} V_{TN} &= \rho_T \times V_{TP} \\ &= \left[1+\left(\lambda_C^2-1\right)\frac{\rho_{Cl}}{100}+\left(\lambda_D^2-1\right)\frac{\rho_{Dl}}{100}+\left(\lambda_F^2-1\right)\frac{\rho_{Fl}}{100}+\left(\lambda_B^2-1\right)\frac{\rho_{Bl}}{100}+\left(\lambda_E^2-1\right)\frac{\rho_{El}}{100} \right]\times V_{TP} \\ &= \left[1+\left((\tfrac{1}{2})^2-1\right)\frac{26{,}37}{100}+\left((\tfrac{1}{2})^2-1\right)\frac{33{,}82}{100}+\left((\tfrac{1}{2})^2-1\right)\frac{39{,}07}{100}+\left(2^2-1\right)\frac{0{,}42}{100}+\left(2^2-1\right)\frac{0{,}15}{100} \right]\times 0{,}007\,478 \\ &= \frac{27{,}29}{100}\times 0{,}007\,478 \\ &= 0{,}002\,041 \\ &= (0{,}045)^2 \end{aligned} \tag{37}$$

The new total variances and standard deviations for the three cases are summarized in Table 12.

Table 12 — New total variance V_T and standard deviation σ_T for cases 1, 2, and 3

	Optimum RPD condition	Case 1	Case 2	Case 3
ρ_T (%)	100,00	25,55	101,74	27,29
V_T (V^2)	0,007 478	0,001 911	0,007 608	0,002 041
σ_T (V)	0,086	0,044	0,087	0,045
NOTE Values are rounded to appropriate number of decimal places.				

Since there is no information on cost, only quality improvement is discussed here. These results demonstrate that using RTD can reduce the variability of circuit output and thus achieve technical improvement in circuit quality.

In case 1, the standard deviation of the output is 0,044 V, which is less than the target variability of 0,050 V. This leads to the improvement in quality.

In case 2, the standard deviation of the output is 0,087 V, which is almost the same as that for the optimum RPD condition. Even if the tolerances of design parameters B and E are twice those assumed in the RPD experiment, the increase in the total output variance is not so much. This can lead to cost reduction.

In case 3, the standard deviation of the output is 0,045 V, which is slightly larger than that in case 1 but still smaller than the target variability of 0,050 V. This demonstrates that both variability and cost can be improved.

6 RTD case study (2) — Stabilizing the piston by using a simulation experiment

6.1 Experimentation

6.1.1 Objective

The demand for passenger cars equipped with small direct injection diesel engines is increasing because such engines have high heat efficiency and low carbon dioxide emissions. It is essential to stabilize and lower the temperature of the piston lip to improve the durability of piston performance. The objective is to set optimum tolerances for improving piston reliability by minimizing variability in piston lip temperature and thereby efficiently achieve the target variance.

6.1.2 Experimental design for data collection and analysis of variance

RTD is used to determine the optimum tolerances of the piston lip design parameters, after RPD is used to determine the optimum nominal values of design parameters for improving robustness[5]. The optimum RPD condition is shown in Table 13. Second level represents the current level.

Table 13 — The optimum RPD condition for piston lip

Factor (design parameter)		Optimum RPD condition	Level in RPD
A	Width of cooling hole	Std.	2
B	Distance of hole from top	Down	1
C	Distance of hole from centre	Down	1
D	Thickness of wall	Std.	2
E	Roundness of wall	More up	1
F	Distance of ring groove	Down	1
G	Temperature of cooling oil	Down	1
H	Temperature of cylinder wall	Down	1

The factors shown in Table 13 are control factors in RPD. The RPD experiment identified the optimum RPD condition as the combination of optimum levels of the factors from the viewpoint of robustness, and set the optimum levels' values to the nominal values. Factors A to D are dimensional design parameters, as shown in Figure 3 a), and factors G and H are piston lip temperatures set by a control device in the engine. Cooling oil runs through the cooling hole and controls the piston lip temperature. Temperature of the piston lip is calculated using CAE (computer aided engineering) software for FEM (finite element method) heat analysis in the model depicted in Figure 3 b).

a) Factors in piston lip

b) FEM analysis model

Key

1 cooling hole

Figure 3 — Target product: piston lip

The experimental factors in RTD are the same design parameters as those in RPD. However, the design parameters in RTD are treated as noise factors, not control factors, because the errors in the design parameters are not controllable in manufacturing. RTD is used to estimate the variance of the effects of the experimental factors. Table 14 shows the factors and their levels for RTD. For the three-level factors, level 2 has the value *m*, where *m* is the nominal value which is identified as the optimum RPD condition. Level width *d* is set in accordance with the standard deviation *σ* in the current condition.

Table 14 — Experimental factors for RTD

Factor		*m* by RPD	Level 1	Level 2	Level 3
A	Width of cooling hole	Std.	$m_A - d_A$	$m_A + d_A$	—
B	Distance of hole from top	Down	$m_B - d_B$	m_B	$m_B + d_B$
C	Distance of hole from centre	Down	$m_C - d_C$	m_C	$m_C + d_C$
D	Thickness of wall	Std.	$m_D - d_D$	m_D	$m_D + d_D$
E	Roundness of wall	More up	$m_E - d_E$	m_E	$m_E + d_E$
F	Distance of ring groove	Down	$m_F - d_F$	m_F	$m_F + d_F$
G	Temperature of cooling oil	Down	$m_G - d_G$	m_G	$m_G + d_G$
H	Temperature of wall	Down	$m_H - d_H$	m_H	$m_H + d_H$

In RTD, product output is the temperature of the piston lip, the same as in RPD. It is calculated in a simulation experiment using CAE software for FEM heat analysis. FEM heat analysis is carried out using boundary conditions based on the results of actual experimental data for the current piston design. The factors are assigned to orthogonal array L_{18}. The temperatures of the piston lip is calculated by FEM heat analysis under the conditions determined using L_{18}. Table 15 shows the assignment of factors and the results of the FEM heat analysis.

Table 15 — Assignment of factors to L_{18} and output data calculated by FEM analysis

Column	1	2	3	4	5	6	7	8	Data (°C)
No.	A	B	C	D	E	F	G	H	
1	1	1	1	1	1	1	1	1	292,090
2	1	1	2	2	2	2	2	2	294,435
3	1	1	3	3	3	3	3	3	296,931
4	1	2	1	1	2	2	3	3	298,361
NOTE Data values are rounded to appropriate number of decimal places.									

Table 15 *(continued)*

Column No.	1 A	2 B	3 C	4 D	5 E	6 F	7 G	8 H	Data (°C)
5	1	2	2	2	3	3	1	1	294,042
6	1	2	3	3	1	1	2	2	293,420
7	1	3	1	2	1	3	2	3	298,816
8	1	3	2	3	2	1	3	1	294,672
9	1	3	3	1	3	2	1	2	294,553
10	2	1	1	3	3	2	2	1	293,125
11	2	1	2	1	1	3	3	2	295,432
12	2	1	3	2	2	1	1	3	291,883
13	2	2	1	2	3	1	3	2	295,097
14	2	2	2	3	1	2	1	3	294,217
15	2	2	3	1	2	3	2	1	293,474
16	2	3	1	3	2	3	1	2	295,602
17	2	3	2	1	3	1	2	3	295,294
18	2	3	3	2	1	2	3	1	294,183
NOTE Data values are rounded to appropriate number of decimal places.									

ANOVA calculation is carried out on the temperature data in Table 15, and the linear effect of each factor is calculated. The pure sum of squares and the contribution ratios of each effect are also calculated. The pooled ANOVA is shown in Table 16 with the pure sum of squares and the contribution ratio. All the quadratic terms of factors are small compared with the corresponding linear terms and comparable to the error term. They are thus pooled into the error term.

Table 16 — Pooled ANOVA for piston lip temperature

Source	*f*	*SS*	*V*	*S'*	*ρ*
A	1	4,513 0	4,513 0	4,503 5	7,70
B*l*	1	7,090 2	7,090 2	7,080 7	12,10
C*l*	1	6,230 9	6,230 9	6,221 4	10,63
D*l*	1	0,127 5	0,127 5	0,118 1	0,20
E*l*	1	0,065 1	0,065 1	0,055 7	0,10
F*l*	1	11,684 1	11,684 1	11,674 6	19,95
G*l*	1	12,585 0	12,585 0	12,575 5	21,49
H*l*	1	16,137 9	16,137 9	16,128 5	27,56
e	9	0,085 2	0,009 5	0,160 9	0,27
T	17	58,518 9	3,442 3	—	100,00
NOTE Values are rounded to appropriate number of decimal places.					

The contribution ratio $\rho_{Bl,}$ of the linear effect of factor B, for example, is calculated as follows:

$$\begin{aligned}\rho_{Bl} &= \frac{S'_{Bl}}{S_T}\times 100 = \frac{S_{Bl} - f_{Bl}\times V_e}{S_T}\times 100 \quad (\%)\\ &= \frac{7{,}0902 - 1\times 0{,}0095}{3{,}4423}\times 100\\ &= \frac{7{,}0807}{3{,}4423}\times 100\\ &= 12{,}10 \quad (\%)\end{aligned} \tag{38}$$

The contribution ratios of the other factors are calculated in the same way.

The contribution ratio of the error term is calculated as follows:

$$\begin{aligned}\rho_e &= \frac{S'_e}{S_T}\times 100 = \frac{17\times 0{,}0095}{3{,}4423}\times 100 \quad (\%)\\ &= 0{,}27 \quad (\%)\end{aligned} \tag{39}$$

Table 16 shows that the contribution ratios of the linear effects of factors G and H are high while those of factors D and E are low. The effect of a tolerance change is considered in the next step.

6.2 Tolerance determination

Three cases are examined for tolerance determination to improve the quality and/or reduce the cost of a piston lip on the basis of the results shown in Table 16.

Case 1: To improve quality, the permissible differences of factors G and H are reduced to half the current permissible differences to reduce the total variance in the output temperature ($\lambda = 1/2$).

Case 2: To reduce cost, the permissible differences of factors D and E are increased to twice the current permissible differences ($\lambda = 2$).

Case 3: To improve quality and reduce cost, the changes made in cases 1 and 2 are applied simultaneously.

The new variances are calculated as follows:

For case 1:

$$\begin{aligned}V_{TN1} &= \left[1 + \left(\lambda_G^2 - 1\right)\frac{\rho_{Gl}}{100} + \left(\lambda_H^2 - 1\right)\frac{\rho_{Hl}}{100}\right]\times V_{TC}\\ &= \left[1 + \left(\left(\tfrac{1}{2}\right)^2 - 1\right)\frac{21{,}49}{100} + \left(\left(\tfrac{1}{2}\right)^2 - 1\right)\frac{27{,}56}{100}\right]\times 3{,}4423\\ &= \frac{63{,}21}{100}\times 3{,}4423\\ &= 2{,}1759\\ &= (1{,}48)^2\end{aligned} \tag{40}$$