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Applications of statistical and related methods to new technology and product development process — Robust parameter design (RPD)

Application de méthodologies statistiques et connexes pour le développement de nouvelles technologies et de nouveaux produits — Modèle paramétrique robuste

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Foreword

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The committee responsible for this document is ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 8, *Application of statistical and related methodology for new technology and product development*.

Introduction

Robust parameter design, also called parameter design, can be applied in product design stage to identify the optimum nominal values of design parameters based on the assessment of robustness of its function. Robustness assessment is performed as a consideration of overall loss during the product's life cycle. The overall loss is composed of costs and losses at each stage of the product's life. It includes all the costs incurred during not only its production stage, but also its disposal stages.

When a product is not robust, the product causes many environmental and social economic losses (including losses to the manufacturer and the users) due to its poor quality caused by functional variability throughout its usable lifetime from shipping to final disposal. Product suppliers have responsibilities and obligations to supply robust products to the market to avert losses and damages resulting from defects in the products.

The aim of applying parameter design in product design is to prevent defects, failures, and quality problems that can occur during the usage of the product. A robust product, an output of parameter design, is a product which is designed in such a way as to minimize user's quality losses caused by defects, failures, and quality problems. Note that defects, failures, and quality problems are caused by functional variability of a non-robust product. In parameter design, optimum nominal values of a product's design parameters can be selected by treating a product's design parameters as control factors and by assessing robustness under noise factors. The use of parameter design at development and design stages makes it possible to determine the optimum product design and specification so that the product is robust in the market.

At manufacturing stage, the product suppliers manufacture their products that meet the product specifications. One can optimize manufacturing processes to produce the products that meet the specifications. However, robustness against customer's environment and products' aging can be addressed only by product design.

Robust parameter design methodology provides effective methods for achieving robustness through its design of specification determination, and it is a preventive countermeasure against various losses in the market.

In practice, many product's defects and failures occur due to the product's response that deviates from or varies around the designed target values by the change in usage environment and deterioration, i.e. noise conditions. The variability of product's response due to noises can be used as a measure of robustness, because market losses increase in proportion to the magnitude of variability of product's response. SN ratio, corresponding to the inverse of the variability measure, is used as a measure of goodness in robustness. In other words, the higher the SN ratio is, the less the market losses are.

For the experimental plan of parameter design, direct product of inner array and outer arrays is proposed. Control factors are assigned to the inner array, and signal and noise factors are assigned to the outer array. By using a direct product plan, all the first level interactions between control factors and noise factors can be assessed and can be utilized to select the optimum level of control factors from the point of view of robustness.

Assessing robustness through SN ratio is a key of parameter design. The outer array is for evaluating SN ratio, robustness, for each combination of levels of control factors indicated by the inner array. The inner array is for comparing SN ratios and selecting the optimum combination of system's design parameters. As for the inner array, an orthogonal array L_{18}, is recommended as an efficient plan, and then only the applications of an orthogonal array L_{18} are discussed in this International Standard. Applications of experimental layout other than orthogonal array L_{18} can be found in the examples in references in the Bibliography. More detailed discussions on inner array and orthogonal arrays can be found in the references.

Robust parameter design (RPD), and thus this International Standard, is directly targeted at the losses incurred at the usage stage. Where possible, losses at other stages are also investigated so that the results of parameter design can be applied to perform the optimum product design for the whole stages of the product's life cycle.

Applications of statistical and related methods to new technology and product development process — Robust parameter design (RPD)

1 Scope

This International Standard gives guidelines for applying the optimization method of robust parameter design, also called as parameter design, an effective methodology for optimization based on Taguchi Methods, to achieve robust products.

This International Standard prescribes signal-to-noise ratio (hereafter SN ratio) as a measure of robustness, and the procedures of parameter design to design robust products utilizing this measure. The word “robust” in this International Standard means minimized variability of product’s function under various noise conditions, that is, insensitivity of the product’s function to the changes in the levels of noises. For robust products, their responses are sensitive to signal and insensitive to noises.

The approach of this International Standard can be applied to any products that are designed and manufactured, including machines, chemical products, electronics, foods, consumer goods, software, new materials, and services. Manufacturing technologies are also regarded as products that are used by manufacturing processes.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-3, *Statistics — Vocabulary and symbols — Part 3: Design of experiments*

3 Terms and definitions and symbols

3.1 Term and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1 and ISO 3534-3, and the following apply.

3.1.1
function
work which a system performs in order to fulfil its objective

Note 1 to entry: A function can be expressed by the mathematical form of input-output relation.

3.1.2
robustness
degree of smallness in variability of a system’s function under various noise conditions

Note 1 to entry: System’s performance can be assessed by robustness. SN ratio is a quantitative measure of robustness.

3.1.3
signal-to-noise ratio
SN ratio
ratio of useful effects to harmful effects in response variations

Note 1 to entry: SN ratio is usually expressed in db value. The notation of db is used instead of dB for SN ratios of robustness measurements.

Note 2 to entry: The anti-logarithm value of an SN ratio, real number, is the inverse of a variation measure such as a variance or a coefficient of variation, and inversely proportional to monetary loss.

Note 3 to entry: The change in response caused by intentional change of input signal value is a useful effect. In case of the ideal function being zero point proportional, the linear slope forced through the zero point is a useful term.

Note 4 to entry: The change in response caused by noise factors is a harmful effect. Effects of noise factors and deviation from the ideal function are examples.

Note 5 to entry: SN ratio should contain the variability under noise factors and the discrepancy from the ideal function under average usage condition.

3.1.4
sensitivity
amount of change in response caused by unit change of input

Note 1 to entry: Sensitivity is usually expressed in db value.

Note 2 to entry: For dynamic characteristic cases, the sensitivity shows the magnitude of linear coefficient due to input signal, β^2, where β is a proportional constant.

Note 3 to entry: For the nominal-the-best response, the sensitivity shows the magnitude of mean, m^2, where m is an average of responses.

3.1.5
noise
variable which disturbs a system's function

Note 1 to entry: Any variable in the user's conditions for operating is either a signal or a noise.

Note 2 to entry: Noise is composed of internal noise and external noise. They are sometimes called as capacity and demand, respectively. Changes of internal constant of the system or its parts over time, such as deterioration, aging. and wear, and manufacturing variations are examples of internal noises. Usage conditions and environment conditions of the product are examples of external noises.

3.1.6
signal
input variable to the system, which is intentionally changed by the user to get an intended value of response in input-output relation

Note 1 to entry: Any variable in the user's conditions for operating is either a signal or a noise

Note 2 to entry: There are two kinds of signal: active signal and passive signal. Active signal is operated by user to get intended response, for example, rotating angle of a steering wheel to change the vehicle's direction. Passive signal is used by user to know the value of input from response reading, for example, temperature in thermal measurement. In both cases, output will change by changing the value of the signal but the user wants to get response value in the active case, and the user wants to know the value of signal in the passive case.

3.1.7
dynamic characteristics
output response which has multiple ideal target values depending on the value of a signal

Note 1 to entry: The relation between dynamic characteristics and a signal can be expressed by input-output functional form. The output of a system's function is dynamic characteristics in many cases.

3.1.8
static characteristics
non-dynamic characteristics
output response which has a fixed target value

Note 1 to entry: Static characteristics can be categorized into three groups depending on the target value; nominal-the-best, smaller-the-better, and larger-the-better characteristics, where the target value is a finite value, zero, and infinity, respectively.

3.1.9
inner array
experimental plan where design parameters are assigned as control factors or indicative factors

Note 1 to entry: Each treatment run will be assessed for robustness using SN ratio and sensitivity.

Note 2 to entry: Orthogonal arrays are recommended for the inner array because many design parameters can be taken into consideration in one set of experiments as control factors.

Note 3 to entry: Experimental factors should be categorized by their roles and assigned separately to inner array or outer array based on their roles in parameter design. Control factors and indicative factors should be assigned to the inner array.

3.1.10
outer array
experimental plan where variables in users' conditions are assigned as noise factors or signal factors for evaluating SN ratio and sensitivity

Note 1 to entry: Any variable in user's conditions for operating is either a signal or a noise.

Note 2 to entry: Experimental factors should be categorized by their roles and assigned separately to inner array or outer array based on their roles in parameter design. Noise factors and signal factors should be assigned to the outer array.

3.2 Symbols

f degree of freedom

k number of levels of signal factor

L linear form

L_i linear form for level of i

M signal factor/input signal

Mi signal level of i

M_i value of signal level of i

N noise factor

n number of levels of noise factor

Ni noise level of i

p_0 standardized error rate

r sum of squares of input signal levels/effective divisor

S sensitivity

S_T total sum of squares

S_m sum of squares due to mean

S_β sum of squares due to linear slope β

$S_{N \times \beta}$ sum of squares due to the variation of linear slope β between noise levels

S_e sum of squares due to error

S_{opt} estimated value of sensitivity for optimum condition

S_{base} estimated value of sensitivity for baseline condition

S_{cur} estimated value of sensitivity for current condition

V_e variance due to error/error variance

V_N variance due to pooled error/variance due to error and noise

y output response

β sensitivity coefficient/linear slope

ΔS gain in sensitivity

$\Delta\eta$ gain in SN ratio

η SN ratio

η_{opt} estimated value of SN ratio for optimum condition

η_{base} estimated value of SN ratio for baseline condition

η_{cur} estimated value of SN ratio for current condition

ρ_0 standardized contribution ratio

4 Robust parameter design — Overview

4.1 Requirements

Robust parameter design is a rational and efficient assessment for discovering technical means to improve robustness in the designing process. It is, therefore, necessary to provide the following two procedures:

a) a procedure for accurate and simple evaluation of robustness;

b) a procedure for efficient assessment of multiple technical means.

This clause provides the approach to the goal of parameter design, and more detailed and specific steps of a robustness evaluation and a parameter design experiment are described in Clauses 5 and 6.

4.2 Assessing the robustness of a system

How can the robustness of a system be accurately assessed by the SN ratio? The robustness of a system is associated with many usage conditions of the system, so it cannot be assessed by a simple measurement. To clarify hidden factors associated with the robustness, the assessment should be approached from the following two viewpoints.

a) Use of an ideal function: The ideal function is a target function of the system. Actual function of the system should be measured and compared with the ideal function of the system in the robustness evaluation. It is important to avoid defects, failure modes, or quality problems for achieving the ideal function of the system.

b) Use of noise factors: Actual system in usage is working under various noise conditions. Noise effects should be intentionally introduced in the experiment by changing noise levels and the actual function of the system should be measured and evaluated under those predetermined noise conditions. Evaluation of the robustness strongly depends on the choice of noise factors and their levels. It is essential to apply effective noise strategies.

The function of a system is a work that it performs in order to fulfil its objective. For example, the function of an electric lamp is to transform electric energy into light energy, and the function of a wind turbine is to transform natural wind energy into rotating energy to perform a work such as water pumping. The function is normally expressed in a mathematical functional form of a relationship between input and output energies. The mathematical functional form can be expressed in many ways. Zero-point proportional formula is common in energy transformation of real physical systems. The details will be discussed in Clause 5.

Input and output characteristics are fixed based on the system's ideal function. Input characteristic is called as a signal in input-output relationship; this is because the changes in output are acquired by the user's intentional changing of the input in real usage and also in the experiment of parameter design. The signal is associated with energy or information necessary to perform its function. The signal factor is one of the user's conditions for changing the input when the users of the system try to control the output of the system. The signal factor has three or more levels in the experiment for dynamic characteristic so that the straightness of the actual input-output relationship could be evaluated. There is no signal factor for static characteristic because it has only one target output. Output characteristic is called as an output response or simply a response.

It is important to identify a suitable measurement method of output response. In time-dependent phenomena, for example, detection of output response is difficult in some cases. New measurement methods should be developed in those cases. The output response is associated with the purpose of a system. In the case of illumination, for example, the output response is magnitude of light, and in the case of a water pump, it is quantity of water.

Noise condition is a source that makes the system's actual function deviate from the ideal function. Examples include environmental conditions in actual working, such as temperature and humidity, an actual supplied voltage, electrical noise conditions, frequency of operations, and stress. They are called as external noises. On the other hand, there are noise conditions which are called internal noises, such as aging and wear. Examples include operating and/or idling time length after started, deterioration of system's parts after long operation, and manufacturing variability of a system and/or its parts. These noise conditions always reduce the system's functional performance to lower level than the level expected at the time of design. Since the purpose of robustness assessment is to clarify the performance by measuring the extent of this reduction, the variation of the system's function under noise conditions should be estimated in robustness evaluation. This is the reason why noise conditions should be taken into the parameter design experiment as noise factors. Three categories of noise factors are a) environment, b) aging and changes over time, and c) manufacturing variations. For the effective noise strategy, various types of noise should be examined in actual usage and environmental conditions.

Figure 1 shows an overview of the evaluation of robustness using a noise factor. Here, multiple data from X_1 to X_n should be obtained for the objective system under noise levels, N1 to Nn, and SN ratio η should be calculated using the data from X_1 to X_n as a robustness measure. Formulations of SN ratio are shown in Clause 5. When more than two systems are compared, the same levels of the same noise factor should be applied for all objective systems

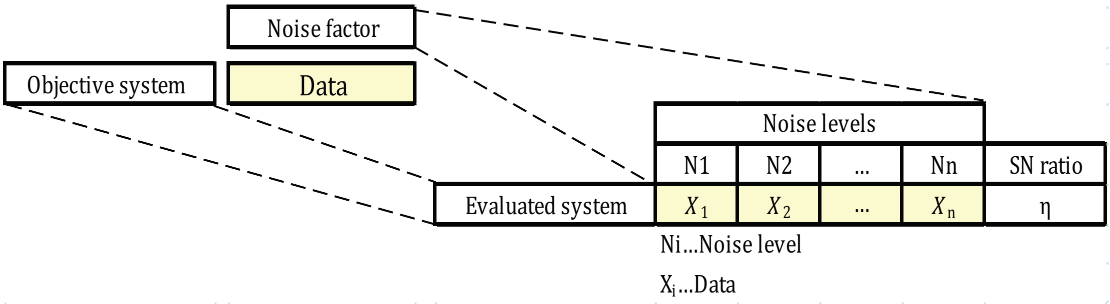

Figure 1 — Robustness assessment with a noise factor

When multiple different types of noise factors are applied in an experiment, an orthogonal array can be applied for determining the noise levels. In Figure 2, noise levels, N1 to Nn, are determined by the combination of levels of noise factors, such as A, B, and C, indicated by the orthogonal array. Experimental layout other than orthogonal arrays can be applicable for determining the noise levels.

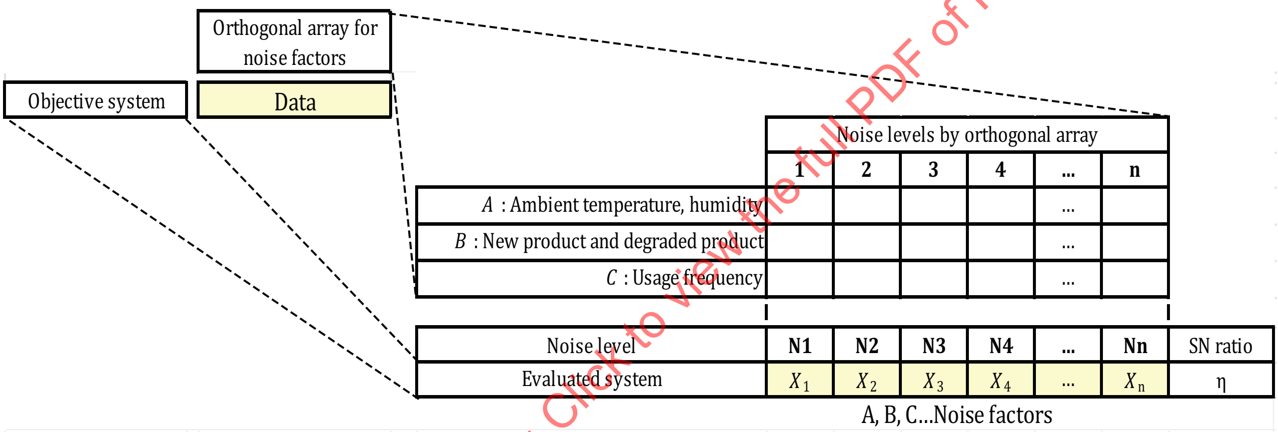

Figure 2 — Robustness assessment with noise levels assigned by an orthogonal array

4.3 Robustness assessment through SN ratio

The ideal function and the noise strategy are the key issues for the robustness assessment of robust parameter design. It is essential to measure and evaluate the variability and efficiency of the actual function of the system. The intent of doing this is that the evaluation covers the whole technical issues of the system's operations to prevent technical problems. Robustness evaluation also involves assessing the effects of noise factors that inhibit the required function. The results are expressed by SN ratio and sensitivity.

The SN ratio can discriminate the true difference in robustness between designs. Only relative comparison of SN ratios is meaningful to perform, because absolute values of SN ratios are affected by setting levels of noise factors. Thus, it is preferable to perform benchmarking in assessing robustness.

A feature of this approach is that the only information needed to evaluate SN ratios is just that on the knowledge of the function of the system and the noise conditions. No detailed technical information about the objective system is needed. SN ratios can be calculated in the same way for the objective systems as long as they have the same function, that is, the same input-output relationship, even if they have different technical constituents. Since the robustness of systems can be accurately evaluated through SN ratios, then the robustness of various systems with different design concepts can be assessed and compared.

The comparison of various systems based on different technologies or different design concepts can be performed in the same way through SN ratio. Systems, such as conventional systems and newly developed systems, one's own systems and one's competitor's systems, can be evaluated and assessed in the same way through SN ratio, when they have the same function. This is the idea to conduct benchmarking on various designs in the robustness assessment through SN ratio.

4.4 An efficient method for assessing technical ideas — Parameter design

Basic technologies and mechanisms should be selected as a design concept first to start designing a system of industrial products. When there are multiple system design concepts to be benchmarked, the robustness assessment introduced in the previous subclause can be applied to select the best design concept.

After selecting the best design concept, the next step is to perform a detailed design by selecting values of system's design parameters. In this detailed design step, designers can optimize the system by selecting the optimum nominal values so that the function of the designed system becomes the most robust and efficient. The system design optimization method performed at this step is called parameter design, because design optimization is performed by setting design parameters to the optimum nominal values.

Consider what sort of states might be significant. When a system is in the optimum state, it achieves the best overall performance in all conceivable usage conditions. More specifically, an industrial system can stably perform its intended function anytime, even when, for example, it is working under a wide range of temperature and humidity, and when it is used in many different ways and in different environments. The optimum design conditions are taken as a combination of design parameters' values that maximize the robustness of the product. Since optimization by parameter design implies optimizing for maximized robustness, that is, minimized variability and maximized efficiency, judgments should be made by using robustness measure, SN ratio, and sensitivity.

The basis of the system design optimization by robustness assessment through SN ratio is a criterion for optimization in parameter design. The robustness assessment should be performed with regard to all the possible designs in design space, but in practice this is impossible. This is because a vast number of tests would have to be performed to take all possible combinations of design parameters into consideration.

As a more practical method for applying in development and design stages, an experiment using an orthogonal array is recommended where the combinations of many design parameters can be tested under a limited number of experimental runs. An orthogonal array plan is recommended not only because it can reduce the number of experimental runs comparing with a full factorial plan with the same number of control factors, but also because it can assign maximum number of control factors in a plan under the situation of same number of experimental runs. Reliability of experimental results should be confirmed in the confirmation experimental run for reproducibility check. Clause 6 describes a specific method for performing the confirmation experiment to check the reproducibility.

Procedure of a parameter design experiment should be as follows:

— (Step 1) Clarify the system's ideal function;

— (Step 2) Select signal factor and its range;

— (Step 3) Select measurement method of output response;

— (Step 4) Develop noise strategy, and select noise factors and their levels;

— (Step 5) Select control factors and their levels from design parameters

— (Step 6) Assign experimental factors to inner or outer array;

— (Step 7) Conduct experiment and collect data;

— (Step 8) Calculate SN ratio, η, and sensitivity, S;

— (Step 9) Generate factorial effect diagrams on SN ratio and sensitivity;

— (Step 10) Select the optimum condition;

— (Step 11) Estimate the improvement in robustness by the gain;

— (Step 12) Conduct a confirmation experiment and check the gain and "reproducibility".

4.5 Two-step optimization (Strategy of parameter design)

Figure 3 presents an overview of parameter design as described above. The experiment in this figure includes two orthogonal arrays; one orthogonal array for control factors, that is, for design parameters (inner array) and the other orthogonal array for noise factors (outer array). This layout is called a direct product plan. The number of experimental data corresponds to the product of the numbers of runs respectively specified in two orthogonal arrays. For example, in the case of the combination of inner array, L_{18}, and outer array, L_{12}, each array has runs of m = 18 and n = 12, and the number of total runs comes to 18 × 12 = 216.

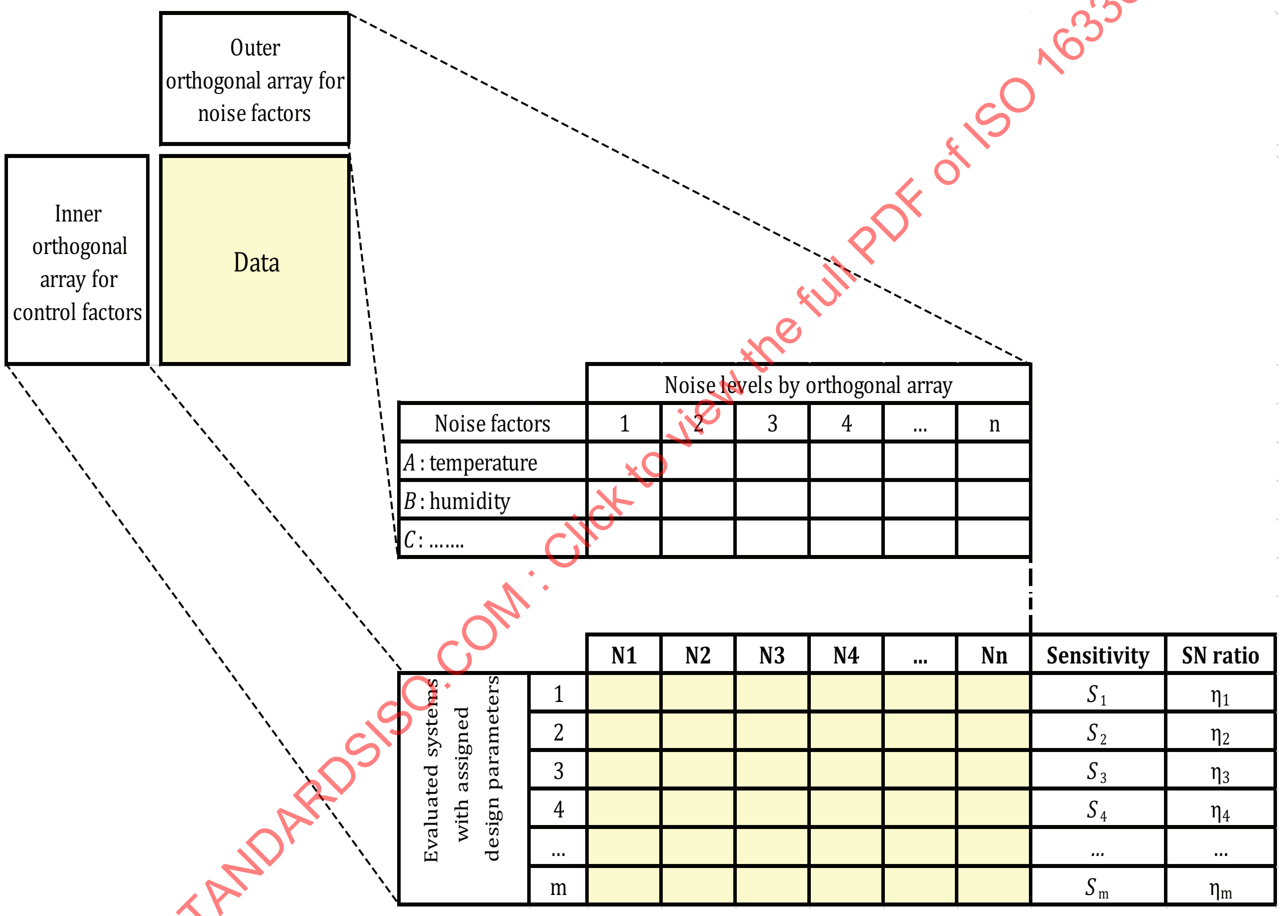

Figure 3 — Direct product plan for parameter design

Full factorial plan can be used as an outer array plan for noise and signal factors instead of an orthogonal array plan in some cases. In the case of physical tests, it is recommended to compound many noise factors into one compounded noise factor. However, it is always recommended to use an orthogonal array plan as an inner array for design parameters, because many design parameters can be assigned in one orthogonal array.

The experimental data obtained for each combination of levels of control factors consist of multiple data with the corresponding number of noise levels. To find out the optimum values of design parameters for robustness, the sensitivity (mean value in case of nominal-the-best response), and the SN ratio should be calculated for each row of inner array, that is, for the combination of values of design parameters. Then the factorial effects of control factors on sensitivity and SN ratio should also be calculated, and they are summarized in factorial effect diagrams as shown in Figures 4 and 5. Specific calculation formulae are described in Clause 6. The optimum values of design parameters are selected using the diagrams on sensitivity and SN ratio. The sensitivity represents the mean value of the data set (in case of static characteristics), and the SN ratio represents robustness.

The factorial effect diagram shows how the system's function is affected by each design parameter incorporated into the experiment. If a factor has a large gradient, it has a large effect on the system's function. Two types of factorial effect diagram represent the degree of influence relating to SN ratio and sensitivity. An important point in two-step design is to pay more attention to SN ratio than to sensitivity. In the first step, the optimum level of control factor should be selected to maximize SN ratio in the factorial effect diagram on SN ratio (see Figure 4), and then, in the second step, typically just one design parameter should be used to adjust a mean value or linear slope, i.e. sensitivity (see Figure 5), to the target value. For this adjustment, it is desired to select one factor with maximal effect on sensitivity and minimal effect on SN ratio. The first step is to optimize the design for robustness by SN ratio, and the second step is to adjust the magnitude to the target value by sensitivity. This two-step design procedure is a very important concept for the design of robustness. This is the reason why parameter design for robustness is also called as a two-step optimization.

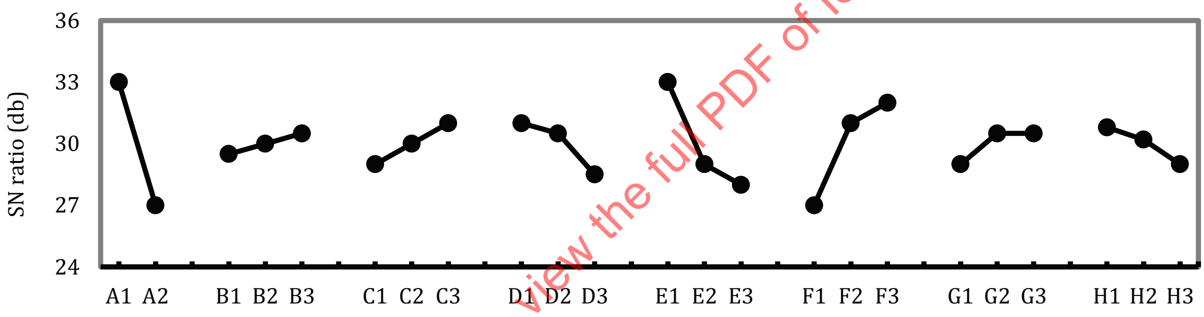

Figure 4 — Factorial effect diagram on SN ratio (robustness)

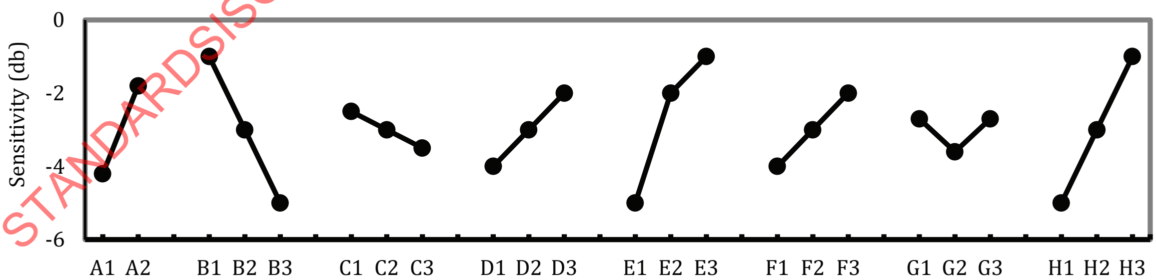

Figure 5 — Factorial effect diagram on sensitivity (mean value or linear slope)

Why is the two-step optimization important? Which is more difficult, the robustness optimization by SN ratio or the magnitude adjustment by sensitivity?

The order of the optimizations is important to design a robust system efficiently. Consider an example of the case of voice recording. If audio data are recorded with high background noise, then adjusting the volume will hardly make the recorded sound any easier to listen to when it is played back. To extract information buried in the noise, techniques such as noise reduction to cancel the effects of noises or using a microphone that is less sensitive to the background noise shall be employed. Improving SN ratio requires advanced technical ability and counter-measures in recording. On the other hand, when the average recording level is too low, it can easily be improved by adjusting the volume during playback. The controlling of the average value of the playback sound by a volume can often be accomplished by a relatively simple method in this way. Adjusting the magnitude can be performed by sensitivity.

Another example is visual imaging function such as photographs and video pictures. The average tone level can easily be corrected, but a picture taken in dark conditions is often very noisy and results in a low-quality picture. There are also limits to how much the picture quality can be improved by image processing.

It is relatively easy to achieve adjusting the magnitude, because it needs just one adjusting parameter. It is easy to adjust the average energy level. On the other hand, it is not easy to improve robustness. It is desired to have as many control factors as possible. In designing a system, it is therefore wise to give greater priority to setting the optimum levels of design parameters to maximize SN ratio. This forms the foundation of the two-step optimization, where robustness optimization by the SN ratio should have the first priority.

4.6 Determination of the optimum design

Once parameter design experiment clarifies which design parameters affect SN ratio and which design parameters affect sensitivity as in factorial effect diagrams, one can select a set of optimum values of design parameters based on robustness. Then, the final optimum design can be determined by considering other constraints such as cost and delivery requirements.

Since a system's final optimum design should be determined by the overall balance of many constraints, it is preferable to select an experimental plan where each experimental factor covers a wider range in a design space. There is a possibility that the optimum value might exist in the range far from experiences; then it is recommended that the levels of control factors should cover a range as widely as possible.

In the parameter design, robustness optimization is performed through maximizing SN ratio. The SN ratio is a quantitative measure of the user's quality loss due to defects, failures, and quality problems caused by lack of robustness. The user's quality loss can include losses by malfunctions, by defects, by additional maintenance costs, etc.

According to Taguchi's quality loss function, the SN ratio can be transformed to user's quality loss in monetary unit. Total loss to the society caused by the product can be derived from the user's quality loss by adding other cost, such as product development cost, material cost, production cost, shipping cost, normal maintenance cost, disposal cost, etc. The total social loss should be a quality measure of the product. In product design stage, a product designer should consider the total social loss from the viewpoint of technology. However, it is difficult for the designer to forecast the total social loss in the design stage, but he should, at least, assess and optimize the product design from the viewpoint of robustness. Robust parameter design focuses on the user's quality loss from the viewpoint of robust engineering, that is, the variability in function of the product.

5 Assessment of robustness by SN ratio

5.1 Concepts of SN ratio

The variability in function of a system should be evaluated and optimized through parameter design for designing a robust product. When a subsystem is to be assessed for robustness, one should consider noise conditions at the whole system level in the users' hand. It is critical to ensure the robustness at whole system level.

A system's function can be defined as a functional form of input-output relationship at usage stage of the system. Users manipulate a signal to get an intended output response of the system. Signal is an input characteristic that is intentionally set to change the output of the system. A functional form that represents the ideal input-output relationship of the system's function is called as a system's ideal function. However, the ideal function might not be perfectly realized in a manufactured product, and also at actual usage stage; the function of the product can deviate from the ideal function due to noise conditions. Deviation from the ideal function should be evaluated at usage stage and expressed by one numerical measure of SN ratio in the first step of parameter design.

The user's conditions, under which the system is actually used, contain only signal and noises. As mentioned above, signal is an input to the system for intentional change of the output of the system. The signal should have a large effect on the system's output. On the other hand, the effect of noises has a negative impact on the system's output. The effect of the signal should be maximized and the effect of the noises should be minimized. In the experiment for assessing the robustness through SN ratio, the characteristic for input should be treated as a signal factor, and the noise sources should be treated as noise factors. Categorizing the variables in the user's conditions is important to clarify the purpose of the experiment.

The SN ratio is a measure that quantitatively expresses how close the actual input-output relationship is to the ideal function under various noise conditions. As SN ratio increases, the actual input-output relationship becomes closer to the ideal, and the loss to society will decrease. In the opposite case, it becomes farther to the ideal, and the loss to society will increase.

5.2 Types of SN ratio

There are three types of SN ratio in robustness assessment: SN ratio for dynamic characteristics, SN ratio for static or non-dynamic characteristics, and SN ratio for digital systems.

The SN ratio for dynamic characteristic represents the stability of the relationship between the signal and the corresponding outputs. The SN ratios for dynamic characteristics can be subdivided into three types by functional forms of the system's ideal function; zero-point proportional ideal function, reference-point proportional ideal function, and linear formula ideal function. The choice of the functional form of the ideal function depends on the physics of the objective system. In many cases, the ideal function can be expressed by a zero-point proportional formula because of proportional nature of physics.

The SN ratio for static or non-dynamic characteristic represents the stability of the output of the system. The output target is fixed and the signal is constant. The SN ratios for static characteristics can be subdivided into three types by the values of system's fixed target; nominal-the-best, smaller-the-better, and larger-the-better characteristics. The choice of the fixed target depends on the system's intent. The value of fixed target is finite for nominal-the-best, zero for smaller-the-better, and infinity for larger-the-better.

SN ratio for a digital system can be applied to evaluate the performance of a digital system which has binary inputs and outputs that can only take a value of 0 or 1. In a digital system, when an input is 0 or 1, the output of the system should be 0 or 1 respectively. This input-output relationship is the ideal function of a digital system. SN ratio for a digital system represents the ability of a digital system after threshold calibration.

Procedures to formulate each type of SN ratio will be shown in the following subclauses.

5.3 Procedure of the quantification of robustness

The procedure to formulate SN ratio and sensitivity for robustness should be as follows.

— **(Step 1) Clarify the system's ideal function.**

  Function is a work that a system performs in order to fulfil its objective. A function has an input signal to represent the operator's intention in a dynamic case. Output response of the system is varied by the input signal to fulfil the system's objective. Function can be expressed by a mathematical form of the relation between input signals and the output responses.

  Define ideal function, i.e. intended relationship between input signals and output responses based on the objective of the system's function. Ideal function represents the system's work expected.

In case of static/non-dynamic case, define the ideal output, the target value of the system's output, then go to step 3.

— **(Step 2) Select signal factor and its range.**

A signal for a dynamic characteristic case is an input to a system that is set actively or passively to vary the output response of the system as intended. In an experiment, the characteristic manipulating the input signal should be treated as a signal factor. A signal factor is an input variable with which the operator in the experiment sets to obtain the system's output response. The range of the signal factor should cover all the range used by users in the market.

— **(Step 3) Select measurement method of output response.**

Select an appropriate measuring method for the output characteristic. Output characteristic is also called as an output response. In dynamic function case, output is the quantity which users expect to get. There can be some difficulties in measuring time-depending responses. In such cases, appropriate measuring method should be developed for the purpose of robustness assessment.

— **(Step 4) Develop noise strategy, and select noise factors and their levels.**

Conditions that disrupt the ideal function under actual usage conditions during system's operation are called as noise conditions or error conditions. In an experiment to evaluate SN ratios, noise conditions should be generated by noise factors. Noise factors are variables that vary the output response in actual operating stages. It is recommended to discuss all possible noise conditions and to define efficient and effective noise conditions for assessing robustness. Noise factors come from "usage environment", "aging/wear", and "manufacturing variation". Noise factors should preferably include as many different types of factors as possible. Also, the levels of values of noise factors should be spread across a wide range by taking the usage conditions of ordinary users into consideration. It is possible to expand the system's lifetime, by taking noise factors due to wear through prolonged use.

— **(Step 5) Conduct experiment and collect data.**

Experimental layout for evaluating robustness by SN ratio is now defined. The layout shows which combination of levels of signal factor and noise factors will be applied in the experiment. Typical assignment is a two-way full factorial layout of a signal factor and a noise factor. In other words, the output response will be measured under various combinations of signal and noise levels. Then conduct experiment and collect data in the experimental layout. Then a dynamic SN ratio is ready to be calculated. In a static/non-dynamic characteristic case, there is no signal factor so obtain data under the noise conditions. The experimental layout for evaluating robustness corresponds to the outer array of a direct product plan of robust parameter design.

— **(Step 6) Calculate SN ratio, η, and sensitivity, S.**

SN ratio and sensitivity are calculated based on the data obtained in Step 5. Calculation formulae are shown in 5.4. The calculation formula is based on the system's ideal function defined in Step 1. SN ratio is to assess the functional variation due to noise conditions. SN ratio is a robustness measure of a system, and sensitivity, S, is an indicator representing the magnitude of the efficiency.

5.4 Formulation of SN ratio: Calculation using decomposition of total sum of squares

5.4.1 Zero-point proportional formula (dynamic characteristic)

When the signal is zero, then the output response is zero, and the output response proportionally increases as the signal increases. If this is the ideal state, as in many cases, the ideal function should be expressed by a zero-point proportional formula such as Formula (1):

$$y = \beta \mathrm{M} \tag{1}$$

where the output response and the input signal are denoted by "y" and "M", respectively.

This function is called a zero-point proportional ideal function. The coefficient β denotes the sensitivity coefficient.

a) Data set for computing SN ratio for zero-point proportional ideal function

The data set for zero-point proportional formula are shown in Table 1, where the signal factor has k levels and the noise factor has n levels.

Table 1 — Data set for zero-point proportional ideal function

Signal		M1	M2		Mk	Linear form
Noise level	N1	y_{11}	y_{12}		y_{1k}	L_1
	N2	y_{21}	y_{22}		y_{2k}	L_2
	...	...	...		...	...
	Nn	y_{n1}	y_{n2}		y_{nk}	L_n

b) Decomposition of total sum of squares for zero-point proportional formula

Total sum of squares

$$S_{\mathrm{T}} = {y_{11}}^2 + {y_{12}}^2 + \cdots + {y_{nk}}^2 \quad (f_{\mathrm{T}} = n \times k) \tag{2}$$

Sum of squares of input signal levels/effective divisor

$$r = M_1^2 + M_2^2 + \cdots + M_k^2 \tag{3}$$

Linear forms for noise levels:

$$\begin{aligned} L_1 &= M_1 y_{11} + M_2 y_{12} + \cdots + M_k y_{1k} \\ L_2 &= M_1 y_{21} + M_2 y_{22} + \cdots + M_k y_{2k} \\ &\ldots \\ L_n &= M_1 y_{n1} + M_2 y_{n2} + \cdots + M_k y_{nk} \end{aligned} \tag{4}$$

Sum of squares due to linear slope β

$$S_\beta = \frac{(L_1 + L_2 + \cdots + L_n)^2}{n \times r} \quad (f_\beta = 1) \tag{5}$$

Sum of squares due to the variation of linear slope β between noise levels

$$S_{N\times\beta}=\frac{L_1^2+L_2^2+\cdots+L_n^2}{r}-S_\beta \qquad (f_{N\times\beta}=n-1) \tag{6}$$

Sum of squares due to error

$$S_e=S_T-S_\beta-S_{N\times\beta} \quad (f_e=f_T-f_\beta-f_{N\times\beta}=n\times k-n) \tag{7}$$

Variance due to error/error variance

$$V_e=\frac{S_e}{f_e}=\frac{S_e}{n\times k-n} \tag{8}$$

Variance due to pooled error/variance due to error and noise

$$V_N=\frac{S_{N\times\beta}+S_e}{f_{N\times\beta}+f_e} \tag{9}$$

SN ratio, η, and sensitivity, S

$$\eta=10\log\frac{\frac{1}{n\times r}(S_\beta-V_e)}{V_N}\ \text{(db)} \tag{10}$$

$$S=10\log\frac{1}{n\times r}(S_\beta-V_e)\ \text{(db)} \tag{11}$$

5.4.2 Linear formula (dynamic characteristic)

The zero-point proportional formula has very wide applications. When input energy is zero, the output energy will be zero. However, there are some other cases where the origin point is not defined and/or only linearity between input and output is desired. In such cases, one can assume the ideal function as the linear formula such as Formula (12):

$$y=\alpha+\beta M \tag{12}$$

This function is called a linear formula ideal function. The coefficient β denotes sensitivity coefficient and parameter α denotes zero-section.

a) Data set for computing SN ratio for linear formula ideal function

The data set for a linear formula are shown Table 2, where the signal factor has k levels and the noise factor has n levels.

Table 2 — Data set for linear formula ideal function

Signal		M1	M2		Mk	sum
Noise level	N1	y_{11}	y_{12}		y_{1k}	N_1
	N2	y_{21}	y_{22}		y_{2k}	N_2
	...	...	...		...	...
	Nn	y_{n1}	y_{n2}		Y_{nk}	N_n
	sum	Y_1	Y_2		Y_k	

b) Decomposition of sum of squares for linear formula

Total sum of squares

$$S_{\mathrm{T}} = {y_{11}}^2 + {y_{12}}^2 + \cdots {y_{nk}}^2 \quad (f_{\mathrm{T}} = n \times k) \tag{13}$$

Sum of squares due to mean

$$S_{\mathrm{m}} = \frac{(y_{11} + y_{12} + \cdots + y_{nk})^2}{n \times k} \quad (f_{\mathrm{m}} = 1) \tag{14}$$

The mean of signal levels

$$\overline{M} = \frac{(M_1 + M_2 + \cdots + M_k)}{k} \tag{15}$$

Sum of squares due to input signal levels from the mean

$$r = (M_1 - \overline{M})^2 + (M_2 - \overline{M})^2 + \cdots + (M_k - \overline{M})^2 \tag{16}$$

Sum of squares due to linear slope β

$$S_{\beta} = \frac{\left[(M_1 - \overline{M})Y_1 + (M_2 - \overline{M})Y_2 + \cdots + (M_k - \overline{M})Y_k\right]^2}{n \times r} \quad (f_{\beta} = 1) \tag{17}$$

Sum of squares due to the main effect of noise

$$S_{\mathrm{N}} = \frac{N_1^2 + N_2^2 + \cdots + N_n^2}{k} - S_{\mathrm{m}} \quad (f_{\mathrm{N}} = n - 1) \tag{18}$$

Sum of squares due to error

$$S_{\mathrm{e}} = S_{\mathrm{T}} - S_{\mathrm{m}} - S_{\beta} - S_{\mathrm{N}} \quad (f_{\mathrm{e}} = n \times k - 1 - n) \tag{19}$$

Variance due to error/error variance

$$V_{\mathrm{e}} = \frac{S_{\mathrm{e}}}{f_{\mathrm{e}}} = \frac{S_{\mathrm{e}}}{n \times k - 1 - n} \tag{20}$$

Variance due to pooled error

$$V_N = \frac{S_N + S_e}{f_N + f_e} \tag{21}$$

SN ratio, η, and sensitivity, S

$$\eta = 10 \log \frac{\frac{1}{n \times r}(S_\beta - V_e)}{V_N} \quad \text{(db)} \tag{22}$$

$$S = 10 \log \frac{1}{n \times r}(S_\beta - V_e) \quad \text{(db)} \tag{23}$$

NOTE 1 In the computation of SN-ratio, pure replication trial is not applied, but multiple trials are performed under the situation where the combinations of levels of noise factor(s) are changed and other conditions are the same. Noise factors are intentionally introduced in the replication rather than random noise conditions. Evaluation of variability under intentionally arranged noise conditions is a feature of the assessment of SN ratio.

NOTE 2 It should be noted that the formulae for the decomposition of sum of squares vary with the cases of formulae's forms. For example, sum of squares due to mean (S_m) is calculated for a linear formula case, but not for a zero-point proportional formula case. There are some other differences in the formulae depending on the functional forms.

5.4.3 Reference-point proportional formula (dynamic characteristic)

One fixed point, which is called the reference point, is treated like an origin point in the space. SN ratio for reference-point proportional ideal function can be calculated from the data (M', y') transformed as follows:

$$(M' y') = (M, y) - (M_0, y_0) \tag{24}$$

Subtracting the reference point (M_0, y_0) from each data (M, y), then the ideal function can be described by a reference-point proportional formula such as,

$$(y - y_0) = \beta (M - M_0) \tag{25}$$

This ideal function is called a reference-point proportional ideal function. The coefficient β denotes linear slope or sensitivity coefficient.

Calculation formulae of SN ratio are the same as those for a zero-point proportional ideal function when the transformed data (M', y') are treated as the original data (M, y).

5.4.4 Nominal-the-best response (static/non-dynamic characteristic)

When a system has one finite fixed target, the system's output response is called nominal-the-best characteristic or nominal-the-best response. SN ratio for nominal-the-best response can be applied in this case in robustness assessment.

a) Data set for computing SN ratio for nominal-the-best response

Data set for static/non-dynamic response is shown in Table 3, where noise factor has n levels, and there is no signal factor. Data is collected under one design specification, such as design A. This data configuration is common for all the types of static/non-dynamic characteristic.

Table 3 — Data set for static/non-dynamic characteristic

Signal		Design specification A
Noise level	N1	y_1
	N2	y_2
	...	...
	Nn	y_n

b) Decomposition of sum of squares for computing SN ratio for nominal-the-best response

Total sum of squares

$$S_T = y_1^2 + y_2^2 + \cdots + y_n^2 \quad (f_T = n) \tag{26}$$

Sum of squares due to mean

$$S_m = \frac{(y_1 + y_2 + \cdots + y_n)^2}{n} \quad (f_m = 1) \tag{27}$$

Sum of squares due to error

$$S_e = S_T - S_m \quad (f_e = n - 1) \tag{28}$$

Variance due to noise and error

$$V_e = \frac{S_e}{f_e} = \frac{S_e}{n-1} \tag{29}$$

SN ratio, η, and sensitivity, S

$$\eta = 10 \log \frac{\frac{1}{n}(S_m - V_e)}{V_e} \quad \text{(db)} \tag{30}$$

$$S = 10 \log \frac{1}{n}(S_m - V_e) \quad \text{(db)} \tag{31}$$

5.4.5 Smaller-the-better response (static/non-dynamic characteristic)

When a system has a non-negative output response and its ideal value is zero, the system's output response is called smaller-the-better characteristic or smaller-the-better response. SN-ratio for smaller-the-better response should be applied in this case in robustness assessment.

a) Data set for computing SN ratio for smaller-the-better response

The data set for smaller-the-better response has the common configuration of the data set for static/non-dynamic characteristic, as shown in Table 3, where the noise factor has n levels.

b) Computing SN ratio for smaller-the-better response

The mean squared deviation from the ideal value of zero is given by

$$\hat{\sigma}^2 = \frac{1}{n}(y_1^2 + y_2^2 + \cdots + y_n^2) \tag{32}$$

In this International Standard, the notation $\hat{\sigma}^2$ is used for an estimate of a mean squared deviation (MSD).

SN ratio, η is calculated as

$$\eta = -10 \log \hat{\sigma}^2 = 10 \log \frac{1}{\hat{\sigma}^2} \quad \text{(db)} \tag{33}$$

5.4.6 Larger-the-better response (static/non-dynamic characteristic)

When a system's output response is desired to be as large as possible, this means the ideal value is infinity. The system's output response is called larger-the-better characteristic or larger-the-better response. SN ratio for larger-the-better response can be applied in the robustness assessment in this case.

In cases where the limit value y_0 is theoretically fixed, after data transformation to the difference from the limit value, $y' = y_0 - y$, the new variable y' can be considered as a smaller-the-better characteristic or nominal-the-best characteristic. It is recommended to transform the data and apply the static/non-dynamic formulation. Percentage data is an example.

a) Data set for computing SN ratio for larger-the-better response

The data set for larger-the-better response has the common configuration of the data set for static/non-dynamic characteristic, as shown in Table 3, where the noise factor has n levels.

b) Computing SN ratio for larger-the-better response

For a larger-the-better response, the inverse of y, $1/y$, is treated as a smaller-the-better characteristic. Therefore, the estimate of mean squared deviation $\hat{\sigma}^2$ is given by Formula (33):

$$\hat{\sigma}^2 = \frac{1}{n}\left(\frac{1}{y_1^2} + \frac{1}{y_2^2} + \cdots + \frac{1}{y_n^2}\right) \tag{34}$$

SN ratio, η, is calculated as

$$\eta = -10 \log \hat{\sigma}^2 = 10 \log \frac{1}{\hat{\sigma}^2} \quad \text{(db)} \tag{35}$$

5.4.7 SN ratio for digital characteristics

In computers, control systems, digital communication systems, and the like, the input and the resulting output data consist of only two digital sates, namely ones and zeroes. In such digital input and digital output systems, robustness should be evaluated by standardized SN ratio. Standardized SN ratio means "SN ratio after optimized by adjusting the threshold level". This adjustment is also called as calibration or levelling.

a) Data set for computing standardized SN ratio for digital characteristic

Considering a communication function, receiving a 0 when a 1 is transmitted constitutes an error. The rate of such errors is denoted by p. Receiving a 1 when a 0 is transmitted is also an error, and this error rate is denoted by q. This can be summarized as shown in Table 4.

Table 4 — Table of two types of error rate

		Output (Determination result)		**Number of trials**
		1	0	
Input (Samples)	1	$1 - p$	p	n_1
	0	q	$1 - q$	n_2

b) Computing standardized SN ratio for digital characteristic

When the communication system is optimized by adjusting the threshold level so that the error rates p and q are the same, standardized error rate p_0 is calculated as

$$p_0 = \frac{1}{1 + \sqrt{\left(\frac{1}{p} - 1\right) \times \left(\frac{1}{q} - 1\right)}} \tag{36}$$

because the following formula is held in this case:

$$\left(\frac{1}{p_0} - 1\right) \times \left(\frac{1}{p_0} - 1\right) = \left(\frac{1}{p} - 1\right) \times \left(\frac{1}{q} - 1\right) \tag{37}$$

Standardized contribution ratio ρ_0 should be calculated from the standardized error rate p_0 as follows:

$$\rho_0 = \left(1 - 2p_0\right)^2 \tag{38}$$

Standardized SN ratio is

$$\eta_0 = -10 \log\left(\frac{1}{\rho_0} - 1\right) \quad \text{(db)} \tag{39}$$

Standardized SN ratio for a digital characteristic shows the robustness of the digital system after optimizing it by levelling the threshold.

5.5 Some topics of SN ratio

5.5.1 Using SN ratios in system comparison

The absolute value of an individual SN ratio does not carry any significant meaning, but the difference in the values of SN ratios of two systems evaluated under the same noise levels can be applied as a measure of system comparison. The magnitudes of SN ratios calculated for the same noise levels provide an indicator for the market losses caused by inefficiency and variability of the systems due to noise conditions. This allows comparison of the robustness of designed systems based on various design concepts. This is called “robustness assessment”.

As stated above, SN ratio can be applied to relative comparisons to be made between systems/various design concepts. Robust assessment can be applied not just for new systems introduced in technology/product development, but also for assessing conventional products and competing products in benchmarking to find out which products are superior in their performance in market.

5.5.2 Nonlinear formula cases

Even if the ideal relationship between signal and output response is nonlinear, in some cases, a simple transformation of the variables can linearize the relationship. It makes the function to a zero-point proportional ideal function. SN ratio can be computed accordingly.

EXAMPLE Given "y" is output, "M" is signal, and "α" is a constant.

The system's ideal function is: $y = \alpha \times e^{\beta M}$.

Take the natural logarithm both sides: $\ln y = \ln \alpha + \beta M$.

Assume "$\ln y$" as a new variable of "Y": $Y = \ln \alpha + \beta M$.

Formula for SN ratio for linear formula is applicable in this case.

5.5.3 SN ratios of static/non-dynamic characteristics

It is most effective and, therefore, recommended that SN ratios for dynamic characteristics should be applied at the upstream stage of the technology/product development process, rather than non-dynamic characteristics. In some cases, SN ratio for non-dynamic characteristics can be applied at the downstream stage of the technology/product development process.

6 Procedure of a parameter design experiment

6.1 General

This clause provides the procedure of a parameter design experiment with the case of zero-point proportional ideal function. A case study will be presented in Clause 7.

It is recommended that an orthogonal array is applied for the inner array to explore the design parameter space because an orthogonal array makes it possible to evaluate the combinations of many design parameters simultaneously. The number of design parameters explored becomes large as compared to other plans of experiment in the same number of experimental runs. It means that the possibility of improvement in robustness by selecting optimum levels of design parameters becomes much higher.

The procedures of parameter design experiment mentioned in 4.4 will be discussed in more detail in the following subclauses.

6.2 (Step 1) Clarify the system's ideal function

A function is work that a system performs to fulfil its objective. A function has an input signal to actuate the operator's intention. The input signal can change the output response of the system to fulfil the system's objective based on a function. The function can be expressed by a mathematical form of the relation between input signal and output response.

Define the ideal state of the function, i.e. ideal relationship between input signal and output response based on the system's function. The ideal function represents the system's work intended.

Asking the following questions is helpful to identify the ideal function:

— What is the intended function of the system?

— How does the system deliver the intended response?

— What is the physics of the system?

— What is the energy transformation of the system, in case of hardware system?

— What is the transformation of information, in case of software/service system?

— What is the input signal that changes the output response?

— What is the output response delivered by the system?

— What is the ideal relationship between the input signal and the output response?

— What is the formula that can be applied to express the function?

When the response should be zero at the zero input (input = 0, then output = 0), and the input and the output should be proportional, then the function can be expressed by a zero-point proportional formula. In many physical systems, phenomena are proportional and zero-point proportional formula is applicable to their ideal function from the viewpoint of physics. A linear system is easy for users to understand and operate it.

Such characteristics expressed in the input-output relationship are also called dynamic characteristics.

There are mainly three types of formula for dynamic ideal function: zero-point proportional, reference-point proportional, and linear formulas as shown in Clause 5. In some cases, the relation between input and output can be expressed by a simple linear formula after some transformation. See more discussions in Clause 5.

6.3 (Step 2) Select a signal factor and its range

Identify the input signal of system. The input signal is supposed to change the value of output responses as intended. The signal factor is the user's manipulating conditions for the purpose of inputting the signal to the system. Sometimes, the user indirectly changes the input, for example, by foot pedal, handle, or lever. Select the signal and a signal factor for changing the signal.

Levels of the signal factor should cover the entire range of customer's usage conditions. In case of active dynamic characteristics, the output range is more important because the output response is the user's demand itself. So check if the range of output response changed by the signal covers the customer's demand. In case of passive dynamic characteristics, such as temperature for a thermometer, the actual range of signal should be covered by the levels of the signal factor in the experiment.

The overall range of the signal should be taken rather wider because results of the experiment can be applicable to wider situations. The number of signal levels should be three or more levels because nonlinearity and higher order distortions can be assessed as a noise effect. When each width between signal levels is set equal, the calculation of sum-of-squares decomposition becomes easier.

6.4 (Step 3) Select measurement method of output response

The method for measuring the response should be selected. In some cases, a suitable measurement method cannot exist. In general, industries have been making efforts to measure non-dynamic responses because of validation purposes. It is critical to develop an efficient and effective measurement system to measure the response of ideal functions.

6.5 (Step 4) Develop noise strategy and select noise factors and their levels

Select noise conditions to be tested in the experiment as noise factors. One or two noise factors can be sufficient to make a comparison of the robustness of a system for an entire noise space. When many noise factors are desired and easy to vary, noise factors can be assigned to an orthogonal array.

Noise factors are classified into mainly two kinds: inner noise and outer noise. An inner noise comes from the deviation of system parameters from the designed value, for example, aging degradation, part precision, manufacturing variation, and assembly variation. An outer noise comes from outside of the system, for example, environmental conditions in usage such as temperature, humidity, vibration, etc.

When many noise factors are taken into the experiment, experimental results can be applicable to wider situations, but on the other hand, the number of experimental runs increases much. Therefore, if the direction of each noise factor's effect on the output response is relatively known, a noise compounding method can be applicable. A compounded noise factor has two extreme levels: a compounded noise condition level where the output response tends to be low, and a compounded noise condition level where the output response tends to be high. Then, the robustness can be assessed by only two noise levels, making the test for assessment extremely efficient. The smaller the effect of the compounded noise factor is, the more robust the system is.

6.6 (Step 5) Select control factors and their levels from design parameters

Select design parameters tested in the experiment as control factors. It is recommended to use three levels where one of the levels is the baseline level. In the final step of the optimization, the optimum levels of control factors will be selected and will be verified. It is recommended that the range of design parameters be determined by how the design space should be explored. It is usual that it should be taken as wide as possible. Sometimes optimum value of a design parameter is found in an unexpected range.

Control factors can be continuous variables, such as length and mass, or it can be attributed, such as material type and shapes, depending on the nature of design parameters.

First of all, control factors should be defined such that they can be varied independently. Secondly, it is recommended to define control factors in such a way that their effects are independent. In some cases, where control factor effects are interactive, a technique called sliding level can be applied, or redefining control factors such that their effects become more independent.

For example, when volume, specific gravity, and mass of a cubic part are selected as control factors in one experiment, they are correlated and not independent. Two of them can be selected as independent control factors. Instead of taking time and temperature as control factors, you can redefine them as heat energy and time, or simply call it time and temp profile.

6.7 (Step 6) Assign experimental factors to inner or outer array

Assign the control factors selected in step 5 to inner array, and the noise factor(s) selected in step 4 and the signal factor selected in step 2 to outer array. Inner array should be an orthogonal array such as an orthogonal array L_{18}, and outer array can be a factorial plan. In simulation experiment, outer array can be an orthogonal array because repeated runs in outer array are rather easy to perform in this case.

Orthogonal array L_{18} has eight columns. One two-level control factor (A) and seven three-level factors (B-H) can be assigned to columns in an orthogonal array L_{18} shown in Table 5. Two-level control factor A should be assigned to the first column. Rows represent the experimental run number. The numbers in each cell represent the level of the factors. The experimental run of row No. 1 should be performed under the condition of the combination of factorial levels A1B1C1D1E1F1GH1. The condition in the run of row No. 2 should be performed under the combination of factorial levels A1B1C2D2E2F2G2H2.

Table 5 — Orthogonal array L_{18}

No.	A	B	C	D	E	F	G	H
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2

Table 5 *(continued)*

No.	A	B	C	D	E	F	G	H
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

As for the inner array in parameter design, an orthogonal array is recommended to explore the design space efficiently. Omitting design parameters due to a large number of experimental runs is contrary to the purpose of parameter design. Therefore, performing experiments based on an orthogonal array is strongly recommended so that it is possible to evaluate combinations of many design parameters simultaneously. It is also essential to assess a design parameter under the conditions where other parameters are varied.

The number of rows of an orthogonal array is the number of experimental runs. The number of columns is the number of assignable factors. If the number of control factors does not exceed the number of columns of an orthogonal array, all factors could be assigned to the orthogonal array. It is recommended to fill up all columns with control factors for the sake of efficiency.

An orthogonal array L_{18} is usually recommended, where the interaction between any two three-level columns are compounded almost uniformly to other three-level columns. Relatively strong main effects can be identified without specific compounded interaction in this family of orthogonal arrays, such as L_{12}, L_{18}, L_{36}, and L_{54}. In contrast, in the 2^n, 3^n, and 4^n systems of orthogonal arrays, the interaction between any two columns is compounded in specific column(s), so it is not robust against strong interactions between control factors. See more discussion on this topic in the Bibliography.

6.8 (Step 7) Conduct experiment and collect data

Collect the outer array data for each row of inner array. SN ratio and sensitivity for each row of inner array should be calculated through each set of corresponding data of outer array.

Note that collection of the outer array data should be repeated for each row of the inner orthogonal array. In other words, robustness assessment should be done for each combination of design parameters indicated by the inner orthogonal array.

Table 6 shows an example of outer array layout for assessing robustness by SN ratio and sensitivity for each row of the inner orthogonal array. While the outer array in this example is a full factorial plan of signal and noise factors, it can be an orthogonal array.

Table 6 — Example of outer array layout for SN ratio and sensitivity (2-way layout)

Signal		M1	M2	Mk	Linear form
Noise factor	N1	y_{11}	y_{12}	y_{1k}	L_1
	N2	y_{21}	y_{22}	y_{2k}	L_2
	...	...	...	...	...
	Nn	y_{n1}	y_{n2}	y_{nk}	L_n

6.9 (Step 8) Calculate SN ratio, η, and sensitivity, S

SN ratio and sensitivity should be calculated for each row of inner orthogonal array L_{18}. Formulations of SN ratio and sensitivity are shown in 5.4 depending on the types of ideal function.

In the case of zero-point proportional formula, calculation of the data set in Table 6, for example, is as follows.

Total sum of squares

$$S_T = y_{11}^2 + y_{12}^2 + \cdots + y_{nk}^2 \quad (f_T = n \times k)$$

Sum of squares of input signal levels/effective divisor

$$r = M_1^2 + M_2^2 + \cdots + M_k^2$$

Linear forms for each noise level

$$L_1 = M_1 \times y_{11} + M_2 \times y_{12} + \cdots + M_k \times y_{1k}$$
$$L_2 = M_1 \times y_{21} + M_2 \times y_{22} + \cdots + M_k \times y_{2k}$$
$$L_n = M_1 \times y_{n1} + M_2 \times y_{n2} + \cdots + M_k \times y_{nk}$$

Sum of squares due to linear slope β

$$S_\beta = \frac{(L_1 + L_2 + \cdots + L_n)^2}{n \times r} \quad (f_\beta = 1)$$

Sum of squares due to the variation of linear slope β between noise levels

$$S_{N \times \beta} = \frac{L_1{}^2 + L_2{}^2 + \cdots + L_n{}^2}{r} - S_\beta \quad (f_{N \times \beta} = n - 1)$$

Sum of squares due to error

$$S_e = S_T - S_\beta - S_{N \times \beta} \quad (f_e = n \times k - n)$$

Variance due to error/error variance

$$V_e = \frac{S_e}{f_e} = \frac{S_e}{n \times k\text{-}n}$$

Variance due to pooled error

$$V_N = \frac{S_e + S_{N \times \beta}}{f_e + f_{N \times \beta}} = \frac{S_e + S_{N \times \beta}}{n \times k - 1}$$

SN ratio

$$\eta = 10 \log \frac{\frac{1}{n \times r}(S_\beta - V_e)}{V_N} \quad \text{(db)}$$

Sensitivity

$$S = 10 \log \frac{1}{n \times r}(S_\beta - V_e) \quad \text{(db)}$$

Table 7 shows the calculated results of SN ratio and sensitivity for each row of inner array.

Table 7 — SN ratio and sensitivity for each row of inner array

No.	SN ratio db	Sensitivity db
1	η_1	S_1
2	η_2	S_2
3	η_3	S_3
4	η_4	S_4
5	η_5	S_5
6	η_6	S_6
7	η_7	S_7
8	η_8	S_8
9	η_9	S_9
10	η_{10}	S_{10}
11	η_{11}	S_{11}
12	η_{12}	S_{12}
13	η_{13}	S_{13}
14	η_{14}	S_{14}
15	η_{15}	S_{15}
16	η_{16}	S_{16}
17	η_{17}	S_{17}
18	η_{18}	S_{18}

From the calculated results of SN ratio and sensitivity for each row of inner array shown in Table 7, the averages of SN ratio and sensitivity are calculated for each level of control factors (design parameters) in the inner orthogonal array to identify the factorial effects.

The average of SN ratio for the level 1 of factor A is calculated by averaging SN ratios of lows from No. 1 to No. 9. Likewise, the average of SN ratios for the level 2 of factor A is calculated by averaging SN ratio of lows from No. 10 to No.18. Similar calculations should be conducted for the control factors B to H, as indicated as follows.

The average of SN ratios for each level of control factors

$$\eta_{A1} = \frac{(\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6 + \eta_7 + \eta_8 + \eta_9)}{9}$$

$$\eta_{A2} = \frac{(\eta_{10} + \eta_{11} + \eta_{12} + \eta_{13} + \eta_{14} + \eta_{15} + \eta_{16} + \eta_{17} + \eta_{18})}{9}$$

$$\eta_{B1} = \frac{(\eta_1 + \eta_2 + \eta_3 + \eta_{10} + \eta_{11} + \eta_{12})}{6} \quad (40)$$

...

$$\eta_{H3} = \frac{(\eta_3 + \eta_4 + \eta_7 + \eta_{12} + \eta_{14} + \eta_{17})}{6}$$

In the same way, the averages of sensitivity for each level of control factors should also be calculated as follows.

The average of sensitivity for each level of control factors

$$S_{A1} = \frac{(S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8 + S_9)}{9}$$

$$S_{A2} = \frac{(S_{10} + S_{11} + S_{12} + S_{13} + S_{14} + S_{15} + S_{16} + S_{17} + S_{18})}{9} \qquad (41)$$

...

$$S_{H3} = \frac{(S_3 + S_4 + S_7 + S_{12} + S_{14} + S_{17})}{6}$$

Table 8 shows the summary of the calculated averages of SN ratio and sensitivity for each factor's level of control factors in inner orthogonal array.

Table 8 — Averages of SN ratio and sensitivity

Control factor	SN ratio db			Sensitivity db		
	Level 1	**Level 2**	**Level 3**	**Level 1**	**Level 2**	**Level 3**
A	η_{A1}	η_{A2}	-	S_{A1}	S_{A2}	-
B	η_{B1}	η_{B2}	η_{B3}	S_{B1}	S_{B2}	S_{B3}
C	η_{C1}	η_{C2}	η_{C3}	S_{C1}	S_{C2}	S_{C3}
D	η_{D1}	η_{D2}	η_{D3}	S_{D1}	S_{D2}	S_{D3}
E	η_{E1}	η_{E2}	η_{E3}	S_{E1}	S_{E2}	S_{E3}
F	η_{F1}	η_{F2}	η_{F3}	S_{F1}	S_{F2}	S_{F3}
G	η_{G1}	η_{G2}	η_{G3}	S_{G1}	S_{G2}	S_{G3}
H	η_{H1}	η_{H2}	η_{H3}	S_{H1}	S_{H2}	S_{H3}

6.10 (Step 9) Generate factorial effect diagrams on SN ratio and sensitivity

Plot factorial effect diagrams on both SN ratio and sensitivity based on the data of averages shown in Table 8. These diagrams show how each control factor affects SN ratio and sensitivity. SN ratio represents variability while sensitivity represents linear slope or mean of the response.

Figure 6 shows examples of factorial effect diagrams on SN ratio and sensitivity.

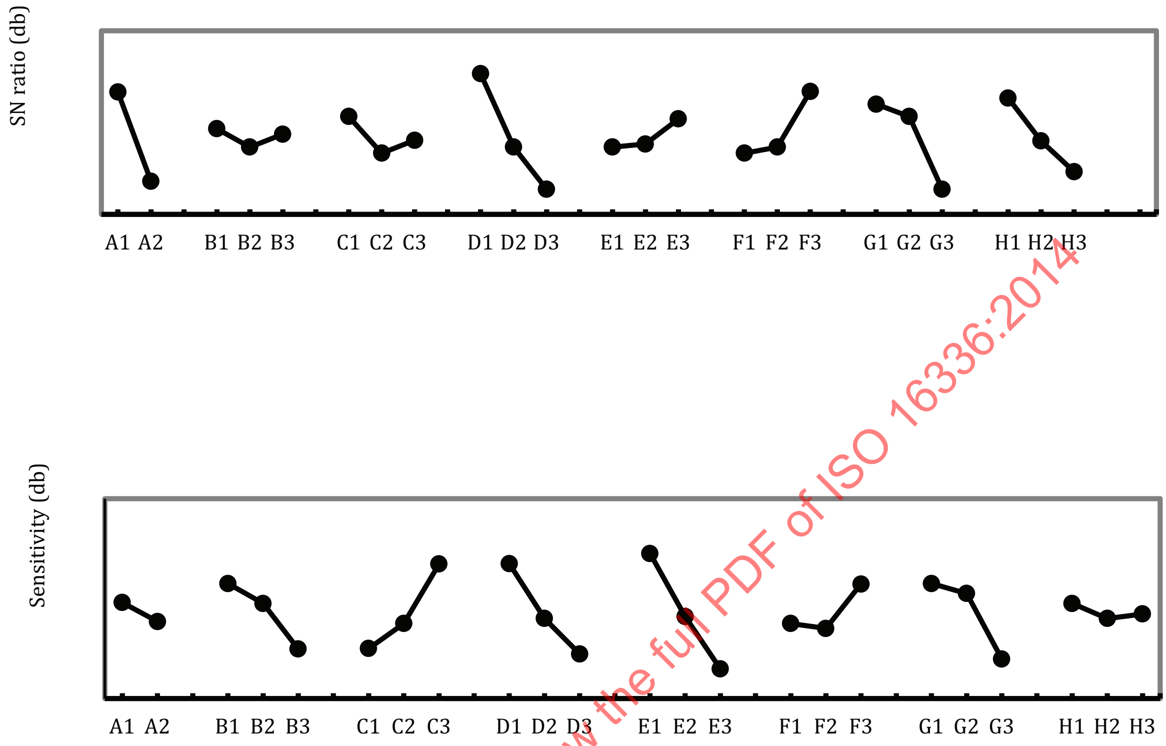

Figure 6 — Examples of factorial effect diagrams on SN ratio and sensitivity

Since the average of each level of control factors (design parameters) is calculated from the orthogonal array, the grand average of SN ratio and sensitivity should be the same as the averages for each control factor (design parameter). When the factorial effect diagrams are generated, it is easy to check for mistakes in calculations.

Grand average of SN ratio

$$\overline{T}_{\text{SN}} = \frac{\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6 + \eta_7 + \eta_8 + \eta_9 + \eta_{10} + \eta_{11} + \eta_{12} + \eta_{13} + \eta_{14} + \eta_{15} + \eta_{16} + \eta_{17} + \eta_{18}}{18} \quad (42)$$

Average of SN ratio for each control factor should be the same as the grand average:

$$\frac{\eta_{\text{A1}} + \eta_{\text{A2}}}{2} = \cdots = \frac{\eta_{\text{H1}} + \eta_{\text{H2}} + \eta_{\text{H3}}}{3} = \overline{T}_{\text{SN}} \quad (43)$$

Grand average of sensitivity

$$\overline{T}_{\beta} = \frac{S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8 + S_9 + S_{10} + S_{11} + S_{12} + S_{13} + S_{14} + S_{15} + S_{16} + S_{17} + S_{18}}{18}$$

(44)

Average of sensitivity for each control factor should be the same as the grand average

$$\frac{S_{A1}+S_{A2}}{2}=\cdots=\frac{S_{H1}+S_{H2}+S_{H3}}{3}=\bar{T}_{\beta} \tag{45}$$

For a continuous variable of control factor, when the plot in the factorial effect diagram is not monotonically increasing or decreasing, it can be an indication of some interaction between control factors, and it is expected to have poor results in confirmation experiment because of such interactions. For a continuous control factor effect on SN ratio, it is not realistic to have middle level being not robust and low and high level being robust. So it is good practice to see if all continuous control factor effects are monotonically increasing or decreasing.

6.11 (Step 10) Select the optimum condition

The strategy of two-step optimization should be applied when selecting the optimum condition.

First, observe the factorial effect diagram on SN ratio and select the level of each control factor with the highest SN ratio average as an optimum level for robustness.

Second, observe the factorial effect diagram on sensitivity. If adjustment of the output is needed, anticipate which factor(s) can be used to adjust the linear slope or mean, after confirming the first step.

Since parameter design implies optimizing robustness, it is essential to select the control factor levels with highest SN ratio. This means that the resulting system can be expected to exhibit the smallest variability of system's function in the market.

For example, assuming the factorial effect diagrams of Figure 6, the optimum condition should be selected as A1B1C1D1E3F3G1H1. However, if choosing another level of the control factor causes a small difference in the db value of SN ratio, another level can be selected by consideration of sensitivity and other criteria than SN ratio. In Figure 6, control factor E has small effect on SN ratio but has a strong linearity on sensitivity. If the level 2 is selected for sensitivity reason rather than the optimum level 3 for SN ratio, it results only in a small difference in SN ratio, that is, robustness. This kind of trade-off can be applied in two-step optimization. In any case, the optimum combination can be strategically selected by considering many different criteria, but robustness should be the most important and it should have the first priority.

6.12 (Step 11) Estimate the improvement in robustness by the gain

As the improvement in robustness, gain in SN ratio should be estimated by the difference in SN ratio between the optimum design condition and some reference design condition, usually the baseline design condition.

For the example of Figure 6, the optimum design condition is A1B1C1D1E3F3G1H1. The gain should be calculated using the following procedures.

Calculate the estimated value of SN ratio for the optimum design condition as follows.

Estimated value of SN ratio for the optimum design condition

$$\eta_{opt}=\eta_{A1}+\eta_{B1}+\eta_{C1}+\eta_{D1}+\eta_{E3}+\eta_{F3}+\eta_{G1}+\eta_{H1}-7\bar{T}_{SN} \tag{46}$$

Calculate the estimated value of SN ratio for the baseline condition in the similar way where the baseline design combination is used, e.g. A1B2C2D2E2F2G2H2.

Estimated value of SN ratio for the baseline design condition

$$\eta_{base}=\eta_{A1}+\eta_{B2}+\eta_{C2}+\mu_{D2}+\eta_{E2}+\eta_{F2}+\eta_{G2}+\eta_{H2}-7\bar{T}_{SN} \tag{47}$$

Calculate the gain in SN ratio as the difference between the SN ratios.

Gain in SN ratio

$$\Delta\eta = \eta_{opt} - \eta_{base} \text{ (db)} \quad (48)$$

The gain of sensitivity can be calculated in the same way.

Estimated value of sensitivity for the optimum design condition

$$S_{opt} = S_{A1} + S_{B1} + S_{C1} + S_{D1} + S_{E3} + S_{F3} + S_{G1} + S_{H1} - 7\overline{T}_{\beta} \quad (49)$$

Estimated value of sensitivity for the baseline design condition

$$S_{base} = S_{A1} + S_{B2} + S_{C2} + S_{D2} + S_{E2} + S_{F2} + S_{G2} + S_{H2} - 7\overline{T}_{\beta} \quad (50)$$

Gain in sensitivity

$$\Delta S = S_{opt} - S_{base} \text{ (db)} \quad (51)$$

6.13 (Step 12) Conduct a confirmation experiment and check the gain and “reproducibility”

Confirmation runs under the baseline design and the optimum design conditions should be conducted to confirm the gains. This experiment is called a confirmation experiment. Just take two runs on the outer array, that is, the run under the baseline design condition and the run under the optimum design condition. Then compute the SN ratios, the sensitivities, and the gains as confirmed values.

Table 9 summarizes the estimated db values from the factorial effect in the parameter design experiment and the confirmed db values in the confirmation experiment for both of SN ratio and sensitivity under the baseline and the optimum design conditions.

Table 9 — Results of confirmation

	SN ratio db		Sensitivity db	
	Estimated value	**Confirmed value**	**Estimated value**	**Confirmed value**
Optimum condition	η_{opt}	η'_{opt}	S_{opt}	S'_{opt}
Baseline condition	η_{cur}	η'_{cur}	S_{cur}	S'_{cur}
Gain	$\Delta\eta$	$\Delta\eta'$	ΔS	$\Delta S'$

When the gains in SN ratio and sensitivity are the almost same between the estimated values and the confirmed ones respectively, then the experimental results of the parameter design can be highly reproducible and will reproduce in actual situation. If not, there are some problems in reproducibility because of poor additivity of factorial effects. One will need to re-examine items in the parameter design experimental plan, such as definition of ideal function, input and output characteristics, formulation of SN ratio, noise strategy, definition of control factors, measurement method, and so on.

When the results of the gain in the confirmation experiment are in highly reproducible, there might be the case where the absolute values of SN ratio and/or sensitivity are not the same between the estimated value and the confirmed one. In that situation, there can be other unknown noise factors which have strong effects on the output response or SN ratio. However if the gains are reproducible, the effects of control factors are considered to be reproduced and the improvement in robustness by selecting the optimum condition relative to the baseline condition will be reproduced in the market.

The profit of improvement from the current or baseline design to the optimized design can be estimated as the decreasing of user's quality loss and can be compared with cost of the improvement. In many cases, choice of the optimum value of design parameter causes no or very little increasing in cost.

7 Case study — Parameter design of a lamp cooling system

Case study of applying parameter design to a lamp cooling system is presented in this clause. In a lighting machine that uses a lamp light source, a cooling system, such as a cooling fan is necessary to prevent overheating. It is not easy to measure temperature in the machine and surroundings because it needs to control the ambient temperature during the experiment. Moreover, the selection of material shall be considered in heat transfer, radiation, and convection. Therefore, this assessment will take a long time and will cost much if different materials are tried in the experiment. In this study, in order to assess the performance of a cooling system, robustness is evaluated based on the function of the cooling system that is described as a relationship between fan motor voltage and air flow speed.

— **(Step 1) Clarify the system's ideal function.**

The cooling system should keep the heat source and its surroundings cool by turning a fan by a motor to expel hot air that has been heated up by the lamp used as a light source. The function of this cooling system can be defined, "To remove hot air by generating air flow by a motor and a fan." The ideal function is defined with thought process as follows: The input is electric energy presented by voltage, and it is consumed by the motor to drive the fun. The output is the speed of air flow to remove hot air. Then the ideal function can be expressed as

$$y = \beta M$$

where

M denotes input signal, "voltage";

y denotes output response, "air flow speed".

Figure 7 shows the ideal function of the cooling system. As the motor voltage increases, the air flow speed increases proportional to the voltage. Any kind of air leak, whirlpool, or turbulence will cause the deviation from the ideal function in actual situation and the inefficiency in energy transformation.

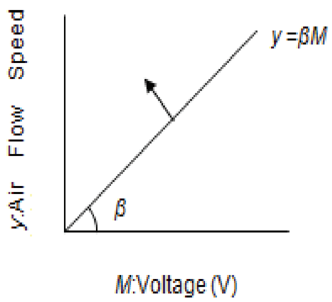

Figure 7 — Ideal function of the cooling system

— **(Step 2) Select signal factor and its range.**

Input signal of the cooling system is applied voltage to the motor to change the cooling air flow. The motor voltage is set usually in the range from 0 to 25 (V). Then the motor voltage was selected as a signal factor and three levels of it was set to 5, 15, or 25 (V) as shown in Table 10.

Table 10 — Signal factor and its levels

Levels	M1	M2	M3
Motor voltage (V)	5	15	25

— **(Step 3) Select measurement method of output response.**

The output response is air flow speed. It was measured by wind velocity or air speed. It can be directly measured with an anemometer.

— **(Step 4) Develop noise strategy, and select noise factors and their levels.**

Noise factors should be selected from noise conditions in the actual use in market, such as environment conditions and deterioration of parts of the system. One or two typical noise conditions are enough to be selected if they have strong effects.

In this study, existence of obstacle at the exhaust port was selected as a noise factor as shown in Table 11. When there is an obstacle at the exhaust port, air flow will be disturbed and cooling efficiency will fall. However the system should work in the same way whether an obstacle exists at the exhaust port or not.

Table 11 — Noise factor and its levels

Levels	N1	N2
Obstacle	No	Yes

— **(Step 5) Select control factors and their levels from the design space.**

One two-level factor and seven three-level factors can be assigned to an orthogonal array L_{18}. Table 12 shows the selected control factors and their levels.

Table 12 — Control factors and their levels for the cooling system

Control factors		Level 1	Level 2	Level 3
A	Baffle plate	No	Yes	-
B	Distance between equipment enclosure and air inlet (mm)	20	40	60
C	Distance between air inlet and heat source (mm)	110	60	40
D	Height of opening (mm)	30	15	0
E	Height of exhaust duct (mm)	30	15	0
F	Hole diameter at the top of heat source (mm)	Large	Medium	No
G	Hole diameter at the bottom of heat source (mm)	No	Medium	Large
H	Distance between heat source and exhaust duct (mm)	60	50	40

— **(Step 6) Assign experimental factors to inner or outer array.**

Control factors should be assigned to inner array. In this study, inner array is an orthogonal array L_{18}. Table 13 shows the assignment of control factors to the columns of an orthogonal array L_{18}. Each row shows the combination of the levels of control factors for each run from No. 1 to No. 18.

Table 13 — Control factor assignment to inner array

No.	A	B	C	D	E	F	G	H
1	No	20	110	30	30	Large	No	60
2	No	20	60	15	15	Medium	Medium	50
3	No	20	40	0	0	No	Large	40
4	No	40	110	30	15	Medium	Large	40
5	No	40	60	15	0	No	No	60
6	No	40	40	0	30	Large	Medium	50
7	No	60	110	15	30	No	Medium	40
8	No	60	60	0	15	Large	Large	60
9	No	60	40	30	0	Medium	No	50
10	Yes	20	110	0	0	Medium	Medium	60
11	Yes	20	60	30	30	No	Large	50
12	Yes	20	40	15	15	Large	No	40
13	Yes	40	110	15	0	Large	Large	50
14	Yes	40	60	0	30	Medium	No	40
15	Yes	40	40	30	15	No	Medium	60
16	Yes	60	110	0	15	No	No	50
17	Yes	60	60	30	0	Large	Medium	40
18	Yes	60	40	15	30	Medium	Large	60

Noise factor and signal factor should be assigned to outer array. In this study, outer array is a two-way full factorial plan.

— **(Step 7) Conduct experiment and collect data.**

Table 14 shows the measured data of the air flow speed in the experimental run for each row of inner array.

Table 14 — Measurement results of air flow speed (m/s)

No.	M1		M2		M3	
	N1	N2	N1	N2	N1	N2
1	0,12	0,09	0,31	0,26	0,44	0,41
2	0,18	0,15	0,28	0,23	0,44	0,32
3	0,36	0,31	1,20	0,96	1,56	1,46
4	0,25	0,22	0,77	0,66	1,24	1,20
5	0,24	0,19	0,84	0,73	1,26	1,08
6	0,23	0,20	0,79	0,67	1,24	1,02
7	0,13	0,08	0,14	0,34	0,30	0,56
8	0,23	0,19	0,57	0,26	0,91	0,56
9	0,24	0,19	0,86	0,68	1,32	1,12
10	0,26	0,17	0,86	0,67	1,30	0,98
11	0,06	0,04	0,23	0,28	0,37	0,27
12	0,36	0,34	1,14	1,04	1,70	1,58
13	0,21	0,12	0,77	0,60	1,18	1,04
14	0,31	0,30	1,12	0,93	1,66	1,42
15	0,10	0,04	0,33	0,24	0,56	0,47
16	0,28	0,23	1,10	0,82	1,66	1,24
17	0,27	0,23	0,83	0,72	1,30	1,08
18	0,28	0,19	0,76	0,57	1,06	0,71

— **(Step 8) Calculate SN ratio, η, and sensitivity, S.**

When the motor voltage is 0, the air flow speed should be 0. Accordingly, formulas for a zero-point proportional ideal function are applied to the calculations of SN ratio and sensitivity as shown in Clause 6.

Computations for the data of row No. 1 of the inner array L_{18} are shown as follows.

Total sum of squares

$$S_T = 0{,}12^2 + 0{,}09^2 + 0{,}31^2 + 0{,}26^2 + 0{,}44^2 + 0{,}41^2 = 0{,}547\ 900\ (f_T = 6)$$

Sum of squares of input signal levels/Effective divisor

$$r = 5^2 + 15^2 + 25^2 = 875$$

Linear forms for each noise level

$$L_1 = 5 \times 0{,}12 + 15 \times 0{,}31 + 25 \times 0{,}44 = 16{,}250\ 000$$

$$L_2 = 5 \times 0{,}09 + 15 \times 0{,}26 + 25 \times 0{,}41 = 14{,}600\ 000$$

Sum of squares due to linear slope, β

$$S_\beta = \frac{(16{,}25 + 14{,}60)^2}{2 \times 875} = 0{,}543\ 841\ \ (f_\beta = 1)$$

Sum of squares due to the variation of linear slope β between N1 and N2

$$S_{N\times\beta} = \frac{16{,}25^2 + 14{,}60^2}{875} - S_\beta = 0{,}001\ 556\ \left(f_{N\times\beta} = 1\right)$$

Sum of squares due to error

S_e = 0,547 900 – 0,543 841 – 0,001 556=0,002 503 (f_e = 4)

Error variance/variance due to error

$$V_e = \frac{0{,}002\ 503}{4} = 0{,}000\ 626$$

Variance due to pooled error/variation due to error and noise

$$V_N = \frac{0{,}002\ 503 + 0{,}001\ 556}{1+4} = 0{,}000\ 812$$

SN ratio, η, and sensitivity, S

$$\eta = 10\log\frac{\frac{1}{2\times 875}\left(0{,}543\ 841 - 0{,}000\ 626\right)}{0{,}000\ 812} = -4{,}17 \quad \text{(db)}$$

$$S = 10\ \log\frac{1}{2\times 875}\left(0{,}543\ 841 - 0{,}000\ 626\right) = -35{,}08 \quad \text{(db)}$$

Similarly, calculations for each row of the inner array were performed.

Table 15 shows the results of calculations of SN ratio and sensitivity for each row of the inner array.

Table 15 — SN ratio and sensitivity for inner array

No.	SN ratio db	Sensitivity db
1	−4,17	−35,08
2	−12,77	−35,86
3	−5,99	−23,94
4	1,76	−26,29
5	−4,81	−26,36
6	−5,35	−26,74
7	−15,93	−35,41
8	−14,45	−30,67
9	−5,35	−26,15
10	−8,82	−26,58
11	−11,40	−37,24
12	−1,08	−23,41
13	−5,57	−27,06
14	−4,92	−23,97
15	−8,00	−33,99
16	−9,13	−24,54
17	−4,89	−26,25
18	−11,99	−28,41

— **(Step 9) Generate factorial effect diagrams on SN ratio and sensitivity.**

The average values of SN ratio and sensitivity for levels of control factors assigned in the inner orthogonal array L_{18} should be calculated based on the data in Table 15 to generate factorial effect diagrams.

The averages of SN ratio for each level of control factors assigned to the inner orthogonal array are calculated by the formulae presented in Clause 6 as follows:

$$\eta_{A1} = \frac{(-4{,}17 - 12{,}77 - 5{,}99 + 1{,}76 - 4{,}81 - 5{,}35 - 15{,}93 - 14{,}45 - 5{,}35)}{9} = -7{,}45$$

$$\eta_{A2} = \frac{(-8{,}82 - 11{,}40 - 1{,}08 - 5{,}57 - 4{,}92 - 8{,}00 - 9{,}13 - 4{,}89 - 11{,}99)}{9} = -7{,}31$$

$$\eta_{B1} = \frac{(-4{,}17 - 12{,}77 - 5{,}99 - 8{,}82 - 11{,}40 - 1{,}08)}{6} = -7{,}37$$

…

$$\eta_{H3} = \frac{(-5{,}99 + 1{,}76 - 15{,}93 - 1{,}08 - 4{,}92 - 4{,}89)}{6} = -5{,}18$$

The averages of sensitivity are similarly calculated.

The results of average calculations are summarized in Table 16.

Table 16 — Averages of SN ratios and sensitivities

Control factor		SN ratio db			Sensitivity db		
		Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
A	Baffle plate	−7,45	−7,31	-	−29,61	−27,94	-
B	Distance between equipment enclosure and air inlet	−7,37	−4,48	−10,29	−30,35	−27.40	−28,57
C	Distance between air inlet and heat source	−6,98	−8,87	−6,29	−29,16	−30,06	−27,11
D	Height of opening	−5,34	−8,69	−8,11	−30,83	−29,42	−26,07
E	Height of exhaust duct	−8,96	−7,28	−5,91	−31,14	−29,13	−26,05
F	Hole diameter at the top of heat source	−5,92	−7,01	−9,21	−28,20	−27,88	−30,24
G	Hole diameter at the bottom of heat source	−4,91	−9,29	−7,94	−26,58	−30,80	−28,94
H	Distance between heat source and exhaust duct	−8,71	−8,26	−5,18	−30,18	−29,60	−26,55

Figure 8 shows the factorial effect diagrams on SN ratio and sensitivity for the cooling system.

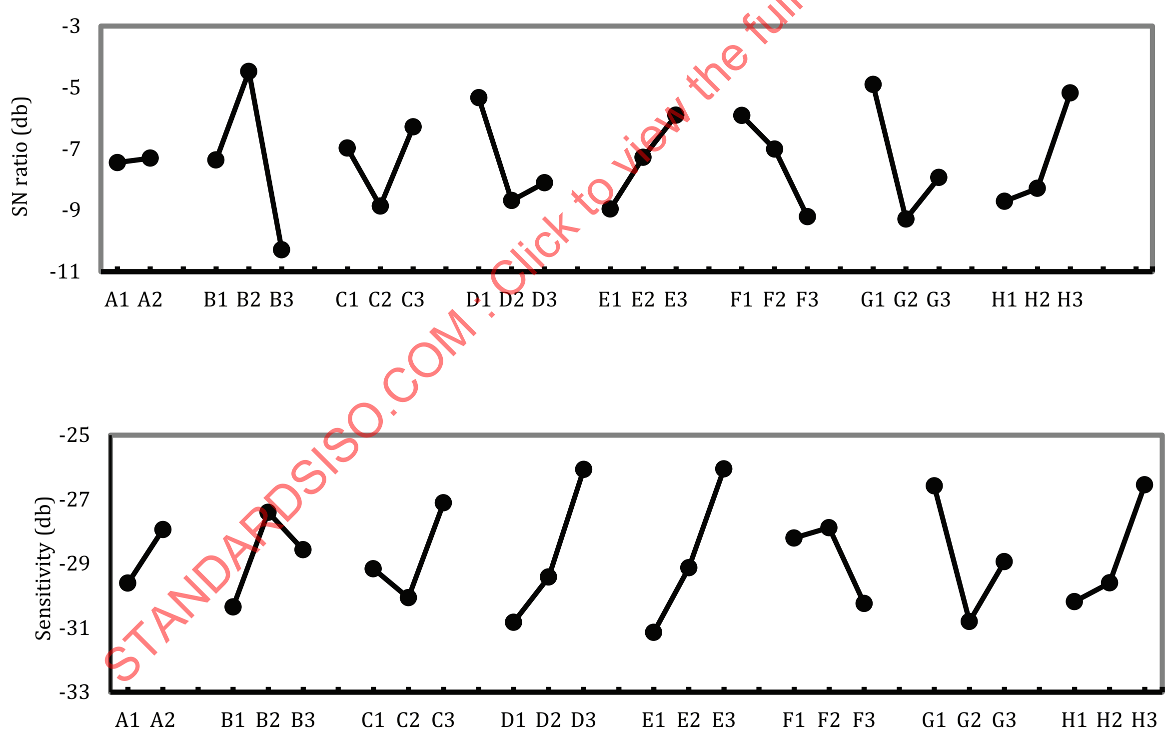

Figure 8 — Factorial effect diagrams for the cooling system

— **(Step 10) Select the optimum condition.**

In order to maximize SN ratio, a factorial level with higher SN ratio should be selected as an optimum level for each control factor. The optimum combination for SN ratio becomes A2B2C3D1E3F1G1H3. From the factorial effect diagrams, the factor whose optimum level is conflicted between SN ratio and sensitivity is factor D. To increase the sensitivity, selecting the level D3 is considered, but finally the level D1 is selected as the optimum condition because robustness with higher SN ratio is more important to the system.

— **(Step 11) Estimate the improvement in robustness by the gain.**

The estimates of SN ratio and sensitivity for optimum and baseline designs are calculated.

The estimation using all the factorial effects tends to be excess in some cases. For the moderate estimations of SN ratio and sensitivity, some factorial effects that are relatively large can be selected for the estimation.

In this case study, factors B, D, G, and H are selected to estimate SN ratio, and factors D, E, G, and H are selected to estimate sensitivity.

The baseline design is the initial design of the cooling system that has a combination of control factor's levels of A1B1C1D1E1F1G1H1.

Using the averages of SN ratio shown in Table 16, the calculations of SN ratios for the optimum and baseline designs are performed as follows.

Grand average of SN ratio

$$\overline{T}_{SN} = \text{ Average of 18 SN ratios} = -7{,}38$$

Estimated value of SN ratio for the optimum design

$$\eta_{opt} = \eta_{B2} + \eta_{D1} + \eta_{G1} + \eta_{H3} - 3\overline{T}_{SN} = -4{,}48 - 5{,}34 - 4{,}91 - 5{,}18 - 3 \times (-7{,}38) = 2{,}23$$

Estimated value of SN ratio for the baseline design

$$\eta_{base} = \eta_{B1} + \eta_{D1} + \eta_{G1} + \eta_{H1} - 3\overline{T}_{SN} = -7{,}37 - 5{,}34 - 4{,}91 - 8{,}71 - 3 \times (-7{,}38) = -4{,}19$$

The gain in SN ratio is calculated as the difference between the estimates for the optimum and the baseline designs.

Gain in SN ratio

$$\Delta\eta = \eta_{opt} - \eta_{base} = 2{,}23 - (-4{,}19) = 6{,}42 \text{ (db)}$$

The gain in sensitivity is also estimated in a similar way. It shows as follows.

Grand average of sensitivity

$$\overline{T}_{\beta} = \text{Average of 18 sensitivities} = -28{,}77$$

Estimated value of sensitivity for the optimum design

$$S_{opt} = S_{D1} + S_{E3} + S_{G1} + S_{H3} - 3\overline{T}_{\beta} = -30{,}83 - 26{,}05 - 26{,}58 - 26{,}55 - 3 \times (-28{,}77) = -23{,}70$$

Estimated value of sensitivity for the baseline design

$$S_{base} = S_{D1} + S_{E1} + S_{G1} + S_{H1} - 3\overline{T}_{\beta} = -30{,}83 - 31{,}14 - 26{,}58 - 30{,}18 - 3 \times (-28{,}77) = -32{,}42$$

The gain in sensitivity is calculated as the difference between the estimates for the optimum and the baseline designs.

Gain in sensitivity

$\Delta S = S_{opt} - S_{base} = 23{,}70 - (-32{,}42) = 8{,}72$ (db)

— **(Step 12) Conduct a confirmation experiment and check the gains and "reproducibility".**

The confirmation experiment was conducted under the baseline design and the optimum design. SN ratio and sensitivity are computed from the data of the confirmation experiment. From the results of the confirmation experiments shown in Table 17, it is confirmed that the gains in db show excellent reproducibility.

Table 17 — Results of the confirmation experiment

	SN ratio db		Sensitivity db	
	Estimated value	**Confirmed value**	**Estimated value**	**Confirmed value**
Optimum	2,23	1,66	−23,70	−24,03
Baseline	−4,19	−4,17	−32,42	−35,08
Gain	6,42	5,83	8,72	11,05

Figure 9 shows the input-output relationships for the optimum design and the baseline design.

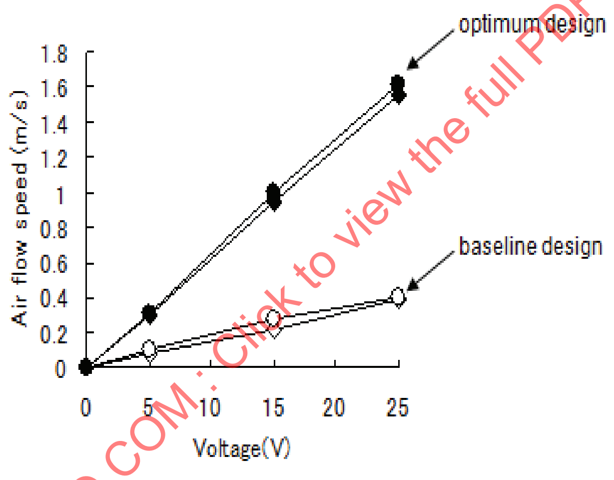

Figure 9 — Functions observed in the confirmation experiment

It can be seen that the robustness and the efficiency of the cooling system are greatly improved by the parameter design. Substantially, higher air flow speed can be achieved with the same motor voltage in the optimum design. It can be predicted by the gain in sensitivity. Due to the improved cooling capacity, these results show that a reduction of temperature in the lighting machine can also be expected.

Finally, the temperature test was performed on the optimum design and on the baseline design. The results of the temperature test are shown in Figure 10.The improvement was confirmed.

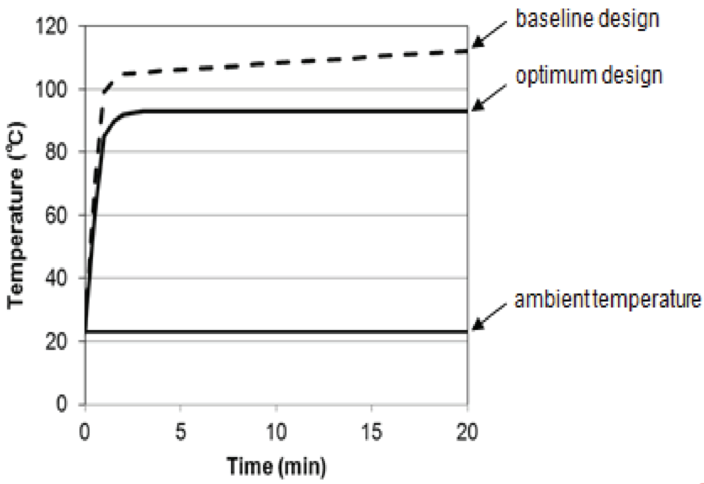

Figure 10 — The temperature change of a thermal part

As shown in Figure 10, for the baseline design, the temperature rising continues 20 min after the test started. However, for the optimum design, the temperature is stabilized after 3 min and does not rise thereafter. It is kept below the critical temperature of 100 °C.

Because testing cooling performance by measuring temperature is very tedious and time consuming, an easy and quick assessment method is conducted in this study. It focuses on the air flow generating function of the cooling system and the robustness optimization through SN ratio. Remember, SN ratio is a measure of performance where the numerator is efficiency of energy transformation and the denominator is the variability of energy transformation.

Annex A
(informative)

Comparison of a system's robustness using SN ratio

A.1 Assessing robustness of design concepts

A.1.1 General

As mentioned in Clause 6 of this International Standard, a parameter design can optimize one design concept by assigning many design parameters of control factors to an inner orthogonal array. However, in product development, it will become necessary to compare the robustness of various design concepts. For instance, it is necessary to benchmark competitors' designs against one's own designs for robustness.

In such cases, as long as the products have the same function (even if there are multiple systems based on different design concepts), robustness assessment can be conducted to compare various design concepts. The procedure described in Clause 5 of this International Standard can be applied as a robustness assessment. Basically, for each design concept, the SN ratio and sensitivity will be evaluated under noise strategy for the common ideal function.

Two examples are presented in the following subclauses.

A.1.2 Case study 1: Robustness assessment of mechanical components

The robustness assessment of ball bearings is presented as a case study. Ball bearings form rotating parts of various mechanical products, where they perform the task (objective function) of supporting a shaft while allowing it to rotate smoothly. Conventionally, a bearing's ability to rotate smoothly is assessed by, for example, using sensory assessment by listening for rotation noise, or by assessing the audible noise level. However, in this example, the smoothness of rotation is assessed based on the rotation function of a bearing as described below.

The input-output relationship for a ball bearing can identified as follows. Assume that the input is applied pre-load and the output response is rotational torque, the rotational torque y should be proportional to the applied pre-load M. Then the ideal function is given by a zero-point proportional formula,

$$y = \beta M \tag{A.1}$$

where y and M represent the rotational torque and the applied pre-load, respectively.

If this torque remains low and stable under all usage conditions, the bearing is highly robust and should also exhibit a low level of quality problems, such as audible noise, vibration, and poor reliability. Therefore, the relation between the rotational torque and the pre-load was chosen as the function of a bearing to be assessed. Using the pre-load as an input signal, the rotational torque was measured while changing the pre-load in three levels, M1 = 20 N, M2 = 30 N, and M3 = 40 N, covering the range encountered in the use of bearings.

Factors that cause variability and inefficiency in this function are called noise factors. Here, rotation speed and elapsed time are selected as noise factors. These factors have a large effect on the deterioration of the bearing. These two noise factors are taken into account for robustness assessment. The rotation speed was taken at two levels, 1 r/min and 3 r/min. The elapsed time is taken at two levels, starting point and 1 min after starting. Then these noise levels are compounded into two combinations: one is to have an effect of relatively smaller deterioration (relatively smaller torque), and the other is to have an effect of relatively larger deterioration (relatively larger torque).

— N1: Combination of lower seed and shorter elapsed time levels (1 r/min and starting point)

— N2: Combination of higher speed and longer elapsed time levels (3 r/min and 1 min after starting)

When the tendency of the effects of each noise factor on the output is known, it is possible to reduce the number of test replications by compounding noise conditions into one compounded noise factor. Also, the noise conditions in the assessment do not have to come from or to be the same as the conditions of test standards or lifetime tests of the product. Develop a noise strategy, such that it is efficient and effective. With regard to the rotation speed, there is also no need to perform assessment at the high rotation speed used in practice, and assessment can be performed under these evaluation conditions. A set of data of rotational torque is obtained as shown in Table A.1 for the example of two design specifications A and B. Figure A.1 shows a plot of the acquired data.

Table A.1 — Measurement results of rotational torque

Dimensions in newton metres

		M1	M2	M3
		20	30	40
Design specification A	N1	15,0	25,5	39,0
	N2	28,5	49,5	65,5
Design specification B	N1	13,5	24,5	42,5
	N2	42,0	65,0	92,5

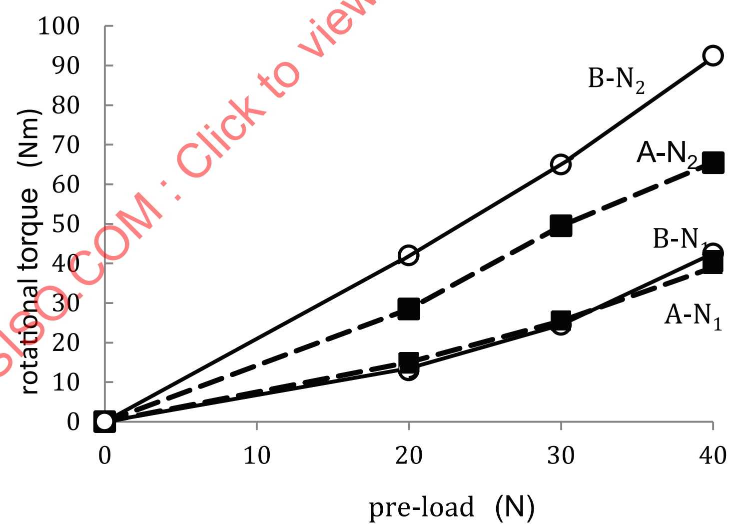

Figure A.1 — Scatter plot of measurement data

It is ideal that the torque is proportional to the pre-load and there is no effect of noise. The response under N1 and N2 should be the same when noise factor has no effect. In reality, as shown in Figure A.1, the torque is not necessarily proportional to the pre-load and the effect of noise factor is significant. The SN ratio will assess the proportionality and how small the effect of noise factor is, i.e. it will assess the robustness. The sensitivity measures nothing but the linear slope, β. In this case, the sensitivity has a unit of torque per pre-load, and represents the amount equivalent to the coefficient of friction. Accordingly, if a combination of the SN ratio and sensitivity is obtained based on the same noise conditions for two design specifications A and B, the difference in the robustness between A and B can be comprehensively and quantitatively expressed.

The SN ratio and sensitivity should be calculated from the data in Table A.1 as follows. The following computation for the design specification A is shown.

Total sum of squares

$$S_T = 15{,}0^2 + 25{,}5^2 + 39{,}0^2 + 28{,}5^2 + 49{,}5^2 + 65{,}5^2 = 9\,949{,}00 \quad (f_T = 6)$$

Sum of squares of input signal levels/effective divisor

$$r = 20^2 + 30^2 + 40^2 = 2\,900$$

Linear forms for each noise level

$$L_1 = 15{,}0 \times 20 + 25{,}5 \times 30 + 39{,}0 \times 40 = 2\,625{,}0$$

$$L_2 = 28{,}5 \times 20 + 49{,}5 \times 30 + 65{,}5 \times 40 = 4\,675{,}0$$

Sum of squares due to linear slope, β

$$S_\beta = \frac{(L_1 + L_2)^2}{2r} = \frac{(2\,625{,}0 + 4\,675{,}0)^2}{2 \times 2\,900} = 9\,187{,}931\,0 \quad (f_\beta = 1)$$

Sum of squares due to the variation of linear slope β between N1 and N2

$$S_{N \times \beta} = \frac{(L_1 - L_2)^2}{2r} = \frac{(2\,625{,}0 - 4\,675{,}0)^2}{2 \times 2\,900} = 724{,}569\,0 \quad (f_{N \times \beta} = 1)$$

Sum of squares due to error

$$S_e = S_T - S_\beta - S_{N \times \beta} = 9\,949{,}00 - 9\,187{,}931\,0 - 724{,}569\,0 = 36{,}500\,0 \quad (f_e = 4)$$

Variance due to error/error variance

$$V_e = \frac{S_e}{f_e} = \frac{36{,}500\,0}{4} = 9{,}125\,0$$

Variance due to pooled error

$$V_N = \frac{S_e + S_{N \times \beta}}{f_e + f_{N \times \beta}} = \frac{36{,}500\,0 + 724{,}569\,0}{4 + 1} = 152{,}213\,8$$

Here, V_e represents the variance due to higher order effects other than linear effect within N1 and N2, or variance due to the deviation from the proportional input-output relationship within N1 and N2. V_N represents the variance due to the difference between N1 and N2 in addition to the variance due to error and/or higher order effects.

SN ratio, η, and sensitivity, S

$$\eta = 10 \log \frac{\frac{1}{2r}(S_\beta - V_e)}{V_N} = 10 \log \frac{\frac{1}{2 \times 2\,900}(9\,187{,}931\,0 - 9{,}125\,0)}{152{,}213\,8} = -19{,}82 \quad \text{(db)}$$

$$S = 10 \log \frac{1}{2r}(S_\beta - V_e) = 10 \log \frac{1}{2 \times 2\,900}(9\,187{,}931\,0 - 9{,}125\,0) = 1{,}99 \quad \text{(db)}$$

Similar calculations are done for the design specification B. The results are summarized in Table A.2.

Table A.2 — SN ratios and sensitivities (db)

	Design specification A	Design specification B	Gain
SN ratio	-19,82	-23,09	3,27
Sensitivity	1,99	4,00	-2,01

From these results, the SN ratio and sensitivity of the design specification A are, respectively, 3,27 db higher and 2,01 db lower as compared to the design specification B. From the anti-logarithm scale, the gain of 3,27 db in the SN ratio is equivalent to a factor of 2,12 times, so it can be concluded that the robustness of the design specification A is approximately twice that of the design specification B.

Also, the anti-logarithm of the sensitivity gain of −2,01 db is 0,63, and its square root is 0,79. This shows that the average value of rotational torque for design specification A is about 20 % lower than that for design specification B with the same pre-load.

The difference in robustness and averages between two design specifications A and B has been expressed quantitatively by gains in SN ratio and sensitivity. Furthermore, based on the benchmarking results, if the target values of new product are, for example, the SN ratio of −20,0 db and sensitivity of 2,0 db, it can be concluded that the design specification A satisfies the targets, while the design specification B does not.

A.1.3 Case study 2: Robustness assessment on a measurement system

A measuring instrument can be defined to have a dynamic ideal function with a passive signal with the expression of $y = \beta M$, where M is the value of measurand (measured quantity) as a passive signal and y is its indicated value by the instrument. Signal in this case is called passive signal as opposed to active signal. In case of car steering, for example, the steering angle would be an active signal, because customers actively manipulate the steering and change the value of steering angle to get desired turning radius of the vehicle. The output response, the turning radius, is important to the customers. In case of measurement system, the value of input signal, the value of measurand, is already fixed when measurement is started and it is what customers want to know. The function of the measurement system is to reproduce the value of input signal from the output response of indicated value. So this kind of signal is called a passive signal.

An example of robustness assessment on three-dimensional coordinate measuring machine is shown.

Three-dimensional measuring machines are widely used for the measurement outside standard rooms recently. The usage conditions of three-dimensional measuring machines differ from one measurement process to another. A sensor attached to the measuring machine is important for reserving measurement accuracy and required to exhibit adequate performance under any conditions. Robustness assessment was applied for selecting an adequate sensor from three kinds of sensor design.

Conventional assessment method for a measuring instrument involves performing repeated measurements under a certain set of usage conditions and calculating a standard deviation in order to determine the magnitude of measurement error of the machine. In this method, the robustness under other usage conditions cannot be known, so it is not possible to guarantee whether the same performance will always be ensured under different usage conditions.

In contrast, this case study presents a method for evaluating the overall robustness of a measuring instrument by conducting measurements under a number of different usage conditions as noise conditions. The function of measurement system including a sensor is expressed as a zero-point proportional formula $y = \beta M$, where M denotes the value of a measured quantity and y denotes its indicated value. Robustness of measuring system is expressed by the smallness of variation of the proportional relationship under various noise conditions, this means the smallness of measurement error in the customers' situation.

First, usage conditions such as measurement speed, probe rotation angle, etc. were studied for selecting noise factors. As a result, four noise factors, each having three levels, were identified. These four factors at three levels were assigned to an orthogonal array L_9, and nine noise conditions were generated by L_9. When many noise factors are dominant and it is not easy to compound them or does not make sense to compound them, assigning them to an orthogonal array is recommended.

Next, for signal factor M, three different test pieces whose values were already known with satisfactory uncertainty were selected to set the three levels of signal factor as shown in Table A.3.

Table A.3 — Signal factor levels and sensor A measurement results

Dimensions in millimetres

	M1	M2	M3
	9,999 6	109,998 9	209,999 2
N1	9,999 0	109,998 9	210,000 1
N2	9,998 7	109,999 1	210,000 6
N3	9,998 0	109,998 6	209,999 6
N4	9,999 1	109,998 7	210,000 3
N5	9,997 0	109,997 0	209,997 8
N6	9,998 9	109,998 8	210,000 0
N7	9,997 3	109,997 1	209.998 8
N8	10,000 6	110,000 5	210,002 3
N9	10,000 0	109,999 9	210,001 4

Under these experimental factors, measurements were conducted using three different sensors A, B, and C, and the robustness of these three sensors was compared. Table A.3 shows the measurement results obtained using sensor A. Using 3 × 9 = 27 data from three levels of passive signal and nine noise conditions from the L_9, SN ratio and sensitivity were computed as follows:

Total sum of squares

$$S_T = 9{,}999\ 0^2 + 109{,}998\ 9^2 + 210{,}000\ 1^2 + \cdots + 10{,}000\ 0^2 + 109{,}999\ 9^2$$
$$+ 210{,}001\ 4^2 = 506\ 697{,}642\ 064\ 29 \quad (f_T = 27)$$

Sum of squares of input signal levels/effective divisor

$$r = 9{,}999\ 6^2 + 109{,}998\ 9^2 + 209{,}999\ 2^2 = 562\ 99{,}414\ 002\ 01$$

Linear forms for each noise level

$$L_1 = 9{,}999\,0 \times 9{,}999\,6 + 109{,}998\,9 \times 109{,}998\,9 + 210{,}000\,1 \times 209{,}999\,2 = 562\,99{,}597\,001\,53$$
$$L_2 = 9{,}998\,7 \times 9{,}999\,6 + 109{,}999\,1 \times 109{,}999\,1 + 210{,}000\,6 \times 209{,}999\,2 = 562\,99{,}721\,001\,03$$
$$\ldots$$
$$L_9 = 10{,}000\,0 \times 9{,}999\,6 + 109{,}999\,9 \times 109{,}998\,9 + 210{,}001\,4 \times 209{,}999\,2 = 562\,99{,}989\,998\,99$$

Sum of squares due to linear slope, β

$$S_\beta = \frac{(L_1 + L_2 + \cdots + L_9)^2}{9r}$$
$$= \frac{(56\ 299{,}597\ 001\ 53 + 56\ 299{,}721\ 001\ 03 + \cdots + 56\ 299{,}989\ 998\ 99)^2}{9 \times 56\ 299{,}414\ 002\ 01}$$
$$= 506\ 697{,}642\ 018\ 87 \quad (f_\beta = 1)$$

Sum of squares due to the difference of linear slope, β, between noise levels

$$S_{N\times\beta}=\frac{L_1{}^2+L_2{}^2+\cdots+L_9{}^2}{r}-S_\beta$$

$$=\frac{56\ 299{,}597\ 001\ 53^2+56\ 299{,}721\ 001\ 03^2+\cdots+56\ 299{,}989\ 998\ 99^2}{56\ 299{,}414\ 002\ 01}$$

$$-506\ 697{,}642\ 018\ 87=0{,}000\ 024\ 19\quad (f_{N\times\beta}=8)$$

Sum of squares due to error

$$S_\mathrm{e}=S_\mathrm{T}-S_\beta-S_{N\times\beta}$$

$$=506\ 697{,}642\ 064\ 29-506\ 697{,}642\ 018\ 87-0{,}000\ 024\ 19\ =0{,}000\ 021\ 23\quad (f_\mathrm{e}=18)$$

Error variance/Variance due to error

$$V_\mathrm{e}=\frac{S_\mathrm{e}}{f_\mathrm{e}}=\frac{0{,}000\ 002\ 123}{18}=0{,}000\ 001\ 18$$

Variance due to pooled error

$$V_\mathrm{N}=\frac{S_\mathrm{e}+S_{N\times\beta}}{f_\mathrm{e}+f_{N\times\beta}}=\frac{0{,}000\ 021\ 23+0{,}000\ 024\ 19}{18+8}=0{,}000\ 001\ 75$$

SN ratio and sensitivity

$$\eta=10\ \log\frac{\frac{1}{9r}(S_\beta-V_\mathrm{e})}{V_\mathrm{N}}$$

$$=10\ \log\frac{\frac{1}{9\times 56\ 299{,}414\ 002\ 01}(506\ 697{,}642\ 018\ 87-0{,}000\ 00118)}{0{,}000\ 001\ 75}=57{,}57\quad (\mathrm{db})$$

$$S=10\ \log\frac{1}{9r}(S_\beta-V_\mathrm{e})$$

$$=10\ \log\frac{1}{9\times 56\ 299{,}414\ 002\ 01}(506\ 697{,}642\ 018\ 87-0{,}000\ 00118)=0{,}000\ 02499\quad (\mathrm{db})$$

Similar computations were done to calculate SN ratios and sensitivities for the sensors B and C. SN ratio of a measurement system is very important because it shows the magnitude of measurement error. However, it is not for sensitivity since it can easily be calibrated, i.e. two-step optimization is easily applicable to measurement systems.

Table A.4 shows the comparison of SN ratios for three sensors. From these results, the robustness of sensor A is better than that of sensor B in 3,14 db, and better than that of sensor C in 8,93 db. This means that after calibration, the standard deviation for measurement error of sensor A is 0,68 times smaller than that of sensor B, and 0,35 times smaller than that of sensor C. By performing this sort of evaluation, it is possible to comprehensively assess the robustness of a measurement system.

Table A.4 — Comparison of SN ratios (db)

	Sensor A	**Sensor B**	**Sensor C**
SN ratio	57,57	54,43	48,64

A.2 Layout other than orthogonal array for inner array

As discussed in Clause 6, parameter design involves a large number of design parameters (control factors), and the design parameters are assigned to an inner orthogonal array to prescribe the plan for experimentation. Even if there are four or fewer design parameters, they can still be assigned to an orthogonal array, but it is also possible to evaluate all possible combinations of options instead of an orthogonal array. This is called a multi-way layout or a full factorial plan.

In any case, it is important to take control factors simultaneously into one experiment so that each factor level can be evaluated under various combinations of other factors' levels rather than a fixed condition.

For example, let control factors A and B have two levels and let C have three levels. In this case, the full factorial layout will consist of 2 × 2 × 3 = 12 combinations of factor levels as follows.

— No. 1: A1B1C1

— No. 2: A1B1C2

— No. 3: A1B1C3

— No. 4: A1B2C1

— No. 5: A1B2C2

— No. 6: A1B2C3

— No. 7: A2B1C1

— No. 8: A2B1C2

— No. 9: A2B1C3

— No. 10: A2B2C1

— No. 11: A2B2C2

— No. 12: A2B2C3

Robustness evaluation is performed for each combination, yielding 12 SN ratios and sensitivities. The average value for each level of control factors is calculated to determine the factorial effect of each parameter.

Calculations are the same as in Clauses 5 and 6 in this International Standard.

Annex B
(informative)

Case studies and SN ratio in various technical fields

B.1 Case studies of dynamic characteristics

B.1.1 Case study 1: Application of dynamic characteristic in electromechanical system (Optimization of a small DC motor based on energy transformation)

An automobile uses many small DC motors in various subsystems, such as window regulator, wiper system, electric calliper, etc. OEMs are demanding to improve performance and quality requirements, such as audible noise, heat generation, torque and rpm requirement, and energy efficiency and reliability.

It is typical to test a motor to check if it meets those requirements one by one, and conduct trade-off if necessary. This is nothing but a validation, not a robustness assessment or optimization.

In this study, the ideal function for robustness assessment was identified based on the energy transformation of DC motor. It is very much in line with today's strong demand for energy efficiency.

This approach is to improve multiple requirements simultaneously by optimizing the ideal function based on energy transformation.

— **(Step 1) Clarify the system's ideal function.**

A DC motor takes electric power consumption to get amount of rotational power needed for doing some works as input. A DC motor's function can be described as transformation of electric power to rotational mechanical power to provide intended displacement.

The ideal function of DC motor should be defined as a zero-proportional ideal function of energy transfer, based on energy thinking that says "to provide the required motive power with low power consumption." It's ideal function can be expressed as

$$y=\beta M$$

where input M is needed motive power and output y is power consumption for providing the motive power to do some works. Coefficient β is defined as the power consumption rate, which corresponds to the electric power consumption needed per unit of motive power. The criteria for optimization is to reduce variability due to noises, i.e. increase "robustness" of this relationship, and to minimize the power consumption rate β as shown in Figure B.1.

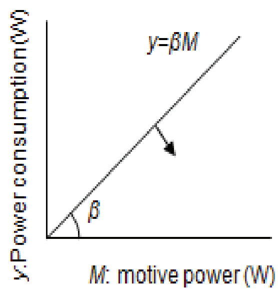

Figure B.1 — DC motor robustness

— **(Step 2) Select signal factor and its range**

In the experiments, the motor was loaded with three levels of torque, 2, 3, and 4 (N · m), to simulate the driving load, and was connected to a 12 (V) supply, which is corresponding to an automobile battery. Needed rotational power can be expressed as mechanical energy, that is, 2π × rotation speed n (rps) × torque T (N · m), that is, $M = 2\pi nT$.

The rotation speed and the internal current were measured for 180 s at sampling intervals of 0,1 s. The motive power was computed to identify the input signal levels. The signal levels are computed as the product of the rotation speed and the load torque as described in Table B.1. Rotation speed is measured at the sampling interval of 0,1 s, because its instantaneous speed changes from time to time due to noise effects, such as heat generation even under a constant torque condition.

Table B.1 — Signal factor for DC motor and its levels

Signal factor	Load torque T_1 M1	Load torque T_2 M2	Load torque T_3 M3
Motive power ($2\pi nT$)	Level values set from the product of the calculated rotation speed and the three load torque levels corresponding to working conditions		

— **(Step 3) Select measurement method of output response.**

The output response is the electrical power consumption. Electric power consumption can be expressed as the product of current and voltage of electrical input, that is, current I (A) × voltage E (V), that is, $y = IE$. Therefore, the ideal function can be expressed as the relationship

$$IE = \beta \times 2\pi nT$$

From the motor operation over time, it seems that the overall trend should be characterized three elapsed time periods: the initial start-up and the operating states at 90 s and 180 s after starting-up. And 10 data samples were extracted during 1,0 s intervals over 10 s at each of “Initial”, “90 s”, and “180 s”. Since the number of data samples at each point was set to ten, making a total of 30 data samples for each loading condition, the number of data samples for SN ratio calculation per each run is 90. With regard to the orthogonal array No. 1 combination of control factors, the left side of Figure B.2 shows the entire power consumption measured under each load conditions, and the right side of Figure B.2 shows the example of the results of input-output relationship of the ideal function.

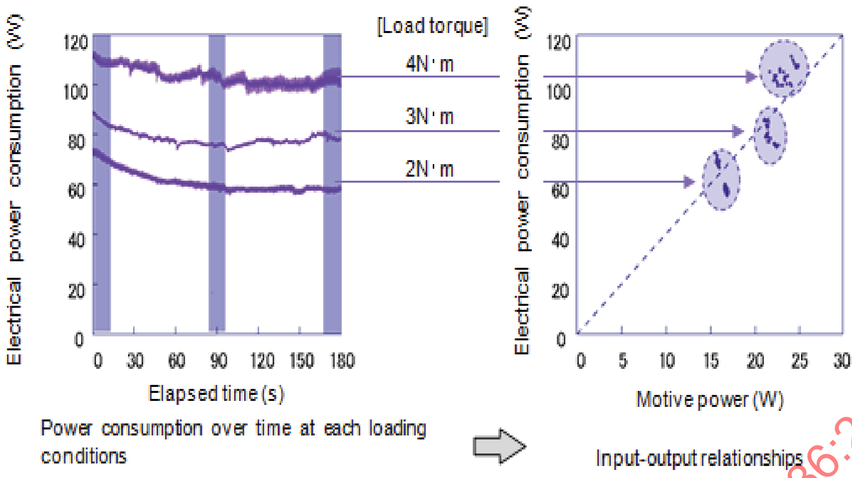

Figure B.2 — Change of power consumption in time and input-output relationships

— **(Step 4) Development noise strategy and select noise factors and their levels.**

Recognize that the noise factor is those three levels of elapsed time, namely, initial, 90 s, and 180 s as shown in Table B.2.

This noise factor results in undesirable effects such as variability, inefficiency, and nonlinear relation between the input and output power.

Noise factors often account for such things as the customer's usage conditions, the ambient temperature and degradation/aging of the product, but in this case study, it was decided to actively incorporate into the data the effects of reduced efficiency caused by heat generation by intentionally running the motor continuously to allow it to heat up, since these effects greatly surpass the effects of actual usage conditions (it can be safely said that no one uses a window regulator continuously for 180 s). Therefore, the noise factor was selected as the elapsed time, and was set to three levels. This is a very creative noise strategy.

Table B.2 — Noise factor for DC motor

Noise factor	Level 1 N1	Level 2 N2	Level 3 N3
Elapsed time	Initial start-up	After 90 s	After 180 s

— **(Step 5) Select control factors and their levels from design parameters.**

Eight control factors from the DC motor design were selected. Control factors and levels are summarized in Table B.3.

Table B.3 — DC motor control factors and levels

Control factor		Level 1	Level 2	Level 3
A	Fixing method for part A	Current	Rigid	–
B	Sheet thickness of part B	Small	Medium	Large
C	Shape of part B	Shape 1	Shape 2	Shape 3
D	Width of part D	Small	Medium	Large
E	Shape of part E	Shape 1	Shape 2	Shape 3
F	Inner R of part F	Small	Medium	Large
G	Shape of part G	Shape 1	Shape 2	Shape 3
H	Sheet thickness of part H	Small	Medium	Large

— **(Step 6) Assign signal factor and its range.**

The assignment of control factors to an inner orthogonal array L_{18} is shown in Table B.4.

Table B.4 — Control factor assignments

	A	B	C	D	E	F	G	H
1	Current	Small	Shape 1	Small	Shape 1	Small	Shape 1	Small
2	Current	Small	Shape 2	Medium	Shape 2	Medium	Shape 2	Medium
3	Current	Small	Shape 3	Large	Shape 3	Large	Shape 3	Large
4	Current	Medium	Shape 1	Small	Shape 2	Medium	Shape 3	Large
5	Current	Medium	Shape 2	Medium	Shape 3	Large	Shape 1	Small
6	Current	Medium	Shape 3	Large	Shape 1	Small	Shape 2	Medium
7	Current	Large	Shape 1	Medium	Shape 1	Large	Shape 2	Large
8	Current	Large	Shape 2	Large	Shape 2	Small	Shape 3	Small
9	Current	Large	Shape 3	Small	Shape 3	Medium	Shape 1	Medium
10	Rigid	Small	Shape 1	Large	Shape 3	Medium	Shape 2	Small
11	Rigid	Small	Shape 2	Small	Shape 1	Large	Shape 3	Medium
12	Rigid	Small	Shape 3	Medium	Shape 2	Small	Shape 1	Large
13	Rigid	Medium	Shape 1	Medium	Shape 3	Small	Shape 3	Medium
14	Rigid	Medium	Shape 2	Large	Shape 1	Medium	Shape 1	Large
15	Rigid	Medium	Shape 3	Small	Shape 2	Large	Shape 2	Small
16	Rigid	Large	Shape 1	Large	Shape 2	Large	Shape 1	Medium
17	Rigid	Large	Shape 2	Small	Shape 3	Small	Shape 2	Large
18	Rigid	Large	Shape 3	Medium	Shape 1	Medium	Shape 3	Small

— **(Step 7) Conduct experiment and collect data.**

Table B.5 shows the data obtained in outer array for each row of inner array.

Table B.5 — Data obtained in outer array

Experimental conditions	Voltage	*E* (fixed)					
Measurement data	Load	T1			T3		
	Measure-ment point	Initial start-up	After 90 s	After 180 s	Initial start-up	After 90 s	After 180 s
	Rotation speed	$n_1 \ldots n_{10}$	$n_{11} \ldots n_{20}$	$n_{21} \ldots n_{30}$	$n_{61} \ldots n_{70}$	$n_{71} \ldots n_{80}$	$n_{81} \ldots n_{90}$
	Current	$I_1 \ldots I_{10}$	$I_{11} \ldots I_{20}$	$I_{21} \ldots I_{30}$	$I_{61} \ldots I_{70}$	$I_{71} \ldots I_{80}$	$I_{81} \ldots I_{90}$
Signal *M*	Motive power: *W*	$n_1T_1 \ldots n_{10}T_1$	$n_{11}T_1 \ldots n_{20}T_1$	$n_{21}T_1 \ldots n_{30}T_1$	$n_{61}T_3 \ldots n_{70}T_3$	$n_{71}T_3 \ldots n_{80}T_3$	$n_{81}T_3 \ldots n_{90}T_3$
Output *y*	Electrical power: *W*	$I_1E \ldots I_{10}E$	$I_{11}E \ldots I_{20}E$	$I_{21}E \ldots I_{30}E$	$I_{61}E \ldots I_{70}E$	$I_{71}E \ldots I_{80}E$	$I_{81}E \ldots I_{90}E$

— **(Step 8) Calculate SN ratio, η, and sensitivity, S.**

Since the input signal and output response are physical values associated with energy, the square root of each value was taken based on a consideration of the additive nature of factorial effects obtained from a sum of squares decomposition. Accordingly, a zero-point proportional relationship of the dynamic ideal function was analysed for the signal factor: $2\pi n_1 T_1$ ... $2\pi n_{90} T_3$, square root M_1 ... M_{90}; output: $I_1 E$ ... $I_{90} T_3$ square root y_1 ... y_{90}. The calculation procedure is the same as the procedure discussed in Clause 6.

Total sum of squares

$$S_T = y_1^2 + y_2^2 + \cdots + y_{90}^2 \ (f_T = 90)$$

Sum of squares due to input signal levels/effective divisor

$$r = M_1^2 + M_2^2 + \cdots + M_{90}^2$$

Linear form

$$L = M_1 y_1 + M_2 y_2 + \cdots + M_{90} y_{90}$$

Sum of squares due to the linear slope

$$S_\beta = \frac{L^2}{r} \quad \left(f_\beta = 1\right)$$

Sum of squares due to error/noise

$$S_e = S_T - S_\beta \ \ (f_e = 89)$$

Error variance/variance due to noise

$$V_e = \frac{S_e}{89}$$

SN ratio, η, and sensitivity, S

$$\eta = 10 \log \frac{\frac{1}{r}\left(S_\beta - V_e\right)}{V_e} \quad \text{(db)}$$

$$S = 10\log\frac{1}{r}\left(S_\beta - V_e\right) \quad \text{(db)}$$

The SN ratio and sensitivity calculated by the above formulations are shown in Table B.6.

Table B.6 — Calculated SN ratio and sensitivity

No.	SN ratio db	Sensitivity db
1	11,20	6,00
2	8,99	6,64
3	14,61	5,99
4	14,04	6,46
5	9,33	6,65
6	14,78	5,98
7	11,95	6,21
8	10,86	6,51
9	9,72	6,81
10	7,34	6,78
11	12,22	6,47
12	8,99	6,17
13	11,90	6,21
14	7,92	6,32
15	12,54	6,66
16	9,68	6,64
17	14,92	6,20
18	8,99	6,61

Response tables of Table B.7 show the average for each level of each factor.

Table B.7 — Average of SN ratios and sensitivities

Control factor		SN ratio db			Sensitivity db		
		Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
A	Fixing method for part A	11,72	10,50	-	6,36	6,45	-
B	Sheet thickness of part B	10,56	11,75	11,02	6,34	6,38	6,50
C	Shape of part B	11,02	10,71	11,61	6,38	6,47	6,37
D	Width of part D	12,44	10,03	10,87	6,43	6,42	6,37
E	Shape of part E	11,18	10,85	11,30	6,27	6,51	6,44
F	Inner R of part F	12,11	9,50	11,72	6,18	6,60	6,44
G	Shape of part G	9,47	11,75	12,10	6,43	6,41	6,38
H	Sheet thickness of part H	10,04	11,22	12,07	6,54	6,46	6,23

— **(Step 9) General factorial effect diagrams on SN ratio and sensitivity.**

Figure B.3 shows the factorial effect diagram on SN ratio and sensitivity for DC-motors.

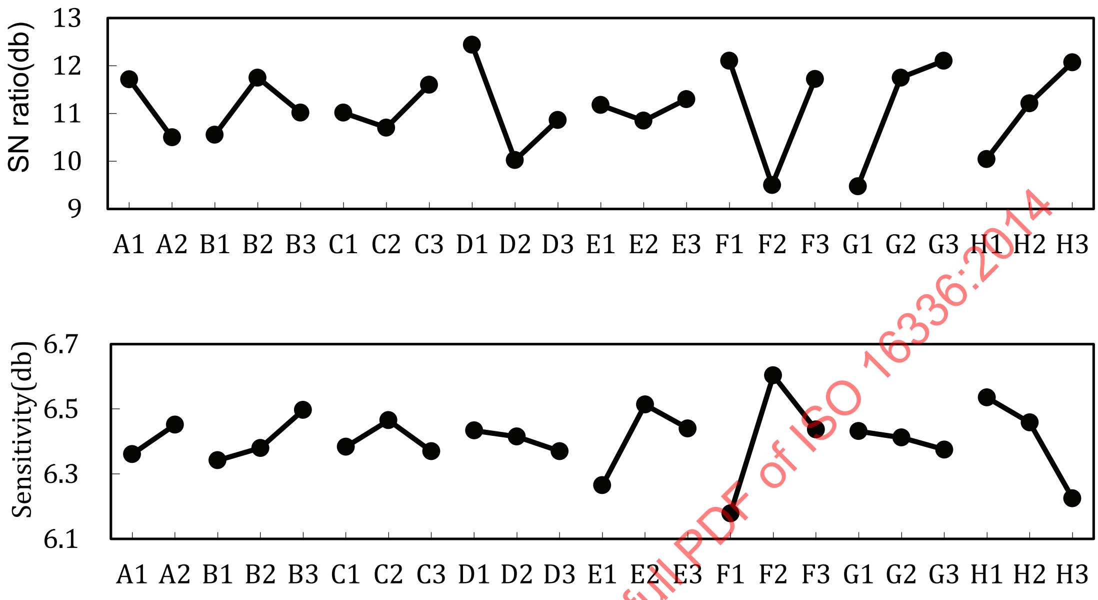

Figure B.3 — Factorial effect diagrams

— **(Step 10) Select the optimum condition.**

Factorial effect diagrams of Figure B.3 show that all the factors except B and D show that sensitivity is low when SN ratio is high. In other words, the better the robustness is, the lower the power consumption is, as expected. The optimum condition just for maximizing SN ratio is A1B2C3D1E3F1G3H3. However, E2 was selected for sensitivity and cost consideration because there is a small difference in SN ratios between E2 and E3. As a result, the optimum condition is selected as A1B2C3D1E2F1G3H3.

— **(Step 11) Estimate the improvement in robustness by the gain.**

Before the confirmation run, SN ratios and sensitivities for the optimum condition and the current conditions are calculated as follows. Here, the effects of all factors are used for the calculations. The current condition is A1B2C1D3E2F1G1H1.

The overall average of the SN ratio is calculated from 18 SN ratios from L_{18}.

Grand average of SN ratio

$$\overline{T}_{SN} = \frac{\eta_1 + \eta_2 + \cdots + \eta_{17} + \eta_{18}}{18} = \frac{11{,}20 + 8{,}99 + \cdots + 8{,}99}{18} = 11{,}174$$

Estimate of the SN ratio for the optimum condition is calculated as follows

$$\begin{aligned}\eta_{opt} &= \eta_{A1} + \eta_{B2} + \eta_{C3} + \eta_{D1} + \eta_{E2} + \eta_{F1} + \eta_{G3} + \eta_{H3} - 7\overline{T}_{SN} \\ &= 11{,}72 + 11{,}75 + 11{,}61 + 12{,}44 + 10{,}85 + 12{,}11 + 12{,}10 + 12{,}07 - 7 \times 11{,}174 = 16{,}43\end{aligned}$$

Similarly, estimate of SN ratio for the current design η_{cur} is calculated.

$$\eta_{cur} = \eta_{A1} + \eta_{B2} + \eta_{C1} + \eta_{D3} + \eta_{E2} + \eta_{F1} + \eta_{G1} + \eta_{H1} - 7\overline{T}_{SN}$$
$$= 11{,}72 + 11{,}75 + 11{,}02 + 10{,}87 + 10{,}85 + 12{,}11 + 9{,}47 + 10{,}04 - 7 \times 11{,}174 = 9{,}61$$

Predicted gain is

$\Delta\eta = \eta_{opt} - \eta_{cur} = 16{,}43 - 9{,}61 = 6{,}82$ (db)

Next, calculation of sensitivities is done in a similar fashion. The overall average of sensitivity is calculated from the 18 sensitivities from L_{18}.

$$\overline{T}_{\beta} = \frac{S_1 + S_2 + \cdots + S_{18}}{18} = \frac{6{,}00 + 6{,}64 + \cdots + 6{,}61}{18} = 6{,}397$$

Estimate of sensitivity for the optimum condition is calculated as follows

$$S_{opt} = S_{A1} + S_{B2} + S_{C3} + S_{D1} + S_{E2} + S_{F1} + S_{G3} + S_{H3} - 7\overline{T}_{\beta}$$
$$= 6{,}36 + 6{,}38 + 6{,}37 + 6{,}43 + 6{,}51 + 6{,}18 + 6{,}38 + 6{,}23 - 7 \times 6{,}397 = 6{,}06$$

Similarly, estimate of sensitivity for the current design S_{cur} is calculated.

$$S_{cur} = S_{A1} + S_{B2} + S_{C1} + S_{D3} + S_{E2} + S_{F1} + S_{G1} + S_{H1} - 7\overline{T}_{\beta}$$
$$= 6{,}36 + 6{,}38 + 6{,}38 + 6{,}37 + 6{,}51 + 6{,}18 + 6{,}43 + 6{,}54 - 7 \times 6{,}397 = 6{,}37$$

The gain in sensitivity is calculated as follows

$\Delta S = S_{opt} - S_{cur} = 6{,}06 - 6{,}37 = -0{,}31$ (db)

A smaller sensitivity or a smaller linear slope means lower power consumption in this case. This means that a minus value of the gain in sensitivity indicates the improvement in power consumption.

— **(Step 12) Conduct confirmation experiment and check the gain and “reproducibility”.**

The confirmation run is conducted under the optimum and the current conditions. The results of SN ratio and sensitivity are summarized in Table B.8. There are some differences between the prediction and the confirmation but improvement in robustness and power consumption is confirmed by choosing the optimum design. We can conclude it is exhibiting good reproducibility.

Table B.8 — Results of confirmation run

	SN ratio db		**Sensitivity** db	
	Estimated value	**Confirmed value**	**Estimated value**	**Confirmed value**
Optimum	16,43	16,43	6,06	6,11
Current	9,61	11,73	6,37	6,93
Gain	6,82	4,70	−0,31	−0,82

The input-output relationships for the optimum and the current conditions are shown in Figure B.4.

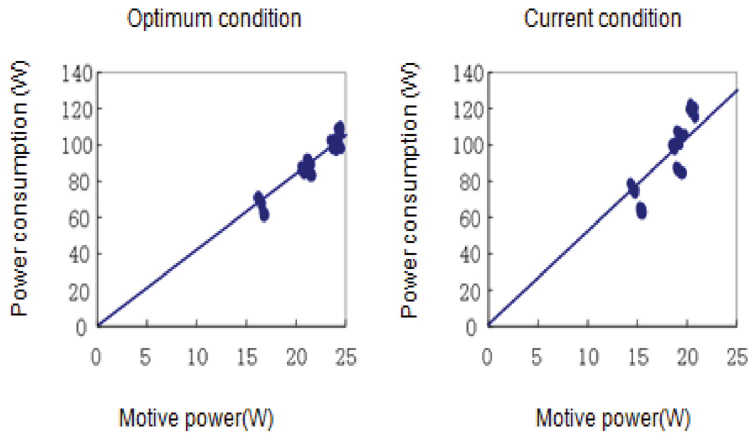

Figure B.4 — Input-output relationship of each condition

Under the optimum condition, there is much less variability in the electrical power consumption. It is also confirmed that the power consumption rate β is 4,08, which is approximately 17 (%) lower than 4,93 of the current conditions.

This means that the energy transformation from electric power to mechanical power becomes more efficient and smooth. We can expect less vibration and audible noise which are nothing but symptoms of poor function. At the same rpm, audible noise was measured and resulted in 8 (db) reduction of noise level at the optimum design compared to the current design.

Moreover, reduction of power consumption means it takes less electric energy to produce the same output torque. This will lead to an opportunity to downsizing the motor without affecting performance. As a result, a cost reduction of 30 million yen per year will be realized.

In this case study of robust optimization, the improvement is achieved by defining the ideal function of DC motor based on its energy transformation, i.e. electric power consumption and mechanical power generation. As a result, symptoms of poor function, such as audible noise and vibration, are improved drastically. Because the improvement of robustness exceeds performance requirements, cost reduction is achieved instead of performance. This improvement is not possible by typical one-factor-at-a-time experiment measuring performance requirements and conventional quality characteristics. This leads to reduction of natural resource and energy consumption and contributes to global ecology.

B.1.2 Case study 2: Application of dynamic SN-ratio in food industry (Optimization of bean sprouting by parameter design)

Bean sprout grows up in a production process where small beans are soaked in water, germinated, and grown in a lightless environment without soil. The function of this production process can be expressed as a growth curve of bean sprout weight. SN ratio for dynamic ideal function can be applied to assess efficiency and variability of the growth curve. Parameter design with SN ratio was applied to optimize the bean sprout production process in food industry.

— **(Step 1) Clarify the system's ideal function.**

Bean sprout growing process is that small beans are soaked in water, germinated, and grown in a lightless environment without soil. This process is divided into three periods as shown in Figure B.5. They are germination, growth, and decay periods.

a) Germination period

Bean of bean sprout is a kind of soybean or other type of small bean, such as black mappe (ketsuru adzuki) and green gram. Small beans are usually in dry and hibernating conditions, so they have to be soaked in water and heated for a certain period for germination. Conditions of germination period have important roles for germination rate and sterilizing bacteria.

b) Growth period

Mappe grows up rapidly after germination with absorbing a considerable amount of water. There is no clear difference between germination mechanism and growth mechanism, and it is difficult to distinguish between germination period and growth period. SN ratio and sensitivity are applied to assess the growth of bean sprout in growth period.

c) Decay period

Plants need photosynthesis for growing up after growth period. Bean sprouts normally decay and rot after taking all the bean's nutrition because no light is supplied in the production environment. Bean sprout is shipped before decay period, which starts typically 7 days or 8 days after the soak started.

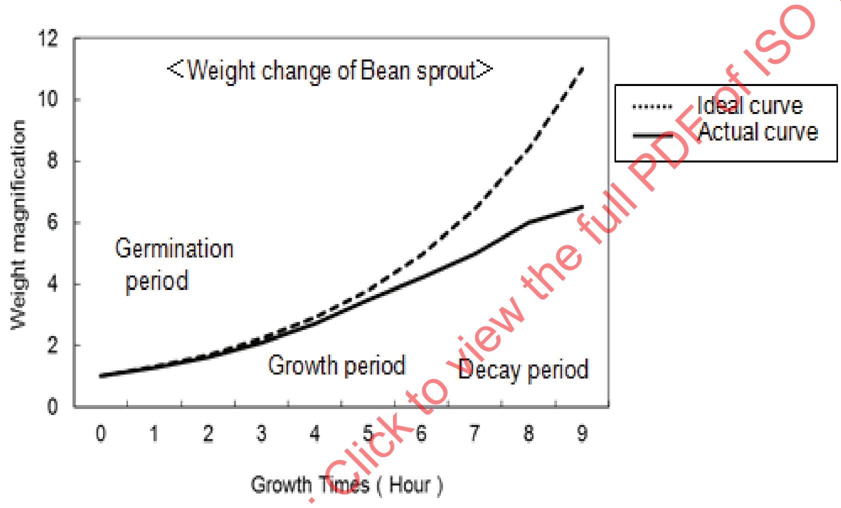

Figure B.5 — Growth curve of bean sprouts (Weight growth over time)

Ideal growth process from the theory of plant growth can be expressed as

$$Y_S = Y_0 e^{\beta M} \tag{B.1}$$

where Y_0 represents the initial weight of bean at the starting point (M = time = 0), and Y_s represents weight of bean sprouts at time M. This is the generic function of bean sprouting. This formula can be used as the ideal function. In order to linearize this formula, a natural logarithm transformation is applied. As a result, this formula can be written as follows:

$$\ln\left(\frac{Y_s}{Y_0}\right) = \beta M \tag{B.2}$$

Let y be

$$y = \ln(Y_s / Y_0) \tag{B.3}$$

then, the formula can be rewritten as

$$y = \beta M$$

Then we can express the ideal function of bean sprout growth as a zero-point proportional formula $y = \beta M$, where the input signal M is time, and the output response y is the natural log of the weight ratio at time M to the initial weight. SN ratio and sensitivity for zero-point proportional ideal function can be applied to assess the efficiency and variability of bean sprout growth.

— **(Step 2) Select signal factor and its range.**

As discussed in Step 1, elapsed time from the starting point (starting the soaking) was selected as a signal factor M. Three levels of the signal factor M were selected as three days in growth period, from 5 d to 7 d, as shown in Table B.9. Recall that it typically takes 7 d to complete sprouting.

Table B.9 — Signal factor's level for growth

Signal	M1	M2	M3
Days (24 h)	5	6	7

— **(Step 3) Select measurement method of output response.**

Output response y_i is computed from the weight Y_S at the time M_i. Weight balance was used to measure the weight at the time M_i. The ratio of the measured data Y_S to the initial weight Y_0 was calculated, and then the natural log of it was taken to derive the output response y_i.

— **(Step 4) Develop noise strategy, and select noise factors and their levels.**

Humidity in the growing room was selected as a noise factor because humidity is easy to control in the experiment. It is plausible to select humidity and temperature in growing room as noise factors, but usually, it is difficult to control humidity and temperature simultaneously in a room. Other conditions than humidity in the room were fixed for each run of experiment to avoid disturbances, because germination conditions usually have strong influences on germination rate and sterilization.

Table B.10 — Noise factor and its levels

Noise factor	Level 1: N1	Level 2: N2
Humidity (%)	60	80

— **(Step 5) Select control factors and their levels from design parameters.**

Table B.11 shows the selected control factors and their levels. Germination conditions, such as temperature or soaking time, are not selected as control factors and, therefore, were fixed. Design parameters from the growing period were taken in this study. It is known that ethylene gas has an effect on bean sprout growing like plant hormone and is strongly related to growth, decay, and rot. Conditions related to the ethylene gas bathing were selected as control factors, such as factors C and D. Control factors E, F, and G are the conditions related to sparkling water.

Table B.11 — Control factors and their levels

Control factors		Level 1	Level 2	Level 3
A	Type of seed	Black mappe	Green gram	-
B	Room temperature (°C)	18	24	30
C	Number of ethylene gas bathing per day (timing)	1 (morning)	2 (morning and noon)	3 (morning, noon, and evening)
D	Ethylene gas concentration	10	20	30
E	Number of sprinkling per day (timing)	1 (morning)	2 (morning and noon)	3 (morning, noon, and evening)
F	Number of mist sprays in one sprinkling (0,5 ml/spray)	1	2	3
G	Addition of mineral to sprinkling water (%)	0	0,1	1,0

— **(Step 6) Assign experimental factors to inner or outer array.**

Table B.12 shows the assignment of control factors in inner array, an orthogonal array L_{18}. Experiment of outer array was performed under each combination of control factors' levels assigned by each row of the inner array.

Table B.12 — Inner array L_{18}: assignment of control factors

column	1	2	3	4	5	6	7
low	A	B	C	D	E	F	G
1	Black mappe	18	1	10	1	1	0
2	Black mappe	18	2	20	2	2	0,1
3	Black mappe	18	3	30	3	3	1
4	Black mappe	24	1	10	2	2	1
5	Black mappe	24	2	20	3	3	0
6	Black mappe	24	3	30	1	1	0,1
7	Black mappe	30	1	20	1	3	0,1
8	Black mappe	30	2	30	2	1	1
9	Black mappe	30	3	10	3	2	0
10	Green gram	18	1	30	3	2	0,1
11	Green gram	18	2	10	1	3	1
12	Green gram	18	3	20	2	1	0
13	Green gram	24	1	20	3	1	1
14	Green gram	24	2	30	1	2	0
15	Green gram	24	3	10	2	3	0,1
16	Green gram	30	1	30	2	3	0
17	Green gram	30	2	10	3	1	0,1
18	Green gram	30	3	20	1	2	1

— **(Step 7) Conduct experiment and collect data.**

Experiment was performed under the experimental plan of inner and outer arrays. Weight of bean sprout was measured under each factorial combination of the outer array. Table B.13 shows the output response y_i for each row of orthogonal array L_{18}. The data shown are only those after the natural logarithm transformation of weight ratio for analysis.