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Microbeam analysis — Electron probe microanalysis — Guidelines for the determination of experimental parameters for wavelength dispersive spectroscopy

Analyse par microfaisceaux — Analyse par microsonde électronique (Microsonde de Castaing) — Lignes directrices pour la détermination des paramètres expérimentaux pour la spectrométrie à dispersion de longueur d'onde

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 202, *Microbeam analysis*, Subcommittee SC 2, *Electron probe microanalysis*.

This second edition cancels and replaces the first edition (ISO 14594:2003), of which it constitutes a minor revision. It also incorporates the Technical Corrigendum ISO 14594:2003/Cor 1:2009.

Microbeam analysis — Electron probe microanalysis — Guidelines for the determination of experimental parameters for wavelength dispersive spectroscopy

1 Scope

This International Standard gives the general guidelines for the determination of experimental parameters relating to the primary beam, the wavelength spectrometer, and the sample that need to be taken into account when carrying out electron probe microanalysis. It also defines procedures for the determination of beam current, current density, dead time, wavelength resolution, background, analysis area, analysis depth, and analysis volume.

This International Standard is intended for the analysis of a well-polished sample using normal beam incidence, and the parameters obtained can only be indicative for other experimental conditions.

This International Standard is not designed to be used for energy dispersive X-ray spectroscopy.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/IEC 17025:2005, *General requirements for the competence of testing and testing laboratories*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1
analysis area
two-dimensional region of sample surface from which the full signal or a specified percentage of that signal is detected

3.2
analysis depth
distance from the sample surface to the bottom normal to the surface from which the full signal or a specified percentage of that signal is detected

3.3
analysis volume
three-dimensional region of a sample from which the full signal or a specified percentage of that signal is detected

3.4
background
non-characteristic component of an X-ray spectrum arising from the X-ray continuum

3.5
beam current
electron current contained within the focused beam

3.6
beam current density
beam current incident on the sample per unit area

3.7
dead time
time associated with the measurement of a signal photon in a detector and/or counting system, representing the time that the system is unavailable to process the next photon

3.8
wavelength resolution
full peak width at half maximum of a characteristic X-ray peak

4 Abbreviated terms

EPMA	Electron Probe Microanalysis
FWHM	Full Width at Half Maximum
WDX	Wavelength Dispersive X-ray

5 Experimental parameters

5.1 General

The parameters given in 5.2.1, 5.2.3, and 5.2.4 should be known and recorded. Checking the calibration of beam energy, beam current, and magnification together with counter dead time should be included in the maintenance schedule of the instrument.

5.2 Parameters related to the primary beam

5.2.1 Beam energy

The beam energy typically ranges from 2 keV to 30 keV. In most cases, the calibration of the beam energy is not critical for qualitative analysis.

NOTE Calibration is very critical in the case of use of low overvoltage ratio or during measurements relating to layer thickness or elemental depth distributions.

5.2.2 Beam current

Because X-ray peak intensity is directly proportional to beam current, the precision of the measurement of the beam current should be better than the precision required for quantitative analysis.

The beam current stability over long periods of time is absolutely essential for consistent quantitative analysis. The beam current stability should be tested periodically, especially prior to quantitative calibration and analysis. It is possible to compensate for small changes in beam current if this is recorded prior to and following each measurement. Then all X-ray peak and background measurements should be scaled appropriately by I_i / I_m, where I_i is the initial beam current and I_m is the beam current at the time of the measurement.

5.2.3 Beam current density

Beam current density is especially important when analyzing beam sensitive materials. The current density in a focused probe can exceed 10^4 A m^{-2}. The effective current density can be reduced for a measurement by lowering the incident electron beam current or, where lateral resolution is not critical, by either defocusing or rastering the probe. If a rastered probe is used, a similar scan should be used for comparative measurements on standards and other specimens because the effective spectrometer efficiency for the selected wavelength decreases as a function of the beam deflection. See 5.3.5.

5.2.4 Magnification

To properly define the dimensional scale for line-scans and images acquired by deflecting the primary electron beam, it is essential to calibrate the magnification scale while operating in the scanning electron mode.

5.3 Parameters related to wavelength dispersive X-ray spectrometers

5.3.1 General

An instrument may be fitted with one or more WDX spectrometers, each with a number of diffracting crystals that may be selected to cover a particular range of X-ray wavelengths depending on the line of the analysed element. The following parameters are important for the proper operation of WDX spectrometers.

5.3.2 Take-off angle

The take-off angle affects quantitative analysis. Any comparison of measurements from instruments with different take-off angles should be taken into account and the procedures used be noted in the analysis report.

NOTE The value of this angle, which is normally fixed, is provided by the instrument manufacturer.

5.3.3 Wavelength resolution

The spectral resolution depends on the following parameters:

- crystal material (and Miller indices of the crystal planes);
- the radius of curvature of the diffracting crystal (fully focusing vs. semi-focusing crystal);
- the presence of a crystal mask (if semi-focusing crystal);
- the size and position of the counter entrance window or of the entrance slit if present.

All these settings determine the wavelength resolution of the measured X-ray spectrum and the observed line-width (FWHM) of the characteristic X-ray peaks.

Resolution can also influence the ability of the system to discriminate against overlapping peaks, background signals, and the sensitivity of measurements to specimen height and beam position on the specimen.

5.3.4 X-ray detector and counting chain

Many spectrometers use a gas-filled proportional counter to detect X-rays. The magnitude of the output pulses from these detectors is determined by the incident X-ray energy and/or the voltage applied to the counters. Two discriminators are used to select the pulse of interest. A low discriminator setting is used to eliminate pulses due to noise, while a high discriminator setting excludes pulses from high order reflections of more energetic X-rays. Optimum settings depend on the X-ray lines of interest.

It is important to set the discriminator to ensure that any unintended shift in pulse amplitude, for example, due to high count rates or changes in atmospheric temperature and pressure (flow counter), has no significant effect on the measured count rate.

Because X-ray counting efficiency decreases with increasing count rate, it is important to correct the measured count rate for the effect of the dead time. In an automated system, the discriminator settings can be set automatically. These settings should be routinely checked to ensure proper automatic operation.

5.3.5 Peak location (wavelength)

Under normal circumstances, the wavelength which has the maximum peak intensity is used to define the location of an X-ray peak. It is necessary, using suitable reference materials, to periodically check and correct for the difference in a peak's theoretical position and its actual measured position on a given spectrometer and diffraction crystal. The time between checks will depend on the stability of the instrument spectrometers.

The measured maximum intensities of peaks which have narrow FWHM values are strongly affected by the errors in peak location. The peak intensity can be changed due to the chemical state and polarization effects.

NOTE 1 If the element in the sample of interest is in a different chemical state than that of the reference material, then the shape of the characteristic X-ray peak might be different for specimen and standard. In this case, the peak maximum might not provide a reliable measure of the total peak intensity and an alternative approach, such as peak area measurements, might be required to obtain reliable results. These chemical state effects are particularly important for X-ray peaks with low energy values.

NOTE 2 If a crystalline sample causes the polarization effects in relation to the position between the sample and the analysis crystal, the peak shape and location can be changed. This can be checked by rotating the sample around an axis perpendicular to the electron beam and observing the effect on peak shape and location. The problem might occur in systems with symmetry lower than cubic and higher than triclinic and is worst when the Bragg angle is close to 45°. The phenomenon has been found in graphite[1] and certain borides.[2] The effect can be much reduced by using peak area measurements.

The position of the peak maximum varies with deviation of the probe from the focal point of the spectrometer on the sample. Calibration measurements and quantitative analysis on the sample should normally be made with the probe in the same position relative to this focal point, and using the same beam defocus or raster setting, if applied. For all quantitative and qualitative analyses carried out using a defocused and scanned beam, the area of the sample surface irradiated should not be so large as to cause a significant fall in X-ray counts from that obtained with the static focused electron beam.

5.3.6 Background

The characteristic X-ray peaks are superimposed on a background of continuum X-rays.

To properly calculate the intensities of characteristic X-rays, the magnitude of this background needs to be determined and corrected if it is statistically significant.

5.4 Parameters related to the specimen

5.4.1 Specimen stage

High precision X, Y, and Z stages allow the sample and standards to be accurately positioned under the electron beam by using an attached optical microscope; the user can set the height of the sample so the axis of the WDX spectrometer and the primary beam position coincide at the surface of the sample. Orthogonality between the electron beam (the optical axis) and the specimen stage is essential in order to perform a proper quantitative analysis. A check on the adjustment of the optical microscope should be included in the routine instrument maintenance schedule.

In an automatic mode of operation, where the measurements are to be made at preset points on the standards and the specimen, it is important to know the reproducibility with which the stage retrieves preset points and to adopt appropriate strategies to overcome any obvious limitations.

5.4.2 Surface roughness

For best results, the surface roughness of the specimen should be minimized.

5.4.3 Analysis volume

Analysis volume is determined by the incident beam area, the depth of beam penetration, the spread of the incident beam within the sample, and the energy of the characteristic X-ray line. This analysis volume can be significantly increased by unwanted fluorescence effects which are caused by the characteristic and continuum X-rays.

6 Procedures and measurements

6.1 General

The following procedures should be adopted to determine a number of critical parameters.

6.2 Beam current

6.2.1 Measurement

Measure the beam current using a Faraday cup. It should be positioned after the final aperture. If the measurement is carried out at another position, the relationship between the above-mentioned position and this position shall be shown.

6.2.2 Density

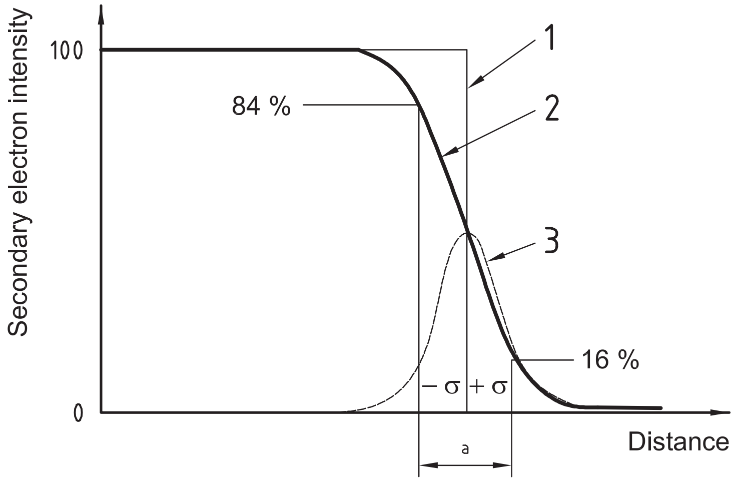
This procedure gives an average current density within the beam. The local current density can be calculated assuming a Gaussian beam profile and using the value measured in 6.2.2.1 a) or 6.2.2.1 b).

6.2.2.1 The diameter of the beam shall be defined by one of the following methods:

a) The diameter of electron beam shall be defined as the interval where the emitted secondary electron intensity drops from 84 % to 16 % of the maximum peak intensity, which is equivalent to two standard deviations ($2\ \sigma$) of the error curve (see Figure 1). This measurement should be done such that the primary electron beam crosses a knife edge at a right angle.

b) The diameter of area exhibiting optical fluorescence for a material such as aluminum oxide, zirconium oxide, or thorium oxide, whereby that diameter is determined by using an optical microscope. This measurement should be done when the beam diameter is more than 5 µm.

6.2.2.2 The beam current density shall be calculated by dividing the incident beam current (as defined in 6.2.1) by the area of the electron beam. For a round defocused beam, the area would be $\pi d^2/4$, where d is the beam diameter.



Key

1	true boundary
2	measured curve
3	error function
a	diameter

Figure 1 — Method for measuring beam diameter

6.3 Parameters related to measured peaks

6.3.1 Dead time correction

Measure the beam current, *i*, in accordance with 6.2.1 and the count rate (N).

6.3.1.1 As shown in Figure 2, make a calibration curve by using the count rate, N, divided by the beam current, i, values as the ordinate values and the count rate, N, as the abscissa values.

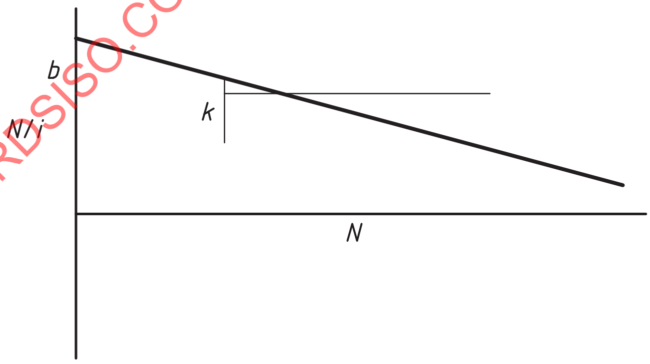


Figure 2 — Counting loss by dead time

In order to confirm the linearity of the current measuring device, it is advisable to monitor the count rate, n, of a low intensity line at the same time as the count rate, N, of a high intensity line; the ratio n:i should be constant for the whole range of measurements.

Typically, a Kα line can be used to determine N and the corresponding Kβ line for n.

NOTE If the probe current cannot be measured accurately, the dead time can be determined by measuring the ratio of two X-ray intensities as a function of count rate on two X-ray spectrometers.[2][3]

6.3.1.2 Determine the gradient, k, and the intercept value, b, at the ordinate axis from the calibration curve.

Calculate the dead time, τ, by using Formula (1):

$$\tau = \frac{-k}{b} \tag{1}$$

6.3.1.3 Calculate the true count rate, N_o, by using Formula (2):

$$N_o = \frac{N}{1 - N \times \tau} \tag{2}$$

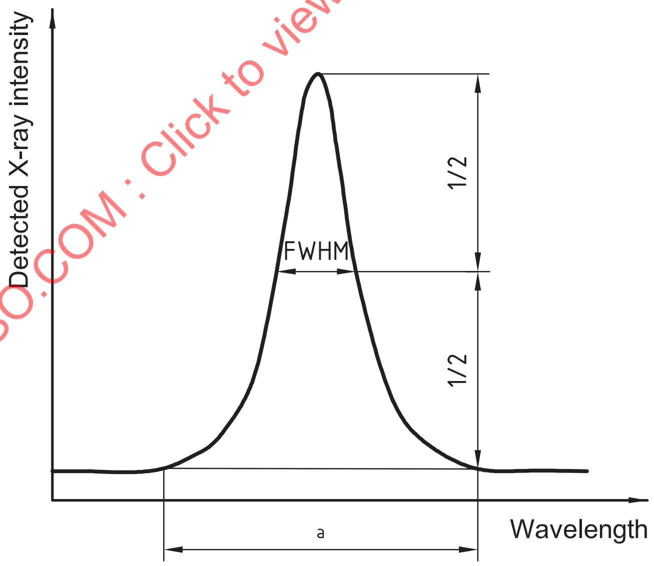
For accurate measurements, the count rate should be restricted so that the correction for dead time does not exceed 5 %.

6.3.2 Wavelength resolution of detected characteristic X-ray peaks

6.3.2.1 Measure a characteristic X-ray intensity versus wavelength spectrum for the elements of interest by measuring the intensity of the X-ray signals while scanning over the wavelengths of interest.

6.3.2.2 Calculate the wavelength resolution of the detected characteristic X-rays by using the following definition.

After subtracting the background, (see 6.3.3) wavelength resolution is equal to FWHM as shown in Figure 3.



Key

a background

Figure 3 — Definition of FWHM

6.3.3 Background subtraction

6.3.3.1 The net intensity of a characteristic X-ray peak is obtained by subtracting the intensity of the background from the intensity of the observed X-ray peak because the observed X-ray peak is the sum of the true intensity of the characteristic X-ray peak and the background intensity.

6.3.3.2 Measure X-ray intensity versus wavelength spectrum in accordance with 6.3.2.1.

6.3.3.3 Select wavelength positions on both sides of the peak of interest as near as possible to the peak, but where any residual intensity from the peak is less than 1 % of the maximum peak intensity, avoiding any subsidiary or interfering peak. To obtain the net characteristic peak intensity, the background intensity at the peak position should be estimated by making a linear interpolation between the two selected background positions, and by subtracting that background intensity from the intensity of the measured peak.

The procedure in 6.3.3.3 might be difficult to apply when the intensity of the background is changing rapidly with wavelength, or when some interference prevents the selection of positions that are reasonably close to the peak. In these cases of difficulty, it will be necessary to estimate background by applying a curve fitting procedure or by measuring a material whose average atomic number is as close as possible to the specimen of interest and which does not contain the element that produces the characteristic peak being analysed.

The time used to count the background intensity should be enough for the desired precision to be maintained.

The nature of the measurement being made, e.g. major component or trace element, is of prime importance in this consideration and should be reported along with details of the individual and overall precision which have been achieved.

6.4 Parameters related to the specimen

6.4.1 General

For many routine analyses, an individual calculation of the dimensions of the analysed volume is not necessary and an estimate can be given with reference to previous similar analyses. There are, however, special cases such as inclusions, fine-grained material, or coatings where knowledge of the spatial resolution of the analysis provides an important justification that the analysis is valid. In these cases, the depth, area, or volume of the analysis can be calculated using the information given in 6.4.2, 6.4.3, and 6.4.4. When this is done, the method chosen and the result should be included in the test report.

Methods for the determination of analysis area, analysis depth, and analysis volume are given in Annex A, Annex B, and Annex C, respectively.

6.4.2 Analysis area

Estimate the analysis area by applying an appropriate method, detailing all parameters used.

6.4.3 Analysis depth

Estimate the analysis depth by applying an appropriate method, detailing all parameters used.

6.4.4 Analysis volume

The first estimation of analysis volume is equal to the product of the analysis area (6.4.2) times the analysis depth (6.4.3).

The more precise value of the analysis volume should be calculated using one of the methods given in Annex C.

7 Test report

Records of the instrument and individual investigations should be kept so that, if required, a test report conforming to 5.10.2 of ISO/IEC 17025:2005 can be issued. The test report should contain the following:

a) title of ISO method used;

b) name and address of laboratory that performed the calibration(s) or test(s);

c) name and address of client, where relevant;

d) date of receipt of calibration item or test item and the date(s) that the calibration(s) or test(s) were performed, where relevant;

e) instrument type and reference number;

f) reference to sampling procedure, where relevant;

g) primary beam energy;

h) beam current;

i) X-ray peak(s) detected;

j) take-off-angle of the instrument;

k) estimation method of analysis volume, if relevant; note value of the constant for Formula (A.1) in Annex A;

l) estimate of the uncertainty of the calibration or test result (where relevant);

m) any deviations from, additions to, or exclusions from the calibration method or test method, and any other information relevant to a specific calibration or test, including, e.g. environmental conditions;

n) signature and title of person giving the signature, or an equivalent identification of the person(s) accepting responsibility for the content of the certificate or report, and date when the report was issued.

Annex A
(informative)

Methods of estimating analysis area

Although not always an issue in wavelength dispersive X-ray analysis, determination of the analysis area is often useful, particularly when analyzing particles, fine grained materials, or layered specimens. A variety of methods exists in the literature for estimating the approximate area of electrons having energies sufficient to excite an X-ray line of interest. Any of these procedures can be used as a first estimate of the analysis area.

The analysis area, *A*, is given by Formula (A.1):

$$A = \pi \left(\frac{d}{2} \right)^2 \qquad \text{(A.1)}$$

where *d* is the diameter computed or estimated by one of the following methods:

— Calculate the diameter of the analysis area using the Monte Carlo method (see Annex C).

— Calculate the analysis area after determining the diameter of the analysis area by applying Formula (A.2), which is based on the diffusion model for electron penetration, and Formula (B.1) with a constant equal to 0,025.

$$d = D_e + \frac{2{,}2\gamma}{1+\gamma} \times Z_m \qquad \text{(A.2)}$$

where

$$\gamma = 0{,}187\, Z^{2/3}$$

$$Z_m = 0{,}025\left(E_i^{1{,}7} - E_k^{1{,}7}\right) \times \frac{m_a}{\rho Z}$$

and

d is the diameter of analysis area, in micrometres;

D_e is the diameter of primary electron beam, in micrometres;

Z_m is the X-ray generation depth, in micrometres;

Z is the mean atomic number of the bombarded point;

E_i is the energy of the incident electron, in kilo-electron volts;

E_k is the critical excitation energy, in kilo-electron volts;

m_a is the mean atomic mass of the bombarded point;

ρ is the mass density of the bombarded point, in grams per cubic centimetre.

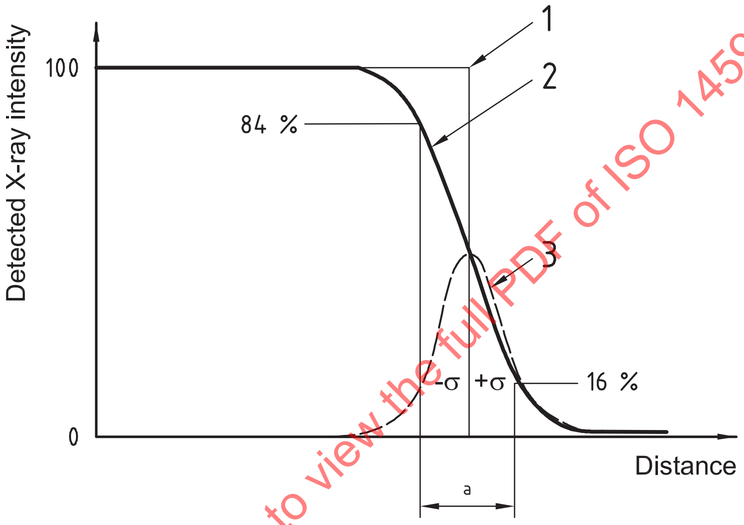
— The following experimental method which estimates the diameter of the analysis area by using a bi-metal system.

Prepare a specimen, formed by plating, which shall be made of two joined materials that have similar mass density, do not suffer inter-diffusion, and have only a small fluorescence effect on each other, e.g. Ni/Cu.

Measure the characteristic X-ray intensity as a function of distance by moving an electron beam perpendicular to the interface between the two different materials or by moving the sample stage.

Determine the interval distance over which the intensity of one characteristic X-ray signal drops from 84 % to 16 % of the maximum signal intensity and use that distance as diameter of the detected X-ray signal. That interval distance is equivalent to two standard deviations (2 *σ*) of the error curve as shown in Figure A 1.

NOTE This approach is only applicable to the particular couple used, unless the beam diameter is much greater than the beam spread.



Key

1 true boundary

2 measured curve

3 error function

a diameter

Figure A.1 — Estimation for the analysis area

Annex B
(informative)

Methods of estimating analysis depth

Although not always an issue in wavelength dispersive X-ray analysis, determination of the analysis depth is often useful, particularly when analyzing particles, fine grained materials, or layered specimens. A variety of methods exists in the literature for estimating the approximate depth of penetration of electrons having energies sufficient to excite an X-ray line of interest. Any of these procedures can be used as a first estimate of the analysis depth. Three such methods are outlined below. However, the exact analysis depth (defined as the depth above which *n* % of the characteristic X-rays detected by the spectrometer are generated, where "*n*" is greater than 90) can be significantly affected by the depth distribution of absorbed X-rays, particularly for low energy radiations or high-*Z* materials, and by X-rays generated by characteristic and continuum fluorescence. Other methods can also be used.

- $\Phi(\rho Z)$ expression[4]
- Monte Carlo simulation (see Annex C)
- Formula (B.1) which is based on many experimental results where

$$Z_\mathrm{m} = \mathrm{const.} \times \left(E_\mathrm{i}^{1,7} - E_\mathrm{k}^{1,7}\right) \times \frac{m_\mathrm{a}}{\rho Z} \qquad \text{(B.1)}$$

where

Z_m is the X-ray generation depth, in micrometres;

E_i is the energy of the incident electron, in kilo-electron volts;

E_k is the critical excitation energy, in kilo-electron volts;

m_a is the mean atomic mass of the bombarded point;

ρ is the mass density of the bombarded point, in grams per cubic centimetre;

Z is the mean atomic number of the bombarded point.

$$m_\mathrm{a} = \sum_\mathrm{i} C_\mathrm{i} m_\mathrm{a,i} \qquad \text{(B.2)}$$

$$Z = \sum_\mathrm{i} C_\mathrm{i} Z_\mathrm{i} \qquad \text{(B.3)}$$

where

C_i is the mass fraction for element *i*;

$m_\mathrm{a,i}$ is the atomic mass of the element *i*;

Z_i is the atomic number of the element *i*.

NOTE The constant in Formula (B.1) is normally either 0,033[5] or 0,025.[6]

If 0,025 and 0,033 are used as the constant, approximately 95 % and 99 % respectively of the directly produced characteristic X-rays are produced within the depth Z_m.

Annex C
(informative)

Method of estimating X-ray analysis volume by applying the Monte Carlo (MC) simulation

C.1 General

When an electron beam is incident on a specimen, X-rays are generated from a large volume due to electron penetration. In X-ray microanalysis, it is important to know this volume in order to know the resolution of this analysis. The volume is dependent upon beam energy, the kind of characteristic X-ray, and so on. A variety of methods such as an experimental method, an analytical method based on the transport formula, and a Monte Carlo method, have so far been proposed.[7] The Monte Carlo method outlined below simulates the electron behaviour based on the electron scattering theory, which is the statistical method due to random sampling through use of pseudo-random numbers. The method can be applied to any solid materials, can handle any type of experimental conditions, and is relatively easy to set in a simulation programme. Here, the simplest model, the single scattering model, is described. Other methods can be also used.

C.2 By applying the following theories and estimations, it is possible to use the Monte Carlo method to simulate the electron trajectory in solid materials.

a) Principal formula

Differential scattering cross section (based on screened Rutherford theory) as shown in Formula (C.1)[8]

$$\frac{\mathrm{d}\sigma_\mathrm{i}}{\mathrm{d}\Omega}=\frac{e^4 Z_\mathrm{i}\left(Z_\mathrm{i}+1\right)}{4E^2\left(1+2\beta_\mathrm{i}-\cos\theta\right)^2} \qquad \text{(C.1)}$$

where

Ω is the scattering solid angle;

e is the elementary electron charge;

Z_i is the atomic number of the element i;

E is the kinetic energy of an electron, in electron volts.

Screening parameter, as shown in Formula (C.2)[9]

$$\beta_\mathrm{i}=\frac{5{,}44{Z_\mathrm{i}}^{2/3}}{E} \qquad \text{(C.2)}$$

Stopping power, as shown in Formula (C.3)[10]

$$\frac{\mathrm{d}E}{\mathrm{d}S}=-2\pi e^4\rho\times\frac{N}{E}\times\sum\frac{C_\mathrm{i}Z_\mathrm{i}}{A_\mathrm{i}}\times\mathrm{In}\frac{1{,}166E}{J_\mathrm{i}} \qquad \text{(C.3)}$$

Mean ionization potential, as shown in Formula (C.4)[11]

$$J_\mathrm{i}=9{,}76Z_\mathrm{i}+58{,}5{Z_\mathrm{i}}^{-0{,}19} \qquad \text{(C.4)}$$

b) Scattering process, as shown in Formula (C.5)

The probability for elastic scattering

$$P(\theta,\varphi)\,\mathrm{d}\Omega=\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\times\frac{1}{\sigma_{\mathrm{i}}}\right)\mathrm{d}\Omega \tag{C.5}$$

where

θ, φ are the scattering angles of incident electrons (see Figure C.1);

σ_{i} is the total elastic scattering cross section

Angle φ is given by

$$\varphi = 2\pi R_1 \tag{C.6}$$

where R_1 is random number, $0 < R_1 < 1$.

Angle θ is given by

$$F(\theta)=\int_{\Omega} P(\theta,\varphi)\mathrm{d}\Omega=\int_{\theta} 2\pi\sin\theta\times P(\theta)\mathrm{d}\theta \tag{C.7}$$

$$\cos\theta = 1-\frac{2\beta_{\mathrm{i}}F(\theta)}{1+\beta_{\mathrm{i}}-F(\theta)}$$

where $F(\theta)$ is the integration of the probability for elastic scattering into an angular range from $\theta = 0$ to a given angle θ, given by random number, $0 \le F(\theta) \le 1$.

The step length between elastic scattering events, as given in Formula (C.8):

$$\Delta S = -L\times \mathrm{In}(R_2) \tag{C.8}$$

where R_2 is a random number, $0 \le R_2 \le 1$.

The energy loss from an elastic scattering event to the next event, as given in in Formula (C.9):

$$\Delta E = \Delta S\times\left|\mathrm{d}E/\mathrm{d}S\right| \tag{C.9}$$

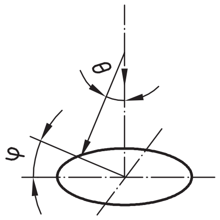


Figure C.1 — Definition of *θ* and *φ* in scattering process