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Statistical interpretation of data —

Part 6: Determination of statistical tolerance intervals

Interprétation statistique des données —

Partie 6: Détermination des intervalles statistiques de tolérance

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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 16269-6 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*.

This first edition of ISO 16269-6 cancels and replaces ISO 3207:1975, which has been technically revised.

ISO 16269 consists of the following parts, under the general title *Statistical interpretation of data*:

— *Part 6: Determination of statistical tolerance intervals*

— *Part 7: Median — Estimation and confidence intervals*

— *Part 8: Determination of prediction intervals*

Introduction

A statistical tolerance interval is an estimated interval, based on a sample, which can be asserted with confidence $1 - \alpha$, for example 95 %, to contain at least a specified proportion p of the items in the population. The limits of a statistical tolerance interval are called statistical tolerance limits. The confidence level $1 - \alpha$ is the probability that a statistical tolerance interval constructed in the prescribed manner will contain at least a proportion p of the population. Conversely, the probability that this interval will contain less than the proportion p of the population is α. This part of ISO 16269 describes both one-sided and two-sided statistical tolerance intervals; a one-sided interval is constructed with an upper or a lower limit while a two-sided interval is constructed with both an upper and a lower limit.

Tolerance intervals are functions of the observations of the sample, i.e. statistics, and they will generally take different values for different samples. It is necessary that the observations be independent for the procedures provided in this part of ISO 16269 to be valid.

Two types of tolerance interval are provided in this part of ISO 16269, parametric and distribution-free. The parametric approach is based on the assumption that the characteristic being studied in the population has a normal distribution; hence the confidence that the calculated statistical tolerance interval contains at least a proportion p of the population can only be taken to be $1 - \alpha$ if the normality assumption is true. For normally distributed characteristics, the statistical tolerance interval is determined using one of the Forms A, B, C or D given in Annex A.

Parametric methods for distributions other than the normal are not considered in this part of ISO 16269. If departure from normality is suspected in the population, distribution-free statistical tolerance intervals may be constructed. The procedure for the determination of a statistical tolerance interval for any continuous distribution is provided in Forms E and F of Annex A.

The tolerance limits discussed in this part of ISO 16269 can be used to compare the natural capability of a process with one or two given specification limits, either an upper one U or a lower one L or both in statistical process management. An indication of this is the fact that these tolerance limits have also been called natural process limits. See ISO 3534-2:1993, 3.2.4, and the general remarks in ISO 3207 which will be cancelled and replaced by this part of ISO 16269.

Above the upper specification limit U there is the upper fraction nonconforming p_U (ISO 3534-2:—, 3.2.5.5 and 3.3.1.4) and below the lower specification limit L there is the lower fraction nonconforming p_L (ISO 3534-2:—, 3.2.5.6 and 3.3.1.5). The sum $p_U + p_L = p_T$ is called the total fraction nonconforming. (ISO 3534-2:—, 3.2.5.7). Between the specification limits U and L there is the fraction conforming $1 - p_T$.

In statistical process management the limits U and L are fixed in advance and the fractions p_U, p_L and p_T are either calculated, if the distribution is assumed to be known, or otherwise estimated. There are many applications of statistical tolerance intervals, although the above shows an example to a quality control problem. Wider applications and more statistical intervals are introduced in many textbooks such as Hahn and Meeker [10].

In contrast, for the tolerance intervals considered in this part of ISO 16269, the confidence level for the interval estimator and the proportion of the distribution within the interval (corresponding to the fraction conforming mentioned above) are fixed in advance, and the limits are estimated. These limits may be compared with U and L. Hence the appropriateness of the given specification limits U and L can be compared with the actual properties of the process. The one-sided tolerance intervals are used when only either the upper specification limit U or the lower specification limit L is relevant, while the two-sided intervals are used when both the upper and the lower specification limits are considered simultaneously.

The terminology with regard to these different limits and intervals has been confusing as the “specification limits” were earlier also called “tolerance limits” (see the terminology standard ISO 3534-2:1993, 1.4.3, where both these terms as well as the term “limiting values” were all used as synonyms for this concept). In the latest revision of ISO 3534-2:—, only the term specification limits have been kept for this concept. Furthermore, the *Guide for the expression of uncertainty in measurement* [5] uses the term “coverage factor” defined as a “numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty”. This use of “coverage” differs from the use of the term in this part of ISO 16269.

Statistical interpretation of data —

Part 6:
Determination of statistical tolerance intervals

1 Scope

This part of ISO 16269 describes procedures for establishing tolerance intervals that include at least a specified proportion of the population with a specified confidence level. Both one-sided and two-sided statistical tolerance intervals are provided, a one-sided interval having either an upper or a lower limit while a two-sided interval has both upper and lower limits. Two methods are provided, a parametric method for the case where the characteristic being studied has a normal distribution and a distribution-free method for the case where nothing is known about the distribution except that it is continuous.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 3534-2:—[1)], *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purposes of this document, the terms and definition given in ISO 3534-1, ISO 3534-2 and the following apply.

3.1.1
statistical tolerance interval
interval determined from a random sample in such a way that one may have a specified level of confidence that the interval covers at least a specified proportion of the sampled population

NOTE The confidence level in this context is the long-run proportion of intervals constructed in this manner that will include at least the specified proportion of the sampled population.

3.1.2
statistical tolerance limit
statistic representing an end point of a statistical tolerance interval

NOTE Statistical tolerance intervals can be either one-sided, in which case they have either an upper or a lower statistical tolerance limit, or two-sided, in which case they have both.

1) To be published. (Revision of ISO 3534-2:1993)

3.1.3
coverage
proportion of items in a population lying within a statistical tolerance interval

NOTE This concept is not to be confused with the concept coverage factor used in the *Guide for the expression of uncertainty in measurement* (GUM) [5].

3.1.4
normal population
normally distributed population

3.2 Symbols

For the purposes of this part of ISO 16269, the following symbols apply.

i	suffix of an observation
$k_1(n; p; 1-\alpha)$	factor used to determine x_L or x_U when the value of σ is known for one-sided tolerance interval
$k_2(n; p; 1-\alpha)$	factor used to determine x_L and x_U when the value of σ is known for two-sided tolerance interval
$k_3(n; p; 1-\alpha)$	factor used to determine x_L or x_U when the value of σ is unknown for one-sided tolerance interval
$k_4(n; p; 1-\alpha)$	factor used to determine x_L and x_U when the value of σ is unknown for two-sided tolerance interval
n	number of observations in the sample
p	minimum proportion of the population claimed to be lying in the statistical tolerance interval
u_p	p-fractile of the standard normal distribution
x_i	ith observed value $(i = 1, 2, \ldots, n)$
$x_{\max}$	maximum value of the observed values: $x_{\max} = \max\{x_1, x_2, \ldots, x_n\}$
$x_{\min}$	minimum value of the observed values: $x_{\min} = \min\{x_1, x_2, \ldots, x_n\}$
x_L	lower limit of the statistical tolerance interval
x_U	upper limit of the statistical tolerance interval
$\bar{x}$	sample mean, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
s	sample standard deviation; $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2} = \sqrt{\frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}}$
$1-\alpha$	confidence level for the claim that the proportion of the population lying within the tolerance interval is greater than or equal to the specified level p
μ	population mean
σ	population standard deviation

4 Procedures

4.1 Normal population with known variance and known mean

When the values of the mean, μ, and the variance, σ^2, of a normally distributed population are known, the distribution of the characteristic under investigation is fully determined. There is exactly a proportion p of the population:

a) to the right of $x_L = \mu - u_p \times \sigma$ (one-sided interval);

b) to the left of $x_U = \mu + u_p \times \sigma$ (one-sided interval);

c) between $x_L = \mu - u_{(1+p)/2} \times \sigma$ and $x_U = \mu + u_{(1+p)/2} \times \sigma$ (two-sided interval).

NOTE As such statements are known to be true, they are made with 100 % confidence.

In the above equations, u_p is p-fractile of the standard normal distribution. Numerical values of u_p may be read from the bottom line of the Tables B.1 to B.6 and Tables C.1 to C.6.

4.2 Normal population with known variance and unknown mean

Forms A and B, given in Annex A, are applicable to the case where the variance of the normal population is known while the mean is unknown. Form A applies to the one-sided case, while Form B applies to the two-sided case.

4.3 Normal population with unknown variance and unknown mean

Forms C and D, given in Annex A, are applicable to the case where both the mean and the variance of the normal population are unknown. Form C applies to the one-sided case, while Form D applies to the two-sided case.

4.4 Any continuous distribution of unknown type

If the characteristic under investigation is a continuous variable from a population of unknown form, and if a sample of n independent random observations of the characteristic has been taken, then a statistical tolerance interval can be determined from the ranked observations. The procedure given in Forms E and F of Annex A provide the determination of the coverage or sample size needed for tolerance intervals determined from the extreme values x_{min} or x_{max} of the sample of observations with given confidence level $1 - \alpha$.

NOTE Statistical tolerance intervals that do not depend on the shape of the sampled population are called *distribution-free* tolerance intervals.

This part of ISO 16269 does not provide procedures for distributions of known type other than the normal distribution. However, if the distribution is continuous, the distribution-free method may be used. Selected references to scientific literature that may assist in determining tolerance intervals for other distributions are also provided at the end of this document.

5 Examples

5.1 Data

Forms A to D, given in Annex A, are illustrated by examples using the numerical values of ISO 2854:1976, Clause 2, paragraph 1 of the introductory remarks, Table X, yarn 2: 12 measures of the breaking load of cotton yarn. It should be noted that the number of observations, $n = 12$, given here for these examples is considerably lower than the one recommended in ISO 2602 [1]. The numerical data and calculations in the different examples are expressed in centi-newtons (see Table 1).

Table 1 — Data for Examples 1 to 4

Values in centi-newtons

x	228,6	232,7	238,8	317,2	315,8	275,1	222,2	236,7	224,7	251,2	210,4	270,7

These measurements were obtained from a batch of 12,000 bobbins, from one production job, packed in 120 boxes each containing 100 bobbins. Twelve boxes have been drawn at random from the batch and a bobbin has been drawn at random from each of these boxes. Test pieces of 50 cm length have been cut from the yarn on these bobbins, at about 5 m distance from the free end. The tests themselves have been carried out on the central parts of these test pieces. Previous information makes it reasonable to assume that the breaking loads measured in these conditions have virtually a normal distribution. It is demonstrated in ISO 2954:1976 that the data do not contradict the assumption of a normal distribution.

These results yield the following:

Sample size: $n = 12$

Sample mean: $\bar{x} = 3\,024{,}1/12 = 252{,}01$

Sample standard deviation: $s = \sqrt{\dfrac{n\sum x^2 - \left(\sum x\right)^2}{n(n-1)}} = \sqrt{\dfrac{166\,772{,}27}{12 \times 11}} = \sqrt{1\,263{,}426\,3} = 35{,}545$

The formal presentation of the calculations will be given only for Form C in Annex A (one-sided interval, unknown variance).

5.2 Example 1: One-sided statistical tolerance interval under known variance

Suppose that previously obtained measurements have shown that the dispersion is constant from one batch to another from the same supplier, and is represented by a standard deviation $\sigma = 33{,}150$, although the mean is not constant. A limit x_L is required such that it is possible to assert with confidence level $1 - \alpha = 0{,}95$ (95 %) that at least 0,95 (95 %) of the breaking loads of the items in the batch, when measured under the same conditions, are above x_L.

Table B.4 gives

k_1 (12; 0,95; 0,95) = 2,120

whence

$$x_L = \bar{x} - k_1(n; p; 1-\alpha) \times \sigma = 252{,}01 - 2{,}120 \times 33{,}150 = 181{,}732$$

A smaller value of the lower limit x_L would be obtained if a larger proportion of the population (for example $p = 0{,}99$) and/or a higher confidence level (for example $1 - \alpha = 0{,}99$) were required.

5.3 Example 2: Two-sided statistical tolerance interval under known variance

Under the same conditions as in Example 1, suppose that limits x_L and x_U are required such that it is possible to assert with a confidence level $1 - \alpha = 0{,}95$ that at least a proportion of $p = 0{,}90$ (90 %) of the breaking load of the batch falls between x_L and x_U.

Table C.4 gives

k_2 (12; 0,90; 0,95) = 1,889

whence

$$x_L = \overline{x} - k_2(n;\ p; 1-\alpha)\times\sigma = 252{,}01 - 1{,}889\times 33{,}150 = 189{,}390$$

$$x_U = \overline{x} + k_2(n;\ p; 1-\alpha)\times\sigma = 252{,}01 + 1{,}889\times 33{,}150 = 314{,}630$$

Comparison with Example 1 should make it clear that assuring that at least 90 % of a population lies between the limits x_L and x_U is not the same thing as assuring that no more than 5 % lies beyond each limit.

5.4 Example 3: One-sided statistical tolerance interval under unknown variance

Here, it is supposed that the standard deviation of the population is unknown and has to be estimated from the sample. The same requirements will be assumed as for the case where the standard deviation is known (Example 1), thus, $p = 0{,}95$ and $1 - \alpha = 0{,}95$. The presentation of the results is given in detail below.

Determination of the statistical tolerance interval of proportion p: a) one-sided interval “to the right” Determined values: b) proportion of the population selected for the tolerance interval: $p = 0{,}95$ c) chosen confidence level: $1 - \alpha = 0{,}95$ d) sample size: $n = 12$ Value of tolerance factor from Table D.4: $k_3(n;\ p;\ 1 - \alpha) = 2{,}737$
Calculations: $\overline{x} = \sum x / n = 252{,}01$ $s = \sqrt{\frac{n\sum x^2 - \left(\sum x\right)^2}{n(n-1)}} = 35{,}545$ $k_3(n;\ p; 1-\alpha)\times s = 97{,}286\ 7$
Results: one-sided interval “to the right” The tolerance interval which will contain at least a proportion p of the population with confidence level $1 - \alpha$ has a lower limit $x_L = \overline{x} - k_3(n;\ p; 1-\alpha)\times s = 154{,}723$

5.5 Example 4: Two-sided statistical tolerance interval under unknown variance

Under the same conditions as in Example 2, suppose it is required to calculate the limits x_L and x_U such that it is possible to assert with a confidence level $1 - \alpha = 0{,}95$ that in a proportion of the batch at least equal to $p = 0{,}90$ (90 %) the breaking load falls between x_L and x_U.

Table E.4 gives

$$k_4(n;\ p;\ 1-\alpha) = 2{,}671$$

whence

$$x_L = \bar{x} - k_4(n;\ p;\ 1-\alpha)\times s = 252{,}01 - 2{,}671 \times 35{,}545 = 157{,}069$$

$$x_U = \bar{x} + k_4(n;\ p;\ 1-\alpha)\times s = 252{,}01 + 2{,}671 \times 35{,}545 = 346{,}951$$

It will be noted that the value of x_L is smaller and the value of x_U higher than in Example 2 (known variance), because the use of s instead of σ requires a larger value of the tolerance factor to allow for the extra uncertainty. It is necessary to have to pay a penalty for not knowing the population standard deviation σ and the extension of the statistical tolerance interval takes this into account. Of course, it is not quite sure that the value $\sigma = 33{,}150$ used in Examples 1 and 2 is correct. Therefore, it is wiser to use the estimate, s, together with Tables D.4 or E.4.

5.6 Example 5: Distribution-free statistical tolerance interval for continuous distribution

In a fatigue test by rotational stress carried out on a component of an aeronautical engine, a sample of 15 items has given the results (measurement of endurance), shown in ascending order of values in Table 2.

Table 2 — Data for Example 5

x	0,200	0,330	0,450	0,490	0,780	0,920	0,950	0,970	1,040	1,710	2,220	2,275	3,650	7,000	8,800

A graphical examination of checking normality, such as probability plot, shows that the hypothesis of normality for the population of components should almost certainly be rejected (see ISO 5479). The methods of Form E, given in Annex A, for determination of a statistical tolerance interval are therefore applicable.

The extreme values from the sample of $n = 15$ measurements are:

$x_{\text{min}} = 0{,}200$, $x_{\text{max}} = 8{,}800$

Suppose that the required confidence level $1 - \alpha$ is 0,95.

a) What is the maximum proportion of the population of components that will fall below $x_{\text{min}} = 0{,}200$? Table F.1, for $1 - \alpha = 0{,}95$, gives for the minimum proportion above x_{min} a value of p slightly higher than 0,75 (75 %). Hence, for the maximum proportion below x_{min} a value of $1 - p$ slightly lower than 0,25 (25 %).

b) What sample size is necessary for it to be possible to assert, at a confidence level 0,95, that a proportion at least $p = 0{,}90$ (90 %) of the population of components will be found below the largest of the values from that sample? Table F.1, for $1 - \alpha = 0{,}95$ and $p = 0{,}90$, gives $n = 29$.

c) At a confidence level of 0,95, what is the minimum proportion of the population of components that fall between $x_{min} = 0{,}200$ and $x_{max} = 8{,}800$? Table G.1, for $1 - \alpha = 0{,}95$ and $n = 15$, gives p slightly below 0,75 (75 %).

d) What sample size is necessary for it to be possible to assert at a confidence level 0,95 that a proportion of at least $p = 0{,}90$ (90 %) of the population of components will be found to fall between the smallest and the largest values from that sample? Table G.1, for $1 - \alpha = 0{,}95$ and $p = 0{,}90$, gives $n = 46$.

e) In general, if a check for normality (see ISO 5479) indicates a departure from the normal distribution, some transformation will be recommended based on the knowledge of the collected data. For example, fatigue data are often approximated lognormally distributed. In such cases, the data could be transformed to normality. Tolerance intervals are then calculated and finally transformed back into the original units.

See Annex H for the construction of a statistical tolerance interval for distribution-free tolerance intervals for any type of distribution. Annex I gives the computation of factors for two-sided parametric statistical tolerance intervals.

Annex A
(informative)

Forms for tolerance intervals

Form A — One-sided statistical tolerance interval (known variance)

Determination of a one-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$

a) One-sided interval "to the left"

b) One-sided interval "to the right"

Known values:

c) the variance: $\sigma^2 =$

d) the standard deviation: $\sigma =$

Determined values:

e) proportion of the population selected for the tolerance interval: $p =$

f) chosen confidence level: $1 - \alpha =$

g) sample size: $n =$

Tabulated factor:

$k_1(n; p; 1 - \alpha) =$

This value can be read from the tables given in Annex B for a range of values of n, p and $1 - \alpha$.

Calculations:

$\bar{x} = \sum x / n =$

$k_1(n; p; 1 - \alpha) \times \sigma =$

Results:

a) One-sided interval "to the left"

The one-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$ has upper limit
$x_U = \bar{x} + k_1(n; p; 1-\alpha) \times \sigma =$

b) One-sided interval "to the right"

The one-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$ has lower limit
$x_L = \bar{x} - k_1(n; p; 1-\alpha) \times \sigma =$

Form B — Two-sided statistical tolerance interval (known variance)

Determination of a two-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$

Known values:

a) the variance: $\sigma^2 =$

b) the standard deviation: $\sigma =$

Determined values:

c) proportion of the population selected for the tolerance interval: $p =$

d) chosen confidence level: $1 - \alpha =$

e) sample size: $n =$

Tabulated factor:

$k_2(n; p; 1 - \alpha) =$

This value can be read from the tables given in Annex C for a range of values of n, p and $1 - \alpha$.

Calculations:

$$\bar{x} = \sum x / n =$$

$$k_2(n; p; 1 - \alpha) \times \sigma =$$

Results:

The two-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$ has limits

$$x_L = \bar{x} - k_2(n; p; 1 - \alpha) \times \sigma =$$

$$x_U = \bar{x} + k_2(n; p; 1 - \alpha) \times \sigma =$$

Form C — One-sided statistical tolerance interval (unknown variance)

Determination of a one-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$

a) One-sided interval “to the left”

b) One-sided interval “to the right”

Determined values:

c) proportion of the population selected for the tolerance interval: $p =$

d) chosen confidence level: $1 - \alpha =$

e) sample size: $n =$

Tabulated factor:

$$k_3(n; p; 1 - \alpha) =$$

This value can be read from the tables given in Annex D for a range of values of n, p and $1 - \alpha$.

Calculations:

$$\overline{x} = \sum x / n =$$

$$s = \sqrt{\frac{n\sum x^2 - (\sum x)^2}{n(n-1)}} =$$

$$k_3(n; p; 1 - \alpha) \times s =$$

Results:

a) One-sided interval “to the left”

The tolerance interval with coverage p at confidence level $1 - \alpha$ has upper limit

$$x_U = \overline{x} + k_3(n; p; 1-\alpha) \times s =$$

b) One-sided interval “to the right”

The tolerance interval with coverage p at confidence level $1 - \alpha$ has lower limit

$$x_L = \overline{x} - k_3(n; p; 1-\alpha) \times s =$$

Form D — Two-sided statistical tolerance interval (unknown variance)

Determination of a two-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$

Determined values:

a) proportion of the population selected for the tolerance interval: $p =$

b) chosen confidence level: $1 - \alpha =$

c) sample size: $n =$

Tabulated factor:

$k_4(n; p; 1 - \alpha) =$

This value can be read from the tables given in Annex E for a range of values of n, p and $1 - \alpha$.

Calculations:

$$\bar{x} = \sum x_i / n =$$

$$s = \sqrt{\frac{n\sum x^2 - (\sum x)^2}{n(n-1)}} =$$

$$k_4(n; p; 1 - \alpha) \times s =$$

Results:

The two-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$ has limits

$$x_L = \bar{x} - k_4(n; p; 1-\alpha) \times s =$$

$$x_U = \bar{x} + k_4(n; p; 1-\alpha) \times s =$$

Form E — One-sided statistical tolerance interval for any distribution

Determination of a one-sided distribution-free statistical tolerance interval with coverage p at confidence level $1 - \alpha$

a) One-sided interval "to the left"

b) One-sided interval "to the right"

Determined values:

c) proportion of the population selected for the tolerance interval: $p =$

d) chosen confidence level: $1 - \alpha =$

e) sample size: $n =$

(Either p or n is to be determined.)

Tabulated value

— p for given n and $1 - \alpha$.

— n for given p and $1 - \alpha$.

This value can be read from Table F.1 for a range of values of n, p and $1 - \alpha$.

Calculations and results

The one-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$ has either

— lower limit $x_L = x_{\text{min}} =$

— or upper limit $x_U = x_{\text{max}} =$

Form F — Two-sided statistical tolerance interval for any distribution

Determination of a two-sided distribution-free statistical tolerance interval with coverage p at confidence level $1 - \alpha$

Determined values:

a) proportion of the population selected for the tolerance interval: $p =$

b) chosen confidence level: $1 - \alpha =$

c) sample size: $n =$

(Either p or n is to be determined.)

Tabulated value

— p for given n and $1 - \alpha$.

— n for given p and $1 - \alpha$.

This value can be read from Table G.1 for a range of values of n, p and $1 - \alpha$.

Calculations and results

The two-sided statistical tolerance interval with coverage p at confidence level $1 - \alpha$ has

— lower limit $x_L = x_{\text{min}} =$

— and upper limit $x_U = x_{\text{max}} =$

Annex B
(normative)

One-sided statistical tolerance limit factors, $k_1(n; p; 1 - \alpha)$, for known σ

Table B.1 — Confidence level 50,0 %
($1 - \alpha = 0,50$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	0,000	0,675	1,282	1,645	2,327	3,091
3	0,000	0,675	1,282	1,645	2,327	3,091
4	0,000	0,675	1,282	1,645	2,327	3,091
5	0,000	0,675	1,282	1,645	2,327	3,091
6	0,000	0,675	1,282	1,645	2,327	3,091
7	0,000	0,675	1,282	1,645	2,327	3,091
8	0,000	0,675	1,282	1,645	2,327	3,091
9	0,000	0,675	1,282	1,645	2,327	3,091
10	0,000	0,675	1,282	1,645	2,327	3,091
11	0,000	0,675	1,282	1,645	2,327	3,091
12	0,000	0,675	1,282	1,645	2,327	3,091
13	0,000	0,675	1,282	1,645	2,327	3,091
14	0,000	0,675	1,282	1,645	2,327	3,091
15	0,000	0,675	1,282	1,645	2,327	3,091
16	0,000	0,675	1,282	1,645	2,327	3,091
17	0,000	0,675	1,282	1,645	2,327	3,091
18	0,000	0,675	1,282	1,645	2,327	3,091
19	0,000	0,675	1,282	1,645	2,327	3,091
20	0,000	0,675	1,282	1,645	2,327	3,091
22	0,000	0,675	1,282	1,645	2,327	3,091
24	0,000	0,675	1,282	1,645	2,327	3,091
26	0,000	0,675	1,282	1,645	2,327	3,091
28	0,000	0,675	1,282	1,645	2,327	3,091
30	0,000	0,675	1,282	1,645	2,327	3,091
35	0,000	0,675	1,282	1,645	2,327	3,091
40	0,000	0,675	1,282	1,645	2,327	3,091
45	0,000	0,675	1,282	1,645	2,327	3,091
50	0,000	0,675	1,282	1,645	2,327	3,091
60	0,000	0,675	1,282	1,645	2,327	3,091
70	0,000	0,675	1,282	1,645	2,327	3,091
80	0,000	0,675	1,282	1,645	2,327	3,091
90	0,000	0,675	1,282	1,645	2,327	3,091
100	0,000	0,675	1,282	1,645	2,327	3,091
150	0,000	0,675	1,282	1,645	2,327	3,091
200	0,000	0,675	1,282	1,645	2,327	3,091
250	0,000	0,675	1,282	1,645	2,327	3,091
300	0,000	0,675	1,282	1,645	2,327	3,091
400	0,000	0,675	1,282	1,645	2,327	3,091
500	0,000	0,675	1,282	1,645	2,327	3,091
1 000	0,000	0,675	1,282	1,645	2,327	3,091
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table B.2 — Confidence level 75,0 %
($1 - \alpha = 0,75$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	0,477	1,152	1,759	2,122	2,804	3,568
3	0,390	1,064	1,671	2,035	2,716	3,480
4	0,338	1,012	1,619	1,983	2,664	3,428
5	0,302	0,977	1,584	1,947	2,628	3,392
6	0,276	0,950	1,557	1,921	2,602	3,366
7	0,255	0,930	1,537	1,900	2,582	3,346
8	0,239	0,913	1,521	1,884	2,565	3,329
9	0,225	0,900	1,507	1,870	2,552	3,316
10	0,214	0,888	1,495	1,859	2,540	3,304
11	0,204	0,878	1,485	1,849	2,530	3,294
12	0,195	0,870	1,477	1,840	2,522	3,285
13	0,188	0,862	1,469	1,832	2,514	3,278
14	0,181	0,855	1,462	1,826	2,507	3,271
15	0,175	0,849	1,456	1,820	2,501	3,265
16	0,169	0,844	1,451	1,814	2,495	3,259
17	0,164	0,839	1,446	1,809	2,490	3,254
18	0,159	0,834	1,441	1,804	2,486	3,250
19	0,155	0,830	1,437	1,800	2,482	3,245
20	0,151	0,826	1,433	1,796	2,478	3,242
22	0,144	0,819	1,426	1,789	2,471	3,235
24	0,138	0,813	1,420	1,783	2,465	3,228
26	0,133	0,807	1,414	1,778	2,459	3,223
28	0,128	0,802	1,410	1,773	2,454	3,218
30	0,124	0,798	1,405	1,768	2,450	3,214
35	0,115	0,789	1,396	1,759	2,441	3,205
40	0,107	0,782	1,389	1,752	2,433	3,197
45	0,101	0,776	1,383	1,746	2,427	3,191
50	0,096	0,770	1,377	1,741	2,422	3,186
60	0,088	0,762	1,369	1,732	2,414	3,178
70	0,081	0,756	1,363	1,726	2,407	3,171
80	0,076	0,750	1,357	1,721	2,402	3,166
90	0,072	0,746	1,353	1,716	2,398	3,162
100	0,068	0,742	1,350	1,713	2,394	3,158
150	0,056	0,730	1,337	1,700	2,382	3,146
200	0,048	0,723	1,330	1,693	2,375	3,138
250	0,043	0,718	1,325	1,688	2,370	3,133
300	0,039	0,714	1,321	1,684	2,366	3,130
400	0,034	0,709	1,316	1,679	2,361	3,124
500	0,031	0,705	1,312	1,676	2,357	3,121
1 000	0,022	0,696	1,303	1,667	2,348	3,112
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table B.3 — Confidence level 90,0 %
(1 − α = 0,90)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	0,907	1,581	2,188	2,552	3,233	3,997
3	0,740	1,415	2,022	2,385	3,067	3,831
4	0,641	1,316	1,923	2,286	2,968	3,732
5	0,574	1,248	1,855	2,218	2,900	3,664
6	0,524	1,198	1,805	2,169	2,850	3,614
7	0,485	1,159	1,766	2,130	2,811	3,575
8	0,454	1,128	1,735	2,098	2,780	3,544
9	0,428	1,102	1,709	2,073	2,754	3,518
10	0,406	1,080	1,687	2,051	2,732	3,496
11	0,387	1,061	1,668	2,032	2,713	3,477
12	0,370	1,045	1,652	2,015	2,697	3,461
13	0,356	1,030	1,637	2,001	2,682	3,446
14	0,343	1,017	1,625	1,988	2,669	3,433
15	0,331	1,006	1,613	1,976	2,658	3,422
16	0,321	0,995	1,602	1,966	2,647	3,411
17	0,311	0,986	1,593	1,956	2,638	3,402
18	0,303	0,977	1,584	1,947	2,629	3,393
19	0,295	0,969	1,576	1,939	2,621	3,385
20	0,287	0,962	1,569	1,932	2,613	3,377
22	0,274	0,948	1,555	1,919	2,600	3,364
24	0,262	0,937	1,544	1,907	2,588	3,352
26	0,252	0,926	1,533	1,897	2,578	3,342
28	0,243	0,917	1,524	1,888	2,569	3,333
30	0,234	0,909	1,516	1,879	2,561	3,325
35	0,217	0,892	1,499	1,862	2,543	3,307
40	0,203	0,878	1,485	1,848	2,529	3,293
45	0,192	0,866	1,473	1,836	2,518	3,282
50	0,182	0,856	1,463	1,827	2,508	3,272
60	0,166	0,840	1,447	1,811	2,492	3,256
70	0,154	0,828	1,435	1,799	2,480	3,244
80	0,144	0,818	1,425	1,789	2,470	3,234
90	0,136	0,810	1,417	1,780	2,462	3,226
100	0,129	0,803	1,410	1,774	2,455	3,219
150	0,105	0,780	1,387	1,750	2,431	3,195
200	0,091	0,766	1,373	1,736	2,417	3,181
250	0,082	0,756	1,363	1,726	2,408	3,172
300	0,074	0,749	1,356	1,719	2,401	3,165
400	0,065	0,739	1,346	1,709	2,391	3,155
500	0,058	0,732	1,339	1,703	2,384	3,148
1 000	0,041	0,716	1,323	1,686	2,367	3,131
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table B.4 — Confidence level 95,0 %
(1 − α = 0,95)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	1,164	1,838	2,445	2,808	3,490	4,254
3	0,950	1,625	2,232	2,595	3,277	4,040
4	0,823	1,497	2,104	2,468	3,149	3,913
5	0,736	1,411	2,018	2,381	3,062	3,826
6	0,672	1,346	1,954	2,317	2,998	3,762
7	0,622	1,297	1,904	2,267	2,949	3,712
8	0,582	1,257	1,864	2,227	2,908	3,672
9	0,549	1,223	1,830	2,194	2,875	3,639
10	0,521	1,195	1,802	2,166	2,847	3,611
11	0,496	1,171	1,778	2,141	2,823	3,587
12	0,475	1,150	1,757	2,120	2,802	3,566
13	0,457	1,131	1,738	2,102	2,783	3,547
14	0,440	1,115	1,722	2,085	2,766	3,530
15	0,425	1,100	1,707	2,070	2,752	3,515
16	0,412	1,086	1,693	2,057	2,738	3,502
17	0,399	1,074	1,681	2,044	2,726	3,490
18	0,388	1,063	1,670	2,033	2,715	3,478
19	0,378	1,052	1,659	2,023	2,704	3,468
20	0,368	1,043	1,650	2,013	2,695	3,459
22	0,351	1,026	1,633	1,996	2,678	3,441
24	0,336	1,011	1,618	1,981	2,663	3,426
26	0,323	0,998	1,605	1,968	2,649	3,413
28	0,311	0,986	1,593	1,956	2,638	3,402
30	0,301	0,975	1,582	1,946	2,627	3,391
35	0,279	0,953	1,560	1,923	2,605	3,369
40	0,261	0,935	1,542	1,905	2,587	3,351
45	0,246	0,920	1,527	1,891	2,572	3,336
50	0,233	0,908	1,515	1,878	2,559	3,323
60	0,213	0,887	1,494	1,858	2,539	3,303
70	0,197	0,872	1,479	1,842	2,523	3,287
80	0,184	0,859	1,466	1,829	2,511	3,275
90	0,174	0,848	1,455	1,819	2,500	3,264
100	0,165	0,839	1,447	1,810	2,491	3,255
150	0,135	0,809	1,416	1,780	2,461	3,225
200	0,117	0,791	1,398	1,762	2,443	3,207
250	0,105	0,779	1,386	1,749	2,431	3,195
300	0,095	0,770	1,377	1,740	2,422	3,186
400	0,083	0,757	1,364	1,728	2,409	3,173
500	0,074	0,749	1,356	1,719	2,400	3,164
1 000	0,053	0,727	1,334	1,697	2,379	3,143
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table B.5 — Confidence level 99,0 %
(1 − α = 0,99)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	1,645	2,320	2,927	3,290	3,972	4,736
3	1,344	2,018	2,625	2,988	3,670	4,434
4	1,164	1,838	2,445	2,809	3,490	4,254
5	1,041	1,715	2,322	2,686	3,367	4,131
6	0,950	1,625	2,232	2,595	3,277	4,040
7	0,880	1,554	2,161	2,525	3,206	3,970
8	0,823	1,497	2,105	2,468	3,149	3,913
9	0,776	1,450	2,058	2,421	3,102	3,866
10	0,736	1,411	2,018	2,381	3,063	3,826
11	0,702	1,376	1,983	2,347	3,028	3,792
12	0,672	1,347	1,954	2,317	2,998	3,762
13	0,646	1,320	1,927	2,291	2,972	3,736
14	0,622	1,297	1,904	2,267	2,949	3,712
15	0,601	1,276	1,883	2,246	2,928	3,691
16	0,582	1,257	1,864	2,227	2,908	3,672
17	0,565	1,239	1,846	2,210	2,891	3,655
18	0,549	1,223	1,830	2,194	2,875	3,639
19	0,534	1,209	1,816	2,179	2,861	3,624
20	0,521	1,195	1,802	2,166	2,847	3,611
22	0,496	1,171	1,778	2,141	2,823	3,587
24	0,475	1,150	1,757	2,120	2,802	3,566
26	0,457	1,131	1,738	2,102	2,783	3,547
28	0,440	1,115	1,722	2,085	2,766	3,530
30	0,425	1,100	1,707	2,070	2,752	3,515
35	0,394	1,068	1,675	2,039	2,720	3,484
40	0,368	1,043	1,650	2,013	2,695	3,459
45	0,347	1,022	1,629	1,992	2,674	3,438
50	0,329	1,004	1,611	1,974	2,656	3,420
60	0,301	0,975	1,582	1,946	2,627	3,391
70	0,279	0,953	1,560	1,923	2,605	3,369
80	0,261	0,935	1,542	1,905	2,587	3,351
90	0,246	0,920	1,527	1,891	2,572	3,336
100	0,233	0,908	1,515	1,878	2,559	3,323
150	0,190	0,865	1,472	1,835	2,517	3,281
200	0,165	0,839	1,447	1,810	2,491	3,255
250	0,148	0,822	1,429	1,792	2,474	3,238
300	0,135	0,809	1,416	1,780	2,461	3,225
400	0,117	0,791	1,398	1,762	2,443	3,207
500	0,105	0,779	1,386	1,749	2,431	3,195
1 000	0,074	0,749	1,356	1,719	2,400	3,164
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table B.6 — Confidence level 99,9 %
(1 − α = 0,999)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	2,186	2,860	3,467	3,830	4,512	5,276
3	1,785	2,459	3,066	3,430	4,111	4,875
4	1,546	2,220	2,827	3,190	3,872	4,636
5	1,382	2,057	2,664	3,027	3,709	4,473
6	1,262	1,937	2,544	2,907	3,588	4,352
7	1,168	1,843	2,450	2,813	3,495	4,259
8	1,093	1,768	2,375	2,738	3,419	4,183
9	1,031	1,705	2,312	2,675	3,357	4,121
10	0,978	1,652	2,259	2,623	3,304	4,068
11	0,932	1,607	2,214	2,577	3,259	4,022
12	0,893	1,567	2,174	2,537	3,219	3,983
13	0,858	1,532	2,139	2,502	3,184	3,948
14	0,826	1,501	2,108	2,471	3,153	3,917
15	0,798	1,473	2,080	2,443	3,125	3,889
16	0,773	1,448	2,055	2,418	3,099	3,863
17	0,750	1,424	2,032	2,395	3,076	3,840
18	0,729	1,403	2,010	2,374	3,055	3,819
19	0,709	1,384	1,991	2,354	3,036	3,800
20	0,691	1,366	1,973	2,336	3,018	3,782
22	0,659	1,334	1,941	2,304	2,986	3,750
24	0,631	1,306	1,913	2,276	2,958	3,722
26	0,607	1,281	1,888	2,251	2,933	3,697
28	0,584	1,259	1,866	2,229	2,911	3,675
30	0,565	1,239	1,846	2,210	2,891	3,655
35	0,523	1,197	1,804	2,168	2,849	3,613
40	0,489	1,164	1,771	2,134	2,815	3,579
45	0,461	1,136	1,743	2,106	2,788	3,551
50	0,438	1,112	1,719	2,082	2,764	3,528
60	0,399	1,074	1,681	2,044	2,726	3,490
70	0,370	1,044	1,651	2,015	2,696	3,460
80	0,346	1,020	1,628	1,991	2,672	3,436
90	0,326	1,001	1,608	1,971	2,653	3,416
100	0,310	0,984	1,591	1,954	2,636	3,400
150	0,253	0,927	1,534	1,898	2,579	3,343
200	0,219	0,894	1,501	1,864	2,545	3,309
250	0,196	0,870	1,477	1,841	2,522	3,286
300	0,179	0,853	1,460	1,824	2,505	3,269
400	0,155	0,830	1,437	1,800	2,481	3,245
500	0,139	0,813	1,420	1,784	2,465	3,229
1 000	0,098	0,773	1,380	1,743	2,425	3,188
∞	0,000	0,675	1,282	1,645	2,327	3,091

Annex C
(normative)

Two-sided statistical tolerance limit factors, $k_2(n; p; 1-\alpha)$, for known σ

Table C.1 — Confidence level 50,0 %
($1-\alpha=0{,}50$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	0,755	1,282	1,823	2,164	2,822	3,575
3	0,727	1,238	1,766	2,100	2,749	3,496
4	0,714	1,216	1,737	2,067	2,710	3,451
5	0,706	1,203	1,719	2,046	2,685	3,423
6	0,701	1,195	1,707	2,033	2,668	3,403
7	0,697	1,188	1,698	2,023	2,656	3,388
8	0,694	1,184	1,692	2,015	2,646	3,377
9	0,692	1,180	1,686	2,009	2,639	3,368
10	0,690	1,177	1,682	2,004	2,633	3,361
11	0,689	1,175	1,679	2,000	2,628	3,355
12	0,688	1,173	1,676	1,997	2,624	3,350
13	0,687	1,171	1,674	1,994	2,620	3,346
14	0,686	1,170	1,672	1,992	2,617	3,342
15	0,685	1,168	1,670	1,990	2,614	3,339
16	0,685	1,167	1,669	1,988	2,612	3,336
17	0,684	1,166	1,667	1,986	2,610	3,333
18	0,684	1,165	1,666	1,985	2,608	3,331
19	0,683	1,165	1,665	1,984	2,607	3,329
20	0,683	1,164	1,664	1,983	2,605	3,327
22	0,682	1,163	1,662	1,981	2,602	3,324
24	0,681	1,162	1,661	1,979	2,600	3,321
26	0,681	1,161	1,660	1,977	2,599	3,319
28	0,680	1,160	1,659	1,976	2,597	3,317
30	0,680	1,160	1,658	1,975	2,596	3,315
35	0,679	1,158	1,656	1,973	2,593	3,312
40	0,679	1,157	1,655	1,972	2,591	3,309
45	0,678	1,157	1,654	1,970	2,589	3,307
50	0,678	1,156	1,653	1,969	2,588	3,306
60	0,678	1,155	1,652	1,968	2,586	3,303
70	0,677	1,155	1,651	1,967	2,585	3,302
80	0,677	1,154	1,650	1,966	2,584	3,300
90	0,677	1,154	1,650	1,965	2,583	3,299
100	0,677	1,153	1,649	1,965	2,582	3,298
150	0,676	1,153	1,648	1,963	2,580	3,296
200	0,676	1,152	1,647	1,963	2,579	3,295
250	0,676	1,152	1,647	1,962	2,579	3,294
300	0,676	1,152	1,647	1,962	2,578	3,294
400	0,675	1,152	1,646	1,962	2,578	3,293
500	0,675	1,151	1,646	1,961	2,578	3,293
1000	0,675	1,151	1,646	1,961	2,577	3,292
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table C.2 — Confidence level 75,0 %
($1-\alpha=0{,}75$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	0,919	1,520	2,106	2,464	3,142	3,905
3	0,834	1,402	1,971	2,323	2,996	3,756
4	0,792	1,340	1,897	2,244	2,911	3,669
5	0,768	1,303	1,850	2,194	2,856	3,611
6	0,752	1,278	1,818	2,158	2,816	3,568
7	0,741	1,260	1,794	2,132	2,786	3,536
8	0,732	1,246	1,776	2,112	2,763	3,511
9	0,726	1,236	1,762	2,096	2,745	3,491
10	0,721	1,227	1,751	2,083	2,730	3,474
11	0,716	1,220	1,742	2,073	2,717	3,459
12	0,713	1,214	1,734	2,064	2,706	3,447
13	0,710	1,209	1,727	2,056	2,697	3,437
14	0,707	1,205	1,722	2,050	2,689	3,427
15	0,705	1,202	1,717	2,044	2,682	3,419
16	0,703	1,198	1,712	2,039	2,676	3,412
17	0,702	1,196	1,708	2,034	2,670	3,406
18	0,700	1,193	1,705	2,030	2,665	3,400
19	0,699	1,191	1,702	2,027	2,661	3,395
20	0,698	1,189	1,699	2,024	2,657	3,390
22	0,695	1,185	1,694	2,018	2,650	3,382
24	0,694	1,183	1,690	2,013	2,644	3,375
26	0,692	1,180	1,687	2,009	2,639	3,369
28	0,691	1,178	1,684	2,006	2,635	3,364
30	0,690	1,176	1,681	2,003	2,631	3,359
35	0,688	1,173	1,676	1,997	2,623	3,350
40	0,686	1,170	1,672	1,992	2,618	3,343
45	0,685	1,168	1,669	1,989	2,613	3,337
50	0,684	1,166	1,667	1,986	2,610	3,333
60	0,682	1,164	1,663	1,982	2,604	3,326
70	0,681	1,162	1,661	1,979	2,600	3,321
80	0,681	1,160	1,659	1,977	2,597	3,318
90	0,680	1,159	1,657	1,975	2,595	3,315
100	0,679	1,158	1,656	1,973	2,593	3,312
150	0,678	1,156	1,653	1,969	2,588	3,305
200	0,677	1,155	1,651	1,967	2,585	3,302
250	0,677	1,154	1,650	1,966	2,583	3,300
300	0,676	1,153	1,649	1,965	2,582	3,298
400	0,676	1,153	1,648	1,964	2,581	3,296
500	0,676	1,152	1,648	1,963	2,580	3,295
1 000	0,675	1,152	1,646	1,962	2,578	3,293
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table C.3 — Confidence level 90,0 %
($1 - \alpha = 0{,}90$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	1,187	1,842	2,446	2,809	3,490	4,254
3	1,013	1,640	2,236	2,597	3,277	4,040
4	0,924	1,527	2,114	2,473	3,151	3,913
5	0,872	1,456	2,034	2,390	3,065	3,827
6	0,837	1,407	1,977	2,330	3,003	3,764
7	0,813	1,371	1,935	2,285	2,955	3,715
8	0,795	1,344	1,902	2,250	2,917	3,675
9	0,781	1,323	1,875	2,222	2,886	3,643
10	0,770	1,306	1,854	2,198	2,861	3,616
11	0,761	1,292	1,836	2,179	2,839	3,593
12	0,754	1,281	1,821	2,162	2,821	3,573
13	0,748	1,271	1,809	2,148	2,804	3,556
14	0,742	1,262	1,797	2,136	2,790	3,541
15	0,738	1,255	1,788	2,125	2,778	3,527
16	0,734	1,248	1,779	2,115	2,767	3,515
17	0,730	1,243	1,772	2,107	2,757	3,504
18	0,727	1,237	1,765	2,099	2,748	3,494
19	0,724	1,233	1,759	2,092	2,740	3,485
20	0,722	1,229	1,753	2,086	2,733	3,477
22	0,717	1,222	1,744	2,075	2,720	3,463
24	0,714	1,216	1,736	2,066	2,709	3,450
26	0,711	1,211	1,729	2,058	2,699	3,439
28	0,708	1,207	1,723	2,052	2,691	3,430
30	0,706	1,203	1,718	2,046	2,684	3,422
35	0,701	1,195	1,708	2,034	2,670	3,405
40	0,698	1,190	1,700	2,025	2,659	3,392
45	0,695	1,185	1,694	2,018	2,650	3,382
50	0,693	1,182	1,689	2,012	2,643	3,373
60	0,690	1,177	1,682	2,004	2,632	3,360
70	0,688	1,173	1,677	1,998	2,625	3,351
80	0,686	1,170	1,673	1,993	2,619	3,344
90	0,685	1,168	1,670	1,990	2,614	3,338
100	0,684	1,166	1,667	1,987	2,610	3,334
150	0,681	1,161	1,660	1,978	2,599	3,320
200	0,680	1,159	1,656	1,974	2,594	3,313
250	0,679	1,157	1,654	1,971	2,590	3,309
300	0,678	1,156	1,653	1,969	2,588	3,306
400	0,677	1,155	1,651	1,967	2,585	3,302
500	0,677	1,154	1,650	1,966	2,583	3,300
1 000	0,676	1,152	1,648	1,963	2,580	3,295
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table C.4 — Confidence level 95,0 %
($1 - \alpha = 0{,}95$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	1,393	2,062	2,668	3,031	3,713	4,477
3	1,160	1,812	2,415	2,777	3,459	4,222
4	1,036	1,668	2,265	2,627	3,307	4,071
5	0,960	1,574	2,165	2,525	3,204	3,967
6	0,910	1,509	2,093	2,451	3,129	3,891
7	0,875	1,460	2,039	2,395	3,070	3,832
8	0,849	1,423	1,996	2,350	3,024	3,785
9	0,828	1,394	1,961	2,313	2,985	3,746
10	0,812	1,370	1,933	2,283	2,953	3,713
11	0,799	1,351	1,909	2,258	2,926	3,685
12	0,788	1,334	1,889	2,236	2,903	3,660
13	0,779	1,320	1,872	2,218	2,882	3,639
14	0,772	1,308	1,857	2,201	2,864	3,620
15	0,765	1,298	1,844	2,187	2,848	3,603
16	0,759	1,289	1,832	2,174	2,834	3,588
17	0,754	1,281	1,822	2,163	2,821	3,574
18	0,749	1,274	1,812	2,152	2,809	3,561
19	0,745	1,267	1,804	2,143	2,799	3,550
20	0,742	1,261	1,797	2,135	2,789	3,540
22	0,736	1,251	1,783	2,120	2,772	3,521
24	0,730	1,243	1,772	2,108	2,758	3,505
26	0,726	1,236	1,763	2,097	2,745	3,491
28	0,722	1,230	1,755	2,088	2,735	3,479
30	0,719	1,225	1,748	2,080	2,725	3,469
35	0,713	1,214	1,733	2,063	2,706	3,446
40	0,708	1,206	1,723	2,051	2,691	3,429
45	0,704	1,200	1,714	2,041	2,679	3,416
50	0,701	1,195	1,708	2,033	2,669	3,404
60	0,697	1,188	1,697	2,022	2,655	3,387
70	0,694	1,182	1,690	2,013	2,644	3,374
80	0,691	1,178	1,684	2,007	2,636	3,365
90	0,689	1,175	1,680	2,002	2,629	3,357
100	0,688	1,173	1,677	1,998	2,624	3,351
150	0,684	1,166	1,666	1,985	2,609	3,332
200	0,681	1,162	1,661	1,979	2,601	3,322
250	0,680	1,160	1,658	1,975	2,596	3,316
300	0,679	1,158	1,656	1,973	2,593	3,312
400	0,678	1,156	1,653	1,970	2,589	3,307
500	0,678	1,155	1,652	1,968	2,586	3,304
1 000	0,676	1,153	1,649	1,964	2,581	3,297
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table C.5 — Confidence level 99,0 %
(1 − α = 0,99)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	1,822	2,496	3,103	3,467	4,148	4,912
3	1,491	2,163	2,769	3,133	3,814	4,578
4	1,301	1,965	2,570	2,933	3,615	4,379
5	1,177	1,831	2,435	2,798	3,479	4,243
6	1,092	1,735	2,336	2,698	3,379	4,142
7	1,031	1,662	2,259	2,621	3,301	4,064
8	0,984	1,605	2,198	2,559	3,238	4,002
9	0,948	1,558	2,148	2,508	3,186	3,950
10	0,919	1,521	2,107	2,465	3,143	3,906
11	0,896	1,489	2,071	2,429	3,105	3,868
12	0,876	1,462	2,041	2,397	3,073	3,835
13	0,860	1,439	2,015	2,370	3,044	3,806
14	0,846	1,420	1,992	2,346	3,019	3,780
15	0,834	1,402	1,971	2,324	2,997	3,757
16	0,824	1,387	1,953	2,305	2,976	3,736
17	0,815	1,374	1,937	2,288	2,958	3,718
18	0,806	1,361	1,922	2,272	2,941	3,700
19	0,799	1,351	1,909	2,258	2,926	3,685
20	0,793	1,341	1,897	2,245	2,912	3,670
22	0,782	1,324	1,876	2,222	2,887	3,644
24	0,772	1,310	1,858	2,203	2,866	3,622
26	0,765	1,297	1,843	2,186	2,847	3,602
28	0,758	1,287	1,830	2,172	2,831	3,585
30	0,752	1,278	1,818	2,159	2,817	3,569
35	0,741	1,260	1,795	2,133	2,787	3,537
40	0,732	1,246	1,777	2,113	2,764	3,512
45	0,726	1,236	1,763	2,097	2,745	3,491
50	0,721	1,227	1,751	2,084	2,730	3,474
60	0,713	1,215	1,734	2,064	2,706	3,447
70	0,707	1,205	1,722	2,050	2,689	3,428
80	0,703	1,199	1,712	2,039	2,676	3,412
90	0,700	1,193	1,705	2,031	2,666	3,400
100	0,698	1,189	1,699	2,024	2,657	3,390
150	0,690	1,176	1,681	2,003	2,631	3,359
200	0,686	1,170	1,672	1,993	2,618	3,343
250	0,684	1,166	1,667	1,986	2,610	3,333
300	0,682	1,164	1,663	1,982	2,604	3,326
400	0,681	1,160	1,659	1,977	2,597	3,318
500	0,679	1,158	1,656	1,973	2,593	3,312
1 000	0,677	1,155	1,651	1,967	2,585	3,302
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table C.6 — Confidence level 99,9 %
(1 − α = 0,999)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	2,327	3,002	3,609	3,972	4,654	5,417
3	1,900	2,575	3,182	3,545	4,227	4,991
4	1,647	2,320	2,927	3,291	3,972	4,736
5	1,476	2,147	2,754	3,117	3,798	4,562
6	1,353	2,020	2,626	2,989	3,670	4,434
7	1,260	1,921	2,526	2,889	3,571	4,334
8	1,187	1,843	2,446	2,809	3,490	4,254
9	1,130	1,778	2,380	2,743	3,424	4,188
10	1,083	1,725	2,325	2,687	3,368	4,131
11	1,045	1,679	2,277	2,639	3,319	4,083
12	1,013	1,640	2,236	2,597	3,277	4,041
13	0,986	1,606	2,200	2,560	3,240	4,003
14	0,962	1,577	2,168	2,528	3,207	3,970
15	0,942	1,551	2,140	2,499	3,178	3,941
16	0,924	1,527	2,114	2,473	3,151	3,914
17	0,909	1,507	2,091	2,449	3,127	3,889
18	0,895	1,488	2,070	2,428	3,104	3,867
19	0,883	1,471	2,051	2,408	3,084	3,846
20	0,872	1,456	2,034	2,390	3,065	3,827
22	0,853	1,430	2,003	2,358	3,032	3,793
24	0,838	1,407	1,977	2,330	3,003	3,764
26	0,824	1,388	1,954	2,306	2,978	3,738
28	0,813	1,372	1,935	2,285	2,955	3,715
30	0,804	1,357	1,917	2,267	2,935	3,694
35	0,784	1,328	1,882	2,228	2,894	3,651
40	0,770	1,306	1,854	2,198	2,861	3,616
45	0,759	1,289	1,832	2,174	2,834	3,588
50	0,751	1,275	1,815	2,155	2,812	3,564
60	0,738	1,255	1,788	2,125	2,778	3,527
70	0,729	1,240	1,768	2,103	2,752	3,499
80	0,722	1,229	1,753	2,086	2,733	3,477
90	0,716	1,220	1,742	2,073	2,717	3,459
100	0,712	1,213	1,732	2,062	2,704	3,445
150	0,700	1,192	1,704	2,029	2,664	3,398
200	0,693	1,182	1,689	2,012	2,643	3,373
250	0,690	1,176	1,681	2,002	2,630	3,358
300	0,687	1,172	1,675	1,995	2,621	3,347
400	0,684	1,166	1,667	1,987	2,610	3,334
500	0,682	1,163	1,663	1,982	2,604	3,326
1 000	0,679	1,157	1,654	1,971	2,590	3,309
∞	0,675	1,151	1,645	1,960	2,576	3,291

Annex D
(normative)

One-sided statistical tolerance limit factors, $k_3(n; p; 1-\alpha)$, for unknown σ

Table D.1 — Confidence level 50,0 %
($1-\alpha=0{,}50$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	0,000	0,888	1,785	2,339	3,376	4,527
3	0,000	0,774	1,499	1,939	2,765	3,689
4	0,000	0,739	1,419	1,830	2,601	3,465
5	0,000	0,722	1,382	1,780	2,526	3,363
6	0,000	0,712	1,361	1,751	2,483	3,304
7	0,000	0,706	1,347	1,732	2,456	3,266
8	0,000	0,701	1,337	1,719	2,436	3,240
9	0,000	0,698	1,330	1,710	2,422	3,220
10	0,000	0,695	1,325	1,702	2,411	3,205
11	0,000	0,693	1,320	1,696	2,402	3,193
12	0,000	0,692	1,317	1,691	2,395	3,184
13	0,000	0,690	1,314	1,687	2,389	3,176
14	0,000	0,689	1,311	1,684	2,384	3,169
15	0,000	0,688	1,309	1,681	2,380	3,163
16	0,000	0,687	1,307	1,679	2,376	3,158
17	0,000	0,686	1,306	1,677	2,373	3,154
18	0,000	0,686	1,304	1,675	2,370	3,150
19	0,000	0,685	1,303	1,673	2,368	3,147
20	0,000	0,685	1,302	1,672	2,366	3,144
22	0,000	0,684	1,300	1,669	2,362	3,139
24	0,000	0,683	1,298	1,667	2,359	3,134
26	0,000	0,682	1,297	1,665	2,356	3,131
28	0,000	0,682	1,296	1,664	2,354	3,128
30	0,000	0,681	1,295	1,662	2,352	3,125
35	0,000	0,680	1,293	1,660	2,348	3,120
40	0,000	0,680	1,292	1,658	2,346	3,116
45	0,000	0,679	1,290	1,657	2,343	3,113
50	0,000	0,679	1,290	1,655	2,342	3,111
60	0,000	0,678	1,288	1,654	2,339	3,108
70	0,000	0,678	1,287	1,652	2,337	3,105
80	0,000	0,677	1,287	1,652	2,336	3,103
90	0,000	0,677	1,286	1,651	2,335	3,102
100	0,000	0,677	1,286	1,650	2,334	3,101
150	0,000	0,676	1,285	1,649	2,332	3,097
200	0,000	0,676	1,284	1,648	2,330	3,096
250	0,000	0,676	1,284	1,647	2,330	3,095
300	0,000	0,676	1,283	1,647	2,329	3,094
400	0,000	0,675	1,283	1,647	2,329	3,093
500	0,000	0,675	1,283	1,646	2,328	3,093
1 000	0,000	0,675	1,282	1,646	2,328	3,092
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table D.2 — Confidence level 75,0 %
($1-\alpha=0{,}75$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	0,708	2,225	3,993	5,122	7,267	9,673
3	0,472	1,465	2,502	3,152	4,396	5,806
4	0,383	1,256	2,134	2,681	3,726	4,911
5	0,332	1,152	1,962	2,464	3,422	4,508
6	0,297	1,088	1,860	2,336	3,244	4,274
7	0,272	1,044	1,791	2,251	3,127	4,119
8	0,252	1,011	1,740	2,189	3,042	4,008
9	0,236	0,985	1,702	2,142	2,978	3,925
10	0,223	0,964	1,671	2,104	2,927	3,858
11	0,212	0,947	1,646	2,074	2,886	3,805
12	0,202	0,933	1,625	2,048	2,852	3,760
13	0,193	0,920	1,607	2,026	2,823	3,722
14	0,186	0,909	1,591	2,008	2,797	3,690
15	0,179	0,900	1,578	1,991	2,776	3,662
16	0,173	0,891	1,566	1,977	2,756	3,637
17	0,168	0,884	1,555	1,964	2,739	3,615
18	0,163	0,877	1,545	1,952	2,724	3,595
19	0,158	0,870	1,536	1,942	2,710	3,577
20	0,154	0,865	1,529	1,932	2,697	3,561
22	0,147	0,854	1,514	1,916	2,675	3,533
24	0,140	0,846	1,503	1,902	2,657	3,509
26	0,135	0,838	1,492	1,889	2,641	3,488
28	0,130	0,831	1,483	1,879	2,626	3,470
30	0,125	0,825	1,475	1,869	2,614	3,454
35	0,116	0,813	1,458	1,850	2,588	3,421
40	0,108	0,803	1,445	1,834	2,568	3,396
45	0,102	0,795	1,435	1,822	2,552	3,375
50	0,097	0,789	1,426	1,811	2,539	3,358
60	0,088	0,778	1,412	1,795	2,518	3,331
70	0,082	0,770	1,401	1,783	2,502	3,311
80	0,076	0,763	1,393	1,773	2,489	3,295
90	0,072	0,758	1,386	1,765	2,479	3,282
100	0,068	0,753	1,380	1,758	2,470	3,271
150	0,056	0,738	1,361	1,736	2,442	3,235
200	0,048	0,730	1,350	1,723	2,425	3,214
250	0,043	0,724	1,342	1,714	2,414	3,200
300	0,039	0,719	1,337	1,708	2,406	3,190
400	0,034	0,713	1,329	1,699	2,395	3,176
500	0,031	0,709	1,324	1,693	2,387	3,167
1 000	0,022	0,699	1,311	1,679	2,369	3,144
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table D.3 — Confidence level 90,0 %
(1 − α = 0,90)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	2,177	5,843	10,253	13,090	18,501	24,582
3	1,089	2,603	4,259	5,312	7,341	9,652
4	0,819	1,973	3,188	3,957	5,439	7,130
5	0,686	1,698	2,743	3,400	4,666	6,112
6	0,603	1,540	2,494	3,092	4,243	5,556
7	0,545	1,436	2,333	2,894	3,973	5,202
8	0,501	1,360	2,219	2,755	3,783	4,955
9	0,466	1,303	2,133	2,650	3,642	4,772
10	0,438	1,257	2,066	2,569	3,532	4,629
11	0,414	1,220	2,012	2,503	3,444	4,515
12	0,394	1,189	1,967	2,449	3,371	4,421
13	0,377	1,162	1,929	2,403	3,310	4,341
14	0,361	1,139	1,896	2,364	3,258	4,274
15	0,348	1,119	1,867	2,329	3,212	4,216
16	0,336	1,101	1,842	2,299	3,173	4,164
17	0,325	1,085	1,820	2,273	3,137	4,119
18	0,315	1,071	1,800	2,249	3,106	4,079
19	0,306	1,058	1,782	2,228	3,078	4,042
20	0,297	1,046	1,766	2,208	3,052	4,009
22	0,283	1,026	1,737	2,174	3,007	3,952
24	0,270	1,008	1,713	2,146	2,970	3,904
26	0,259	0,993	1,692	2,121	2,937	3,862
28	0,249	0,979	1,674	2,099	2,909	3,826
30	0,240	0,967	1,658	2,080	2,884	3,795
35	0,221	0,943	1,624	2,041	2,833	3,730
40	0,207	0,923	1,598	2,011	2,794	3,679
45	0,194	0,907	1,577	1,986	2,762	3,639
50	0,184	0,894	1,560	1,966	2,735	3,605
60	0,168	0,873	1,533	1,934	2,694	3,553
70	0,155	0,857	1,512	1,910	2,663	3,513
80	0,145	0,845	1,495	1,890	2,638	3,482
90	0,137	0,834	1,482	1,875	2,618	3,457
100	0,130	0,825	1,471	1,862	2,601	3,436
150	0,106	0,796	1,433	1,819	2,546	3,366
200	0,091	0,779	1,412	1,794	2,515	3,326
250	0,082	0,768	1,397	1,777	2,493	3,299
300	0,075	0,760	1,387	1,765	2,478	3,280
400	0,065	0,748	1,372	1,748	2,457	3,253
500	0,058	0,740	1,362	1,737	2,442	3,235
1 000	0,041	0,721	1,338	1,709	2,407	3,191
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table D.4 — Confidence level 95,0 %
(1 − α = 0,95)

n	*p*					
	0,50	0,75	0,90	0,95	0,99	0,999
2	4,465	11,763	20,582	26,260	37,094	49,276
3	1,686	3,807	6,156	7,656	10,553	13,858
4	1,177	2,618	4,162	5,144	7,043	9,215
5	0,954	2,150	3,407	4,203	5,742	7,502
6	0,823	1,896	3,007	3,708	5,062	6,612
7	0,735	1,733	2,756	3,400	4,642	6,063
8	0,670	1,618	2,582	3,188	4,354	5,688
9	0,620	1,533	2,454	3,032	4,144	5,414
10	0,580	1,466	2,355	2,911	3,982	5,204
11	0,547	1,412	2,276	2,815	3,853	5,037
12	0,519	1,367	2,211	2,737	3,748	4,901
13	0,495	1,329	2,156	2,671	3,660	4,787
14	0,474	1,296	2,109	2,615	3,585	4,691
15	0,455	1,268	2,069	2,567	3,521	4,608
16	0,439	1,243	2,033	2,524	3,464	4,536
17	0,424	1,221	2,002	2,487	3,415	4,472
18	0,411	1,201	1,974	2,453	3,371	4,415
19	0,398	1,183	1,949	2,424	3,331	4,364
20	0,387	1,167	1,926	2,397	3,296	4,319
22	0,367	1,138	1,887	2,349	3,234	4,239
24	0,350	1,114	1,853	2,310	3,182	4,172
26	0,335	1,093	1,825	2,276	3,137	4,115
28	0,322	1,075	1,800	2,246	3,098	4,066
30	0,311	1,059	1,778	2,220	3,064	4,023
35	0,286	1,026	1,733	2,167	2,995	3,934
40	0,267	1,000	1,698	2,126	2,941	3,866
45	0,251	0,978	1,669	2,093	2,898	3,811
50	0,238	0,961	1,646	2,065	2,863	3,766
60	0,216	0,933	1,609	2,023	2,808	3,696
70	0,200	0,912	1,582	1,990	2,766	3,643
80	0,187	0,895	1,560	1,965	2,733	3,602
90	0,176	0,882	1,542	1,944	2,707	3,568
100	0,167	0,870	1,527	1,927	2,684	3,540
150	0,136	0,832	1,478	1,870	2,612	3,448
200	0,117	0,810	1,450	1,838	2,570	3,396
250	0,105	0,795	1,431	1,816	2,543	3,361
300	0,096	0,784	1,417	1,800	2,522	3,336
400	0,083	0,769	1,398	1,778	2,495	3,301
500	0,074	0,759	1,386	1,764	2,476	3,277
1 000	0,053	0,734	1,354	1,728	2,431	3,221
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table D.5 — Confidence level 99,0 %
($1 - \alpha = 0{,}99$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	22,501	58,940	103,029	131,427	185,617	246,558
3	4,021	8,729	13,996	17,371	23,896	31,348
4	2,271	4,716	7,380	9,084	12,388	16,176
5	1,676	3,455	5,362	6,579	8,940	11,650
6	1,374	2,849	4,412	5,406	7,335	9,550
7	1,188	2,491	3,860	4,728	6,412	8,346
8	1,060	2,254	3,498	4,286	5,812	7,565
9	0,966	2,084	3,241	3,973	5,389	7,015
10	0,893	1,955	3,048	3,739	5,074	6,606
11	0,834	1,853	2,898	3,557	4,830	6,289
12	0,785	1,771	2,777	3,410	4,634	6,035
13	0,744	1,703	2,677	3,290	4,473	5,827
14	0,709	1,645	2,594	3,189	4,338	5,653
15	0,678	1,596	2,522	3,103	4,223	5,505
16	0,651	1,553	2,460	3,028	4,124	5,377
17	0,627	1,515	2,406	2,963	4,037	5,266
18	0,606	1,481	2,358	2,906	3,961	5,167
19	0,586	1,451	2,315	2,854	3,893	5,080
20	0,568	1,424	2,276	2,808	3,832	5,002
22	0,537	1,377	2,210	2,729	3,727	4,867
24	0,511	1,337	2,154	2,663	3,640	4,755
26	0,488	1,303	2,107	2,607	3,566	4,661
28	0,468	1,274	2,066	2,558	3,502	4,579
30	0,450	1,248	2,030	2,516	3,447	4,508
35	0,413	1,195	1,958	2,430	3,335	4,365
40	0,384	1,154	1,902	2,365	3,249	4,255
45	0,360	1,122	1,858	2,312	3,181	4,169
50	0,341	1,095	1,821	2,269	3,125	4,098
60	0,309	1,052	1,765	2,203	3,039	3,988
70	0,285	1,020	1,722	2,153	2,974	3,906
80	0,266	0,995	1,689	2,114	2,924	3,843
90	0,250	0,975	1,662	2,083	2,884	3,791
100	0,237	0,957	1,639	2,057	2,850	3,749
150	0,193	0,901	1,566	1,972	2,741	3,611
200	0,166	0,869	1,525	1,923	2,679	3,533
250	0,149	0,847	1,497	1,891	2,638	3,481
300	0,136	0,831	1,477	1,868	2,609	3,444
400	0,117	0,809	1,449	1,836	2,568	3,393
500	0,105	0,795	1,430	1,815	2,541	3,359
1 000	0,074	0,759	1,385	1,763	2,475	3,276
∞	0,000	0,675	1,282	1,645	2,327	3,091

Table D.6 — Confidence level 99,9 %
($1 - \alpha = 0{,}999$)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	225,079	589,447	1 030,337	1 314,316	1 856,232	2 465,649
3	12,891	27,753	44,420	55,106	75,775	99,385
4	5,108	10,360	16,122	19,813	26,980	35,204
5	3,208	6,363	9,782	11,970	16,223	21,114
6	2,406	4,740	7,247	8,849	11,965	15,551
7	1,969	3,881	5,921	7,223	9,754	12,668
8	1,692	3,353	5,113	6,235	8,416	10,926
9	1,501	2,995	4,570	5,573	7,521	9,763
10	1,359	2,736	4,181	5,099	6,881	8,933
11	1,250	2,540	3,886	4,741	6,401	8,310
12	1,162	2,385	3,656	4,463	6,027	7,825
13	1,090	2,259	3,471	4,238	5,726	7,436
14	1,030	2,156	3,318	4,054	5,479	7,117
15	0,978	2,068	3,190	3,899	5,272	6,850
16	0,934	1,993	3,080	3,767	5,096	6,623
17	0,895	1,928	2,986	3,653	4,945	6,427
18	0,860	1,871	2,903	3,554	4,813	6,257
19	0,829	1,820	2,830	3,466	4,696	6,107
20	0,801	1,775	2,765	3,389	4,593	5,974
22	0,752	1,698	2,655	3,256	4,417	5,748
24	0,712	1,634	2,563	3,147	4,273	5,563
26	0,677	1,580	2,487	3,056	4,152	5,408
28	0,647	1,533	2,421	2,978	4,049	5,276
30	0,621	1,493	2,365	2,910	3,961	5,162
35	0,566	1,412	2,251	2,775	3,783	4,935
40	0,524	1,350	2,165	2,674	3,650	4,765
45	0,490	1,300	2,098	2,594	3,545	4,631
50	0,462	1,260	2,043	2,529	3,460	4,523
60	0,418	1,198	1,958	2,429	3,330	4,357
70	0,384	1,152	1,895	2,355	3,235	4,236
80	0,358	1,115	1,847	2,298	3,161	4,142
90	0,336	1,086	1,808	2,252	3,102	4,067
100	0,318	1,062	1,775	2,215	3,053	4,005
150	0,257	0,983	1,671	2,093	2,896	3,806
200	0,222	0,937	1,612	2,025	2,809	3,696
250	0,198	0,907	1,574	1,980	2,751	3,623
300	0,181	0,886	1,546	1,948	2,710	3,571
400	0,156	0,856	1,507	1,904	2,653	3,500
500	0,139	0,836	1,482	1,874	2,616	3,453
1 000	0,098	0,787	1,420	1,803	2,526	3,340
∞	0,000	0,675	1,282	1,645	2,327	3,091

Annex E
(normative)

Two-sided statistical tolerance limit factors, $k_4(n; p; 1-\alpha)$, for unknown σ

Table E.1 — Confidence level 50,0 %
$(1-\alpha=0{,}50)$

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	1,243	2,057	2,870	3,376	4,348	5,457
3	0,943	1,582	2,229	2,635	3,416	4,310
4	0,853	1,441	2,040	2,416	3,144	3,979
5	0,809	1,370	1,946	2,308	3,011	3,818
6	0,782	1,328	1,889	2,243	2,930	3,721
7	0,765	1,300	1,851	2,199	2,876	3,655
8	0,752	1,279	1,823	2,168	2,837	3,608
9	0,743	1,264	1,802	2,143	2,807	3,572
10	0,735	1,252	1,786	2,124	2,783	3,544
11	0,730	1,242	1,772	2,109	2,764	3,521
12	0,725	1,234	1,761	2,096	2,749	3,502
13	0,721	1,227	1,752	2,086	2,735	3,486
14	0,717	1,222	1,744	2,077	2,724	3,472
15	0,714	1,217	1,738	2,069	2,714	3,461
16	0,712	1,212	1,732	2,062	2,706	3,450
17	0,709	1,209	1,727	2,056	2,698	3,441
18	0,707	1,205	1,722	2,051	2,691	3,433
19	0,706	1,202	1,718	2,046	2,685	3,426
20	0,704	1,200	1,714	2,042	2,680	3,419
22	0,701	1,195	1,708	2,034	2,671	3,408
24	0,699	1,191	1,703	2,028	2,663	3,399
26	0,697	1,188	1,698	2,023	2,656	3,391
28	0,696	1,186	1,694	2,018	2,651	3,384
30	0,694	1,183	1,691	2,014	2,646	3,378
35	0,691	1,179	1,685	2,007	2,636	3,366
40	0,689	1,175	1,680	2,001	2,629	3,357
45	0,688	1,172	1,676	1,997	2,623	3,350
50	0,686	1,170	1,673	1,993	2,618	3,344
60	0,684	1,167	1,668	1,988	2,612	3,335
70	0,683	1,165	1,665	1,984	2,607	3,329
80	0,682	1,163	1,662	1,981	2,603	3,324
90	0,681	1,162	1,661	1,979	2,600	3,321
100	0,681	1,160	1,659	1,977	2,598	3,318
150	0,679	1,157	1,654	1,971	2,591	3,309
200	0,678	1,156	1,652	1,969	2,587	3,305
250	0,677	1,155	1,651	1,967	2,585	3,302
300	0,677	1,154	1,650	1,966	2,583	3,300
400	0,676	1,153	1,649	1,965	2,582	3,298
500	0,676	1,153	1,648	1,964	2,581	3,296
1 000	0,676	1,152	1,647	1,962	2,578	3,294
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table E.2 — Confidence level 75,0 %
$(1-\alpha=0{,}75)$

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	2,674	4,394	6,109	7,178	9,231	11,574
3	1,492	2,487	3,489	4,117	5,326	6,710
4	1,211	2,036	2,872	3,397	4,412	5,576
5	1,083	1,829	2,590	3,069	3,996	5,060
6	1,009	1,709	2,425	2,877	3,753	4,760
7	0,961	1,630	2,316	2,750	3,592	4,561
8	0,926	1,573	2,238	2,659	3,476	4,418
9	0,900	1,530	2,179	2,590	3,389	4,309
10	0,880	1,497	2,133	2,536	3,320	4,224
11	0,864	1,469	2,095	2,492	3,264	4,155
12	0,850	1,447	2,064	2,456	3,217	4,097
13	0,839	1,428	2,038	2,425	3,178	4,049
14	0,829	1,412	2,015	2,399	3,145	4,007
15	0,821	1,398	1,996	2,376	3,116	3,971
16	0,814	1,386	1,979	2,356	3,090	3,939
17	0,807	1,375	1,964	2,338	3,067	3,910
18	0,802	1,366	1,950	2,322	3,047	3,885
19	0,797	1,357	1,938	2,308	3,029	3,862
20	0,792	1,349	1,927	2,295	3,012	3,842
22	0,784	1,336	1,908	2,273	2,983	3,806
24	0,777	1,325	1,892	2,254	2,959	3,775
26	0,771	1,315	1,879	2,238	2,938	3,749
28	0,766	1,306	1,867	2,224	2,920	3,727
30	0,762	1,299	1,857	2,211	2,904	3,707
35	0,753	1,284	1,835	2,186	2,872	3,666
40	0,747	1,273	1,819	2,167	2,847	3,634
45	0,741	1,263	1,806	2,152	2,827	3,609
50	0,737	1,256	1,795	2,139	2,810	3,588
60	0,730	1,244	1,779	2,119	2,784	3,556
70	0,725	1,236	1,766	2,105	2,765	3,532
80	0,721	1,229	1,757	2,093	2,750	3,513
90	0,718	1,223	1,749	2,084	2,738	3,497
100	0,715	1,219	1,742	2,076	2,728	3,485
150	0,706	1,204	1,722	2,051	2,696	3,443
200	0,701	1,196	1,710	2,037	2,677	3,420
250	0,698	1,191	1,702	2,028	2,665	3,405
300	0,696	1,187	1,697	2,022	2,657	3,393
400	0,693	1,181	1,689	2,012	2,645	3,378
500	0,691	1,178	1,684	2,006	2,637	3,368
1000	0,686	1,169	1,672	1,992	2,618	3,344
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table E.3 — Confidence level 90,0 %
(1 − α = 0,90)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	6,809	11,166	15,513	18,221	23,424	29,362
3	2,492	4,135	5,789	6,824	8,819	11,104
4	1,766	2,954	4,158	4,913	6,373	8,047
5	1,473	2,478	3,500	4,143	5,387	6,816
6	1,314	2,218	3,141	3,723	4,850	6,146
7	1,213	2,053	2,913	3,456	4,509	5,721
8	1,144	1,939	2,755	3,270	4,271	5,424
9	1,093	1,854	2,637	3,133	4,095	5,204
10	1,053	1,789	2,546	3,026	3,958	5,033
11	1,022	1,737	2,474	2,941	3,849	4,897
12	0,996	1,694	2,414	2,871	3,760	4,785
13	0,975	1,659	2,365	2,813	3,684	4,691
14	0,957	1,628	2,322	2,763	3,621	4,611
15	0,941	1,602	2,286	2,720	3,565	4,542
16	0,928	1,580	2,254	2,683	3,517	4,482
17	0,916	1,560	2,226	2,650	3,475	4,428
18	0,905	1,542	2,201	2,620	3,437	4,381
19	0,896	1,526	2,179	2,594	3,403	4,338
20	0,887	1,512	2,159	2,570	3,372	4,300
22	0,873	1,487	2,124	2,529	3,319	4,233
24	0,861	1,466	2,095	2,494	3,274	4,177
26	0,850	1,449	2,070	2,465	3,236	4,129
28	0,841	1,434	2,048	2,439	3,203	4,087
30	0,833	1,420	2,029	2,417	3,174	4,050
35	0,817	1,393	1,991	2,372	3,115	3,976
40	0,805	1,372	1,962	2,337	3,069	3,918
45	0,795	1,356	1,938	2,309	3,033	3,872
50	0,787	1,342	1,919	2,286	3,003	3,835
60	0,775	1,321	1,889	2,250	2,957	3,776
70	0,766	1,306	1,867	2,224	2,922	3,732
80	0,759	1,294	1,849	2,203	2,895	3,698
90	0,753	1,284	1,835	2,187	2,873	3,670
100	0,748	1,276	1,824	2,173	2,855	3,647
150	0,733	1,249	1,786	2,128	2,796	3,572
200	0,724	1,234	1,765	2,103	2,763	3,530
250	0,718	1,225	1,751	2,086	2,741	3,502
300	0,714	1,217	1,741	2,074	2,725	3,481
400	0,708	1,208	1,727	2,057	2,704	3,454
500	0,705	1,201	1,717	2,046	2,689	3,435
1 000	0,695	1,186	1,695	2,020	2,654	3,391
∞	0,675	1,151	1,645	1,960	2,576	3,291

Table E.4 — Confidence level 95,0 %
(1 − α = 0,95)

n	p					
	0,50	0,75	0,90	0,95	0,99	0,999
2	13,652	22,383	31,093	36,520	46,945	58,844
3	3,585	5,938	8,306	9,789	12,648	15,920
4	2,288	3,819	5,369	6,342	8,221	10,377
5	1,812	3,041	4,291	5,077	6,598	8,346
6	1,566	2,639	3,733	4,423	5,758	7,294
7	1,416	2,392	3,390	4,020	5,242	6,647
8	1,314	2,224	3,157	3,746	4,890	6,207
9	1,240	2,101	2,987	3,546	4,633	5,886
10	1,183	2,008	2,857	3,394	4,437	5,641
11	1,139	1,935	2,754	3,273	4,282	5,446
12	1,103	1,875	2,671	3,175	4,156	5,288
13	1,074	1,825	2,602	3,094	4,051	5,156
14	1,049	1,784	2,543	3,025	3,962	5,045
15	1,027	1,748	2,493	2,965	3,886	4,949
16	1,009	1,717	2,449	2,914	3,819	4,866
17	0,992	1,689	2,411	2,869	3,761	4,792
18	0,978	1,665	2,377	2,829	3,709	4,727
19	0,965	1,644	2,347	2,793	3,663	4,669
20	0,954	1,625	2,319	2,761	3,621	4,617
22	0,934	1,591	2,272	2,705	3,550	4,526
24	0,918	1,563	2,233	2,659	3,489	4,450
26	0,904	1,540	2,200	2,619	3,438	4,386
28	0,892	1,519	2,171	2,585	3,394	4,330
30	0,881	1,502	2,146	2,555	3,355	4,281
35	0,860	1,466	2,095	2,495	3,277	4,182
40	0,844	1,438	2,056	2,449	3,216	4,106
45	0,831	1,417	2,025	2,412	3,168	4,045
50	0,821	1,399	2,000	2,382	3,129	3,996
60	0,804	1,371	1,960	2,336	3,069	3,919
70	0,792	1,351	1,931	2,301	3,023	3,861
80	0,783	1,335	1,909	2,274	2,988	3,816
90	0,776	1,322	1,890	2,252	2,960	3,780
100	0,769	1,312	1,875	2,234	2,936	3,750
150	0,749	1,278	1,826	2,176	2,860	3,653
200	0,738	1,258	1,799	2,143	2,817	3,598
250	0,731	1,246	1,781	2,122	2,788	3,562
300	0,725	1,236	1,768	2,106	2,768	3,536
400	0,718	1,224	1,750	2,085	2,740	3,500
500	0,713	1,216	1,738	2,071	2,721	3,476
1 000	0,701	1,196	1,709	2,037	2,676	3,419
∞	0,675	1,151	1,645	1,960	2,576	3,291