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Methods for the calibration of vibration and shock transducers —

Part 12:
Primary vibration calibration by the reciprocity method

Méthodes pour l'étalonnage des transducteurs de vibrations et de chocs —

Partie 12: Étalonnage primaire de vibrations par méthode réciproque

Reference number
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Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this part of ISO 16063 may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 16063-12 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration and shock*, Subcommittee SC 3, *Use and calibration of vibration and shock measuring instruments*.

ISO 16063 consists of the following parts, under the general title *Methods for the calibration of vibration and shock transducers*:

— *Part 1: Basic concepts*

— *Part 11: Primary vibration calibration by laser interferometry*

— *Part 12: Primary vibration calibration by the reciprocity method*

— *Part 13: Primary shock calibration using laser interferometry*

— *Part 21: Vibration calibration by comparison to a reference transducer*

— *Part 22: Secondary shock calibration*

Annex A forms a normative part of this part of ISO 16063. Annex B is for information only.

Methods for the calibration of vibration and shock transducers —

Part 12:
Primary vibration calibration by the reciprocity method

1 Scope

This part of ISO 16063 specifies the instrumentation and procedures to be used for primary calibration of accelerometers using the reciprocity method and the SI system of units.

It is applicable to the calibration of rectilinear accelerometers over a frequency range of 40 Hz to 5 kHz and a frequency-dependent amplitude range of 10 m/s^2 to 100 m/s^2 and is based on the use of the coil of an electrodynamic vibrator as the reciprocal transducer.

Calibration of the sensitivity of a transducer can be obtained using this part of ISO 16063 provided that the signal conditioner or amplifier used with the transducer during calibration has been adequately characterized. In order to achieve the uncertainties of measurement given in clause 3, it has been assumed that the transducer has been calibrated in combination with its signal conditioner or amplifier (the combination of which in this part of ISO 16063 is referred to as the "accelerometer").

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 16063. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 16063 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 266, *Acoustics — Preferred frequencies*

ISO 16063-1:1998, *Methods for the calibration of vibration and shock transducers — Part 1: Basic concepts*

3 Uncertainty of measurement

At a reference frequency of 160 Hz and a reference amplitude of 100 m/s^2, 50 m/s^2, 20 m/s^2 or 10 m/s^2, the applicable limits of uncertainty are 0,5 % of the modulus (magnitude) of complex sensitivity and 1° of the argument (phase shift) of complex sensitivity. Over the full range of amplitudes and frequencies, the limits of uncertainty in the measured magnitude and phase shift of sensitivity are 1 % and 2°, respectively.

All users of this part of ISO 16063 are expected to make uncertainty budgets according to annex A to document the uncertainty of measurement.

The uncertainty of measurement is expressed as the expanded measurement uncertainty in accordance with ISO 16063-1 (referred to here as "uncertainty").

4 Symbols

A general list of symbols used in this part of ISO 16063 is contained in Table 1. Specific symbols used in formulae are defined following the formulae in which they appear.

Table 1 — General symbols

Symbol	Definition	Unit
f	frequency of vibration	Hz
n	indices of test masses (n = 0 indicates no test mass)	
m_n	mass of the test mass number n	kg
u	complex voltage	V
U	complex voltage ratio	
Y	complex electrical admittance	S
R	electrical resistance	Ω
α	complex intercept of least-squares fit	kg·Ω
β	complex slope of least-squares fit	Ω
S_a	complex sensitivity of the calibrated accelerometer	V/(ms^{-2})
$|S_a|$	modulus (magnitude) of S_a	V/(ms^{-2})
φ_a	argument (phase shift) of S_a	degree
Re	real part of a complex quantity	
Im	imaginary part of a complex quantity	
| |	modulus or absolute value of a complex quantity	
arg	argument of a complex quantity	

5 Requirements for apparatus

5.1 General

The case of the transducer shall be structurally rigid over the frequency range of interest. The sensitivity to base strain and transverse motion and the stability of the accelerometer (transducer in combination with the signal conditioner or amplifier) shall be included in the calculation of the expanded uncertainties in determining the modulus and argument of complex sensitivity (see annex A).

5.2 Frequency generator and indicator or counter

Use equipment having the following characteristics:

a) maximum uncertainty in frequency: 0,01 %;

b) change in frequency: less than 0,01 % over each measurement period;

c) change in amplitude: less than 0,01 % over each measurement period.

5.3 Power amplifier/vibrator combination

Use equipment having the following characteristics for all measurement conditions:

a) maximum total harmonic distortion: 2 %;

b) transverse, bending and rocking acceleration: commensurate with the uncertainty of the measured sensitivity (typically <10 % of the acceleration in the intended direction over the frequency range of interest);

c) minimum ratio of signal to noise at the output of the accelerometer: 30 dB;

d) change in acceleration amplitude: less than 0,05 % over each measurement period.

5.4 Seismic block for vibrator

The vibrator shall be mounted on a massive rigid seismic block so as to minimize the reaction of the vibrator support structure to the motion of the vibrator from significantly affecting the uncertainty in the calibration results. The mass of the seismic block should be at least 2 000 times that of the moving element of the vibrator. Examples of seismic blocks suitable for this use include granite blocks or steel honeycomb optical tables. The seismic block should be vibration isolated with vertical and horizontal suspension resonances of less than 2 Hz if significant seismic vibration exists in the calibration environment.

5.5 Instrumentation for complex voltage ratio measurements

Use equipment having the following characteristics:

a) frequency range: 40 Hz to 5 kHz;

b) maximum uncertainty in the modulus (magnitude) of complex voltage ratio: 0,1 %;

c) maximum uncertainty in the argument of complex voltage ratio: 0,1°.

5.6 Resistor

The resistor shall have a maximum uncertainty in the determination of its resistance of 0,05 % over the calibration frequency range and the range of power dissipated.

Ensure that the value of the impedance of the standard resistor used to determine current does not vary appreciably due to inductive and thermal effects.

5.7 Set of test masses

The test masses shall

a) cover a range of at least five approximately equal intervals, with the largest test mass between approximately 0,5 to 1 times the mass of the moving element of the vibrator, and

b) have a maximum uncertainty in the determination of mass of 0,05 %.

It is recommended that the shape of the test masses be similar to that of a cube or cylinder with a length-to-width ratio of approximately one. The maximum frequency at which the test mass behaves as a rigid body can then be estimated by use of the formula $c/(2L)$ where c is the speed of sound in the material of the test mass and L is its length. The surface finish specifications and the machining tolerances of the mounting hardware of the test masses should meet or exceed the requirements specified for mounting the transducer being calibrated. This is particularly critical if calibrations are performed at high frequencies. The test masses should be machined from a relatively stiff material such as tungsten carbide to maximize the frequencies of the natural resonances occurring in them.

In practice, the number and size of the test masses selected will be a compromise between reducing the statistical uncertainty versus increasing the measurement uncertainty due to thermal effects occurring in the drive coil as a result of making a relatively large number of measurements with large differences in measured electrical admittance.

5.8 Distortion-measuring instrumentation

Use equipment capable of measuring a total harmonic distortion of 0,01 % to 5 % and having the following characteristics:

a) frequency range: 40 Hz to 5 kHz;

b) maximum uncertainty: 10 % of the measured value of distortion.

5.9 Oscilloscope

While an oscilloscope is useful for examining the waveforms of the accelerometer and electrodynamic moving coil, its use is not mandatory.

5.10 Air-handling equipment

This shall be capable of maintaining the ambient conditions within the requirements specified in clause 6.

6 Ambient conditions

Calibrations shall be carried out under the following ambient conditions:

a) room temperature: (23 ± 3) °C;

b) maximum relative humidity: 75 %.

7 Preferred amplitudes and frequencies

The amplitudes and frequencies of acceleration used during calibration should be chosen from the following series:

a) acceleration: 10 m/s^2, 20 m/s^2, 50 m/s^2, 100 m/s^2;

b) reference acceleration: 100 m/s^2, 50 m/s^2, 20 m/s^2 or 10 m/s^2;

c) frequency: selected from the standardized one-third-octave frequencies given in ISO 266 from 40 Hz to 5 kHz;

d) reference frequency: 160 Hz.

Calibrations performed at large acceleration amplitudes could have relatively large uncertainties due to thermal effects occurring in the drive coil.

8 Procedure

8.1 General

Calibration of electromechanical transducers by reciprocity utilizes the linear bilateral relationship between the electrical and mechanical terminals of the transducers being calibrated. Three transducers are required in order to perform an absolute calibration of two of the transducers. One transducer is used only as a vibration sensor, one is used only as a vibration source, and one is used reciprocally as both a vibration sensor and a vibration source (generator). In principle, the electromechanical coupling of the reciprocal transducer can be either electrodynamic or piezoelectric. However, in practice, electrodynamic transducers are much more widely used as the reciprocal transducer in vibration calibrations by reciprocity. Therefore, the methods described in this part of ISO 16063 are based on the use of the coil of an electrodynamic vibrator as the reciprocal transducer with the coil located in close proximity to the transducer being calibrated.

The transducer that is used only as a vibration source may be either a second vibrator mechanically coupled to the moving element containing the reciprocal transducer and the transducer of the accelerometer, or a second coil attached to the same moving element. (See the bibliography for references to practical realizations of systems utilizing either a second vibrator or a second coil.) If a second vibrator is used, it may be relatively rigidly coupled to the moving element via a short threaded stud provided that the reciprocal transducer is otherwise adequately isolated from the second vibrator and that the rectilinear motion of the moving element has not been affected by the presence of the secondary vibration source. Caution should be exercised if the secondary vibration source is electrodynamic so as to prevent mutual coupling between the two electrodynamic elements from unduly affecting the uncertainty in the calibration results. Figures 1 and 2 contain block diagrams of one possible realization of a calibration system based on reciprocity, with the transducer of the accelerometer shown mounted inside the vibrator with the reciprocal transducer and with the second vibration source shown as a second vibrator.

The calibration shall be performed at frequencies well below the resonance frequencies inherent in the moving element containing the reciprocal transducer and supporting the transducer being calibrated. Transverse and axial resonances may be determined using a triaxial accelerometer with sufficiently high resonance frequencies. Departures from rigid-body motion by the moving element may be determined from relative measurements made on the top (mounting) surface of the moving element. Ideally, the transverse and axial resonances should be determined with the triaxial accelerometer mounted on a test fixture with the sum of the masses of the accelerometer and the test fixture equal to that of the largest test mass used to determine $Y_n - Y_0$. A typical upper frequency limit of calibration would be 0,25 times the resonance frequency of the moving element when loaded with the transducer under test and the largest test mass used to determine $Y_n - Y_0$. Attempts to perform calibrations at frequencies where minor resonances occur should be avoided. These minor resonances, which include suspension and structural resonances, are not considered part of the natural resonance(s) inherent in the moving element.

Obtain measurement results with the reciprocal transducer used as a vibration source (driver) and as a vibration sensor (velocity coil) (see 8.2.1 and 8.2.2, respectively). The first case requires that measurements be performed with and without a test mass attached to the moving element. It is important that these measurements be performed under uniform thermal conditions with the coil of the reciprocal transducer in the same static position in the magnetic gap. A typical upper limit in variability in thermal conditions would be between 1 ºC and 2 ºC. An offset in the static position of the reciprocal transducer may be corrected by applying a d.c. bias voltage across the reciprocal coil. Ideally, the instrumentation should be grounded at one point only to avoid ground loops. All voltages measured across the reciprocal coil and standard resistor should be measured as close to the voltage source as possible to minimize induced noise. The standard resistor may either be removed or shorted during the voltage ratio measurements of U_v (see 8.2.2). However, if the standard resistor is shorted, it should be verified that the uncertainty is not degraded at high frequencies due to inductive effects.

After establishing the instrumentation settings, perform a calibration at 160 Hz and the reference amplitude, and then perform calibrations at the other selected frequencies and acceleration amplitudes. The measurement results can then be expressed as the modulus (magnitude) of complex sensitivity, the argument (phase shift) of complex sensitivity, or both. For every combination of frequency and acceleration, the distortion, transverse motion (bending and rocking acceleration), hum and noise shall be appropriate to the uncertainties given in clause 3. During the calibration itself, all instruments not necessary for the calibration shall be disconnected from the measurement apparatus.

8.2 Experimental

8.2.1 Experiment 1: Measurement of the complex electrical admittance Y (complex ratio of driving coil current to accelerometer open-circuit output voltage)

With the reciprocal electrodynamic moving coil operating as a driving coil (vibration source), measure the complex electrical admittance by dividing the complex voltage ratio (U_d) by the standard resistance (R) where U_d is the voltage drop (u_r) across the standard resistance divided by the open-circuit voltage at the output of the accelerometer (u_{a1}), i.e. (see Figure 1):

$$Y = U_d/R = (u_r/u_{a1})(1/R)$$

Perform a series of these measurements with and without test masses added to the moving element. In the equations that follow, the complex electrical admittance without any mass added to the moving element and the complex electrical admittance with test mass m_n added to the moving element have been denoted Y_0 and Y_n, respectively.

When measuring U_d, it is critical to have the accelerometer and the standard resistor at the same ground potential. Experiment 1 shall be performed at all the acceleration amplitudes used during calibration.

8.2.2 Experiment 2: Measurement of the complex open-circuit voltage ratio U_v (complex open-circuit voltage ratio of the output of the accelerometer to the output of the velocity coil)

With the reciprocal electrodynamic moving coil operating as a velocity coil (vibration sensor), measure the complex open-circuit voltage ratio of the output of the accelerometer (u_{a2}) to the output of the moving coil (u_c) using an

external vibration source or a secondary driving coil on the moving element to drive the moving element (see Figure 2). This ratio ($U_v = u_{a2}/u_c$) is determined without any mass added to the moving element.

When measuring U_v, it is critical to have the accelerometer and the reciprocal coil at the same ground potential.

9 Computation of sensitivity

See equations (1) to (10) and annex B.

By means of a least-squares fit of the function

$$F(m_n, Y_n, Y_0) = \frac{m_n}{Y_n - Y_0} \tag{1}$$

obtain the complex intercept and slope of $F(m_n, Y_n, Y_0)$ at each calibration frequency and amplitude using the measured values obtained for m_n, Y_n and Y_0. This fit may be obtained using either uniform ($w_n = 1$) or non-uniform statistical weighting from the following formulae:

$$\text{Re }\alpha = \frac{\Sigma\left(w_n^2 m_n^2\right)\Sigma\text{Re}\left(\dfrac{w_n^2 m_n}{Y_n - Y_0}\right) - \Sigma w_n^2 m_n \Sigma\text{Re}\left(\dfrac{w_n^2 m_n^2}{Y_n - Y_0}\right)}{\Sigma w_n^2 \Sigma\left(w_n^2 m_n^2\right) - \left[\Sigma\left(w_n^2 m_n\right)\right]^2} \tag{2}$$

$$\text{Im}\,\alpha = \frac{\Sigma\left(w_n^2 m_n^2\right)\Sigma\text{Im}\left(\dfrac{w_n^2 m_n}{Y_n - Y_0}\right) - \Sigma w_n^2 m_n \Sigma\text{Im}\left(\dfrac{w_n^2 m_n^2}{Y_n - Y_0}\right)}{\Sigma w_n^2 \Sigma\left(w_n^2 m_n^2\right) - \left[\Sigma\left(w_n^2 m_n\right)\right]^2} \tag{3}$$

$$\text{Re }\beta = \frac{\Sigma w_n^2 \Sigma\text{Re}\left(\dfrac{w_n^2 m_n^2}{Y_n - Y_0}\right) - \Sigma w_n^2 m_n \Sigma\text{Re}\left(\dfrac{w_n^2 m_n}{Y_n - Y_0}\right)}{\Sigma w_n^2 \Sigma\left(w_n^2 m_n^2\right) - \left[\Sigma\left(w_n^2 m_n\right)\right]^2} \tag{4}$$

$$\text{Im }\beta = \frac{\Sigma w_n^2 \Sigma\text{Im}\left(\dfrac{w_n^2 m_n^2}{Y_n - Y_0}\right) - \Sigma w_n^2 m_n \Sigma\text{Im}\left(\dfrac{w_n^2 m_n}{Y_n - Y_0}\right)}{\Sigma w_n^2 \Sigma\left(w_n^2 m_n^2\right) - \left[\Sigma\left(w_n^2 m_n\right)\right]^2} \tag{5}$$

where

α is the complex intercept, in kilogram ohms, of the function $F(m_n, Y_n, Y_0)$;

β is the complex slope, in ohms, of the function $F(m_n, Y_n, Y_0)$;

n is the index corresponding to the test mass m_n;

w_n is the statistical weighting factor applied to the measurement using the test mass m_n;

m_n is the test mass, in kilograms, added;

Y_n is the electrical admittance, in siemens, measured with test mass m_n added to the moving element;

Y_0 is the electrical admittance, in siemens, measured without a test mass attached to the moving element.

NOTE   Depending upon how the accelerometer is being calibrated, it may not be necessary to compute the slope, and it may not be necessary to compute the real and the imaginary parts of the intercept but rather only the magnitude; see equations (8) to (10) [1].

The modulus and argument of the complex sensitivity of the accelerometer can then be obtained as a function of frequency from the following formulations.

In the case of an accelerometer that has a standard reference transducer permanently mounted on the moving element of the vibrator for the purpose of calibrating other transducers by comparison, the sensitivity varies with the mechanical impedance loading the moving element and is determined from the following equations:

$$S_a = \left| \sqrt{\frac{U_v\,\alpha}{\mathrm{j}2\pi f}} \left[\frac{1}{1 - \beta\left(Y_t - Y_0\right)} \right] \right| \ \frac{\mathrm{V}}{\mathrm{m/s}^2} \tag{6}$$

$$\varphi_a = \arg \sqrt{\frac{U_v\,\alpha}{\mathrm{j}2\pi f}} \left[\frac{1}{1 - \beta\left(Y_t - Y_0\right)} \right] \ \deg \tag{7}$$

where

$|S_a|$ is the modulus (magnitude) of the complex sensitivity, in volts per metre per second squared, of the accelerometer at frequency f;

φ_a is the argument (phase shift) of the complex sensitivity of the accelerometer, in degrees, at frequency f;

j   is the imaginary unit, $\mathrm{j}^2 = -1$;

f is the frequency, in hertz;

U_v is the complex open-circuit voltage ratio measured at frequency f with the reciprocal transducer operating as a velocity coil;

α is the complex intercept, in kilogram ohms, of the function $F(m_n, Y_n, Y_0)$ at frequency f;

β is the complex slope, in ohms, of the function $F(m_n, Y_n, Y_0)$ at frequency f;

Y_t is the electrical admittance, in siemens, at frequency f with a particular transducer added to the moving element of the vibrator;

Y_0 is the electrical admittance, in siemens, at frequency f without any added mass attached to the moving element of the vibrator.

In the case of an accelerometer which has a standard transducer that is removed from the moving element, the sensitivity is determined from the following equations:

$$S_a = \left| \sqrt{\frac{U_v\,\alpha}{\mathrm{j}2\pi f}} \right| \ \frac{\mathrm{V}}{\mathrm{m/s}^2} \tag{8}$$

$$\varphi_a = \arg \sqrt{\frac{U_v\alpha}{\mathrm{j}2\pi f}} \ \deg \tag{9}$$

where the symbols are as defined for equations (6) and (7).

At sufficiently low frequencies (typically for frequencies less than 1 kHz), β is approximately 0 Ω, arg (U_v) is approximately 90°, and arg (U_d) is approximately 0°. When these conditions are satisfied, the modulus of complex sensitivity of the accelerometer reduces to:

$$\left| S_a \right| = \sqrt{\frac{\left| U_v \right| \left| \alpha \right|}{2 \pi f}} \quad \frac{\text{V}}{\text{m/s}^2} \tag{10}$$
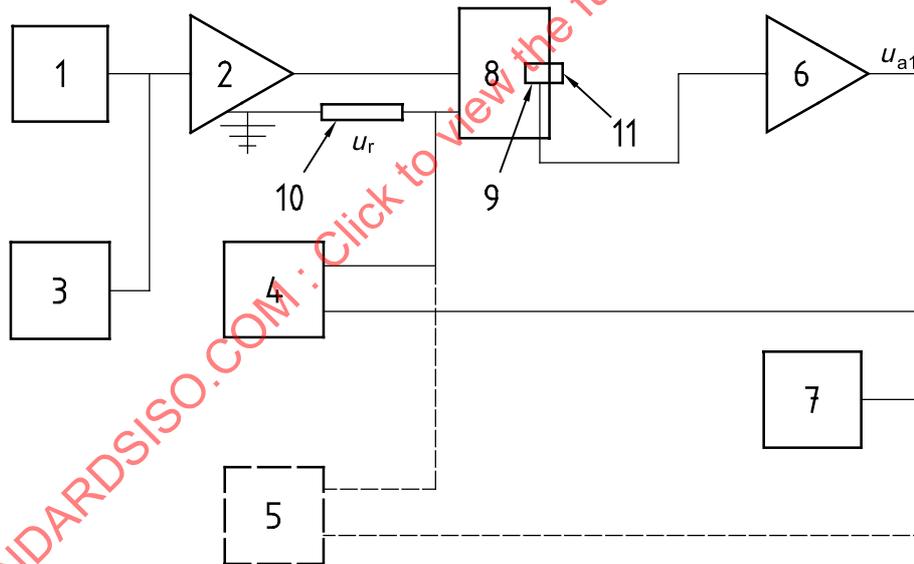
where

$\left| U_v \right|$ is the modulus (magnitude) of the complex open-circuit voltage ratio measured at frequency f with the reciprocal transducer operating as a velocity coil;

$\left| \alpha \right|$ is the modulus (magnitude) of the complex intercept, in kilogram ohms, of the function $F(m_n, Y_n, Y_0)$ at frequency f;

and the other symbols are as defined for equations (6) and (7).

In cases for which equation (10) is applicable, only the modulus (magnitude) of the complex voltage ratios needs to be determined and the modulus of α can be determined from a least-squares fit of $F(m_n, Y_n, Y_0)$ using the moduli of differences in complex admittance.

When the calibration results are reported, the total calibration uncertainty and the corresponding coverage factor shall be calculated according to annex A using a coverage factor $k = 2$.

Key

1   Frequency generator
2   Power amplifier
3   Frequency counter
4   Voltage ratio instrumentation
5   Oscilloscope (optional)
6   Signal conditioner or charge amplifier

7   Distortion analyser
8   Vibrator with reciprocal transducer
9   Transducer
10   Standard resistor
11   Test mass

Figure 1 — Block diagram of the measuring system for experiment 1 with the reciprocal transducer used as a vibration source
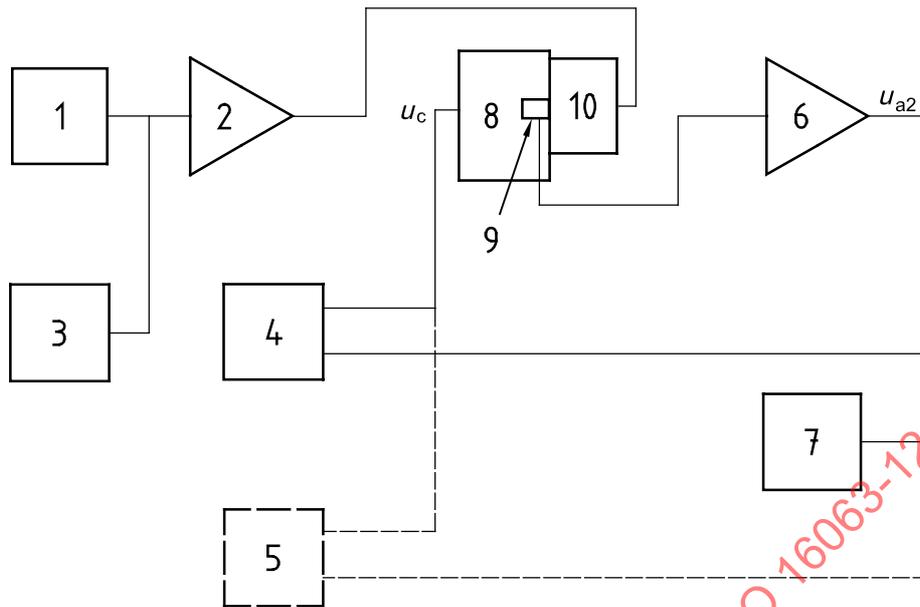
Key

1   Frequency generator
2   Power amplifier
3   Frequency counter
4   Voltage ratio instrumentation
5   Oscilloscope (optional)
6   Signal conditioner or charge amplifier
7   Distortion analyser
8   Vibrator with reciprocal transducer
9   Transducer
10 Secondary vibration source

Figure 2 — Block diagram of the measuring system for experiment 2 with the reciprocal transducer used as a vibration sensor

Annex A
(normative)

Calculation of uncertainty

A.1 Calculation of the expanded uncertainty in the measurement of the modulus (magnitude) of complex sensitivity, and of the expanded uncertainty in the measurement of the argument (phase shift) of complex sensitivity for the frequencies, amplitudes and amplifier settings used at the time of calibration

A.1.1 Calculation of $U(|S|)$

The expanded uncertainty, $U(|S|)$, in the measurement of the modulus (magnitude) of the complex sensitivity for the frequencies, amplitudes and amplifier settings used at the time of calibration is calculated in accordance with annex A of ISO 16063-1:1998 from the following formulae:

$$U(|S|) = ku_c(|S|) \tag{A.1}$$

$$u_c(|S|) = \frac{1}{|S|}\sqrt{\sum_{i=1}^{9} u_i^2(|S|)} \tag{A.2}$$

using the uncertainty components shown in Table A.1 and a coverage factor $k = 2$.

Table A.1 — Uncertainty components in determining $|S|$

i	Standard uncertainty component $u(x_i)$	Source of uncertainty	Uncertainty contribution $u_i(y)$
1	$u(f_{FG})$	Uncertainty in frequency	$u_1(|S|)$
2	$u(u_{Temp})$	Uncertainty in the temperature of the transducer of the reference accelerometer during calibration	$u_2(|S|)$
3	$u(u_D)$	Uncertainty in the modulus (magnitude) of the complex spectral output of the accelerometer due to distortion	$u_3(|S|)$
4	$u(u_T)$	Uncertainty in the modulus (magnitude) of the complex output of the accelerometer due to departures from ideal motion by the moving element of the vibrator (e.g. transverse motion, base strain)	$u_4(|S|)$
5	$u(m_m)$	Uncertainty in the determination of mass	$u_5(|S|)$
6	$u(U_U)$	Uncertainty in the determination of the modulus (magnitude) of the complex open-circuit voltage ratio of the output of the accelerometer to the output velocity coil	$u_6(|S|)$
7	$u(R_Y)$	Uncertainty in resistance when measuring the modulus of the complex admittance	$u_7(|S|)$
8	$u(U_Y)$	Uncertainty in the determination of the modulus (magnitude) of the complex voltage ratio when measuring the modulus of complex admittance	$u_8(|S|)$
9	$u(S_{RE})$	Uncertainty due to residual effects on the determination of the modulus (magnitude) of the complex sensitivity (e.g. random noise, experimental standard deviation)	$u_9(|S|)$

A.1.2 Calculation of $U(\varphi)$

The expanded uncertainty, $U(\varphi)$, in the measurement of the argument (phase shift) of the complex sensitivity for the frequencies, amplitudes, and amplifier settings used at the time of calibration is calculated in accordance with annex A of ISO 16063-1:1998 from the following formulae:

$$U(\varphi) = ku_{\mathrm{c}}(\varphi) \tag{A.3}$$

$$u_{\mathrm{c}}(\varphi) = \sqrt{\sum_{i=1}^{9} u_i^2(\varphi)} \tag{A.4}$$

using the uncertainty components shown in Table A.2 and a coverage factor k = 2.

Table A.2 — Uncertainty components in determining φ

i	Standard uncertainty component $u(x_i)$	Source of uncertainty	Uncertainty contribution $u_i(y)$
1	$u(f_{\mathrm{FG}})$	Uncertainty in frequency	$u_1(\varphi)$
2	$u(u_{\mathrm{Temp}})$	Uncertainty in the temperature of the reference accelerometer during calibration	$u_2(\varphi)$
3	$u(u_{\mathrm{D}})$	Uncertainty in the argument of the complex spectral output of the accelerometer due to distortion	$u_3(\varphi)$
4	$u(u_{\mathrm{T}})$	Uncertainty in the argument of the complex output of the accelerometer due to departures from ideal motion by the moving element of the vibrator (e.g. transverse motion, base strain)	$u_4(\varphi)$
5	$u(m_{\mathrm{m}})$	Uncertainty in the determination of mass	$u_5(\varphi)$
6	$u(U_{\mathrm{U}})$	Uncertainty in the determination of the argument of the complex open-circuit voltage ratio of the output of the accelerometer to the output of the velocity coil	$u_6(\varphi)$
7	$u(R_{\mathrm{Y}})$	Uncertainty in resistance when measuring the argument of the complex admittance	$u_7(\varphi)$
8	$u(U_{\mathrm{Y}})$	Uncertainty in the determination of the argument of the complex voltage ratio when measuring the argument of the complex admittance	$u_8(\varphi)$
9	$u(S_{\mathrm{RE}})$	Uncertainty due to residual effects on the determination of the argument of the complex sensitivity (e.g. random noise, experimental standard deviation)	$u_9(\varphi)$

A.2 Calculation of the expanded uncertainty in the modulus (magnitude) of complex sensitivity and of the expanded uncertainty in the argument (phase shift) of complex sensitivity over the complete frequency and amplitude range

A.2.1 Calculation of $U(|S_{\mathrm{t}}|)$

The expanded uncertainty, $U|S|$, in the measurement of the modulus (magnitude) of the complex sensitivity given in A.1.1 is only valid for the particular frequencies, amplitudes and amplifier settings used at the time of calibration. The expanded uncertainty, $U(|S_{\mathrm{t}}|)$, in the modulus (magnitude) of the complex sensitivity for the complete frequency and amplitude range at any time interval between successive calibrations is calculated from the following formulae:

$$U(|S_{\mathrm{t}}|) = ku_{\mathrm{c}}(|S_{\mathrm{t}}|) \tag{A.5}$$

$$u_c(|S_t|) = \frac{1}{|S_t|}\sqrt{\sum_{i=1}^{10} u_i^2(|S_t|)} \tag{A.6}$$

using the uncertainty components shown in Table A.3 and a coverage factor k = 2.

Table A.3 — Uncertainty components in determining $|S_t|$

i	Standard uncertainty component $u(x_i)$	Source of uncertainty	Uncertainty contribution $u_i(y)$
1	$u_c(|S|)$	Combined uncertainty in the modulus (magnitude) of the complex sensitivity at the frequencies, amplitudes and amplifier settings used in calibration calculated in accordance with A.1.1	$u_1(|S_t|)$
2	$u(e_{GA})$	Uncertainty in the gain of the reference amplifier as a function of amplifier settings	$u_2(|S_t|)$
3	$u(e_{FA})$	Uncertainty in the gain of the reference amplifier as a function of frequency	$u_3(|S_t|)$
4	$u(e_{FT})$	Uncertainty in the magnitude of the sensitivity of the reference transducer as a function of frequency	$u_4(|S_t|)$
5	$u(e_{LA})$	Uncertainty in the amplitude linearity of the reference amplifier	$u_5(|S_t|)$
6	$u(e_{LT})$	Uncertainty in the amplitude linearity of the reference transducer	$u_6(|S_t|)$
7	$u(e_{TA})$	Uncertainty in the gain of the reference amplifier and the output impedance of the reference accelerometer as a function of time (instability over time)	$u_7(|S_t|)$
8	$u(e_{TT})$	Uncertainty in the magnitude of the sensitivity of the reference transducer as a function of time (instability over time)	$u_8(|S_t|)$
9	$u(e_{EA})$	Uncertainty in the gain of the reference amplifier due to environmental effects	$u_9(|S_t|)$
10	$u(e_{ET})$	Uncertainty in the magnitude of the sensitivity of the reference transducer due to environmental effects	$u_{10}(|S_t|)$

A.2.2 Calculation of $U(\varphi_t)$

The expanded uncertainty, $U(\varphi)$, of the measurement of the argument (phase shift) of the complex sensitivity given in A.1.2 is only valid for the particular frequencies, amplitudes and amplifier settings used at the time of calibration. The expanded uncertainty, $U(\varphi_t)$, in the argument (phase shift) of complex sensitivity for the complete frequency and amplitude range at any time interval between successive calibrations is calculated from the following formulae:

$$U(\varphi_t) = k u_c(\varphi_t) \tag{A.7}$$

$$u_c(\varphi_t) = \sqrt{\sum_{i=1}^{10} u_i^2(\varphi_t)} \tag{A.8}$$

using the uncertainty components shown in Table A.4 and a coverage factor k = 2.

Table A.4 — Uncertainty components in determining φ_t

i	Standard uncertainty component $u(x_i)$	Source of uncertainty	Uncertainty contribution $u_i(y)$
1	$u_c(\varphi)$	Combined uncertainty in the argument of the complex sensitivity at the frequencies, amplitudes and amplifier settings used in calibration calculated in accordance with A.1.2	$u_1(\varphi_t)$
2	$u(e_{GA})$	Uncertainty in the phase shift of the reference amplifier as a function of amplifier settings	$u_2(\varphi_t)$
3	$u(e_{FA})$	Uncertainty in the phase shift of the reference amplifier as a function of frequency	$u_3(\varphi_t)$
4	$u(e_{FT})$	Uncertainty in the phase shift of the sensitivity of the reference transducer as a function of frequency	$u_4(\varphi_t)$
5	$u(e_{LA})$	Uncertainty in the phase shift of the reference amplifier as a function of amplitude	$u_5(\varphi_t)$
6	$u(e_{LT})$	Uncertainty in the phase shift of the sensitivity of the reference transducer as a function of amplitude	$u_6(\varphi_t)$
7	$u(e_{TA})$	Uncertainty in the phase shift of the reference amplifier and the output impedance of the reference accelerometer as a function of time (instability over time)	$u_7(\varphi_t)$
8	$u(e_{TT})$	Uncertainty in the phase shift of the sensitivity of the reference transducer as a function of time (instability over time)	$u_8(\varphi_t)$
9	$u(e_{EA})$	Uncertainty in the phase shift of the reference amplifier due to environmental effects	$u_9(\varphi_t)$
10	$u(e_{ET})$	Uncertainty in the phase shift of the sensitivity of the reference transducer due to environmental effects	$u_{10}(\varphi_t)$

Annex B
(informative)

Application of the theory of reciprocity to the calibration of electromechanical transducers

The application of the theory of reciprocity to electromechanical transducers and their calibration assumes that the transducers are linear and bilateral moving with a single degree of freedom. Given these restrictions, the equations describing the electromechanical coupling of such a transducer when modelled as a two-port network using the impedance analogy are as follows:

$$u = z_e\, i + z_{em}\, v \tag{B.1}$$

$$F = z_{me}\, i + z_m\, v \tag{B.2}$$

where

u is the complex voltage across the electrical terminals;

i is the complex current through the electrical terminals;

F is the complex force across the mechanical terminals;

v is the complex velocity through the mechanical terminals;

z_e is the driving point electrical impedance with $v = 0$;

z_m is the driving point mechanical impedance with $i = 0$;

z_{em} and z_{me} are transduction coefficients with $z_{me} = z_{em}$ if the transduction mechanism is piezoelectric or electrostatic and $z_{me} = -z_{em}$ if the transduction mechanism is electromagnetic.

Let S_v, the sensitivity of the transducer as a velocity sensor, be defined as the complex ratio of the open-circuit output voltage to the velocity through the mechanical terminals, and let G_F, the sensitivity of the transducer as a force generator, be defined as the complex ratio of the open-circuit force (blocked mechanical impedance) to the current through the electrical terminals. Then:

$$S_v = \left.\frac{u}{v}\right|_{i=0} = z_{em} \tag{B.3}$$

$$G_F = \left.\frac{F}{i}\right|_{v=0} = z_{me} \tag{B.4}$$

$$G_F = \pm\, S_v \tag{B.5}$$

If there are two linear transducers, at least one of which is reciprocal, that are driven sinusoidally with a single degree of freedom then two measurement protocols can be established to determine the product and ratio of the absolute complex sensitivities of the transducers, S_{v1} and S_{v2}, and then the absolute sensitivity of either or both of the transducers can be determined from these. If the two transducers are rigidly coupled mechanically in juxtaposition such that the velocity when referred to the mechanical ports of the transducers is equal in amplitude but opposite in phase, then $v_1 = -v_2$. If transducer 1 is used as a force generator to drive transducer 2, then the

product of the sensitivities of the transducers may be determined as follows (Experiment 1 in 6.1.1). Let z_T equal the combined mechanical impedance of the two transducers when the mechanical ports of the transducers are coupled directly together. Then from equation (B.4):

$$F = G_{F1} i_1 = v_1 z_T = -v_2 z_T \tag{B.6}$$

and

$$z_T = -G_{F1} i_1 \frac{S_{v2}}{u_2} = \mp S_{v1} S_{v2} Y_0 \tag{B.7}$$

where

$$Y_0 = \frac{i_1}{u_2}$$

If a known mechanical impedance z_n is inserted between the mechanical ports of the transducers, then the driven mechanical impedance becomes:

$$z_T + z_n = \mp S_{v1} S_{v2} Y_n \tag{B.8}$$

where

$$Y_n = \frac{i_1}{u_2} \quad \text{with the known added mechanical impedance.}$$

Subtracting z_T from $z_T + z_n$:

$$z_n = \mp S_{v1} S_{v2} \left(Y_n - Y_0 \right) \tag{B.9}$$

and therefore

$$S_{v1} S_{v2} = \pm \frac{z_n}{\left(Y_n - Y_0 \right)} \tag{B.10}$$

If the added mechanical impedance is a known mass, then for sinusoidal excitation z_n is $j\omega m_n$ and the product of the sensitivities becomes:

$$S_{v1} S_{v2} = \pm \frac{j\omega m_n}{\left(Y_n - Y_0 \right)} \tag{B.11}$$

where

j   is the imaginary unit, $j^2 = -1$;

ω is angular frequency equal to $2\pi f$;

m_n is the added mass.

The ratio of the sensitivities of the transducers may be determined as follows (Experiment 2 in 8.2.2). Again, with the two transducers rigidly coupled mechanically in juxtaposition such that the velocity at the mechanical ports of the transducers is equal in amplitude but opposite in phase ($v_1 = -v_2$), and with the reciprocal transducer acting as a velocity sensor with $i_1 = i_2 = 0$ then by applying equation B.3 to both transducers:

15

$$\frac{S_{\text{v2}}}{S_{\text{v1}}} = -\frac{u_2}{u_1} = -U_{\text{v}} \tag{B.12}$$

where U_{v} is the ratio of the open-circuit voltage of transducer 1 and transducer 2 with transducer 1 operating as a velocity sensor.

The absolute complex sensitivity of either or both of the transducers may then be determined from the product and ratio of the sensitivities. For example, in the case of transducer 2:

$$S_{\text{v2}} = \sqrt{\pm \frac{U_{\text{v}} \, \text{j}\omega \, m_n}{(Y_n - Y_0)}} \qquad \text{for the case of a velocimeter} \tag{B.13}$$

$$S_{\text{a2}} = \sqrt{\pm \frac{U_{\text{v}} \, m_n}{\text{j}\omega (Y_n - Y_0)}} \qquad \text{for the case of an accelerometer} \tag{B.14}$$

where the sign under the radical is positive if the transduction mechanism of the reciprocal transducer is electromagnetic and negative if the transduction mechanism of the reciprocal transducer is piezoelectric or electrostatic.

In principle, sensitivity may be determined using only one added mass. However, in practice much smaller uncertainty is obtained if a series of masses m_1, m_2, . . . m_n and corresponding measured complex electrical admittances Y_1, Y_2, . . . Y_n are used to obtain a linear least-squares fit of the function $m/(Y - Y_0)$ versus m to obtain the equation of the line $\alpha + \beta m$ where α and β are the complex intercept and slope, respectively, of the line corresponding to $m/(Y - Y_0)$ versus m. By substitution, the measured complex sensitivity of transducer 2 then becomes:

$$S_{\text{v2}} = \sqrt{\pm U_{\text{v}} \, \text{j}\omega \, \alpha} \qquad \text{for the case of a velocimeter} \tag{B.15}$$

$$S_{\text{a2}} = \sqrt{\pm \frac{U_{\text{v}} \, \alpha}{\text{j}\omega}} \qquad \text{for the case of an accelerometer} \tag{B.16}$$

Often in the practical realization of systems designed to implement the calibration of electromechanical transducers by the reciprocity method, the two transducers are separated by a mechanical impedance as well as being separated from the attachment point of the requisite added mass by yet another mechanical impedance. When this is the case, it is desirable to calibrate sensitivity in terms of the complex ratio of the open-circuit output voltage of the transducer to the velocity of the surface upon which the mass is added so that the calibrated transducer may then be used to calibrate a second transducer by comparison when the second transducer is mounted on the surface upon which the mass is added. It can be shown (see reference [8]) that when the reciprocal transducer used in the calibration by the reciprocity method is electrodynamic, the equations relating the force and velocity at the mounting surface, the current and voltage at the electrical port of the reciprocal transducer (transducer 1), and the open-circuit output voltage of the velocity sensor (transducer 2) take the form in the mobility analogy as follows:

$$i_1 = y_{\text{e}} \, u_1 - y_{\text{em}} \, F \tag{B.17}$$

$$v = y_{\text{em}} \, u_1 + y_{\text{m}} \, F \tag{B.18}$$

$$u_2 = k_{\text{e}} \, u_1 + k_{\text{m}} \, F \tag{B.19}$$

where

u_1 is the complex voltage across the electrical terminal of transducer 1;

u_2 is the complex voltage across the electrical terminals of transducer 2;