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Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

Part 4: Guidelines to applications

Détermination des limites caractéristiques (seuil de décision, limite de détection et limites de l'intervalle élargi) pour le mesurage des rayonnements ionisants — Principes fondamentaux et applications —

Partie 4: Lignes directrices relatives aux applications

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

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For an explanation on the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 85, *Nuclear energy, nuclear technologies, and radiological protection*, Subcommittee SC 2, *Radiological protection*.

This third edition of ISO 11929-4 cancels and replaces the second edition (ISO 11929-4:2020), of which it constitutes a minor revision.

The main changes are as follows:

— Editorial changes were done in text and formulae

A list of all parts of ISO 11929 can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Measurement uncertainties and characteristic values, i.e. characteristic limits such as the decision threshold, the detection limit and limits of the coverage interval for measurements as well as the best estimate and its associated standard measurement uncertainty, are of importance in metrology, in general, and for radiological protection, in particular. The quantification of the uncertainty associated with a measurement result provides a basis for the trust an individual can have in a measurement result.

NOTE 1 Conformity with regulatory limits, constraints or reference values can only be demonstrated taking into account and quantifying all sources of uncertainty. Characteristic limits provide – in the end – the basis for deciding accepting results under uncertainty.

ISO 11929 (all parts) provides characteristic values of a non-negative measurand of ionizing radiation. It is applicable for a wide range of measuring methods extending beyond measurements of ionizing radiation.

The limits to be provided according to ISO 11929 (all parts) for specified probabilities of wrong decisions allow detection possibilities to be assessed for a measurand and for the physical effect quantified by this measurand as follows:

— the "decision threshold" allows a decision to be made on whether or not the physical effect quantified by the measurand is present;

— the "detection limit" indicates the smallest true quantity value of the measurand that can still be detected with the applied measurement procedure; this gives and allows for a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose;

— the "limits of the coverage interval" enclose, in the case of the physical effect recognized as present, a coverage interval containing the true quantity value of the measurand with a specified probability.

Hereinafter, the limits mentioned are jointly called "characteristic limits".

NOTE 2 According to ISO/IEC Guide 99 updated by JCGM 200:2012, the term "coverage interval" is used here instead of "confidence interval" in order to distinguish the wording of Bayesian terminology from that of conventional statistics.

All the characteristic values are based on Bayesian statistics and on the ISO/IEC Guide 98-3 as well as on the ISO/IEC Guide 98-3:2008/Suppl.1 and ISO/IEC Guide 98-3:2008/Suppl.2. As explained in detail in ISO 11929-2, the characteristic values are mathematically defined by means of moments and quantiles of probability distributions of the possible measurand values.

Since measurement uncertainty plays an important role in all parts of ISO 11929, the evaluation of measurements and the treatment of measurement uncertainties are carried out by means of the general procedures according to the ISO/IEC Guide 98-3 and to the ISO/IEC Guide 98-3:2008/Suppl.1; see also References [21] to [25]. This enables the strict separation of the evaluation of the measurements, on the one hand, and the provision and calculation of the characteristic values, on the other hand. ISO 11929 (all parts) makes use of a theory of uncertainty in measurement[26] to [28] based on Bayesian statistics (e.g. References [29] to [36]) in order to allow taking into account also those uncertainties that cannot be derived from repeated or counting measurements. The latter uncertainties cannot be handled by frequentist statistics.

Because of developments in metrology concerning measurement uncertainty, laid down in the ISO/IEC Guide 98-3, ISO 11929:2010 was drawn up on the basis of ISO/IEC Guide 98-3, but using Bayesian statistics and the Bayesian theory of measurement uncertainty. This theory provides a Bayesian foundation for the ISO/IEC Guide 98-3. Moreover, ISO 11929:2010 was based on the definitions of the characteristic values[21], the standard proposal[22], and the introducing article[23]. It unified and replaced all earlier parts of ISO 11929 and was applicable not only to a large variety of particular measurements of ionizing radiation but also, in analogy, to other measurement procedures. Some explanatory material about the basics of ISO 11929 (all parts), in general, and its application in has been published elsewhere[42][43].

Since the ISO/IEC Guide 98-3:2008/Suppl.1 has been published, the Monte Carlo method has been used to deal comprehensively with a more general treatment of measurement uncertainty in complex measurement evaluations. This development provided an incentive for writing a corresponding Monte Carlo supplement[24] to ISO 11929:2010. The revised ISO 11929 (all parts) is also essentially founded on Bayesian statistics and can serve as a bridge between documents ISO 11929:2010 and the ISO/IEC Guide 98-3:2008/Suppl.1. Moreover, more general definitions of the characteristic values (ISO 11929-2) and the Monte Carlo computation of the characteristic values make it possible to go a step beyond the present state of standardization laid down in ISO 11929:2010 since probability distributions rather than uncertainties can be propagated. It is thus more comprehensive and extending the range of applications.

The revised ISO 11929 (all parts), moreover, is more explicit on the calculation of the characteristic values. Reference [25] gives a survey on the basis of the revision. Further, in ISO 11929-3, it gives detailed advice how to calculate characteristic values in the case of multivariate measurements using unfolding methods. For such measurements, the ISO/IEC Guide 98-3:2008/Suppl.2 provides the basis of the uncertainty evaluation.

Formulae are provided for the calculation of the characteristic values of an ionizing radiation measurand via the "standard measurement uncertainty" of the measurand (hereinafter "standard uncertainty") derived according to the ISO/IEC Guide 98-3 as well as via probability density functions (PDFs) of the measurand derived on the basis of the ISO/IEC Guide 98-3:2008/Suppl.1. The standard uncertainties or probability density functions take into account the uncertainties of the actual measurement as well as those of sample treatment, calibration of the measuring system and other influences. The latter uncertainties are assumed to be known from previous investigations.

Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

Part 4:
Guidelines to applications

1 Scope

This document specifies a procedure, in the field of ionizing radiation metrology, for the calculation of the "decision threshold", the "detection limit" and the "limits of the coverage interval" for a non-negative ionizing radiation measurand when counting measurements with preselection of time or counts are carried out. The measurand results from a gross count rate and a background count rate as well as from further quantities on the basis of a model of the evaluation. In particular, the measurand can be the net count rate as the difference of the gross count rate and the background count rate, or the net activity of a sample. It can also be influenced by calibration of the measuring system, by sample treatment and by other factors.

ISO 11929 has been divided into four parts covering elementary applications in ISO 11929-1, advanced applications on the basis of the ISO/IEC Guide 98-3:2008/Suppl.1 in ISO 11929-2, applications to unfolding methods in ISO 11929-3, and guidance to the application in ISO 11929-4.

ISO 11929-1 covers basic applications of counting measurements frequently used in the field of ionizing radiation metrology. It is restricted to applications for which the uncertainties can be evaluated on the basis of the ISO/IEC Guide 98-3 (JCGM 2008). In ISO 11929-1:2019, Annex A the special case of repeated counting measurements with random influences and in ISO 11929-1:2019, Annex B, measurements with linear analogous ratemeters are covered.

ISO 11929-2 extends ISO 11929-1 to the evaluation of measurement uncertainties according to the ISO/IEC Guide 98-3:2008/Suppl.1. ISO 11929-2 also presents some explanatory notes regarding general aspects of counting measurements and Bayesian statistics in measurements.

ISO 11929-3 deals with the evaluation of measurements using unfolding methods and counting spectrometric multi-channel measurements if evaluated by unfolding methods, in particular, alpha- and gamma-spectrometric measurements. Further, it provides some advice how to deal with correlations and covariances.

ISO 11929-4 gives guidance to the application of ISO 11929 (all parts), summarizing shortly the general procedure and then presenting a wide range of numerical examples. The examples cover elementary applications according to ISO 11929-1 and ISO 11929-2.

The ISO 11929 (all parts) also applies analogously to other measurements of any kind if a similar model of the evaluation is involved. Further practical examples can be found in other International Standards, for example, see References [1 to 20].

NOTE A code system, named UncertRadio, is available allowing for calculations according to ISO 11929-1 to ISO 11929-3. UncertRadio[40][41] can be downloaded for free from https://www.thuenen.de/en/fi/fields-of-activity/marine-environment/coordination-centre-of-radioactivity/uncertradio/. The download contains a setup installation file that copies all files and folders into a folder specified by the user. After installation one has to add information to the PATH of Windows as indicated by a pop-up window during installation. English language can be chosen and extensive "help" information is available. Another tool is the package 'metRology'[44] which is available for programming in R. It contains the two R functions 'uncert' and 'uncertMC' which perform the GUM-conform uncertainty propagation, either analytically or by the Monte Carlo method, respectively. Covariances/correlations of input quantities are included. Applying these two functions within iterations for decision threshold and the detection limit calculations simplifies the programming effort significantly. It is also possible to implement this document in a spreadsheet containing a Monte Carlo add-in or into other commercial mathematics software.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document including any amendments applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 11929-1, *Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application — Part 1: Elementary applications*

ISO 11929-2, *Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application — Part 2: Advanced applications*

ISO 80000-1, *Quantities and units — Part 1: General*

ISO 80000-10, *Quantities and units — Part 10: Atomic and nuclear physics*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 98-3:2008/Suppl.1, *Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" — a Propagation of distributions using a Monte Carlo method, JCGM 101:2008*

ISO/IEC Guide 98-3:2008/Suppl.2, *Evaluation of measurement data — Supplement 2 to the "Guide to the expression of uncertainty in measurement" — Extension to any number of output quantities, JCGM 102:2011*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

IEC/TR 62461, *Radiation protection instrumentation — Determination of uncertainty in measurement, Ed. 2.0, IEC 23.1.2015*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 80000-1, ISO 80000-10, ISO/IEC Guide 98-3, ISO/IEC Guide 98-3:2008/Suppl.1, ISO/IEC Guide 98-3:2008/Suppl.2, ISO/IEC Guide 99, ISO 3534-1, ISO 11929-1, and ISO 11929-2 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform available at https://www.iso.org/obp

— IEC Electropedia available at https://www.electropedia.org/

4 Quantities and symbols

The quantities and symbols below are used throughout this document. Additional quantities and symbols are listed – if necessary – in the respective examples.

m	number of input quantities, also used for the mass of the test sample
n_M	number of Monte Carlo trials performed
X_i	input quantity $(i = 1,...,m)$
Y	measurand, quantity of interest
G	model of evaluation connecting the input quantities with the measurand: $Y = G(X_1,...,X_m)$
x_i	measured value of the input quantity X_i, estimate of the true value $\tilde{x}_i$ of X_i
$\tilde{x}_i$	possible or assumed true values of the input quantity X_i
$u(x_i)$	standard uncertainty of the input quantity X_i associated with the estimate X_i
Δx_i	width of the region of the possible values of the input quantity X_i
$u_\mathrm{rel}(w)$	relative standard uncertainty of a quantity W associated with the estimate w
$\tilde{y}$	possible or assumed true values of the measurand; if the physical effect of interest is not present, then $\tilde{y} = 0$ otherwise, $\tilde{y} > 0$
y	determined value of the measurand Y, estimate of the measurand, primary measurement result of the measurand
y_j	values y from different measurements $(j = 0,1,2, ...)$
$u(y)$	standard uncertainty of the measurand associated with the primary measurement result y
$\tilde{u}(\tilde{y})$	standard uncertainty of an estimator of the measurand Y as a function of an assumed true value $\tilde{y}$ of the measurand
$\hat{y}$	best estimate of the measurand
$u(\hat{y})$	standard uncertainty of the measurand associated with the best estimate $\hat{y}$
y^*	decision threshold of the measurand
$y^\#$	detection limit of the measurand
y_r	guideline value of the measurand
$y^\triangleleft$, $y^\triangleright$	lower and upper limit of the symmetric coverage interval, respectively, of the measurand
$y^<$, $y^>$	lower and upper limit of the shortest coverage interval, respectively, of the measurand

$f(\tilde{y}\vert y)$	posterior probability density function (PDF) for a true value $\tilde{y}$ given the estimate y taking NOT into account the condition that the measurand Y is non-negative
$f(\tilde{y}\vert y, Y \geq 0)$	posterior probability density function (PDF) for a true value $\tilde{y}$ given the estimate y taking into account the condition that the measurand Y is non-negative
$f(y\vert\tilde{y}=0)$	predictive probability density function (PDF) to obtain a measured value y if a true value $\tilde{y}=0$ of the measurand Y is assumed
$f(y\vert\tilde{y}=y^{\#})$	predictive probability density function (PDF) to obtain a measured value y if a true value $\tilde{y}=y^{\#}$ of the measurand Y equal to the detection limit $y^{\#}$ is assumed
$\mathrm{Ga}(\tilde{r};n,1/t)$	Gamma distribution as the probability density function (PDF) of the true value $\tilde{r}$ of a count rate R given n counts obtained during a counting time t; see ISO 11929-2:2019, Annex A, for details.
$\mathrm{N}(\tilde{x};x,u(x))$	Normal or Gaussian distribution as the probability density function (PDF) of the true value $\tilde{x}$ of a quantity X given an estimate x with its associated standard uncertainty $u(x)$
$\mathrm{R}(\tilde{x};x_\mathrm{L},x_\mathrm{U})$	Rectangular distribution as the probability density function (PDF) of the true value $\tilde{x}$ of a quantity X given the lower and upper limits x_L and x_U
$\mathrm{T}(\tilde{x};a,b)$	Triangular distribution as the probability density function (PDF) of the true value $\tilde{x}$ of a quantity X being the sum of two quantities A and B being assigned rectangular probability distributions with the lower and upper limits a_L and b_L respectively a_U and b_L and with $a=a_\mathrm{L}+b_\mathrm{L}$ and $b=a_\mathrm{U}+b_\mathrm{U}$.
$\mathrm{H}(x)$	Heaviside step function: $\mathrm{H}(x)=\begin{cases}0\\1\end{cases}\text{for}\begin{cases}x<0\\x\geq 0\end{cases}$
r_g, r_0	estimate of the gross count rate and of the background count rate, respectively
r_n	estimate of the net count rate
n_i	number of counted pulses obtained from the measurement of the count rate R_i
n_g, n_0	number of counted pulses of the gross measurement and of the background measurement, respectively
t_i	measurement duration of the measurement of the count rate R_i
t_g, t_0	duration of the gross and the background measurement, respectively
r_i	estimate of the count rate R_i
A	activity
a	estimate of the activity A
w	calibration factor
a_m	activity per unit mass
a_s	activity per unit surface
α, β	probability of a false positive and false negative decision, respectively
$1-\gamma$	probability for the coverage interval of the measurand
q_p	quantile of a distribution for the probability p

$\Phi(x)$ distribution function of the standard normal distribution

ω $\omega = \Phi[y/u(y)]$, value of the distribution function of the standard normal distribution at $y/u(y)$

5 Summary of this document

5.1 Procedures according to ISO 11929 (all parts)

ISO 11929-1 standardizes the evaluation of measurements of ionizing radiation for a wide range of models of evaluation and the calculation of characteristic limits (decision threshold, detection limit and limits of coverage intervals) on the basis of the ISO/IEC Guide 98-3. However, important exceptions exist for which the procedures do not provide reliable results and other procedures need to be applied, such as those described in ISO/IEC Guide 98-3:2008/Suppl.1. Such procedures are dealt with in ISO 11929-2. Both the aspects of the procedure and the tools to ascertain whether ISO 11929-1 or ISO 11929-2 is suitable or not for the specific application are described in ISO 11929-1.

It is a characteristic of measurements of ionizing radiation that they have to be performed in the presence of a radiation background, which has to be subtracted from a gross measurement quantity. However, the procedures described in this document likewise are applicable to any measurements where a background or blank contribution has to be subtracted from a gross quantity.

5.2 Survey on the examples

This document gives numerical examples of elementary applications of ISO 11929-1 and ISO 11929-2. The data in the tables are often given with more digits than are meaningful, so that the calculations can also be reconsidered and verified with higher accuracy, in particular for testing computer programs under development.

NOTE 1 The Monte Carlo calculations were performed in this guide with 1 000 000 trials. As a consequence, the results for the characteristic values remain uncertain in the third significant digit. In addition, the user has to be aware that the necessary number of trials depends significantly on the model of evaluation and on the PDFs assigned to the input quantities. In practical applications, a number of 10 000 trials is often sufficient.

A criterion whether ISO 11929-1 or ISO 11929-2 is to be preferred in practice is given in IEC/TR 62461. It recommends “the results of both methods should be given in order to display their difference. When the 95 % coverage intervals of the Monte Carlo method and of the analytical method do not deviate by more than 10 %, then the analytical one may be used for the uncertainty determination in similar cases, i.e. a similar model function and similar or smaller values of the uncertainty of the input quantities”. See also ISO 11929-1:2019, 5.3.

The examples, the models, and the data are exemplary and not normative.

In ISO 11929-1, a quite general model is specified as follows:

$$y = (x_1 - x_2 \cdot x_3 - x_4) \cdot \frac{x_6 \cdot x_8 \cdots}{x_5 \cdot x_7 \cdots} = (x_1 - x_2 x_3 - x_4) \cdot w \qquad (1)$$

with

$$w = \frac{x_6 \cdot x_8 \cdots}{x_5 \cdot x_7 \cdots} \qquad (2)$$

where $x_1 = r_g$ is the gross count rate and $x_2 = r_0$ is the background count rate. The other input quantities, x_i, are calibration, correction or influence quantities, or conversion factors, for instance the emission or response probability. In particular, x_3 is a shielding factor and x_4 an additional background correction quantity. If some of the input quantities are not involved, $x_i = 1 \quad (i = 3; i > 4)$, $x_4 = 0$ and $u(x_i) = 0$ shall be set for them.

NOTE 2 Generally, physical quantities are denoted by uppercase letters and have to be carefully distinguished from their values, denoted by the corresponding lowercase letters. However, for sake of simplicity no distinction is made between uppercase letters for quantities and lowercase letters for values throughout this document. Only lowercase letters are used because the examples deal with values rather than with quantities.

In this document, the simplest form of Formula (1) used for most examples for ease of understanding is:

$$y = (x_1 - x_2) \cdot w = (r_g - r_0) \cdot w \tag{3}$$

This simple model according to Formula (3) of a **counting measurement with small or moderate relative uncertainties** below 25 % such as in the majority of laboratory experiments is dealt with in Clause 6.

Small count numbers provide a particular problem in counting measurements since the information about the associated count rates is scarce. An example describing the measurement of alpha-particles with low count numbers according to ISO 11929-2:2019, A.1 is given in Clause 7.

Clause 8 gives an example of a counting measurement with **uncertain counting geometry** where the uncertainties of the calibration factor are generally large and where, in consequence, a dominating uncertainty exists for a quantity in the numerator of the calibration factor of Formula (2).

Clause 9 gives an example of a counting measurement in form of a **wipe test** where the uncertainties are generally large and where, in particular, a dominating uncertainty exists for a quantity in the denominator of the calibration factor of Formula (2).

Clause 10 is an example where **large uncertainties occur due to shielding of the background** as in the case of measurements with a portal monitor.

Clause 11 exemplifies also the **general model** according to Formula (1) by dealing with the clearance measurement of bulk samples of concrete with a special counting device and with consideration of natural radioactivity in the background.

Clause 12 exemplifies the **general model** according to Formula (1) by dealing with the γ-spectrometry of ^{235}U with an interference from ^{226}Ra.

Clause 13 deals with the case of measurements of ambient dose rates with a measuring device of unknown functionality and algorithm, i.e. a so-called **black box measurement** according to A.4 of ISO 11929-1 where no technical details about the measuring procedure are known and only readings of a device are available.

The particular cases of **counting measurements with unknown random influences** from the sample treatment according to ISO 11929-1:2019, A.2 are dealt with in Clause 14.

The case of **counting measurements with known random influences** from the sample treatment according to of ISO 11929-1:2019, A.3 is dealt with in Clause 15.

The examples in Clause 16 and Clause 17 demonstrate the applicability of ISO 11929-1 and ISO 11929-2 to non-counting measurements. The case of an **active personal dosemeter** is treated according ISO 11929-1 and ISO 11929-2 in Clause 16. The particlular feature of this example is that the standard uncertainty associated with the gross quantity is constant, thereby allowing to calculate the standard uncertainty as a function of an assumed true value of the measurand.

The case of a **dose rate measurement using a neutron area monitor** treated according to ISO 11929-1 and ISO 11929-2 is dealt with in Clause 17. In this example it is known that the measurement is a counting measurement. However, the actual gross and background count numbers are unknown. In spite of that, the standard uncertainty as a function of an assumed true value of the measurand can be calculated using a general feature of the relative uncertainty of the gross indication.

A simple example of the calculation of the calibration factor w is given in Annex A.

The examples are generic and are not intended to standardize measurement procedures for the respective applications. They shall only serve as explanations for the application of ISO 11929-1 and ISO 11929-2.

Calculations are performed according to ISO 11929-1 and ISO 11929-2 for each example in order to allow the reader to judge the appropriateness of both methods. In all examples, the probabilistically symmetric coverage intervals as well as the shortest coverage intervals are calculated. The limits of the coverage intervals and the best estimate and its associated standard uncertainty are also calculated in cases where the primary measurement result is below the decision threshold, in spite of the fact that ISO 11929 (all parts) requires only to calculate the coverage intervals if the primary measurement result exceeds the decision threshold.

Small $y/u(y)$-values, i.e. large relative uncertainties of the primary measurement result, can be due to small count numbers (example in Clause 7) or to large relative uncertainties of the calibration factor. The latter ones can be due to relatively large uncertainty of a component in the numerator of the calibration factor (example in Clause 8) or due to such a component in the denominator of the calibration factor (example in Clause 9).

A general assessment regarding the relevance of the choice of the coverage interval in the case of small $y/u(y)$-values given the decision threshold is not possible. Therefore, all characteristic limits, i.e. the decision threshold, the detection limit and the limits of the probabilistically symmetric and the shortest coverage interval, are calculated in this document for each example in order to allow discussing and judging about the relevance of the different coverage intervals given the actual data.

In models of evaluation with $x_3 = 1$, $u(x_3) = 0$ and $x_4 = 0$, $u(x_4) = 0$, the decision threshold does not depend on the uncertainty of the calibration factor. The decision threshold depends on the uncertainties of x_3 and x_4 in case of $x_3 \neq 1$ or $x_4 > 0$. The example of a portal monitor is one of the cases where the decision threshold depends on the uncertainty of the shielding factor. Also, the example for the general model (gamma-spectrometry of ^{235}U with interference by ^{226}Ra) is such a case as well as the example regarding the clearance measurement with consideration of natural radioactivity.

From the didactic point of view one has to distinguish four cases that often lead to misunderstandings and, therefore, need clarification.

This case is characterized by $y^* < y$ and $y^\# < y_r$. A primary result above the decision threshold was obtained. One decides to conclude that an effect of the sample was recognized. The detection limit is lower than the guideline value. One concludes that the measurement method is suitable for the measurement purpose.

See Clauses 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, and 17 for examples.

A frequent misunderstanding results from comparison of the primary measurement result with the detection limit. Such a comparison is not according to the stipulations of ISO 11929 (all parts). The only characteristic limit to be compared with the primary measurement result is the decision threshold, not the detection limit. If the primary measurement result exceeds the decision threshold one decides to conclude that an effect of the sample was recognized.

This decision to be made does not depend on whether the primary measurement result is below or above the detection limit. The only purpose of the detection limit is to characterize the quality of the measurement procedure by comparison with a guideline value. If the detection limit is lower than the guideline value, one concludes that the measurement method is suitable for the measurement purpose.

This case is characterized by $y < y^*$ and $y^\# < y_r$. A result below the decision threshold was obtained. One decides to conclude that no effect of the sample was recognized, though it might be present. The detection limit is lower than the guideline value. One concludes that the measurement method is suitable for the measurement purpose.

See Clause 7 for an example.

NOTE 3 Also in case that the primary measurement result is smaller than the decision threshold all characteristic values are calculated and presented in the examples.

This case is characterized by $y^* < y$ and $y_r < y^\#$. A result above the decision threshold was obtained. One decides to conclude that an effect of the sample was recognized. However, the detection limit is higher than the guideline value. Consequently, the measurement method is not suitable for the measurement purpose in principle.

Nevertheless, $y > y^*$ is a primary measurement result with its associated standard uncertainty, which needs no justification regarding to the suitability of the procedure.

See Clause 14 for an example.

This case is characterized by $y < y^*$ and $y_r < y^\#$. A primary result below the decision threshold was obtained. One decides to conclude that no effect of the sample was recognized. The detection limit is higher than the guideline value. One concludes that the measurement method is not suitable for the measurement purpose and does not satisfy the measurement purpose. In total, the procedure is not suitable and the measurement was not successful.

There is no example of such a pathological case in this document.

NOTE 4 In this document, the guideline value, y_r, in comparison with the detection limit, $y^\#$, simply serves to demonstrate the decision whether or not the measuring procedure is suitable for the measurement purpose. This document does not provide stipulations whether or not a measurement result, y, conforms to requirement from legal or other sources. Such assessments are beyond the scope of this document. An application of ISO 11929-1 to such conformity assessments can be found elsewhere[45].

5.3 General stipulations

The measurand and the characteristic values are calculated according to ISO 11929-1 and ISO 11929-2. First, the model of evaluation is given; then the input data as well as their PDFs are specified. The evaluation of the measurement and the calculation of the characteristic limits according ISO 11929-1 are presented in detail. The standard uncertainties are calculated according to the mathematical formalism of the ISO/IEC Guide 98-3.

The Monte Carlo calculations according to ISO 11929-2 were performed with the code UncertRadio[40][41]. The results of both evaluations (according to ISO 11929-1 and ISO 11929-2) are summarized. Finally, a comparison of the results obtained by application of ISO 11929-1 and ISO 11929-2 is given together with a documentation of the results and an assessment of the measurement.

For the purpose of comparison, both the probabilistically symmetric coverage interval and the shortest coverage interval are calculated. In a real case the regulator or customer has to decide which coverage interval has to be calculated.

Unless otherwise stated the following stipulations are valid throughout this document.

— The probabilities α and β for the calculation of the decision threshold and the detection limit are chosen $\alpha = \beta = 5$ %. A coverage probability $1 - \gamma = 95$ % is assumed for the coverage interval.

— The standard uncertainties of the durations of the measurements are neglected.

— The nuclear data used in the examples are in Reference [38].

— Before the measurement of a sample the background effect has to be measured. It is assumed that the background effect as well as the calibration factor and its standard uncertainty were determined by independent experiments. The standard uncertainty associated with the calibration factor is evaluated by applying the ISO/IEC Guide 98-3. See Annex A for an example of the determination of a calibration factor.

6 Counting measurements with small or moderate uncertainties

6.1 Definition of the task and general aspects

The activity of a radionuclide in a sample is investigated by a counting measurement. This is a simplified example according to the general model of ISO 11929-1.

The comparison between the primary measurement result and the decision threshold provides information on whether or not an activity of the sample is observed. If the primary measurement result is larger than the decision threshold, it is concluded that an activity of the sample has been observed.

A guideline value $a_r = 3\,\text{Bq}$ is to be compared with the detection limit. If the detection limit is smaller than the guide value, it is concluded that the measuring method is suitable for the measurement purpose.

6.2 Model of evaluation and standard uncertainty

The measurand is the activity in the sample. The primary result a of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation using preselection of the counting times:

$$a = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0}\right) \cdot w = \left(r_g - r_0\right) \cdot w = r_n \cdot w \tag{4}$$

The standard uncertainty $u(a)$ associated with the primary result a is calculated by:

$$u^2(a) = r_n^2 \cdot u^2(w) + w^2 \cdot u^2(r_n) = w^2 \cdot \left(\frac{n_g}{t_g^2} + \frac{n_0}{t_0^2}\right) + a^2 \cdot u_{rel}^2(w) \tag{5}$$

6.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 1. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 1.

Table 1 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	21 670	—	—	1
Count time of the gross measurement	t_g	1 200	—	—	s
Number of counts of the background effect	n_0	73 150	—	—	1
Count time of the background effect	t_0	12 000	—	—	s
Calibration factor	w	4,1	0,6	$\text{N}(\tilde{w}; w, u(w))$	Bq s
Intermediate values					
Gross count rate	r_g	18,06	0,123	$\text{Ga}(\tilde{r}_g; n_g, 1/t_g)$	s^{-1}
Count rate of the background effect	r_0	6,10	0,022 5	$\text{Ga}(\tilde{r}_0; n_0, 1/t_0)$	s^{-1}

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description.

6.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

6.4.1 Background effect

The background effect of the sample has been measured. The number of the counted pulses is $n_0 = 73\ 150$. This gives a count rate of

$$r_0 = \frac{n_0}{t_0} = 6{,}096\ \mathrm{s}^{-1} \tag{6}$$

The standard uncertainty of the count rate is given by:

$$u(r_0) = \sqrt{\frac{n_0}{t_0^2}} = 0{,}022\ 5\ \mathrm{s}^{-1} \tag{7}$$

6.4.2 Primary result and its associated standard uncertainty

The activity of the sample is calculated by using the Formula (4). This relation depends on the calibration factor *w*, which needs to be evaluated first, as well as its standard uncertainty $u(w)$.

$$w = 4{,}1\ \mathrm{Bq \cdot s}\ ,\ u(w) = 0{,}6\ \mathrm{Bq \cdot s}\ ,\ u_{\mathrm{rel}}(w) = 0{,}146 \tag{8}$$

The activity a of the sample is given by

$$a = r_{\mathrm{n}} \cdot w = \left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}} - r_0 \right) \cdot w = 49{,}05\ \mathrm{Bq} \tag{9}$$

with the standard uncertainty

$$u(a) = \sqrt{r_{\mathrm{n}}^2 \cdot u^2(w) + w^2 \cdot \left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}^2} + \frac{n_0}{t_0^2} \right)} = 7{,}20\ \mathrm{Bq} \tag{10}$$

6.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a} = \tilde{r}_{\mathrm{n}} w = \left(\tilde{r}_{\mathrm{g}} - \frac{n_0}{t_0} \right) w$ of the measurand, one expects a gross count rate $\tilde{r}_{\mathrm{g}} = \frac{\tilde{a}}{w} + \frac{n_0}{t_0}$ and with Formula (5) one obtains

$$\tilde{u}^2(\tilde{a}) = \tilde{r}_{\mathrm{n}}^2 \cdot u^2(w) + w^2 \cdot u^2(\tilde{r}_{\mathrm{n}}) = w^2 \cdot \left(\frac{\tilde{a}}{t_{\mathrm{g}} w} + \frac{n_0}{t_{\mathrm{g}} t_0} + \frac{n_0}{t_0^2} \right) + \tilde{a}^2 \cdot u_{\mathrm{rel}}^2(w) \tag{11}$$

6.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1 - \alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$a^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot \sqrt{w^2 \cdot \left(\frac{n_0}{t_{\mathrm{g}} t_0} + \frac{n_0}{t_0^2} \right)} = 0{,}504\ \mathrm{Bq} \tag{12}$$

The measurement result a exceeds the decision threshold a^*.

6.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$\begin{aligned} a^{\#} &= a^{*} + k_{1-\beta} \cdot \tilde{u}\left(a^{\#}\right) \\ &= a^{*} + k_{1-\beta} \cdot \sqrt{a^{\#2} \cdot u_{\mathrm{rel}}^{2}(w) + w^{2} \cdot \left(\frac{a^{\#}}{t_{\mathrm{g}} w} + \frac{n_0}{t_{\mathrm{g}} t_0} + \frac{n_0}{t_0^2}\right)} = 1{,}08\,\mathrm{Bq} \end{aligned} \tag{13}$$

Since $\alpha = \beta$, $k_{1-\alpha} = k_{1-\beta} = k$. The detection limit can also be calculated by the explicit Formula (14):

$$a^{\#} = a^{*} + k_{1-\beta} \cdot \tilde{u}\left(a^{\#}\right) = \frac{2 \cdot a^{*} + (k^{2} \cdot w)/t_{\mathrm{g}}}{1 - k^{2} \cdot u_{\mathrm{rel}}^{2}(w)} = 1{,}08\,\mathrm{Bq} \tag{14}$$

The guideline value a_{r} exceeds the detection limit $a^{\#}$.

6.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a exceeds the decision threshold a^{*}. With $\omega = \Phi[a/u(a)] = 1{,}00$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega \cdot (1 - \gamma/2) = 0{,}975$ and $q = 1 - \omega \cdot \gamma/2 = 0{,}975$, and hence the quantiles k_p and k_q are equal to 1,960. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a^{\triangleleft} = a - k_p \cdot u(a) = 34{,}94\,\mathrm{Bq} \text{ and } a^{\triangleright} = a + k_q \cdot u(a) = 63{,}15\,\mathrm{Bq} \tag{15}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated according to Formula (16). With $\omega = \Phi[a/u(a)] = 1{,}00$, one obtains $p = [1 + \omega \cdot (1 - \gamma)]/2 = 0{,}975$, and hence the quantile k_p is equal to 1,960. With this, the limits of the shortest coverage interval are

$$a^{<} = a - k_p \cdot u(a) = 34{,}94\,\mathrm{Bq} \text{ and } a^{>} = a + k_p \cdot u(a) = 63{,}15\,\mathrm{Bq} \tag{16}$$

6.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample and its associated standard uncertainty $u(\hat{a})$ are given by:

$$\hat{a} = a + \frac{u(a) \cdot \exp\left\{-a^{2}/\left[2u^{2}(a)\right]\right\}}{\omega\sqrt{2\pi}} = 49{,}05\,\mathrm{Bq} \tag{17}$$

and its associated standard uncertainty

$$u(\hat{a}) = \sqrt{u^{2}(a) - (\hat{a} - a) \cdot \hat{a}} = 7{,}20\,\mathrm{Bq} \tag{18}$$

6.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The available information is $\boldsymbol{a} = \{n_{\mathrm{g}}, t_{\mathrm{g}}, n_0, t_0, w, u(w)\}$; see 6.3 and Table 1. The results and characteristic limits are summarized in Table 2.

Applying ISO 11929-2 the marginal posterior for the true value of the measurand taking into account that the measurand is non-negative is given by:

$$f_A(\tilde{a}|\boldsymbol{a}) = C \cdot \mathrm{H}(\tilde{a}) \cdot \int_{-\infty}^{+\infty} \mathrm{Ga}(\tilde{r}_\mathrm{g}; n_\mathrm{g}, 1/t_\mathrm{g}) \cdot \mathrm{Ga}(\tilde{r}_0; n_0, 1/t_0) \cdot \mathrm{N}[\tilde{w}; w, u(w)] \cdot \delta\left[\tilde{a} - (\tilde{r}_\mathrm{g} - \tilde{r}_0) \cdot \tilde{w}\right] \mathrm{d}\tilde{r}_\mathrm{g}\, \mathrm{d}\tilde{r}_0\, \mathrm{d}\tilde{w} \qquad (19)$$

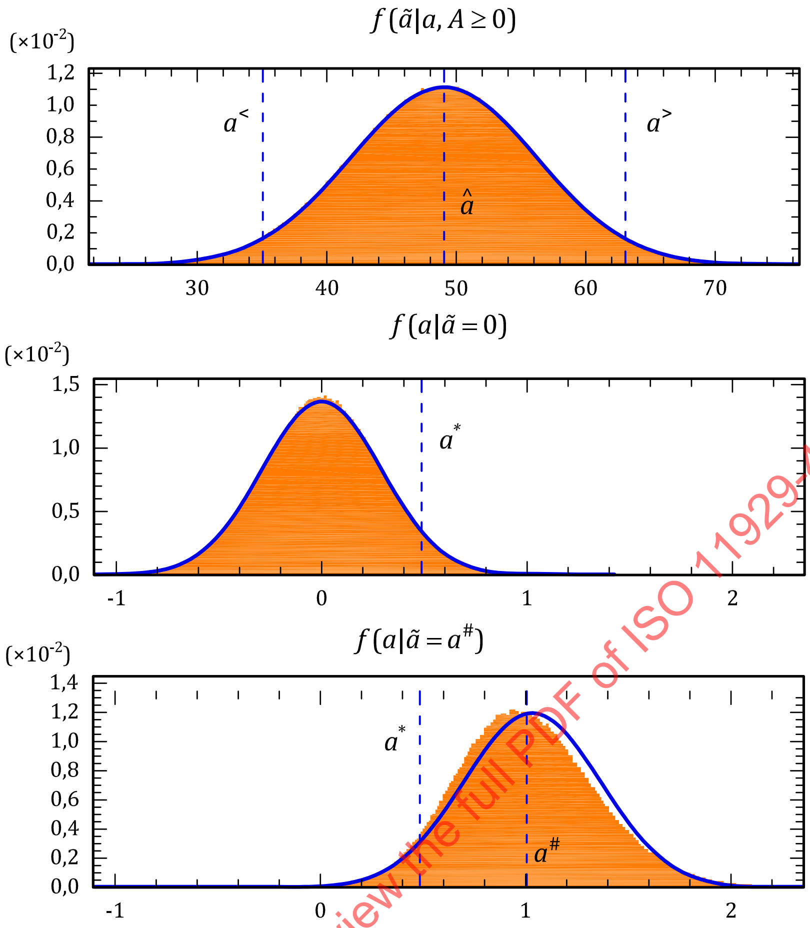
Analogous formulae are valid for the marginal posteriors of the other examples in this document if the PDFs under the integrals are replaced by those indicated in the tables in the clauses "Available information, input data, and specifications." Therefore, the respective posteriors are not repeated for each example.

The integral is solved using Monte Carlo methods using the code UncertRadio[40]. Annex B gives an exemplary description of the procedure following ISO 11929-2. The best estimate and its associated standard uncertainty are calculated as the mean and the square root of the variance of the PDF according to Formula (19); the limits of the coverage intervals are calculated as suitable quantiles of this PDF. The PDFs required for the calculation of the primary measurement result and its associated standard uncertainty as well as for the decision threshold and the detection limit are also given in Annex B.

The PDFs obtained by the Monte Carlo simulations are presented in Figure 1.

Table 2 — Results and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	49,05	49,05
Standard uncertainty associated with the primary result	$u(a)$	Bq	7,20	7,19
Relative standard uncertainty associated with the primary result	$u_\mathrm{rel}(a)$	1	0,147	0,147
Decision threshold	a^*	Bq	0,504	0,510
Detection limit	$a^\#$	Bq	1,08	1,05
Best estimate	$\hat{a}$	Bq	49,05	49,05
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	7,20	7,19
Relative uncertainty associated with the best estimate	$u_\mathrm{rel}(\hat{a})$	1	0,147	0,147
Lower limit of the probabilistically symmetric coverage interval	$a^\triangleleft$	Bq	34,94	34,98
Upper limit of the probabilistically symmetric coverage interval	$a^\triangleright$	Bq	63,15	63,17
Lower limit of the shortest coverage interval	$a^<$	Bq	34,94	34,94
Upper limit of the shortest coverage interval	$a^>$	Bq	63,15	63,13



NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 1 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

6.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a = 49{,}05\,\mathrm{Bq}$ exceeds the decision threshold $a^{*} = 0{,}504\,\mathrm{Bq}$. It is decided to conclude that an effect from the sample was recognized.

— The detection limit $a^{\#} = 1{,}08\,\mathrm{Bq}$ is below the guideline values $a_{\mathrm{r}} = 3\,\mathrm{Bq}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a^{\triangleleft} = 34{,}94\,\mathrm{Bq}$ and $a^{\triangleright} = 63{,}15\,\mathrm{Bq}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a^{<} = 34{,}94\,\mathrm{Bq}$ and $a^{>} = 63{,}15\,\mathrm{Bq}$.

— The best estimate is $\hat{a} = 49{,}05\ \mathrm{Bq}$ with an associated standard uncertainty $u(\hat{a}) = 7{,}20\ \mathrm{Bq}$.

Because the model of evaluation is linear and the relative uncertainty of the primary measurement result is small, the results obtained by application of ISO 11929-2 are practically identical with those obtained by ISO 11929-1:2019, Table 2. The ISO/IEC Guide 98-3, approximations assuming normal PDFs and using a Taylor expansion truncated after the linear term, hold.

The probabilistically symmetric coverage interval is equal to the shortest coverage interval. The best estimate and its associated standard uncertainty equal the primary measurement result and its associated standard uncertainty. According to Formula (B.3) in ISO 11929-2:2019, Annex B, this holds of for all results of measurements with $y/u(y) \geq 4$, i.e. for measurement procedures with relative uncertainties $u_{\mathrm{rel}}(y) \leq 0{,}25$.

For larger relative uncertainties of the primary measurement result deviations between the limits of the two coverage intervals and between the primary measurement result and the best estimate and their respective standard uncertainties occur. This can be due to small count numbers (see Clause 7) or due to large uncertainties of the calibration factor. With respect to uncertainties of the calibration factor according to Formula (2), one has to distinguish whether they result from quantities in the numerator or the denominator of Formula (2). These cases are dealt with in Clause 9 and Clause 10, respectively. A further origin of large uncertainties can be due to those of the shielding factor; see Clause 11 for an example.

7 Counting measurement with small count numbers

7.1 Definition of the task and general aspects

This is an example for the application of ISO 11929-2:2019, Annex A. The activity of a radionuclide in a sample is investigated by a counting measurement in which only very low count numbers are obtained as e.g. in some applications of counting of α-particles. Small count numbers provide a particular problem in counting measurements since the approximation of the PDFs of the count rates by Gaussian distributions does no longer hold.

The comparison between the primary measurement result and the decision threshold provides information on whether or not an activity of the sample is observed. If the primary measurement result is larger than the decision threshold, it is concluded that an activity of the sample has been observed.

A guideline value $a_{\mathrm{r}} = 0{,}1\,\mathrm{Bq}$ is to be compared with the detection limit. If the detection limit is smaller than the guideline value, it is concluded that the measuring method is suitable for the measurement purpose.

7.2 Model of evaluation and standard uncertainty

The measurand is the activity in the sample. The primary result a of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation:

$$a = \left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}} - \frac{n_0}{t_0}\right) \cdot w = \left(r_{\mathrm{g}} - r_0\right) \cdot w = r_{\mathrm{n}} \cdot w \tag{20}$$

The associated standard uncertainty is calculated by:

$$u^2(a) = w^2 \cdot u^2(r_{\mathrm{n}}) + r_{\mathrm{n}}^2 \cdot u^2(w) = w^2 \cdot \left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}^2} + \frac{n_0}{t_0^2}\right) + a^2 \cdot u_{\mathrm{rel}}^2(w) \tag{21}$$

7.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 3. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 3.

Table 3 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	6	—	—	1
Count time of the gross measurement	t_g	1 200	—	—	s
Number of counts of the background effect	n_0	3	—	—	1
Count time of the background effect	t_0	1 200	—	—	s
Calibration factor	w	4,1	0,6	$\mathrm{N}(\tilde{w};w,u(w))$	Bq s
Intermediate values					
Gross count rate	r_g	5,00E-3	2,04E-3	$\mathrm{Ga}(\tilde{r}_g;n_g,1/t_g)$	s^{-1}
Count rate of the background effect	r_0	2,50E-3	1,44E-3	$\mathrm{Ga}(\tilde{r}_0;n_0,1/t_0)$	s^{-1}

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also of ISO 11929-2:2019, 6.3 for a detailed description.

7.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

7.4.1 Background effect

The background effect of the sample has been measured. The number of the counted pulses is $n_0 = 3$. This gives a count rate of

$$r_0 = \frac{n_0}{t_0} = 0{,}002\,50\,\mathrm{s}^{-1} \tag{22}$$

The standard uncertainty of the count rate is given by:

$$u(r_0) = \sqrt{\frac{n_0}{t_0^2}} = 0{,}001\,44\,\mathrm{s}^{-1} \tag{23}$$

7.4.2 Primary result and its associated standard uncertainty

The activity of the sample is calculated by using the Formula (20). This relation depends on the calibration factor *w*, which needs to be evaluated first, as well as its standard uncertainty $u(w)$.

The calibration factor is given by:

$$w = 4{,}1\ \text{Bq}\cdot\text{s} \text{ with } u(w) = 0{,}6\ \text{Bq}\cdot\text{s and } u_{\text{rel}}(w) = 0{,}146 \tag{24}$$

The activity a of the sample is given by:

$$a = r_{\text{n}} \cdot w = \left(\frac{n_{\text{g}}}{t_{\text{g}}} - r_0\right) \cdot w = 0{,}0103\ \text{Bq} \tag{25}$$

with the standard uncertainty

$$u(a) = \sqrt{r_{\text{n}}^2 \cdot u^2(w) + w^2 \cdot \left(\frac{n_{\text{g}}}{t_{\text{g}}^2} + \frac{n_0}{t_0^2}\right)} = 0{,}0104\ \text{Bq} \tag{26}$$

and

$$u_{\text{rel}}(a) = u(a)/a = 1{,}01 \tag{27}$$

7.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a} = \tilde{r}_{\text{n}} \cdot w$ of the measurand, one expects a gross count rate $\tilde{r}_{\text{g}} = \frac{\tilde{a}}{w} + \frac{n_0}{t_0}$ and with Formula (21) one obtains

$$\tilde{u}^2(\tilde{a}) = \tilde{r}_{\text{n}}^{\,2} \cdot u^2(w) + w^2 \cdot u^2(\tilde{r}_{\text{n}}) = w^2 \cdot \left(\frac{\tilde{a}}{t_{\text{g}} w} + \frac{n_0}{t_{\text{g}} t_0} + \frac{n_0}{t_0^2}\right) + \tilde{a}^2 \cdot u_{\text{rel}}^2(w) \tag{28}$$

7.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1 - \alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$a^* = k_{1-\alpha}\, \tilde{u}(0) = k_{1-\alpha} \cdot \sqrt{w^2 \cdot \left(\frac{n_0}{t_{\text{g}} t_0} + \frac{n_0}{t_0^2}\right)} = 0{,}0138\ \text{Bq} \tag{29}$$

The measurement result a is below the decision threshold a^*.

7.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$a^\# = a^* + k_{1-\beta} \cdot \sqrt{a^{\#2} \cdot u_{\text{rel}}^2(w) + w^2 \cdot \left(\frac{a^\#}{t_{\text{g}} w} + \frac{n_0}{t_{\text{g}} t_0} + \frac{n_0}{t_0^2}\right)} = 0{,}039\ \text{Bq} \tag{30}$$

Since $\alpha = \beta$, $k_{1-\alpha} = k_{1-\beta} = k$. The detection limit can also be calculated by the explicit Formula (31):

$$a^\# = a^* + k_{1-\beta} \cdot \tilde{u}\left(a^\#\right) = \frac{2 \cdot a^* + (k^2 \cdot w)/t_{\text{g}}}{1 - k^2 \cdot u_{\text{rel}}^2(w)} = 0{,}039\ \text{Bq} \tag{31}$$

The guideline value a_{r} exceeds the detection limit $a^\#$.

7.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated exemplarily though the measurement result a is below the decision threshold a^*. With $\omega = \Phi(a/u(a)) = 0{,}837$ and $\gamma = 0{,}05$ one obtains the probabilities $p = \omega \cdot (1-\gamma/2) = 0{,}816$ and $q = 1 - \omega \cdot \gamma/2 = 0{,}979$, and hence the quantiles k_p and k_q are equal to 0,908 and 2,034, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a^{\triangleleft} = a - k_p \cdot u(a) = 0{,}000\,854\,\text{Bq} \text{ and } a^{\triangleright} = a + k_q \cdot u(a) = 0{,}031\,3\,\text{Bq} \tag{32}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi(a/u(a)) = 0{,}837$, one obtains $p = (1 + \omega \cdot (1-\gamma))/2 = 0{,}897$ and hence the quantile k_p is equal to 1,27. With this, the limits of the shortest coverage interval according to Formula (33) are

$$a^{<} = -0{,}002\ 91\,\text{Bq} \text{ and } a^{>} = a + k_p \cdot u(a) = 0{,}023\,4\,\text{Bq} \tag{33}$$

Since the lower limit of the shortest coverage interval according to Formula (33) is negative, one has to proceed according to Formula (34) and obtains with $q = 1 - \omega \cdot \gamma = 0{,}958$ the quantile $k_q = 1{,}728$

$$a^{<} = 0\,\text{Bq} \text{ and } a^{>} = a + k_q \cdot u(a) = 0{,}028\,2\,\text{Bq} \tag{34}$$

7.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample is given by:

$$\hat{a} = a + \frac{u(a) \cdot \exp\left(-a^2 / \left(2u^2(a)\right)\right)}{\omega\sqrt{2\pi}} = 0{,}013\ 3\ \text{Bq} \tag{35}$$

with its associated standard uncertainty $u(\hat{a})$

$$u(\hat{a}) = \sqrt{u^2(a) - (\hat{a} - a) \cdot \hat{a}} = 0{,}008\ 20\ \text{Bq} \tag{36}$$

7.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

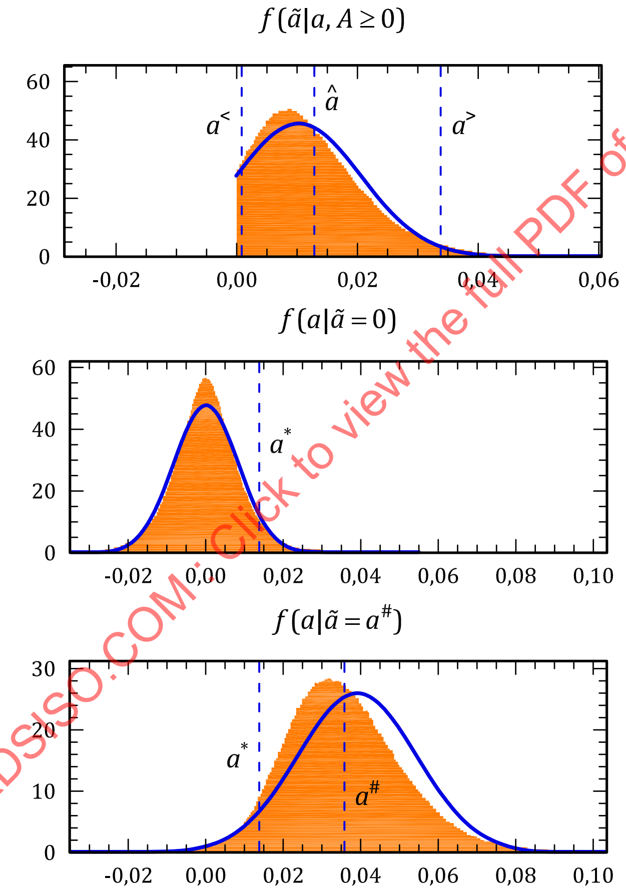
The results and characteristic limits are summarized in Table 4 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 2.

Table 4 — Result and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	1,03E-2	1,02E-2
Standard uncertainty associated with the primary result	$u(a)$	Bq	1,04E-2	1,05E-2
Relative standard uncertainty associated with the primary result	$u_{rel}(a)$	1	1,01	1,03
Decision threshold	a^*	Bq	1,38E-2	1,38E-2
Detection limit	$a^{\#}$	Bq	3,90E-2	3,58E-2
Best estimate	$\hat{a}$	Bq	1,33E-2	1,29E-2
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	8,20E-3	8,75E-3

Table 4 *(continued)*

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{a})$	1	0,62	0,68
Lower limit of the probabilistically symmetric coverage interval	$a^{\lhd}$	Bq	8,54E-4	8,24E-4
Upper limit of the probabilistically symmetric coverage interval	$a^{\rhd}$	Bq	3,13E-2	3,38E-2
Lower limit of the shortest coverage interval	$a^{<}$	Bq	0	0
Upper limit of the shortest coverage interval	$a^{>}$	Bq	2,82E-2	2,93E-2



NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 2 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

7.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a = 0{,}0103\ \mathrm{Bq}$ is below the decision threshold $a^{*} = 0{,}0138\ \mathrm{Bq}$. It is decided to conclude that no effect of the sample was recognized. In spite of that all characteristic values were calculated and documented in this example.

— The detection limit $a^{\#} = 0{,}039\,\text{Bq}$ is below the guideline value $a_r = 0{,}1\,\text{Bq}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a^{\triangleleft} = 0{,}000\,854\,\text{Bq}$ and $a^{\triangleright} = 0{,}031\,3\,\text{Bq}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a^{<} = 0\,\text{Bq}$ and $a^{>} = 0{,}028\,2\,\text{Bq}$.

— The best estimate is $\hat{a} = 0{,}013\,3\ \text{Bq}$ with an associated standard uncertainty $u(\hat{a}) = 0{,}008\,2\ \text{Bq}$.

There are just minor differences between the results obtained by applying ISO 11929-1 and ISO 11929-2, which are due to the deviation between the Gaussian distributions and the actual posterior distributions calculated according to ISO 11929-2; see Figure 2.

If large relative uncertainties of the primary result are due to the uncertainties of the count rates, this does not affect the existence of the detection limit calculated according to ISO 11929-1. Also in this extreme case of low count numbers, the application of ISO 11929-1 is justified.

7.7 An alternative example of a measurement with small count numbers

7.7.1 General

The data of an alternative example are given in Table 5. The input data differ from the example of Table 3 just with respect of the count numbers, which are now $n_0 = 1$ and $n_g = 4$.

Table 5 — Input quantities and data

Quantity	symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	4	—	—	1
Count time of the gross measurement	t_g	1 200	—	—	s
Number of counts of the background effect	n_0	1	—	—	1
Count time of the background effect	t_0	1 200	—	—	s
Calibration factor	w	4,1	0,6	$\text{N}(\tilde{w};w,u^2(w))$	s Bq
Intermediate values					
Gross count rate	r_g	3,33E-3	1,67E-3	$\text{Ga}(\tilde{r}_g;n_g,1/t_g)$	s^{-1}
Count rate of the background effect	r_0	8,33E-4	8,33E-4	$\text{Ga}(\tilde{r}_0;n_0,1/t_0)$	s^{-1}

NOTE This case with just one background count is regarded as a limiting case. In any reasonable single-channel counting measurement it is expected that at least one count is obtained in both, the gross and the background measurement. The case of a single-channel counting measurement with zero background counts is considered as a pathological case where the measurement or even the measurement method is insufficient.

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also of ISO 11929-2:2019, 6.3 for a detailed description.

7.7.2 Background effect

The background effect of the sample has been measured. The number of the counted pulses is $n_0 = 1$. This gives a count rate of

$$r_0 = \frac{n_0}{t_0} = 0{,}000\,833\,\mathrm{s}^{-1} \tag{37}$$

The standard uncertainty of the count rate is given by:

$$u(r_0) = \sqrt{\frac{n_0}{t_0^2}} = 0{,}000\,833\,\mathrm{s}^{-1} \tag{38}$$

7.7.3 Primary result and its associated standard uncertainty

The activity of the sample is calculated by using the Formula (20). This relation depends on the calibration factor w, which needs to be evaluated first, as well as its standard uncertainty $u(w)$.

The calibration factor is given by:

$$w = 4{,}1\ \mathrm{Bq \cdot s}\ \text{ with }\ u(w) = 0{,}6\ \mathrm{Bq \cdot s}\ \text{and}\ u_{\mathrm{rel}}(w) = 0{,}146 \tag{39}$$

The activity a of the sample is given by:

$$a = r_\mathrm{n} \cdot w = \left(\frac{n_\mathrm{g}}{t_\mathrm{g}} - r_0 \right) \cdot w = 0{,}0103\,\mathrm{Bq} \tag{40}$$

with the standard uncertainty

$$u(a) = \sqrt{r_\mathrm{n}^2 \cdot u^2(w) + w^2 \cdot \left(\frac{n_\mathrm{g}}{t_\mathrm{g}^2} + \frac{n_0}{t_0^2} \right)} = 0{,}007\,79\,\mathrm{Bq} \tag{41}$$

and

$$u_{\mathrm{rel}}(a) = u(a)/a = 0{,}756 \tag{42}$$

7.7.4 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a} = \tilde{r}_\mathrm{n} \cdot w$ of the measurand one expects a gross count rate $r_\mathrm{g} = \frac{\tilde{a}}{w} + \frac{n_0}{t_0}$ and with Formula (21) one obtains

$$\tilde{u}^2(\tilde{a}) = \tilde{r}_\mathrm{n}^2 \cdot u^2(w) + w^2 \cdot u^2(\tilde{r}_\mathrm{n}) = \tilde{a}^2 \cdot u_{\mathrm{rel}}^2(w) + w^2 \cdot \left(\frac{\tilde{a}}{t_\mathrm{g} w} + \frac{n_0}{t_\mathrm{g} t_0} + \frac{n_0}{t_0^2} \right) \tag{43}$$

7.7.5 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1 - \alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$a^* = k_{1-\alpha}\,\tilde{u}(0) = k_{1-\alpha} \cdot \sqrt{w^2 \cdot \left(\frac{n_0}{t_g t_0} + \frac{n_0}{t_0^2}\right)} = 0{,}007\ 95\,\mathrm{Bq} \tag{44}$$

The measurement result a is above the decision threshold a^*.

7.7.6 Detection limit

For β = 5 %, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$a^\# = a^* + k_{1-\beta} \cdot \sqrt{a^{\#2} \cdot u_{\mathrm{rel}}^2(w) + w^2 \cdot \left(\frac{a^\#}{t_g w} + \frac{n_0}{t_g t_0} + \frac{n_0}{t_0^2}\right)} = 0{,}0267\,\mathrm{Bq} \tag{45}$$

Since $\alpha = \beta$, $k_{1-\alpha} = k_{1-\beta} = k$. The detection limit can also be calculated by the explicit Formula (46):

$$a^\# = a^* + k_{1-\beta} \cdot \tilde{u}\left(a^\#\right) = \frac{2 \cdot a^* + (k^2 \cdot w)/t_g}{1 - k^2 \cdot u_{\mathrm{rel}}^2(w)} = 0{,}0267\,\mathrm{Bq} \tag{46}$$

The guideline value a_r exceeds the detection limit $a^\#$.

7.7.7 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a exceeds the decision threshold a^*. With $\omega = \Phi[a/u(a)] = 0{,}907$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega \cdot (1 - \gamma/2) = 0{,}884$ and $q = 1 - \omega \cdot \gamma/2 = 0{,}977$, and hence the quantiles k_p and k_q are equal to 1,20 and 2,00, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a^\triangleleft = a - k_p \cdot u(a) = 0{,}000\,970\,\mathrm{Bq} \text{ and } a^\triangleright = a + k_q \cdot u(a) = 0{,}0258\,\mathrm{Bq} \tag{47}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[a/u(a)] = 0{,}907$, one obtains $p = [1 + \omega \cdot (1 - \gamma)]/2 = 0{,}931$, and hence the quantile k_p is equal to 1,48. With this the limits of the shortest coverage interval according to Formula (48) were

$$a^< = a - k_p \cdot u(a) = -0{,}00124\,\mathrm{Bq} \text{ and } a^> = a + k_p \cdot u(a) = 0{,}0218\,\mathrm{kBq} \tag{48}$$

Since the lower limit of the shortest coverage interval according to Formula (48) is negative, one has to proceed according to Formula (49) and obtains with $q = 1 - \omega \cdot \gamma = 0{,}955$ the quantile $k_q = 1{,}692$

$$a^< = 0\,\mathrm{Bq} \text{ and } a^> = a + k_q \cdot u(a) = 0{,}023\ 4\,\mathrm{Bq} \tag{49}$$

7.7.8 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample is given by:

$$\hat{a} = a + \frac{u(a) \cdot \exp\left\{-a^2 / \left[2u^2(a)\right]\right\}}{\omega\sqrt{2\pi}} = 0{,}011\ 7\ \mathrm{Bq} \tag{50}$$

with its associated standard uncertainty $u(\hat{a})$

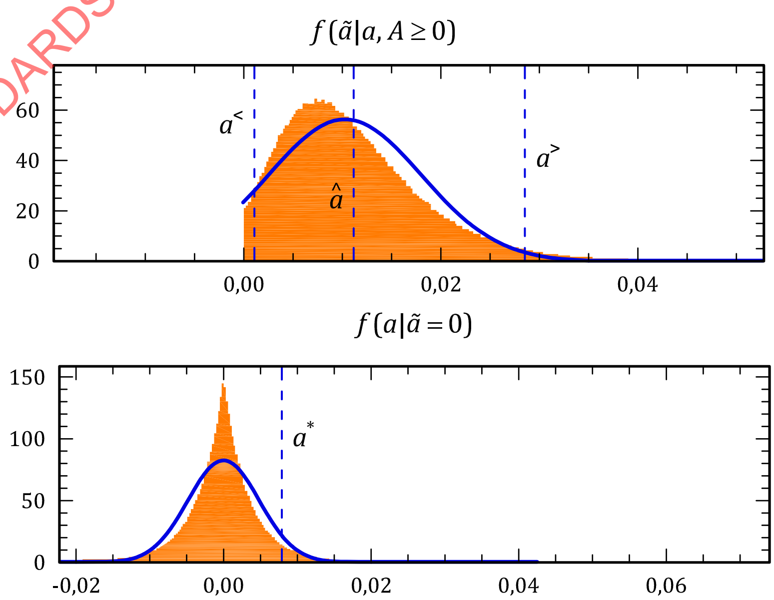
$$u(\hat{a}) = \sqrt{u^2(a) - (\hat{a} - a) \cdot \hat{a}} = 0{,}006\ 62\ \text{Bq} \qquad (51)$$

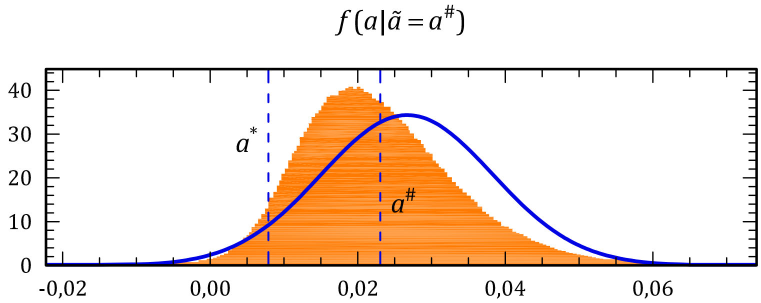
7.8 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The results and characteristic limits are summarized in Table 6 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 3.

Table 6 — Results and characteristic limits

Results	symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	1,03E-2	1,02E-2
Standard uncertainty associated with the primary result	$u(a)$	Bq	7,79E-3	7,86E-3
Relative standard uncertainty associated with the primary result	$u_{rel}(a)$	1	0,756	0,771
Decision threshold	a^*	Bq	7,95E-3	7,91E-3
Detection limit	$a^\#$	Bq	2,67E-2	2,30E-2
Best estimate	$\hat{a}$	Bq	1,17E-2	1,12E-2
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	6,62E-3	7,20E-3
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{a})$	1	0,566	0,643
Lower limit of the probabilistically symmetric coverage interval	$a^\triangleleft$	Bq	9,70E-4	1,06E-3
Upper limit of the probabilistically symmetric coverage interval	$a^\triangleright$	Bq	2,58E-2	2,86E-2
Lower limit of the shortest coverage interval	$a^<$	Bq	0	0
Upper limit of the shortest coverage interval	$a^>$	Bq	2,34E-2	2,47E-2





NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 3 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

7.9 Assessment of the alternative example and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a = 0{,}0103\,\text{Bq}$ exceeds the decision threshold $a^* = 0{,}007\ 95\,\text{Bq}$. It is decided to conclude that an effect of the sample was recognized.

— The detection limit $a^\# = 0{,}026\ 7\,\text{Bq}$ is below the guideline values $a_\text{r} = 0{,}1\,\text{Bq}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a^\triangleleft = 0{,}000\,97\,\text{Bq}$ and $a^\triangleright = 0{,}026\,\text{Bq}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a^< = 0\,\text{Bq}$ and $a^> = 0{,}023\,\text{Bq}$.

— The best estimate is $\hat{a} = 0{,}011\ 7\ \text{Bq}$ with an associated standard uncertainty $u(\hat{a}) = 0{,}006\ 6\ \text{Bq}$.

There are just minor differences between the results obtained by applying ISO 11929-1 and ISO 11929-2, which are due to the deviation between the Gaussian distributions and the actual posterior distributions calculated according to ISO 11929-2; see Figure 3.

If large relative uncertainties of the primary result are due to the uncertainties of the count rates, this does not affect the existence of the detection limit calculated according to ISO 11929-1. Also in this extreme case of low count numbers the application of ISO 11929-1 is justified.

8 Counting measurements with large uncertainties in the numerator of the calibration factor

8.1 Definition of the task and general aspects

The measurand is the activity of a sample with an uncertain counting geometry. Consequently the calibration factor has large uncertainties. This is a simplified example according to the general model of ISO 11929-1.

This measurement is an example for a case where the uncertainty of one of the input quantities is dominating.

The comparison between the primary measurement result and the decision threshold provides information on whether or not an activity of the sample is observed. If the primary measurement result is larger than the decision threshold, it is concluded that an activity has been observed.

A guideline value $a_r = 10\,\text{Bq}$ is to be compared with the detection limit. If the detection limit is smaller than the guide value, it is concluded that the measuring method is suitable for the measurement purpose.

8.2 Model of evaluation and standard uncertainty

The measurand is the activity in the sample. The primary result a of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation:

$$a = \left(r_g - r_0\right) \cdot w = r_n \cdot w = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0}\right) \cdot w \qquad (52)$$

The associated standard uncertainty is calculated by:

$$u^2(a) = w^2 \cdot u^2(r_n) + r_n^2 \cdot u^2(w) = w^2 \cdot \left(\frac{n_g}{t_g^2} + \frac{n_0}{t_0^2}\right) + a^2 \cdot u_{rel}^2(w) \qquad (53)$$

8.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 7. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 7.

Table 7 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	21 670	—	—	1
Count time of the gross measurement	t_g	1 200	—	—	s
Number of counts of the background effect	n_0	73 150	—	—	1
Count time of the background effect	t_0	12 000	—	—	s
Calibration factor	w	11,5	4,907 5	R($\tilde{w}$;3,20)	s Bq
Intermediate values					
Gross count rate	r_g	18,06	0,123	Ga($\tilde{r}_g$; n_g, $1/t_g$)	s^{-1}
Count rate of the background effect	r_0	6,096	0,022 5	Ga($\tilde{r}_0$; n_0, $1/t_0$)	s^{-1}

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description. It is assumed in this example that the calibration is badly defined and that only the limits of a range of calibration factors (3 s·Bq to 20 s·Bq) are known. This leads to assuming a rectangular PDF for the calibration factor and a large relative standard uncertainty of the calibration factor of $u_{rel}(w) = 0,43$.

8.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

8.4.1 Background effect

The count rate of the background effect of the sample measurement has been measured. The number of counted pulses is $n_g = 73\,150$. This gives a background count rate of

$$r_0 = \frac{n_0}{t_0} = 6{,}096\,\mathrm{s}^{-1} \tag{54}$$

Its standard uncertainty associated with the background count rate is given by:

$$u(r_0) = \sqrt{\frac{n_0}{t_0^2}} = 0{,}0225\,\mathrm{s}^{-1} \tag{55}$$

8.4.2 Primary result and its associated standard uncertainty

The activity is calculated from Formula (52). This formula depends on the calibration factor, the input quantities of which and their associated standard uncertainties were determined independently. This yielded a calibration factor of

$$w = 11{,}5\,\mathrm{Bq}\cdot\mathrm{s} \tag{56}$$

with the associated standard uncertainty

$$u(w) = 4{,}9075\ \mathrm{Bq}\cdot\mathrm{s} \tag{57}$$

The primary result of the measurement is given by:

$$a = (n_g / t_g - n_0 / t_0)\cdot w = 137{,}6\,\mathrm{Bq} \tag{58}$$

This yields the standard uncertainty associated with the activity

$$u(a) = \sqrt{w^2 \cdot \left(\frac{n_g}{t_g^2} + \frac{n_0}{t_0^2}\right) + \left(\frac{n_g}{t_g} - \frac{n_0}{t_0}\right)^2 \cdot u^2(w)} = 58{,}7\,\mathrm{Bq} \tag{59}$$

8.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ of the measurand is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a} = (\tilde{r}_g - r_0)\cdot w$ one expects $\tilde{n}_g = t_g \cdot (\tilde{a}/w + r_0)$. This yields with Formula (53)

$$\tilde{u}^2(\tilde{a}) = w^2 \cdot ((\tilde{a}/w + r_0)/t_g + r_0/t_0) + \tilde{a}^2 \cdot u_{\mathrm{rel}}^2(w) \tag{60}$$

8.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$a^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot \sqrt{w^2 \cdot (r_0/t_g + r_0/t_0)} = 1{,}41\,\mathrm{Bq} \tag{61}$$

The measured primary result a exceeds the decision threshold a^*.

8.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$a^{\#} = a^{*} + k_{1-\beta} \cdot \sqrt{w^2 \cdot ((a^{\#} / w + r_0) / t_g + r_0 / t_0) + a^{\#2} u_{rel}^2(w)} = 5{,}63 \text{ Bq} \quad (62)$$

Since $\alpha = \beta$, $k_{1-\alpha} = k_{1-\beta} = k$, the detection limit can also be calculated by the explicit Formula (63):

$$a^{\#} = a^{*} + k_{1-\beta} \cdot \tilde{u}\left(a^{\#}\right) = \frac{2 \cdot a^{*} + (k^2 \cdot w) / t_g}{1 - k^2 \cdot u_{rel}^2(w)} = 5{,}63 \text{ Bq} \quad (63)$$

The guideline value $a_{s,r}$ exceeds the detection limit $a^{\#}$.

8.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a exceeds the decision threshold a^{*} . With $\omega = \Phi[a / u(a)] = 0{,}990$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega \cdot (1 - \gamma / 2) = 0{,}966$ and $q = 1 - \omega \cdot \gamma / 2 = 0{,}975$, and hence the quantiles k_p and k_q are equal to 1,821 and 1,964, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a^{\triangleleft} = a - k_p \cdot u(a) = 30{,}66 \text{ Bq} \text{ and } a^{\triangleright} = a + k_q \cdot u(a) = 252{,}91 \text{ Bq} \quad (64)$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[a / u(a)] = 0{,}990$, one obtains $p = [1 + \omega \cdot (1 - \gamma)] / 2 = 0{,}970$, and hence the quantile k_p is equal to 1,887. With this, the limits of the shortest coverage interval according to Formula (65) were:

$$a^{<} = a - k_p \cdot u(a) = 26{,}73 \text{ Bq} \text{ and } a^{>} = a + k_p \cdot u(a) = 248{,}41 \text{ Bq} \quad (65)$$

8.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample is given by:

$$\hat{a} = a + \frac{u(a) \cdot \exp\left\{-a^2 / \left[2u^2(a)\right]\right\}}{\omega \sqrt{2\pi}} = 139{,}09 \text{ Bq} \quad (66)$$

with its associated standard uncertainty $u(\hat{a})$

$$u(\hat{a}) = \sqrt{u^2(a) - (\hat{a} - a)\hat{a}} = 56{,}89 \text{ Bq} \quad (67)$$

8.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

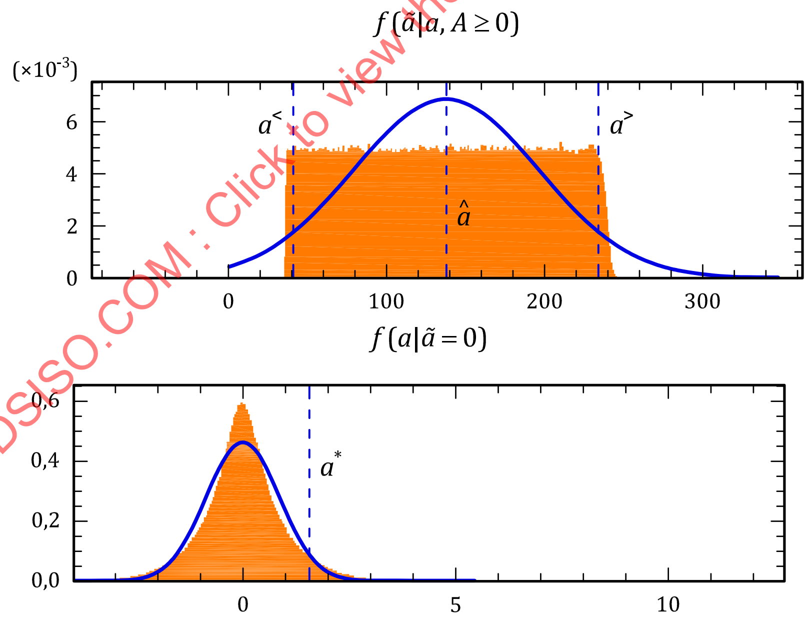
The results and characteristic limits are summarized in Table 8 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 4.

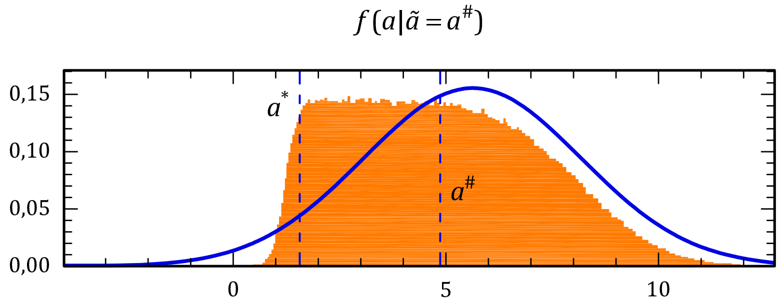
Table 8 — Result and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	138	137

Table 8 *(continued)*

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Standard uncertainty associated with the primary result	$u(a)$	Bq	59	59
Relative standard uncertainty associated with the primary result	$u_{rel}(a)$	1	0,428	0,431
Decision threshold	a^*	Bq	1,41	1,56
Detection limit	$a^\#$	Bq	5,63	4,87
Best estimate	$\hat{a}$	Bq	139	138
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	57	59
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{a})$	1	0,410	0,428
Lower limit of the probabilistically symmetric coverage interval	$a^\triangleleft$	Bq	31	41
Upper limit of the probabilistically symmetric coverage interval	$a^\triangleright$	Bq	253	234
Lower limit of the shortest coverage interval	$a^<$	Bq	27	40
Upper limit of the shortest coverage interval	$a^>$	Bq	248	233





NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 4 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

8.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a = 138\,\text{Bq}$ exceeds the decision threshold $a^* = 1{,}41\,\text{Bq}$. It is decided to conclude that an effect of the sample was recognized.

— The detection limit $a^{\#} = 5{,}63\,\text{Bq}$ is below the guideline values $a_r = 10\,\text{Bq}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a^{\triangleleft} = 30{,}7\,\text{Bq}$ and $a^{\triangleright} = 253\,\text{Bq}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a^{<} = 26{,}7\,\text{Bq}$ and $a^{>} = 248\,\text{Bq}$.

— The best estimate is $\hat{a} = 139$ Bq with an associated standard uncertainty $u(\hat{a}) = 57$ Bq.

The results obtained by applying ISO 11929-1 and ISO 11929-2 differ with regard to the detection limits and the limits of the coverage intervals. This is due to the deviation between the Gaussian distributions and the actual posterior distributions calculated according to ISO 11929-2 (see Figure 4). The rectangular PDF of the calibration factor is responsible for the differences. The differences between the best estimates and their associated standard uncertainties are small because the central estimates of the rectangular distribution of the calibration factor is identical to that of the normal distribution applied in ISO 11929-1.

Considering the differences in the limits of the coverage intervals, the application of ISO 11929-2 is to be preferred if there are large and dominating uncertainties in the numerator of the calibration factor.

9 Counting measurements with large uncertainties in the denominator of the calibration factor

9.1 Definition of the task and general aspects

This example deals with a wipe test. The measurand is the activity per unit surface. This is a generic example of an indirect measurement of a surface contamination using a wipe test. It does not change any stipulations laid down in ISO 7503 (all parts) regarding measurement and evaluation of surface contamination. This is a simplified example according to the general model of ISO 11929-1.

It is assumed that an area s of a surface has been wiped with a wiping efficiency e_{w}. The wipe, i.e. the sample, is measured by an unspecified detector with the counting efficiency e_{D}. For simplicity it is assumed that the surface emission rate measured is equal to the count rate caused by the surface related activity a_{s}. This measurement is an example for a case where the uncertainty of one of the input quantities is dominating.

The comparison between the primary measurement result and the decision threshold provides information on whether or not an activity of the sample is observed. If the primary measurement result is larger than the decision threshold, it is concluded that an activity of the sample has been observed.

A guideline value $a_{\mathrm{s,r}} = 0{,}5\,\mathrm{Bq \cdot cm^{-2}}$ is to be compared with the detection limit. If the detection limit is smaller than the guide value, it is concluded that the measuring method is suitable for the measurement purpose.

9.2 Model of evaluation and standard uncertainty

The measurand is the surface related activity in the sample. The primary result a_{s} of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation:

$$a_{\mathrm{s}} = \frac{n_{\mathrm{g}} / t_{\mathrm{g}} - n_0 / t_0}{e_{\mathrm{D}} \cdot s \cdot e_{\mathrm{w}}} = (n_{\mathrm{g}} / t_{\mathrm{g}} - n_0 / t_0) \cdot w = \frac{r_{\mathrm{g}} - r_0}{e_{\mathrm{D}} \cdot s \cdot e_{\mathrm{w}}} \tag{68}$$

The calibration factor is calculated by:

$$w = \frac{1}{e_{\mathrm{D}} \cdot s \cdot e_{\mathrm{w}}} \tag{69}$$

The relative standard uncertainty of the calibration factor $u_{\mathrm{rel}}(w) = u(w)/w$ is calculated by:

$$u_{\mathrm{rel}}(w) = \sqrt{u_{\mathrm{rel}}^2(e_{\mathrm{D}}) + u_{\mathrm{rel}}^2(s) + u_{\mathrm{rel}}^2(e_{\mathrm{w}})} \tag{70}$$

9.3 Available information, input data, and specifications

The efficiency of the detector e_{D} and its associated standard uncertainty $u(e_{\mathrm{D}})$ were independently determined to be $e_{\mathrm{D}} = 0{,}30\ \mathrm{s^{-1} \cdot Bq^{-1}}$ and $u(e_{\mathrm{D}}) = 0{,}015\,5\ \mathrm{s^{-1} \cdot Bq^{-1}}$.

The area wiped was chosen as $s = 100\ \mathrm{cm^2}$ and its associated relative standard uncertainty was set to $u_{\mathrm{rel}}(s) = 10\ \%$ by expert guess.

For the wiping efficiency e_{w} past experience was that it randomly varied in the interval $e_{\mathrm{w}} \in [0{,}06;0{,}62]$ with an average $e_{\mathrm{w}} = 0{,}34$ and an associated standard uncertainty $u(e_{\mathrm{w}}) = 0{,}162$ derived from a rectangular PDF.

The input data and their associated uncertainties are given in Table 9. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 9.

Table 9 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_{g}	2 591	—	—	1
Count time of the gross measurement	t_{g}	360	—	—	s
Number of counts of the background effect	n_0	41 782		—	1

Table 9 *(continued)*

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Count time of the background effect	t_0	7 200	—	—	s
Efficiency of the detector	e_D	0,30	0,015 5	$N(\tilde{e}_D; e_D, u(e_D))$	$Bq^{-1} s^{-1}$
Wiping efficiency	e_W	0,34	0,162	$R(\tilde{e}_w; 0{,}06, 0{,}62)$	1
Wiped area	s	100	10	$N(\tilde{s}; s, u(s))$	cm^2
Intermediate values					
Gross count rate	r_g	7,20	0,141	$Ga(\tilde{r}_g; n_g, 1/t_g)$	s^{-1}
Count rate of the background effect	r_0	5,80	0,028 4	$Ga(\tilde{r}_0; n_0, 1/t_0)$	s^{-1}

The PDFs of the efficiency of the detector, the wiping efficiency, the wiped area, and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description. It is assumed in this example that for the wiping efficiency only the limits of a range (0,06 to 0,62) are known from experience. This leads to assuming a rectangular PDF for the wiping efficiency and a large relative standard uncertainty of the wiping efficiency of $u_{rel}(e_w) = 0{,}48$.

The particular point of this example compared to that in Clause 8 is that now the quantity with a large uncertainty is in the denominator.

9.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

9.4.1 Background effect

The count rate of the background effect is

$$r_0 = \frac{n_0}{t_0} = 5{,}80\ s^{-1} \quad (71)$$

Its associated standard uncertainty is given by:

$$u(r_0) = \sqrt{\frac{r_0}{t_0}} = 0{,}028\,4\ s^{-1} \quad (72)$$

9.4.2 Primary result and its associated standard uncertainty

With a calibration factor

$$w = \frac{1}{e_D \cdot s \cdot e_w} = 0{,}098\,0\ s \cdot Bq \cdot cm^{-2} \quad (73)$$

and its standard uncertainty

$$u(w) = w \cdot u_{rel}(w) = w \cdot \sqrt{u_{rel}^2(e_D) + u_{rel}^2(s) + u_{rel}^2(e_w)} = 0{,}048\ s\, Bq \cdot cm^{-2} \quad (74)$$

the primary result of the measurement is given by:

$$a_s = (n_g / t_g - n_0 / t_0) \cdot w = 0{,}137\ Bq \cdot cm^{-2} \quad (75)$$

This yields the standard uncertainty

$$u(a_s) = \sqrt{w^2 \cdot (r_g / t_g + r_0 / t_0) + (r_g - r_0)^2 \cdot u^2(w)} = 0{,}068\,3\ Bq \cdot cm^{-2} \quad (76)$$

9.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a}_s)$ as a function of the true value $\tilde{a}_s$ of the measurand is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a}_s = (\tilde{r}_g - r_0) \cdot w$ one expects $\tilde{n}_g = t_g \cdot (\tilde{a}_s / w + r_0)$. This yields with Formula (76)

$$\tilde{u}^2(\tilde{a}_s) = w^2 \cdot ((\tilde{a}_s / w + r_0)/t_g + r_0 / t_0) + \tilde{a}_s^2 u_{rel}^2(w) \tag{77}$$

9.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$a_s^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot \sqrt{w^2 \cdot (r_0 / t_g + r_0 / t_0)} = 0{,}0210\ \text{Bq} \cdot \text{cm}^{-2} \tag{78}$$

The measured primary result a_s exceeds the decision threshold a_s^*.

9.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$a_s^\# = a_s^* + k_{1-\beta} \cdot \sqrt{w^2 \cdot [(a_s^\# / w + r_0)/t_g + r_0 / t_0] + a_s^{\#2} u_{rel}^2(w)} = 0{,}121\ \text{Bq} \cdot \text{cm}^{-2} \tag{79}$$

Since $\alpha = \beta$, $k_{1-\alpha} = k_{1-\beta} = k$, the detection limit can also be calculated by the explicit Formula (80):

$$a_s^\# = a_s^* + k_{1-\beta} \cdot \tilde{u}\left(a_s^\#\right) = \frac{2 \cdot a_s^* + (k^2 \cdot w)/t_g}{1 - k^2 \cdot u_{rel}^2(w)} = 0{,}121\ \text{Bq} \cdot \text{cm}^{-2} \tag{80}$$

The guideline value $a_{s,r}$ exceeds the detection limit $a_s^\#$.

9.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a_s exceeds the decision threshold a_s^*. With $\omega = \Phi[a_s / u(a_s)] = 0{,}977$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega \cdot (1-\gamma/2) = 0{,}953$ and $q = 1 - \omega \cdot \gamma / 2 = 0{,}976$, and hence the quantiles k_p and k_q are equal to 1,674 and 1,970, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a_s^\triangleleft = a_s - k_p \cdot u(a_s) = 0{,}0224\ \text{Bq} \cdot \text{cm}^{-2} \text{ and } a_s^\triangleright = a_s + k_q \cdot u(a_s) = 0{,}271\ \text{Bq} \cdot \text{cm}^{-2} \tag{81}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[a_s / u(a_s)] = 0{,}977$, one obtains $p = [1 + \omega \cdot (1-\gamma)]/2 = 0{,}964$, and hence the quantile k_p is equal to 1,802. With this, the limits of the shortest coverage interval according to Formula (82) are

$$a_s^< = a_s - k_p \cdot u(a_s) = 0{,}0136\ \text{Bq} \cdot \text{cm}^{-2} \text{ and } a_s^> = a_s + k_p \cdot u(a_s) = 0{,}260\ \text{Bq} \tag{82}$$

9.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}_s$ of the activity of the sample is given by:

$$\hat{a}_s = a_s + \frac{u(a_s)\cdot\exp\left\{-a_s^2/\left[2u^2(a_s)\right]\right\}}{\omega\sqrt{2\pi}} = 0{,}140\ \text{Bq}\cdot\text{cm}^{-2} \tag{83}$$

with its associated standard uncertainty $u(\hat{a}_s)$

$$u(\hat{a}_s) = \sqrt{u^2(a_s) - (\hat{a}_s - a_s)\,\hat{a}_s} = 0{,}064\ 3\ \text{Bq}\cdot\text{cm}^{-2} \tag{84}$$

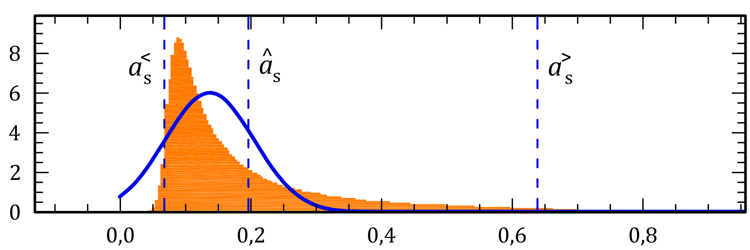
9.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

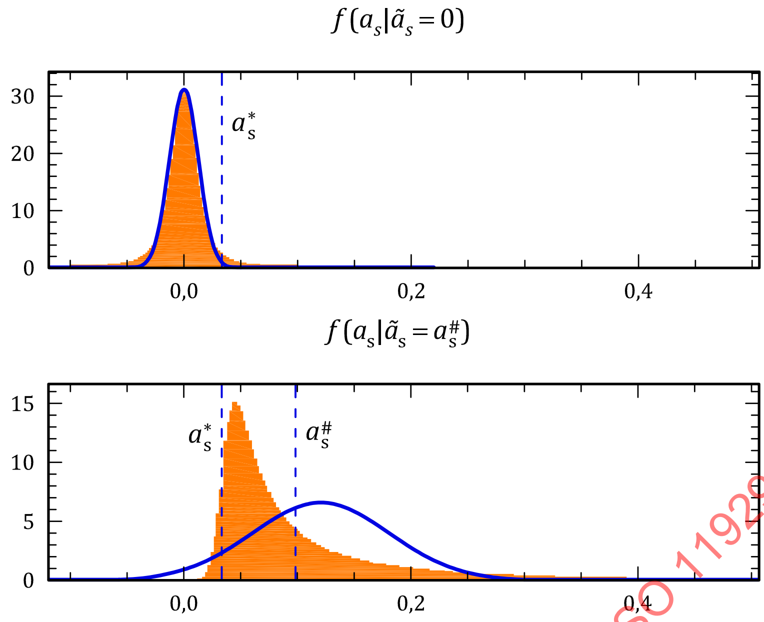
The results and characteristic limits are summarized in Table 10 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 5.

Table 10 — Result and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a_s	Bq cm-2	0,137	0,196
Standard uncertainty associated with the primary result	$u(a_s)$	Bq cm-2	0,068	0,150
Relative standard uncertainty associated with the primary result	$u_{rel}(a_s)$	1	0,50	0,77
Decision threshold	a_s^*	Bq cm-2	0,021	0,035
Detection limit	$a_s^\#$	Bq cm-2	0,121	0,099
Best estimate	$\hat{a}_s$	Bq cm-2	0,140	0,196
Standard uncertainty associated with the best estimate	$u(\hat{a}_s)$	Bq cm-2	0,064	0,150
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{a}_s)$	1	0,46	0,77
Lower limit of the probabilistically symmetric coverage interval	$a_s^\triangleleft$	Bq cm-2	0,022	0,068
Upper limit of the probabilistically symmetric coverage interval	$a_s^\triangleright$	Bq cm-2	0,271	0,640
Lower limit of the shortest coverage interval	$a_s^<$	Bq cm-2	0,014	0,054
Upper limit of the shortest coverage interval	$a_s^>$	Bq cm-2	0,260	0,538

$f(\tilde{a}_s|a_s, A_s \geq 0)$





NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 5 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

9.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a_s = 0{,}137\ \mathrm{Bq \cdot cm^{-2}}$ exceeds the decision threshold $a_s^* = 0{,}021\ \mathrm{Bq \cdot cm^{-2}}$. It is decided to conclude that a contamination of the surface was recognized.

— The detection limit $a_s^{\#} = 0{,}121\ \mathrm{Bq \cdot cm^{-2}}$ is below the guideline values $a_{s,r} = 0{,}5\ \mathrm{Bq \cdot cm^{-2}}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a_s^{\triangleleft} = 0{,}022\ \mathrm{Bq \cdot cm^{-2}}$ and $a_s^{\triangleright} = 0{,}271\ \mathrm{Bq \cdot cm^{-2}}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a_s^{<} = 0{,}014\ \mathrm{Bq \cdot cm^{-2}}$ and $a_s^{>} = 0{,}260\ \mathrm{Bq \cdot cm^{-2}}$.

— The best estimate is $\hat{a}_s = 0{,}140\ \mathrm{Bq \cdot cm^{-2}}$ with an associated standard uncertainty $u(\hat{a}_s) = 0{,}064\ \mathrm{Bq \cdot cm^{-2}}$.

The results obtained by applying ISO 11929-1 and ISO 11929-2 differ strongly with regard numerical values (Table 10) and the PDFs (Figure 5). This effect is mainly due to the rectangular PDF in the denominator. Considering these differences, the application of ISO 11929-2 is to be preferred if there are large and dominating uncertainties in the denominator of the calibration factor.

10 Counting measurements with shielding of the background

10.1 Definition of the task and general aspects

This example deals with a measurement using a portal monitor by which trucks are investigated for radioactive materials. No calibration in terms of activity is made. Rather, the net count rate is used as an indicator whether or not radioactivity is in the truck load. A recognized count rate is simply taken as an alarm indicating that further detailed inspection is necessary. This is a simplified example according to the general model of ISO 11929-1.

Since the background is measured independently without a truck in the portal monitor, a measurement of the gross effect has to take into account that a truck shields the background. The shielding factor is assumed from experience to lie between 0,1 and 0,7. A rectangular PDF is assumed for the shielding factor.

A guideline value $r_{\mathrm{n,r}} = 10\,\mathrm{s}^{-1}$ is assumed, just for completeness of the example.

10.2 Model of evaluation and standard uncertainty

The measurand is the net count rate observed. The primary result r_n of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation:

$$r_\mathrm{n} = n_\mathrm{g} / t_\mathrm{g} - s \cdot n_0 / t_0 = r_\mathrm{g} - s \cdot r_0 \quad (85)$$

The standard uncertainty $u(r_\mathrm{n})$ associated with the primary measurement result r_n is calculated by:

$$u^2(r_\mathrm{n}) = u^2(r_\mathrm{g}) + s^2 \cdot u^2(r_0) + r_0^2 \cdot u^2(s) = n_\mathrm{g} / t_\mathrm{g}^2 + s^2 \cdot n_0 / t_0^2 + (n_0 / t_0)^2 \cdot u^2(s) \quad (86)$$

10.3 Available information, input data, and specifications

It is assumed that the shielding factor was determined in independent measurements and its associated standard uncertainty was evaluated by applying the ISO/IEC Guide 98-3.

The input data and their associated uncertainties are given in Table 11. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 11.

The standard uncertainties of the durations of the measurements are neglected.

Table 11 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	2 900	—	—	1
Count time of the gross measurement	t_g	180	—	—	s
Number of counts of the background effect	n_0	25 000	—	—	1
Count time of the background effect	t_0	1 800	—	—	s
Shielding factor	s	0,400	0,173 21	$\mathrm{R}(\tilde{s};0{,}1{,}0{,}7)$	1
Intermediate values					
Gross count rate	r_g	16,11	0,299	$\mathrm{Ga}(\tilde{r}_\mathrm{g}; n_\mathrm{g}, 1/t_\mathrm{g})$	s^{-1}
Count rate of the background effect	r_0	13,89	0,087 8	$\mathrm{Ga}(\tilde{r}_0; n_0, 1/t_0)$	s^{-1}

The PDFs of the shielding factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description. It is assumed in this example that for the shielding factor only the limits of a range of shielding factors (0,1 to 0,7) are known. This leads to assuming a rectangular PDF for the calibration factor and a large relative standard uncertainty of the calibration factor of $u_{\mathrm{rel}}(w) = 0{,}43$.

10.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

10.4.1 Background effect

The count rate of the background effect is

$$r_0 = \frac{n_0}{t_0} = 13{,}89\ \mathrm{s}^{-1} \tag{87}$$

and its associated standard uncertainty is given by:

$$u(r_0) = \sqrt{\frac{r_0}{t_0}} = 0{,}087\,8\ \mathrm{s}^{-1} \tag{88}$$

10.4.2 Primary result and its associated standard uncertainty

Since there is no calibration of the detector with respect to an activity, the measurand is the net count rate. The primary result of the measurement is given by:

$$r_{\mathrm{n}} = n_{\mathrm{g}} / t_{\mathrm{g}} - s \cdot n_0 / t_0 = 10{,}56\ \mathrm{s}^{-1} \tag{89}$$

and its standard uncertainty by:

$$u(r_{\mathrm{n}}) = \sqrt{r_{\mathrm{g}} / t_{\mathrm{g}} + s^2 \cdot r_0 / t_0 + r_0^2 \cdot u^2(s)} = 2{,}424\ \mathrm{s}^{-1} \tag{90}$$

10.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{r}_{\mathrm{n}})$ as a function of the true value $\tilde{r}_{\mathrm{n}}$ of the measurand is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{r}_{\mathrm{n}} = \tilde{r}_{\mathrm{g}} - s \cdot r_0$ one expects $\tilde{n}_{\mathrm{g}} = t_{\mathrm{g}} \cdot (\tilde{r}_{\mathrm{n}} + s \cdot r_0)$. This yields with Formula (86)

$$\tilde{u}^2(\tilde{r}_{\mathrm{n}}) = (\tilde{r}_{\mathrm{n}} + s \cdot r_0)/t_{\mathrm{g}} + s^2 \cdot r_0 / t_0 + r_0^2 \cdot u^2(s) \tag{91}$$

10.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1 - \alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the net count rate is calculated by:

$$r_{\mathrm{n}}^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot \sqrt{s \cdot r_0 \cdot \left(\frac{1}{t_{\mathrm{g}}} + \frac{s}{t_0}\right) + r_0^2 \cdot u^2(s)} = 3{,}97\ \mathrm{s}^{-1} \tag{92}$$

The measured primary result r_{n} exceeds the decision threshold r_{n}^*.

10.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$r_n^{\#} = r_n^{*} + k_{1-\beta} \cdot \sqrt{\frac{r_n^{\#} + s \cdot r_0}{t_g} + s^2 \cdot \frac{r_0}{t_0} + r_0^2 \cdot u^2(s)} = 7{,}95\ \text{s}^{-1} \tag{93}$$

The guideline value $r_{n,r}$ exceeds the detection limit $r_n^{\#}$.

10.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result r_n exceeds the decision threshold r_n^{*}. With $\omega = \Phi[r_n / u(r_n)] = 1{,}000$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega \cdot (1 - \gamma/2) = 0{,}975$ and $q = 1 - \omega \cdot \gamma/2 = 0{,}975$, and hence the quantiles k_p and k_q are both equal to 1,96. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$r_n^{\triangleleft} = r_n - k_p \cdot u(r_n) = 5{,}80\ \text{s}^{-1} \text{ and } r_n^{\triangleright} = r_n + k_q \cdot u(r_n) = 15{,}31\ \text{s}^{-1} \tag{94}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[r_n / u(r_n)] = 1{,}00$, one obtains $p = [1 + \omega \cdot (1 - \gamma)]/2 = 0{,}975$, and hence the quantile k_p is equal to 1,96. With this the limits of the shortest coverage interval according to Formula (95) are

$$r_n^{<} = r_n - k_p \cdot u(r_n) = 5{,}80\ \text{s}^{-1} \text{ and } r_n^{>} = r_n + k_p \cdot u(r_n) = 15{,}31\ \text{s}^{-1} \tag{95}$$

10.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{r}_n$ of the net count rate of the sample is given by:

$$\hat{r}_n = r_n + \frac{u(r_n) \cdot \exp\left\{-r_n^2 \Big/ \left[2u^2(r_n)\right]\right\}}{\omega\sqrt{2\pi}} = 10{,}56\ \text{s}^{-1} \tag{96}$$

with its associated standard uncertainty $u(\hat{r}_n)$

$$u(\hat{r}_n) = \sqrt{u^2(r_n) - (\hat{r}_n - r_n)\hat{r}_n} = 2{,}42\ \text{s}^{-1} \tag{97}$$

10.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

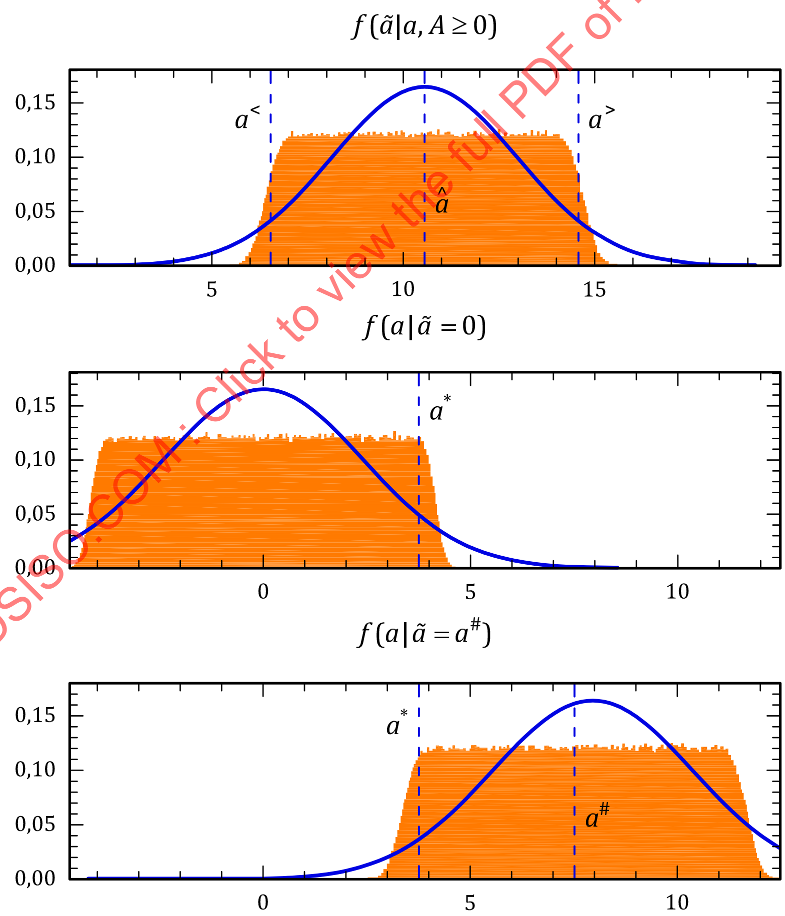
The results and characteristic limits are summarized in Table 12 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 6.

Table 12 — Results and characteristic limits

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	r_n	s^{-1}	10,56	10,56
Standard uncertainty associated with the primary result	$u(r_n)$	s^{-1}	2,42	2,43
Relative standard uncertainty associated with the primary result	$u_{rel}(r_n)$	1	0,23	0,23
Decision threshold	r_n^{*}	s^{-1}	3,97	3,75

Table 12 *(continued)*

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Detection limit	$r_n^{\#}$	s^{-1}	7,95	7,51
Best estimate	$\hat{r}_n$	s^{-1}	10,56	10,56
Standard uncertainty associated with the best estimate	$u(\hat{r}_n)$	s^{-1}	2,42	2,43
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{r}_n)$	1	0,23	0,23
Lower limit of the probabilistically symmetric coverage interval	$r_n^{\triangleleft}$	s^{-1}	5,80	6,53
Upper limit of the probabilistically symmetric coverage interval	$r_n^{\triangleright}$	s^{-1}	15,3	14,57
Lower limit of the shortest coverage interval	$r_n^{<}$	s^{-1}	5,80	6,54
Upper limit of the shortest coverage interval	$r_n^{>}$	s^{-1}	15,3	14,58



NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 6 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

10.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $r_n = 10{,}56\,s^{-1}$ exceeds the decision threshold $r_n^* = 3{,}97\,s^{-1}$. It is decided to conclude that an effect from the sample was recognized.

— The detection limit $r_n^\# = 7{,}95\,s^{-1}$ is below the guideline values $r_{n,r} = 10\,s^{-1}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $r_n^\triangleleft = 5{,}8\,s^{-1}$ and $r_n^\triangleright = 15{,}3\,s^{-1}$.

— The lower and upper limits of the shortest coverage interval are calculated as $r_n^< = 5{,}8\,s^{-1}$ and $r_n^> = 15{,}3\,s^{-1}$.

— The best estimate is $\hat{r}_n = 10{,}6\ s^{-1}$ with an associated standard uncertainty $u(\hat{r}_n) = 2{,}4\ s^{-1}$.

The results obtained by applying ISO 11929-1 and ISO 11929-2 exhibit differences up to 10 %. This is due to the deviation between the Gaussian distributions and the actual posterior distributions calculated according to ISO 11929-2; see Figure 6. The differences are mainly due to the lack of knowledge about the shielding factor and its associated rectangular PDF.

The coverage intervals calculated with ISO 11929-2 are smaller than those based on ISO 11929-1. The differences between the best estimates and their associated standard uncertainties are small because the central estimates of the rectangular distribution of the shielding factor is identical to that of the normal distribution applied in ISO 11929-1.

Considering the differences in the limits of the coverage intervals, the application of ISO 11929-2 is to be preferred if there are large and dominating uncertainties in the numerator of the shielding factor.

11 Counting clearance measurement

11.1 Definition of the task and general aspects

This example deals with a measurement using a large waste monitor for clearance measurements. It is assumed that the monitor is calibrated with ^{60}Co and the results are ^{60}Co-equivalents. Furthermore, it is assumed that the different count rates of the individual detectors are combined into a sum channel. The monitor counts the gamma-radiation emerging from 300 kg concrete located in a 200-l-waste-drum. Such a monitor is typically calibrated for different waste packages and different filling heights with several categories of net masses of the waste (parameters of efficiency calibration).

The example discussed is dealing with one specific calibration and is simplified by using only the sum channel and no nuclide vector. The calibration measurements were performed with a certified point source at 20 different positions in a calibration dummy, which resulted in a representative calibration assuming a homogeneous activity distribution in the waste. The uncertainty in the calibration should cover especially the effect of an inhomogeneous distribution of the activity in the waste.

NOTE 1 Because of a lack of international standardization of clearance measurements and because of the wide variety of respective measurement procedures it is emphasized that this example is informative and neither the model of evaluation nor the data shall be understood as stipulations. This example just serves to exemplify the application of the model of ISO 11929 (all parts) and does not anticipate standardization in the field of clearance measurements.

NOTE 2 These monitors consist of a lead shielding of at least 5 cm thickness surrounding the waste as a cuboid and several plastic scintillation detectors (typically 6 to 24 detectors) in the inner volume in front of the shielding. The detectors and the shielding are built up in a way to cover the material in a so-called 4π-geometry. According to the large volumes of the detectors, uncertainties due to counting statistics can practically be neglected. Due to the gross measurement of gamma radiation no nuclide specific information can be obtained; it is a counting measurement of nuclide equivalents.

NOTE 3 To ensure metrological traceability this leads to the fact, that a lot of efficiency calibrations are performed (typically 10 to 100) for different combinations of the parameters mentioned above. For practical reasons a somewhat conservative parameter is chosen to calculate the results (e. g. if there is a calibration for 250 kg and 300 kg and the net mass is 280 kg, the calibration with 300 kg is used).

NOTE 4 In comparison to spectrometric measurements (e. g. with drum scanners) using HPGe-detectors the disadvantages of non-nuclide-specific measurements stand in contrast to advantages, in particular, of significantly shorter measurement duration times and lower measurement uncertainties due to inhomogeneity of the activity distribution.

A guideline value $a_r = 30\,000\,\text{Bq}$ according to the clearance level for unconditional clearance of 0,1 Bq/g = 30 000 Bq/300 kg is assumed.

11.2 Model of evaluation and standard uncertainty

The measurand is the activity of the sample. The primary result a of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation:

$$a = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0} \cdot x_3 - x_{41} + x_{42} \right) \cdot w = \left(r_g - r_0 \cdot x_3 - x_{41} + x_{42} \right) \cdot w \tag{98}$$

with

x_3 correction of the background count rate for its variability due to work activities near the device for clearance measurements,

x_{41} correction for natural radionuclides, i.e. ^{40}K and the ^{226}Ra and ^{232}Th decay series, in the material to be measured. The concrete was characterized in independent measurements for its natural radioactivity and the resulting contribution x_4 to the gross count rate and its associated standard certainty $u(x_4)$ were determined.

x_{42} correction for shielding of the background by the material to be measured.

Formula (98) is an example for the general model of ISO 11929-1.

The standard uncertainty $u(a)$ associated with the primary measurement result a is calculated by:

$$u^2(a) = w^2 \cdot \left(r_g / t_g + x_3^2 \cdot r_0 / t_0 + r_0^2 \cdot x_3^2 \cdot u^2(x_3) + u^2(x_{41}) + u^2(x_{42}) \right) + a^2 \cdot u_{rel}^2(w) \tag{99}$$

11.3 Available information, input data, and specifications

It is assumed that the calibration factor as well x_3, x_{41} and x_{42} were determined in independent measurements and their associated standard uncertainties were evaluated by applying the ISO/IEC Guide 98-3.

The corrections for the standardized clearance measurement of batches of 300 kg of bulk material were investigated in independent measurements and their standard uncertainties were evaluated according to the ISO/IEC Guide 98-3.

The input data and their associated uncertainties are given in Table 13. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 13.

Table 13 — Input quantities and data

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Number of counts of the gross measurement	n_g	389 589	—	—	1
Count time of the gross measurement	t_g	60	—	—	s
Number of counts of the background effect	n_0	306 000	—	—	1
Count time of the background effect	t_0	180	—	—	s
Background correction for changing work activities	x_3	1	0,011 5	$\mathrm{R}(\tilde{x}_3; 0{,}98, 1{,}02)$	1
Background correction of the natural radioactivity	x_{41}	2 345	220	$\mathrm{N}[\tilde{x}_{41}; x_{41}, u(x_{41})]$	s^{-1}
Shielding correction	x_{42}	276	70	$\mathrm{N}[\tilde{x}_{42}; x_{42}, u(x_{42})]$	s^{-1}
Calibration factor	w	4,58	1,37 4	$\mathrm{N}[\tilde{w}; w, u(w)]$	s Bq
Intermediate values					
Gross count rate	r_g	6 493	10,40	$\mathrm{Ga}(\tilde{r}_g; n_g, 1/t_g)$	s^{-1}
Count rate of the background effect	r_0	1 700	3,07	$\mathrm{Ga}(\tilde{r}_0; n_0, 1/t_0)$	s^{-1}

The PDFs of the calibration factor, of the correction factors, and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description.

11.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

11.4.1 Background effect

The count rate of the background effect is

$$r_0 = \frac{n_0}{t_0} = 1\,700\ \mathrm{s}^{-1} \tag{100}$$

Its associated standard uncertainty is given by:

$$u(r_0) = \sqrt{\frac{r_0}{t_0}} = 3{,}07\ \mathrm{s}^{-1} \tag{101}$$

11.4.2 Primary result and its associated standard uncertainty

The primary result of the measurement is given by:

$$a = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0} \cdot x_3 - x_{41} + x_{42}\right) \cdot w = 12\,477\ \mathrm{Bq} \tag{102}$$

and the associated standard uncertainty by

$$u(a) = \sqrt{w^2 \cdot \left[r_g/t_g + x_3^2 \cdot r_0/t_0 + r_0^2 \cdot u^2(x_3) + u^2(x_{41}) + u^2(x_{42})\right] + a^2 \cdot u_{\mathrm{rel}}^2(w)} \\ = 3\,891\ \mathrm{Bq} \tag{103}$$

11.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a})$ as a function of the true value $\tilde{a}$ of the measurand is needed to calculate the decision threshold and the detection limit.

For a true value $\tilde{a}$ one expects $\tilde{n}_g = \left(\frac{\tilde{a}}{w} + \frac{n_0}{t_0} \cdot x_3 + x_{41} - x_{42} \right) \cdot t_g$. This yields with Formula (99)

$$u^2(\tilde{a}) = w^2 \cdot \left(\begin{array}{l} \left(\frac{\tilde{a}}{w} + \frac{n_0}{t_0} \cdot x_3 + x_{41} - x_{42} \right) / t_g + x_3^2 \cdot r_0 / t_0 \\ + r_0^2 \cdot u^2(x_3) + u^2(x_{41}) + u^2(x_{42}) \end{array} \right) + \tilde{a}^2 \cdot u_{rel}^2(w) \quad (104)$$

11.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$\begin{aligned} a^* &= k_{1-\alpha} \cdot \tilde{u}(0) \\ &= k_{1-\alpha} \cdot w \cdot \sqrt{\left(\frac{n_0}{t_0} \cdot x_3 + x_{41} - x_{42} \right) / t_g + x_3^2 \cdot r_0 / t_0 + r_0^2 \cdot u^2(x_3) + u^2(x_{41}) + u^2(x_{42})} \\ &= 1\,747 \text{ Bq} \end{aligned} \quad (105)$$

The measured primary result exceeds the decision threshold a^*.

11.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by iteration, which leads to

$$\begin{aligned} a^\# &= a^* + k_{1-\beta} \cdot \sqrt{\begin{array}{l} w^2 \cdot \left[\left(\frac{\tilde{a}^\#}{w} + \frac{n_0}{t_0} \cdot x_3 + x_{41} - x_{42} \right) / t_g + x_3^2 \cdot r_0 / t_0 + r_0^2 \cdot u^2(x_3) + u^2(x_{41}) + u^2(x_{42}) \right] \\ + \tilde{a}^{\#2} \cdot u_{rel}^2(w) \end{array}} \\ &= 4\,618 \text{ Bq} \end{aligned}$$

(106)

The guideline value a_r exceeds the detection limit $a^\#$.

11.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a exceeds the decision threshold a^*. With $\omega = \Phi[a/u(a)] = 0{,}999$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega \cdot (1-\gamma/2) = 0{,}974$ and $q = 1 - \omega \cdot \gamma/2 = 0{,}975$, and hence the quantiles k_p and k_q are equal to 1,95 and 1,96, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a^{\triangleleft} = a - k_p \cdot u(a) = 4\,894 \text{ Bq and } a^{\triangleright} = a + k_q \cdot u(a) = 20\,104 \text{ Bq} \quad (107)$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[a/u(a)] = 0{,}999$, one obtains $p = [1 + \omega \cdot (1-\gamma)]/2 = 0{,}975$, and hence the quantile k_p is equal to 1,954. With this the limits of the shortest coverage interval according to Formula (108) were

$$a^{<} = a - k_p \cdot u(a) = 4\,872 \text{ Bq and } a^{>} = a + k_p \cdot u(a) = 20\,081 \text{ Bq} \quad (108)$$

11.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}$ of the activity of the sample is given by:

$$\hat{a} = a + \frac{u(a) \cdot \exp\left\{-a^2 / \left[2u^2(a)\right]\right\}}{\omega\sqrt{2\pi}} = 12\ 486\ \text{Bq} \tag{109}$$

with its associated standard uncertainty $u(\hat{a})$

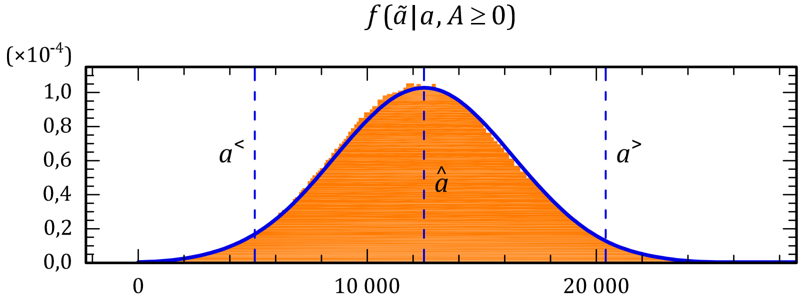
$$u(\hat{a}) = \sqrt{u^2(a) - (\hat{a} - r_\text{n})\hat{a}} = 3\ 876\ \text{Bq} \tag{110}$$

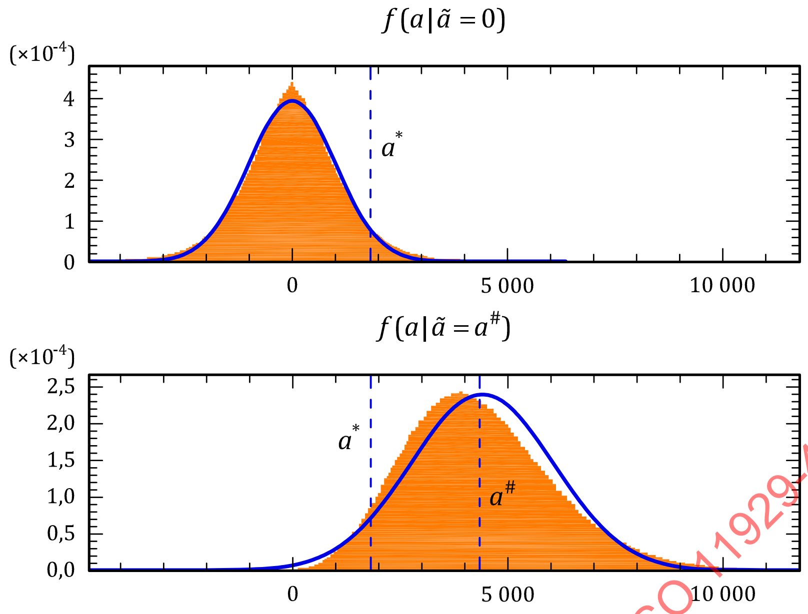
11.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The results and characteristic limits are summarized in Table 14 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 7.

Table 14 — Results and characteristic limits

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a	Bq	12 477	12 477
Standard uncertainty associated with the primary result	$u(a)$	Bq	3 891	3 901
Relative standard uncertainty associated with the primary result	$u_\text{rel}(a)$	1	0,31	0,31
Decision threshold	a^*	Bq	1 747	1 819
Detection limit	$a^\#$	Bq	4 618	4 350
Best estimate	$\hat{a}$	Bq	12 486	12 482
Standard uncertainty associated with the best estimate	$u(\hat{a})$	Bq	3 876	3 895
Relative uncertainty associated with the best estimate	$u_\text{rel}(\hat{a})$	1	0,31	0,31
Lower limit of the probabilistically symmetric coverage interval	$a^\triangleleft$	Bq	4 894	5 076
Upper limit of the probabilistically symmetric coverage interval	$a^\triangleright$	Bq	20 104	20 376
Lower limit of the shortest coverage interval	$a^<$	Bq	4 872	4 946
Upper limit of the shortest coverage interval	$a^>$	Bq	20 081	20 231





NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 7 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

11.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a = 12\,477\ \text{Bq}$ exceeds the decision threshold $a^* = 1\,747\ \text{Bq}$. It is decided to conclude that an effect from the sample was recognized.

— The detection limit $a^\# = 4\,618\ \text{Bq}$ is below the guideline values $a_\text{r} = 30\,000\ \text{Bq}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a^\triangleleft = 4\,894\ \text{Bq}$ and $a^\triangleright = 20\,104\ \text{Bq}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a^< = 4\,872\ \text{Bq}$ and $a^> = 20\,081\ \text{Bq}$.

— The best estimate is $\hat{a} = 12\ 486\ \text{Bq}$ with an associated standard uncertainty $u(\hat{a}) = 3\ 876\ \text{Bq}$.

The results obtained by applying ISO 11929-1 and ISO 11929-2 exhibit differences of less than 10 %. The differences are due to the deviation between the Gaussian distributions and the actual posterior distributions calculated according to ISO 11929-2; see Figure 7.

The coverage intervals calculated with ISO 11929-2 differ only slightly from those based on ISO 11929-1. The differences between the best estimates and their associated standard uncertainties are small because the central estimates of the rectangular distribution of the calibration factor is identical to that of the normal distribution applied in ISO 11929-1.

Considering the differences between the results obtained by ISO 11929-1 and ISO 11929-2 the ISO/IEC Guide 98-3 approximation and consequently ISO 11929-1 are sufficient.

NOTE 1 Since this document does not deal with the problem of conformity with requirements, as e.g. with the clearance level for unconditional clearance, the question whether or not the material measured can be unconditionally cleared is not answered. For an answer to this question see Reference [45].

NOTE 2 It is necessary to take into account that in the present case favourable boundary conditions with regard to the mass of the material to be measured and the photon energies (^{60}Co) been selected. By increasing the mass to several 100 kg or smaller photon energies (e. g. ^{137}Cs) it can be necessary to use an approach as given in A.3.

12 Gamma-spectrometry of Uranium-235 with interference by Radium-226

12.1 Definition of the task and general aspects

When measuring ^{235}U by γ-spectrometry via the gamma-line at 186 keV there is an interference with the gamma-line of ^{226}Ra. If radioactive equilibrium between ^{226}Ra and ^{214}Bi can be assumed, the interference can be corrected for via the gamma-line of ^{214}Bi at 609 keV taking into account the ratios of the different emission probabilities and detector efficiencies. This is an example according to the general model of ISO 11929-1.

In addition, it is assumed in this example that there is an impurity of ^{235}U in the detector material, which was determined by an independent measurement of the background caused by this impurity.

In this example, it is assumed that all measurements were performed with the same count time. In addition, only the Poisson uncertainties of the counted events are taken into account. For simplicity it is assumed that the efficiencies at the different γ-energies were measured independently with suitable reference sources, thus avoiding covariances. A more detailed calculation was performed in Reference [39].

A soil sample is investigated by gamma-spectrometry for ^{235}U via the 186 keV gamma-line. The interference of this line with the gamma-line from ^{226}Ra and a contribution to the background by an impurity ^{235}U in the detector material has to be taken into account. The measurand is the activity of ^{235}U per unit mass.

It is assessed whether the primary measurement result exceeds the decision threshold and an activity of ^{235}U per unit mass is recognized in the soil sample.

No guideline value for the activity per unit mass of ^{235}U is given. Therefore, a comparison of the detection limit with the guideline value is not needed. In spite of that, the detection limit is calculated for completeness.

12.2 Model of evaluation and standard uncertainty

The measurand is activity per unit mass, of ^{235}U in the sample. The primary result of the measurement as an estimate of the true quantity value of the measurand is calculated by the model of evaluation:

$$a_m = \left(\frac{n_{g,186}}{t} - \frac{n_{n,609}}{t} \cdot \frac{e_{Ra,186} \cdot \varepsilon_{186}}{e_{Ra,609} \cdot \varepsilon_{609}} - \frac{n_{U,186}}{t} - \frac{n_{n,186,0}}{t} \right) \cdot \frac{1}{\varepsilon_{186} \cdot e_{U,186} \cdot m}$$
$$= \left(r_{g,186} - r_{n,609} \cdot \frac{e_{Ra,186} \cdot \varepsilon_{186}}{e_{Ra,609} \cdot \varepsilon_{609}} - r_{U,186} - r_{n,186,0} \right) \cdot \frac{1}{\varepsilon_{186} \cdot e_{U,186} \cdot m} \quad (111)$$

In order to avoid a covariance Formula (111) is slightly rearranged and yields the final model of evaluation with the calibration factor $w = \frac{1}{e_{U,186} \cdot m}$ and a correction factor $k = \frac{e_{Ra,186}}{e_{Ra,609} \cdot \varepsilon_{609}}$

$$a_m = \left(\frac{r_{g,186}}{\varepsilon_{186}} - r_{n,609} \cdot \frac{e_{Ra,186}}{e_{Ra,609} \cdot \varepsilon_{609}} - \frac{r_{U,186}}{\varepsilon_{186}} - \frac{r_{n,186,0}}{\varepsilon_{186}} \right) \cdot \frac{1}{e_{U,186} \cdot m}$$
$$= \left(\frac{r_{g,186}}{\varepsilon_{186}} - r_{n,609} \cdot k - \frac{r_{U,186}}{\varepsilon_{186}} - \frac{r_{n,186,0}}{\varepsilon_{186}} \right) \cdot w \quad (112)$$

The standard uncertainty $u(a_m)$ of the measurand associated with the estimate a_m is calculated by:

$$u^2(a_m) = \sum_{i=1}^{n} \left(\frac{\partial a_m}{\partial x_i} \right)^2 \cdot u^2(x_i) \quad (113)$$

which yields

$$u^2(a_m) = w^2 \left[\frac{r_{g,186}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + \frac{r_{n,609}}{t} \cdot k^2 + \frac{r_{U,186}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + \frac{r_{n,186,0}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + \frac{u^2(\varepsilon_{186})}{\varepsilon_{186}^2} \cdot \left(r_{g,186} - r_{U,186} - r_{n,186,0} \right)^2 + u^2(k) \cdot r_{n,609}^2 \right] + a_m^2 \cdot u_{rel}^2(w) \quad (114)$$

The standard uncertainty of the correction factor $k = \frac{e_{Ra,186}}{e_{Ra,609} \cdot \varepsilon_{609}}$ is given by:

$$u^2(k) = k^2 \cdot \left[u_{rel}^2(e_{Ra,186}) + u_{rel}^2(e_{Ra,609}) + u_{rel}^2(\varepsilon_{609}) \right] \quad (115)$$

The relative standard uncertainty of the calibration factor $w = \frac{1}{e_{U,186} \cdot m}$ is given by:

$$u_{rel}^2(w) = u_{rel}^2(e_{U,186}) + u_{rel}^2(m) \quad (116)$$

The following symbols are used in this example:

t	count time
$n_{g,186}$	number of gross counts in the peak at 186 keV
$n_{n,609}$	Number of counts in the net peak of ^{214}Bi at 609 keV
$n_{U,186}$	number of counts in the background under the peak at 186 keV
$n_{n,186,0}$	number of net counts in the peak at 186 keV due to activity of the impurity in the detector
$r_{g,186}$	gross count rate in the 186 keV peak

$r_{n,609}$	net count rate in the 609 keV peak
$r_{U,186}$	background count rate in the 186 keV peak
$r_{n,186,0}$	net count rate due to the activity of the impurity of the detector in the peak at 186 keV
$e_{Ra,186}$	emission probability of the 186 keV gamma-rays of ^{226}Ra
$e_{Ra,609}$	emission probability of the 609 keV gamma-rays of ^{214}Bi
$e_{U,186}$	emission probability of 186 keV the gamma-rays of ^{235}U
ε_{186}	detector efficiency at 186 keV
ε_{609}	detector efficiency at 609 keV
k	correction factor for the detector efficiencies and the emission probabilities
m	mass of the test sample
w	calibration factor

12.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 15. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 15.

Table 15 — Input quantities and measurement data

Quantity	Symbol for x_i	x_i	$u(x_i)$	Unit	PDF
Count time	t	15 000	—	s	
Number of count in the gross peak at 186 keV	$n_{g,186}$	7 468	—	1	
Number of counts in the net peak of ^{214}Bi at 609 keV	$n_{n,609}$	6 957	—	1	
Number of counts in the back-ground under the peak at 186 keV	$n_{U,186}$	6 181	—	1	
Number of counts in the net peak at 186 keV due to the activity of the impurity of the detector	$n_{n,186,0}$	207	—	1	
Emission probability of the 186 keV gamma-line of ^{226}Ra	$e_{Ra,186}$	0,035 1	0,000 6	1	$N[\tilde{e}_{Ra,186}; e_{Ra,186}, u(e_{Ra,186})]$
Detector efficiency at 186 keV	ε_{186}	0,800	0,064	1	$N[\tilde{\varepsilon}_{186}; \varepsilon_{186}, u(\varepsilon_{186})]$
Emission probability of the 609 keV gamma-line of ^{214}Bi	$e_{Ra,609}$	0,446	0,005	1	$N[\tilde{e}_{Ra,609}; e_{Ra,609}, u(e_{Ra,609})]$
Emission probability of the 186 keV gamma-line of ^{235}U	$e_{U,186}$	0,572	0,005	1	$N[\tilde{e}_{U,186}; e_{U,186}, u(e_{U,186})]$
Detector efficiency at 609 keV	ε_{609}	0,551	0,033	1	$N[\tilde{\varepsilon}_{609}; \varepsilon_{609}, u(\varepsilon_{609})]$
Mass of the test sample	m	0,750	0,038	kg	$N[\tilde{m}; m, u(m)]$
Intermediate values					
Gross count rate in the 186 keV peak	$r_{g,186}$	0,498	0,005 76	s^{-1}	$Ga(\tilde{r}_g; n_g, 1/t_g)$
Net count rate in the 609 keV peak	$r_{n,609}$	0,464	0,005 56	s^{-1}	

Table 15 *(continued)*

Quantity	Symbol for x_i	x_i	$u(x_i)$	Unit	PDF
Background count rate of the 186 keV peak	$r_{U,186}$	0,412	0,005 24	s^{-1}	$Ga(\tilde{r};n,1/t)$
Count rate in the net peak at 186 keV due to the activity of the impurity in the detector	$r_{n,186,0}$	0,013 8	0,000 959	s^{-1}	$Ga(\tilde{r};n,1/t)$

The PDFs of the count rates and of the input data, except the given actual numbers of counts, were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description.

12.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

12.4.1 Background effect

The background under the peak at 186 keV is $n_{U,186} = 618\ 1$ which yields a count rate of

$$r_{U,186} = \frac{n_{U,186}}{t} = 0{,}412\,\text{s}^{-1} \tag{117}$$

The standard uncertainty of background under the peak at 186 keV is

$$u(r_{U,186}) = \sqrt{\frac{n_{U,186}}{t^2}} = 0{,}005\,24\,\text{s}^{-1} \tag{118}$$

12.4.2 Primary result and its associated standard uncertainty

For the calculation of the primary measurement result a_m, the activity per unit mass of ^{235}U, first the correction factor k and its associated standard uncertainty $u(k)$ are determined.

The calibration factor w is calculated by:

$$w = \frac{1}{e_{U,186} \cdot m} = 2{,}331\ \mathrm{Bq} \cdot \mathrm{s}^{-1} \cdot \mathrm{kg}^{-1} \quad (119)$$

with the relative standard uncertainty

$$u_{rel}(w) = \sqrt{u_{rel}^2(e_{U,186}) + u_{rel}^2(m)} = 0{,}051\,4 \quad (120)$$

and the standard uncertainty

$$u(w) = u_{rel}(w) \cdot w = 0{,}120\ \mathrm{Bq} \cdot \mathrm{s}^{-1} \cdot \mathrm{kg}^{-1} \quad (121)$$

The correction factor k is calculated with the emission probabilities and the detector efficiencies

$$k = \frac{e_{Ra,186}}{e_{Ra,609} \cdot \varepsilon_{609}} = 0{,}143 \quad (122)$$

and its relative standard uncertainty is

$$u_{rel}(k) = \sqrt{u^2{}_{rel}(e_{Ra,186}) + u^2{}_{rel}(e_{Ra,609}) + u^2{}_{rel}(\varepsilon_{609})} = 0{,}063\,3 \quad (123)$$

and its standard uncertainty is

$$u(k) = u_{rel}(k) \cdot k = 0{,}009\,0 \quad (124)$$

With the correction factor k and the calibration factor w the primary measurement result a_m is

$$a_m = \left(\frac{r_{g,186}}{\varepsilon_{186}} - r_{n,609} \cdot k - \frac{r_{U,186}}{\varepsilon_{186}} - \frac{r_{n,186,0}}{\varepsilon_{186}} \right) \cdot w = 0{,}055\,4\ \mathrm{Bq} \cdot \mathrm{g}^{-1} \quad (125)$$

with the standard uncertainty

$$u(a_m) = \sqrt{w^2 \left[\frac{r_{g,186}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + \frac{r_{n,609}}{t} \cdot k^2 + \frac{r_{U,186}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + \frac{r_{n,186,0}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + \frac{u_{rel}^2(\varepsilon_{186})}{\varepsilon_{186}^2} \cdot \left(r_{g,186} - r_{U,186} - r_{n,186,0} \right)^2 + u^2(k) \cdot r_{n,609}^2 \right] + a_m^2 \cdot u_{rel}^2(w)} \quad (126)$$

$$= 0{,}030\,2\ \mathrm{Bq} \cdot \mathrm{g}^{-1}$$

12.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a}_m)$ as a function of the true value $\tilde{a}_m$ of the measurand is needed to calculate the decision threshold and the detection limit. For a true value $\tilde{a}_m$ one expects $\tilde{r}_{g,186} = \varepsilon_{186} \cdot \frac{\tilde{a}_m}{w} + \varepsilon_{186} \cdot r_{n,609} \cdot k + r_{U,186} + r_{n,186,0}$.

This yields with Formula (127):

$$\tilde{u}^2(\tilde{a}_m) = w^2 \left[\begin{aligned} &\left(\frac{\varepsilon_{186} \cdot \tilde{a}_m}{w} + r_{n,609} \cdot \varepsilon_{186} \cdot k + r_{U,186} + r_{n,186,0}\right) \cdot \frac{1}{\varepsilon_{186}^2 \cdot t} + \frac{r_{n,609}}{t} \cdot k^2 + \frac{r_{U,186}}{t} \cdot \frac{1}{\varepsilon_{186}^2} \\ &+ \frac{r_{n,186,0}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + u_{rel}^2(\varepsilon_{186}) \cdot \left(\frac{\tilde{a}_m}{w} + r_{n,609} \cdot k\right)^2 + u^2(k) \cdot r_{n,609}^2 \end{aligned} \right] + \tilde{a}_m^2 \cdot u_{rel}^2(w) \tag{127}$$

12.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$\begin{aligned} a_m^* &= k_{1-\alpha} \cdot \tilde{u}(0) \\ &= k_{1-\alpha} \cdot w \cdot \sqrt{\begin{aligned} &\left(r_{n,609} \cdot \varepsilon_{186} \cdot k + r_{U,186} + r_{n,186,0}\right) \cdot \frac{1}{\varepsilon_{186}^2 \cdot t} + \frac{r_{n,609}}{t} \cdot k^2 + \frac{r_{U,186}}{t} \cdot \frac{1}{\varepsilon_{186}^2} \\ &+ \frac{r_{n,186,0}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + u_{rel}^2(\varepsilon_{186}) \cdot r_{n,609}^2 \cdot k^2 + u^2(k) \cdot r_{n,609}^2 \end{aligned}} \\ &= 0{,}0455\ \text{Bq} \cdot \text{g}^{-1} \end{aligned} \tag{128}$$

The primary measurement result a_m exceeds the decision threshold a_m^*.

12.4.5 Detection limit

For β = 0,05 is $k_{1-\beta} = 1{,}65$ and by solving the implicit Formula one obtains the detection limit

$$\begin{aligned} a_m^\# &= a_m^* + k_{1-\beta} \cdot \tilde{u}(a_m^\#) \\ &= a_m^* + k_{1-\beta} \cdot w \cdot \sqrt{\begin{aligned} &\left(\frac{\varepsilon_{186} \cdot a_m^\#}{w} + r_{n,609} \cdot \varepsilon_{186} \cdot k + r_{U,186} + r_{n,186,0}\right) \cdot \frac{1}{\varepsilon_{186}^2 \cdot t} \\ &+ \frac{r_{n,609}}{t} \cdot k^2 + \frac{r_{U,186}}{t} \cdot \frac{1}{\varepsilon_{186}^2} + \frac{r_{n,186,0}}{t} \cdot \frac{1}{\varepsilon_{186}^2} \\ &+ u_{rel}^2(\varepsilon_{186}) \cdot \left(\frac{a_m^\#}{w} + r_{n,609} \cdot k\right)^2 + u^2(k) \cdot r_{n,609}^2 + \frac{a_m^{\#2}}{w^2} \cdot u_{rel}^2(w) \end{aligned}} \\ &= 0{,}0992\ \text{Bq} \cdot \text{g}^{-1} \end{aligned} \tag{129}$$

12.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a exceeds the decision threshold a^*. With $\omega = \Phi[a_m / u(a_m)] = 0{,}9667$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega \cdot (1 - \gamma/2) = 0{,}9425$ and $q = 1 - \omega \cdot \gamma/2 = 0{,}9758$, and hence the quantiles k_p and k_q are equal to 1,576 and 1,974, respectively. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a_m^{\triangleleft} = a_m - k_p \cdot u(a_m) = 0{,}007\ 79\,\mathrm{Bq \cdot g^{-1}} \text{ and } a_m^{\triangleright} = a_m + k_q \cdot u(a_m) = 0{,}115\,\mathrm{Bq \cdot g^{-1}} \quad (130)$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[a_m / u(a_m)] = 0{,}966\ 7$, one obtains $p = [1 + \omega \cdot (1 - \gamma)]/2 = 0{,}959\,2$, and hence the quantile k_p is equal to 1,741. With this, the limits of the shortest coverage interval according to Formula (131) are

$$a_m^{<} = a_m - k_p \cdot u(a_m) = 0{,}002\,81\,\mathrm{Bq \cdot g^{-1}} \text{ and } a_m^{>} = a_m + k_p \cdot u(a_m) = 0{,}108\,\mathrm{Bq \cdot g^{-1}} \quad (131)$$

12.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}_m$ of the activity of ^{235}U per unit mass of the sample is given by:

$$\hat{a}_m = a_m + \frac{u(a_m) \cdot \exp\left\{-a_m^2 / \left[2u^2(a_m)\right]\right\}}{\omega\sqrt{2\pi}} = 0{,}057\ 7\ \mathrm{Bq \cdot g^{-1}} \quad (132)$$

with its associated standard uncertainty

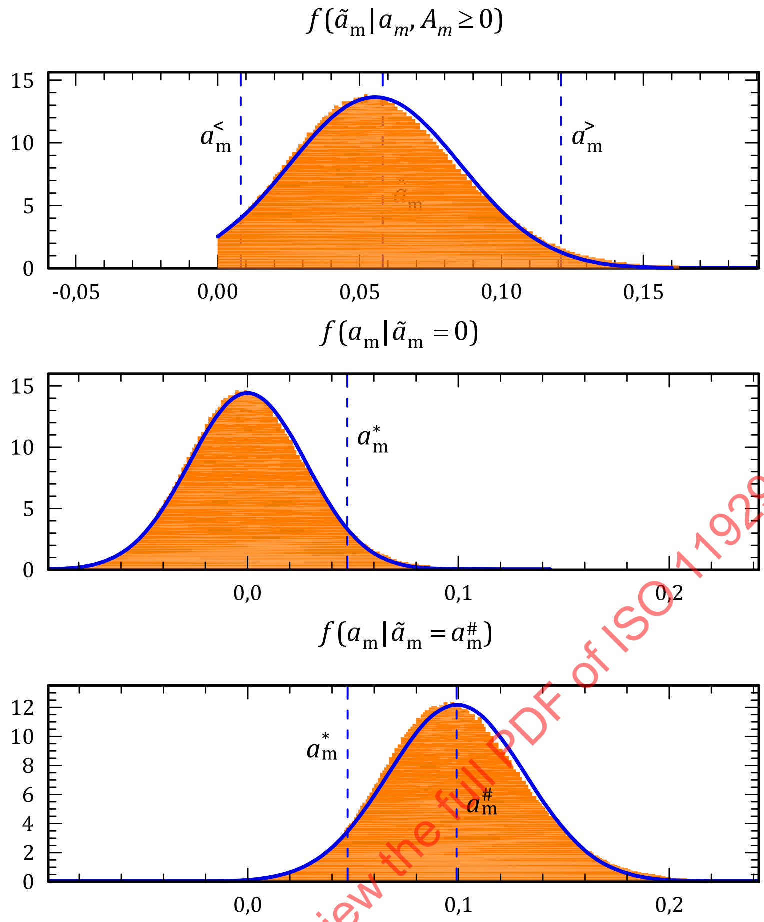
$$u(\hat{a}_m) = \sqrt{u^2(a_m) - (\hat{a}_m - a_m)\hat{a}_m} = 0{,}027\ 9\ \mathrm{Bq \cdot g^{-1}} \quad (133)$$

12.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The results and characteristic limits are summarized in Table 16 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 8.

Table 16 — Result and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a_m	Bq/g	0,055 4	0,056 4
Standard uncertainty associated with the primary result	$u(a_m)$	Bq/g	0,030 2	0,030 8
Relative standard uncertainty associated with the primary result	$u_{rel}(a_m)$	1	0,545	0,546
Decision threshold	a_m^*	Bq/g	0,045 5	0,047 5
Detection limit	$a_m^\#$	Bq/g	0,099 2	0,099 1
Best estimate	$\hat{a}_m$	Bq/g	0,057 7	0,058 1
Standard uncertainty associated with the best estimate	$u(\hat{a}_m)$	Bq/g	0,027 9	0,029 1
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{a}_m)$	1	0,484	0,501
Lower limit of the probabilistically symmetric coverage interval	$a_m^{\triangleleft}$	Bq/g	0,007 79	0,008 18
Upper limit of the probabilistically symmetric coverage interval	$a_m^{\triangleright}$	Bq/g	0,115	0,121
Lower limit of the shortest coverage interval	$a_m^{<}$	Bq/g	0,002 81	0,002 10
Upper limit of the shortest coverage interval	$a_m^{>}$	Bq/g	0,108	0,111



NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 8 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

12.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a_m = 0{,}055\,4\ \text{Bq} \cdot \text{g}^{-1}$ exceeds the decision threshold $a_m^* = 0{,}045\ 5\ \text{Bq} \cdot \text{g}^{-1}$. It is decided to conclude that an activity per unit mass of ^{235}U was recognized in the sample.

— Since no guideline value is given the assessment of the measurement procedure is omitted.

— The lower and upper limits of the probabilistically symmetric coverage interval are calculated as $a_m^{\triangleleft} = 0{,}007\,8\ \text{Bq} \cdot \text{g}^{-1}$ and $a_m^{\triangleright} = 0{,}115\ \text{Bq} \cdot \text{g}^{-1}$.

— The lower and upper limits of the shortest coverage interval are calculated as $a_m^< = 0{,}002\,8\ \text{Bq} \cdot \text{g}^{-1}$ and $a_m^> = 0{,}108\ \text{Bq} \cdot \text{g}^{-1}$.

— The best estimate is calculated as $\hat{a}_m = 0{,}057\ 7\ \text{Bq} \cdot \text{g}^{-1}$ with an associated standard uncertainty $u(\hat{a}_m) = 0{,}027\ 9\ \text{Bq} \cdot \text{g}^{-1}$.

The model of evaluation is linear, and there are no dominating large uncertainties so that the application of ISO 11929-1 is justified. In spite of the fact that the relative uncertainty of the primary estimate exceeds 50 %, many results obtained by application of ISO 11929-2 are practically identical with those obtained by ISO 11929-1:2019, Table 16. Large differences are seen between the limits of the probabilistically symmetric and the shortest coverage intervals, the upper limits being more affected than the lower limits. The ISO/IEC Guide 98-3 approximations, assuming normal PDFs and using a Taylor expansion truncated after the linear term, hold.

13 Black box measurements

13.1 Definition of the task and general aspects

Measurements of ambient dose rates with a large volume plastic detector of unknown functionality and algorithm are taken as an example for so-called black-box measurements (ISO 11929-1:2019, A.4).

Based on repeated measurements of ambient dose rates in an area considered not to be contaminated it is investigated whether an enhanced dose rate can be seen by repeated measurements in another area suspected to be potentially contaminated.

A guideline value $y_r = 50\,\text{nSv}\cdot\text{h}^{-1}$ is assumed for completeness of the example.

13.2 Model of evaluation and standard uncertainty

The measurand is a net dose rate as a difference between series of measurements of a background area and of a gross area potentially contaminated. The characteristic limits, the best estimate, and the associated standard uncertainty are calculated for this measurand.

NOTE Since the physical quantity of the particular dose rate is not specified the symbol y is used.

The model of the evaluation is given by:

$$y = (\bar{x}_g - \bar{x}_b)\cdot w \tag{134}$$

with the means of the gross and background measurements, $\bar{x}_g = \frac{1}{n_g}\sum_{i=1}^{n_g} x_{g,i}$ and $\bar{x}_b = \frac{1}{n_b}\sum_{i=1}^{n_b} x_{b,i}$, and

with the standard deviations of the gross and background measurements, $s_g = \left[\frac{1}{n_g - 1}\sum_{i=1}^{n_g}(x_{g,i} - \bar{x}_g)^2\right]^{1/2}$

and $s_b = \left[\frac{1}{n_b - 1}\sum_{i=1}^{n_b}(x_{b,i} - \bar{x}_b)^2\right]^{1/2}$, respectively.

According to the ISO/IEC Guide 98-3:2008/Suppl.1:2008, 6.4.9 the repeated measurements are assumed to be drawn from a Gaussian distribution with unknown mean and variance. Consequently, the PDF is the *t*-distribution $t_{n-1}(\bar{x}, s^2/n)$. The variance of this distribution then yields the standard uncertainties associated with the means of the gross and background measurements

$$u(\bar{x}_g) = \left(\frac{n_g - 1}{n_g - 3}\right)^{1/2} \cdot \frac{s_g}{\sqrt{n_g}} \quad \text{and} \quad u(\bar{x}_b) = \left(\frac{n_b - 1}{n_b - 3}\right)^{1/2} \cdot \frac{s_b}{\sqrt{n_b}} \tag{135}$$

NOTE Obviously, $n_g, n_b > 3$ is required.

Then the standard uncertainty associated with the primary measurement result y is calculated by:

$$u^2(y) = w^2 \cdot [u^2(\bar{x}_g) + u^2(\bar{x}_b)] + y^2 \cdot u_{rel}^2(w) \quad (136)$$

13.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 17. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 17.

Table 17 — Input data, intermediate values

Quantity	Symbol	Value	PDF
Number of readings related to the gross and to the background measurements	n_g, n_b	28, 27	
Gross effect in nSv h^{-1}	$x_{g,\,i}$	358, 370, 337, 229, 339, 186, 207, 194, 198, 170, 154, 177, 170, 157, 184, 147, 151, 127, 213, 192, 138, 170, 134, 135, 123, 153, 124, 146	$N[\tilde{\bar{x}}_g; \bar{x}_g, u(\bar{x}_g)]$
Background effect in nSv h^{-1}	$x_{b,i}$	95, 82, 69, 80, 83, 73, 70, 75, 76, 79, 80, 78, 77, 71, 74, 71, 72, 69, 68, 68, 71, 75, 80, 81, 74, 76, 77	$N[\tilde{\bar{x}}_b; \bar{x}_b, u(\bar{x}_b)]$
Calibration factor, assumed to be unity with 30 % relative uncertainty	$w, u(w)$	1; 0,3	$N[\tilde{w}; w, u(w)]$

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description and ISO 11929-1:2019, A.4 for the general methodology.

13.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

13.4.1 Background effect

The background effect was obtained by repeated n_b readings. The values of the individual readings $x_{b,i}$ are given in Table 17. One calculates the mean background measurement by

$$\bar{x}_b = \frac{1}{n_b} \sum_{i=1}^{n_b} x_{b,i} = 75{,}70 \text{ nSv} \cdot \text{h}^{-1} \quad (137)$$

with a standard deviation

$$s_b = \left[\frac{1}{n_b - 1} \sum_{i=1}^{n_b} (x_{b,i} - \bar{x}_b)^2 \right]^{1/2} = 5{,}90 \text{ nSv} \cdot \text{h}^{-1} \quad (138)$$

and a standard uncertainty

$$u(\bar{x}_b) = \left(\frac{n_b - 1}{n_b - 3} \right)^{1/2} \cdot \frac{s_b}{\sqrt{n_b}} = 1{,}18 \text{ nSv} \cdot \text{h}^{-1} \quad (139)$$

13.4.2 Primary result and its associated standard uncertainty

The gross measurement was obtained by repeated n_g readings. The values of the individual readings $x_{g,i}$ are given in Table 17. One calculates the mean gross effect by

$$\bar{x}_g = \frac{1}{n_g} \sum_{i=1}^{n_g} x_{g,i} = 192{,}25 \, \text{nSv} \cdot \text{h}^{-1} \tag{140}$$

with a standard deviation

$$s_g = \left[\frac{1}{n_g - 1} \sum_{i=1}^{n_g} (x_{g,i} - \bar{x}_g)^2 \right]^{1/2} = 71{,}72 \, \text{nSv} \cdot \text{h}^{-1} \tag{141}$$

and a standard uncertainty

$$u(\bar{x}_g) = \left(\frac{n_g - 1}{n_g - 3} \right)^{1/2} \cdot \frac{s_g}{\sqrt{n_g}} = 14{,}09 \, \text{nSv} \cdot \text{h}^{-1} \tag{142}$$

This yield the primary result

$$y = (\bar{x}_g - \bar{x}_b) \cdot w = 116{,}55 \, \text{nSv} \cdot \text{h}^{-1} \tag{143}$$

The standard uncertainty associated with the primary measurement result is given by:

$$u(y) = \sqrt{w^2 \cdot [u^2(\bar{x}_g) + u^2(\bar{x}_b)] + y^2 \cdot u_{\text{rel}}^2(w)} = 37{,}71 \, \text{nSv} \cdot \text{h}^{-1} \tag{144}$$

13.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty as a function of an assumed true value is not explicitly available. Therefore, the following procedure is applied.

For a true value of the measurand $\tilde{y} = 0$ one expects with ISO 11929-1:2019, Formula (A.24)

$$\tilde{u}^2(\tilde{y}=0) = w^2 \cdot \left[\frac{n_g - 1}{n_g \cdot (n_g - 3)} + \frac{n_b - 1}{n_b \cdot (n_b - 3)} \right] \cdot s_b^2 \tag{145}$$

and the decision threshold

$$y^* = k_{1-\alpha} \cdot w \cdot s_b \cdot \sqrt{\frac{n_g - 1}{n_g \cdot (n_g - 3)} + \frac{n_b - 1}{n_b \cdot (n_b - 3)}} \tag{146}$$

According to ISO 11929-1:2019, 5.5, the following linear interpolation often suffices, if only $\tilde{u}(0)$, one result, $y > 0$, and its associated standard uncertainty, $u(y)$, are known

$$\tilde{u}^2(\tilde{y}) = \tilde{u}^2(0) \cdot (1 - \tilde{y}/y) + u^2(y) \cdot \tilde{y}/y \tag{147}$$

13.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha})=1-\alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the dose rate is calculated by:

$$y^{*}=k_{1-\alpha}\cdot w\cdot s_{\mathrm{b}}\cdot\sqrt{\frac{n_{\mathrm{g}}-1}{n_{\mathrm{g}}\cdot(n_{\mathrm{g}}-3)}+\frac{n_{\mathrm{b}}-1}{n_{\mathrm{b}}\cdot(n_{\mathrm{b}}-3)}}=2{,}72\,\mathrm{nSv}\cdot\mathrm{h}^{-1} \tag{148}$$

13.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The linear interpolation according to Formula (147) leads with the auxiliary quantity a according to Formula (149)

$$a=k_{1-\alpha}\cdot\tilde{u}(0)+\frac{1}{2}\left\{\left(k_{1-\beta}^{2}/y\right)\left[\left(u^{2}(y)-\tilde{u}^{2}(0)\right)\right]\right\} \tag{149}$$

to the detection limit

$$y^{\#}=a+\sqrt{a^{2}+(k_{1-\beta}^{2}-k_{1-\alpha}^{2})\tilde{u}^{2}(0)}=38{,}39\,\mathrm{nSv}\cdot\mathrm{h}^{-1} \tag{150}$$

The guideline value exceeds the detection limit.

13.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result y exceeds the decision threshold y^{*}. With $\omega=\Phi[y/u(y)]=0{,}999$ and $\gamma=0{,}05$ one obtains the probabilities by $p=\omega\cdot(1-\gamma/2)=0{,}974$ and $q=1-\omega\cdot\gamma/2=0{,}975$, and hence the quantiles are $k_p=1{,}944$ and $k_q=1{,}960$. Then the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$y^{\lhd}=y-k_p\cdot u(y)=43{,}25\,\mathrm{nSv}\cdot\mathrm{h}^{-1}\ \text{and}\ y^{\rhd}=y+k_q\cdot u(y)=190{,}48\,\mathrm{nSv}\cdot\mathrm{h}^{-1} \tag{151}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega=\Phi[y/u(y)]=0{,}999\ 0$, one obtains $p=[1+\omega\cdot(1-\gamma)]/2=0{,}974\ 5$, and hence the quantile k_p is equal to 1,96. With this, the limits of the shortest coverage interval according to Formula (152) are

$$y^{<}=y-k_p\cdot u(y)=42{,}93\,\mathrm{nSv}\cdot\mathrm{h}^{-1}\ \text{and}\ y^{>}=y+k_p\cdot u(y)=190{,}16\,\mathrm{nSv}\cdot\mathrm{h}^{-1} \tag{152}$$

13.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{y}$ of the measurand is given by:

$$\hat{y}=y+\frac{u(y)\cdot\exp\left\{-y^{2}/\left[2u^{2}(y)\right]\right\}}{\omega\sqrt{2\pi}}=116{,}67\ \mathrm{nSv}\cdot\mathrm{h}^{-1} \tag{153}$$

with its associated standard uncertainty $u(\hat{y})$

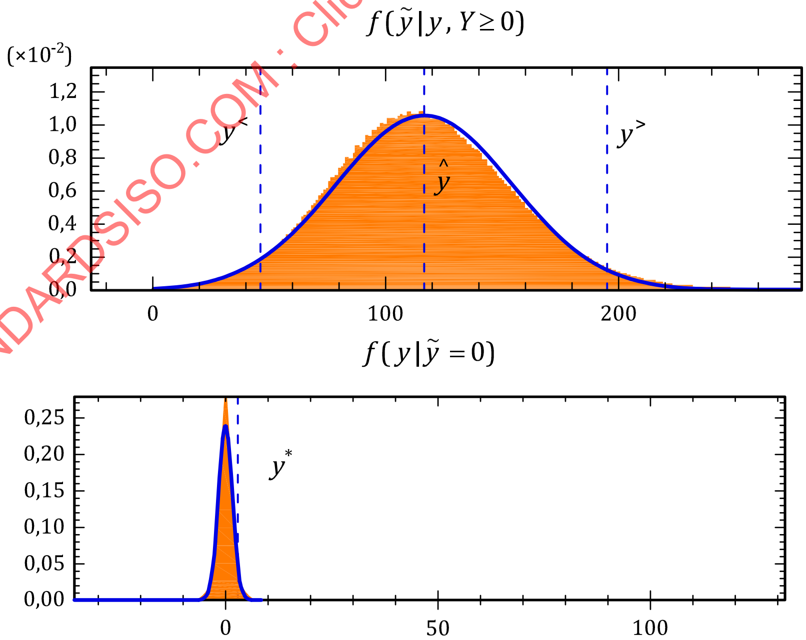
$$u(\hat{y})=\sqrt{u^{2}(y)-(\hat{y}-y)\hat{y}}=37{,}52\ \mathrm{nSv}\cdot\mathrm{h}^{-1} \tag{154}$$

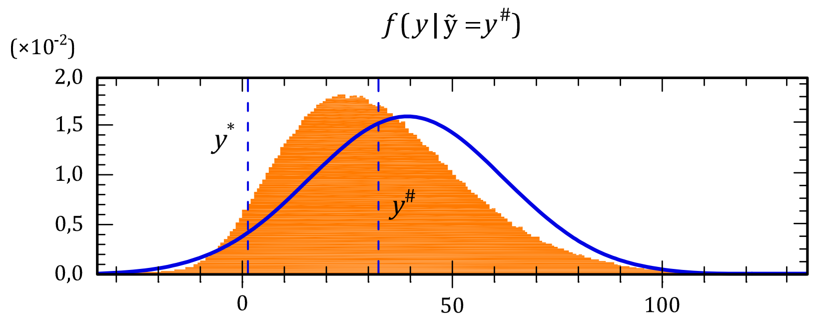
13.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The results and characteristic limits are summarized in Table 18 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 9.

Table 18 — Result and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	y	nSv h^{-1}	116,55	116,6
Standard uncertainty associated with the primary result	$u(y)$	nSv h^{-1}	37,71	37,9
Relative standard uncertainty associated with the primary result	$u_{rel}(y)$	1	0,323	0,325
Decision threshold	y^*	nSv h^{-1}	2,72	2,84
Detection limit	$y^\#$	nSv h^{-1}	38,39	32,00
Best estimate	$\hat{y}$	nSv h^{-1}	116,67	116,60
Standard uncertainty associated with the best estimate	$u(\hat{y})$	nSv h^{-1}	37,52	37,83
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{y})$	1	0,321	0,324
Lower limit of the probabilistically symmetric coverage interval	$y^\triangleleft$	nSv h^{-1}	43,25	46,54
Upper limit of the probabilistically symmetric coverage interval	$y^\triangleright$	nSv h^{-1}	190,48	194,98
Lower limit of the shortest coverage interval	$y^<$	nSv h^{-1}	42,93	43,78
Upper limit of the shortest coverage interval	$y^>$	nSv h^{-1}	190,16	191,80





NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 9 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

13.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $y = 116{,}55\ \text{nSv} \cdot \text{h}^{-1}$ exceeds the decision threshold $y^* = 2{,}72\ \text{nSv} \cdot \text{h}^{-1}$. It is decided to conclude that a difference between the two investigated areas was recognized.

— The detection limit $y^{\#} = 38{,}39\ \text{nSv} \cdot \text{h}^{-1}$ is below the guideline values $y_r = 50\ \text{nSv} \cdot \text{h}^{-1}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $y^{\triangleleft} = 43{,}25\ \text{nSv} \cdot \text{h}^{-1}$ and $y^{\triangleright} = 190{,}48\ \text{nSv} \cdot \text{h}^{-1}$. They are equal to those of the shortest coverage interval.

— The best estimate is $\hat{y} = 116{,}67\ \text{nSv} \cdot \text{h}^{-1}$ with an associated standard uncertainty $u(\hat{y}) = 37{,}52\ \text{nSv} \cdot \text{h}^{-1}$
.

The model of evaluation is linear, and there are no dominating large uncertainties so that the application of ISO 11929-1 is justified. This is a case with moderate relative uncertainties. The results obtained by application of ISO 11929-2 are practically identical with those obtained by ISO 11929-1:2019, Table 18. The ISO/IEC Guide 98-3 approximations, assuming normal PDFs and using a Taylor expansion truncated after the linear term, hold.

14 Counting measurements with unknown random influence of sample treatment

14.1 Definition of the task and general aspects

This is an example for the application of ISO 11929-1:2019, A.2.

A sample of solid material containing a radionuclide is examined by chemical separation of this nuclide and subsequent counting measurement of its radiation. The measurement is randomly influenced by sample treatment because of the chemical separation. To determine and reduce the influence, several samples of the same kind of material and blanks are separately tested. The results for the respective samples are then averaged and analysed regarding the measurement uncertainty. This is an example according to the general model of ISO 11929-1:2019, A.2.

After the counting measurements of the gross effect on m_g samples to be tested and of the background effect on m_0 blanks are carried out with the preselected measurement durations t_g and t_0, respectively, the numbers $\bar{n}_g$ and $\bar{n}_0$ of the recorded events averaged according to Formulae (157, 158) are available. This first yields the estimates $\bar{r}_g = \bar{n}_g / t_g$ and $\bar{r}_0 = \bar{n}_0 / t_0$ of the respective mean count rates.

A guideline value $a_{m,r} = 0,5 \, Bq \cdot kg^{-1}$ is assumed.

14.2 Model of evaluation and standard uncertainty

The measurand is the specific activity a_m (activity of the sample divided by the total mass of the sample) for which the characteristic limits, the best estimate, and the associated standard uncertainty are calculated.

The model of the evaluation is given by:

$$a_m = \left(\frac{\bar{n}_g}{t_g} - \frac{\bar{n}_0}{t_0} \right) \cdot w = \frac{\bar{r}_g - \bar{r}_0}{m \cdot \kappa \cdot \varepsilon} = (\bar{r}_g - \bar{r}_0) \cdot w \qquad (155)$$

where

$$w = \frac{1}{m \cdot \kappa \cdot \varepsilon} \qquad (156)$$

The mean value of the background counts $\bar{n}_0$ and the square of its standard uncertainty $u^2(\bar{n}_0)$ of the values $n_{0,i}$ are given by:

$$\bar{n}_0 = \frac{1}{m_0} \sum_{i=1}^{m_0} n_{0,i} \;;\; u^2(\bar{n}_0) = \frac{1}{m_0} \left[\bar{n}_0 + \frac{m_0 - 1}{m_0 - 3} \bar{n}_0 + \frac{1}{m_0 - 3} \sum_{i=1}^{m_0} (n_{0,i} - \bar{n}_0)^2 \right] \qquad (157)$$

The mean value of the gross counts $\bar{n}_g$ and its uncertainty $u^2(\bar{n}_g)$ of the values $n_{g,i}$ are given by:

$$\bar{n}_g = \frac{1}{m_g} \sum_{i=1}^{m_g} n_{g,i} \;;\; u^2(\bar{n}_g) = \frac{1}{m_g} \left[\bar{n}_g + \frac{m_g - 1}{m_g - 3} \bar{n}_g + \frac{1}{m_g - 3} \sum_{i=1}^{m_g} (n_{g,i} - \bar{n}_g)^2 \right] \qquad (158)$$

NOTE Obviously, $m_0, m_g > 3$ is required.

14.3 Available information, input data, and specifications

The efficiency of the detector κ and its associated standard uncertainty $u(\kappa)$ were independently determined to be $\kappa = 0,51$ $s^{-1} \cdot Bq^{-1}$ and $u(\kappa) = 0,02$ $s^{-1} \cdot Bq^{-1}$.

The mass of the sample was determined by a balance to be $m = 0,1 \, kg$. Its associated relative standard uncertainty was set to $u_{rel}(m) = 1 \; \%$ according to the specifications of the balance.

The chemical yield ε was determined by separate experiments, which resulted in an average $\varepsilon = 0,57$ and an associated standard uncertainty $u(\varepsilon) = 0,04$.

The input data and their associated uncertainties are given in Table 19. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 19.

Table 19 — Input data, intermediate values

Quantity	Symbol	Value	Standard uncertainty	PDF
Number of samples and blank samples	m_g, m_0	5, 5		
Number of recorded events for the samples (gross effect)	$n_{g,i}$	1 832; 2 259; 2 138; 2 320; 1 649		$N[\tilde{\bar{n}}_g ; \bar{n}_g , u(\bar{n}_g)]$

Table 19 *(continued)*

Quantity	Symbol	Value	Standard uncertainty	PDF
Number of recorded events for the blanks (background effect)	$n_{0,i}$	966; 676; 911; 856; 676		$\mathrm{N}[\tilde{\bar{n}}_0; \bar{n}_0, u(\bar{n}_0)]$
Measurement durations	t_g, t_0	30 000 s		-
Sample mass (general)	m with $u(m)$	0,100 kg	0,001 kg	$\mathrm{N}[\tilde{m}; m, u(m)]$
Detection efficiency	κ with $u(\kappa)$	0,510	0,020 s^{-1} Bq	$\mathrm{N}[\tilde{\kappa}; \kappa, u(\kappa)]$
Chemical yield	ε with $u(\varepsilon)$	0,570	0,040	$\mathrm{N}[\tilde{\varepsilon}; \varepsilon, u(\varepsilon)]$
Intermediate values				
Quantity	Symbol		Value	
Mean values	$\bar{n}_0$, $\bar{n}_g$		817,00; 2 039,6	
Empirical standard deviations	s_0, s_g		288,14; 134,46	
Standard uncertainty	$u(\bar{n}_0)$, $u(\bar{n}_g)$		87,88; 185,57	

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3 for a detailed description and ISO 11929-1:2019, A.2 for the general methodology.

14.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

14.4.1 Background effect

The count rate of the background effect has been measured by repeated m_b analyses of blank samples. The numbers of counted pulses $n_{0,i}$ are given in Table 19. One calculates the mean background count rate by

$$\bar{r}_0 = \frac{\bar{n}_0}{t_0} = 0{,}068\,0\ \mathrm{s}^{-1} \tag{159}$$

The standard uncertainty associated with the background count rate is given by:

$$u(\bar{r}_0) = \sqrt{\frac{u^2(\bar{n}_0)}{t_0^2}} = 0{,}002\,93\ \mathrm{s}^{-1} \tag{160}$$

14.4.2 Primary result and its associated standard uncertainty

The mass-related activity is calculated from Formula (155). This Formula depends on the calibration factor, the input quantities of which and their associated standard uncertainties were determined independently.

This yields a calibration factor calculated by:

$$w = \frac{1}{m \cdot \kappa \cdot \varepsilon} = 34{,}40\ \text{s Bq} \cdot \text{kg}^{-1} \tag{161}$$

The relative standard uncertainty of the calibration factor $u_{\text{rel}}(w) = u(w)/w$ is calculated by:

$$u_{\text{rel}}(w) = \sqrt{u_{\text{rel}}^2(m) + u_{\text{rel}}^2(\varepsilon) + u_{\text{rel}}^2(\kappa)} = 0{,}081\,0 \tag{162}$$

which yields

$$u(w) = 2{,}79\ \text{s Bq} \cdot \text{kg}^{-1} \tag{163}$$

The primary result a_{m} of the measurement is given by:

$$a_{\text{m}} = \frac{\bar{r}_{\text{g}} - \bar{r}_0}{m \cdot \kappa \cdot \varepsilon} = \left(\frac{\bar{n}_{\text{g}}}{t_{\text{g}}} - \frac{\bar{n}_0}{t_0} \right) \cdot w = 1{,}40\ \text{Bq} \cdot \text{kg}^{-1} \tag{164}$$

The standard uncertainty $u(a_{\text{m}})$ associated with the primary measurement result a_{m} is calculated by:

$$u(a_{\text{m}}) = \sqrt{w^2 \cdot \left[u^2(\bar{n}_{\text{g}})/t_{\text{g}}^2 + u^2(\bar{n}_0)/t_0^2 \right] + a_{\text{m}}^2 \cdot u_{\text{rel}}^2(w)} = 0{,}261\ \text{Bq} \cdot \text{kg}^{-1}. \tag{165}$$

14.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty as a function of an assumed true value is not explicitly available. Therefore, the following procedure is applied

For a true value of the measurand $\tilde{y} = 0$ one expects $\bar{x}_{\text{g}} = \bar{x}_{\text{b}}$ and hence one receives

$$u^2(0) = 2 \cdot w^2 \cdot u^2(r_0) \tag{166}$$

which allows calculating the decision threshold.

If only $\tilde{u}(0) = u(a_{\text{m},0})$, $a_{\text{m},1} > 0$ and $u(a_{\text{m},1})$ are known, the following linear interpolation can be used

$$\tilde{u}^2(\tilde{a}_{\text{m}}) = \tilde{u}^2(0) \cdot (1 - \tilde{a}_{\text{m}}/a_{\text{m},1}) + u^2(a_{\text{m},1}) \cdot \tilde{a}_{\text{m}}/a_{\text{m},1} \tag{167}$$

14.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1 - \alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is then calculated by:

$$a_{\text{m}}^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot w \cdot \sqrt{2 \cdot u^2(\bar{n}_0)/t_0^2} = 0{,}234\ \text{Bq} \cdot \text{kg}^{-1} \tag{168}$$

14.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The linear interpolation according to Formula (167) and the auxiliary quantity a according to Formula (169)

$$a = k_{1-\alpha} \cdot \tilde{u}(0) + \frac{1}{2}\left\{\left(k_{1-\beta}^2 / a_{m,1}\right)\left[\left(u^2(y_1) - \tilde{u}^2(0)\right]\right\} \tag{169}$$

leads to the detection limit

$$a_\mathrm{m}^{\#} = a + \sqrt{a^2 + (k_{1-\beta}^2 - k_{1-\alpha}^2)\tilde{u}^2(0)} = 0{,}546\ \mathrm{Bq \cdot kg^{-1}} \tag{170}$$

The detection limit exceeds the guideline value.

14.4.6 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a_m exceeds the decision threshold a_m^*. With $\omega = \Phi[a_\mathrm{m} / u(a_\mathrm{m})] = 0{,}999$ and $\gamma = 0{,}05$ one obtains the probabilities $p = \omega \cdot (1 - \gamma / 2) = 0{,}975$ and $q = 1 - \omega \cdot \gamma / 2 = 0{,}975$, and hence the quantiles k_p and k_q are equal to 1,96 and 1,96, respectively. Then, the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a_\mathrm{m}^{\triangleleft} = a_\mathrm{m} - k_p \cdot u(a_\mathrm{m}) = 0{,}890\ \mathrm{Bq \cdot kg^{-1}} \text{ and } a_\mathrm{m}^{\triangleright} = a_\mathrm{m} + k_q \cdot u(a_\mathrm{m}) = 1{,}91\ \mathrm{Bq \cdot kg^{-1}} \tag{171}$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[a_\mathrm{m} / u(a_\mathrm{m})] = 0{,}999$, one obtains $p = [1 + \omega \cdot (1 - \gamma)] / 2 = 0{,}975$, and hence the quantile k_p is equal to 1,96. With this, the limits of the shortest coverage interval according to Formula (172) are

$$a_\mathrm{m}^{<} = a_\mathrm{m} - k_p \cdot u(a_\mathrm{m}) = 0{,}890\ \mathrm{Bq \cdot kg^{-1}} \text{ and } a_\mathrm{m}^{>} = a_\mathrm{m} + k_p \cdot u(a_\mathrm{m}) = 1{,}91\ \mathrm{Bq \cdot kg^{-1}} \tag{172}$$

14.4.7 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}_\mathrm{m}$ of the activity of the sample is given by:

$$\hat{a}_\mathrm{m} = a_\mathrm{m} + \frac{u(a_\mathrm{m}) \cdot \exp\left\{-a_\mathrm{m}^2 / \left[2u^2(a_\mathrm{m})\right]\right\}}{\omega\sqrt{2\pi}} = 1{,}40\ \mathrm{Bq \cdot kg^{-1}} \tag{173}$$

with its associated standard uncertainty $u(\hat{a}_\mathrm{m})$

$$u(\hat{a}_\mathrm{m}) = \sqrt{u^2(a_\mathrm{m}) - (\hat{a}_\mathrm{m} - a_\mathrm{m})\hat{a}_\mathrm{m}} = 0{,}261\ \mathrm{Bq \cdot kg^{-1}} \tag{174}$$

14.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

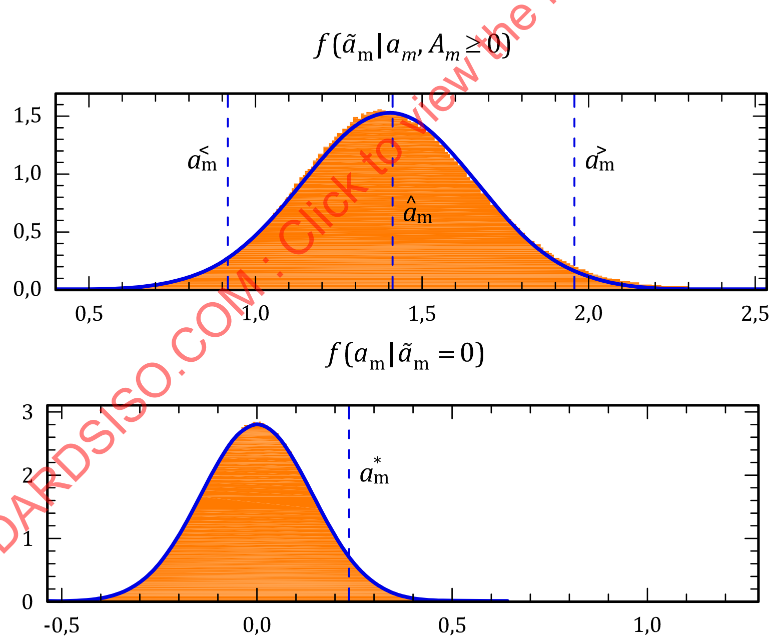
The results and characteristic limits are summarized in Table 20 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 10.

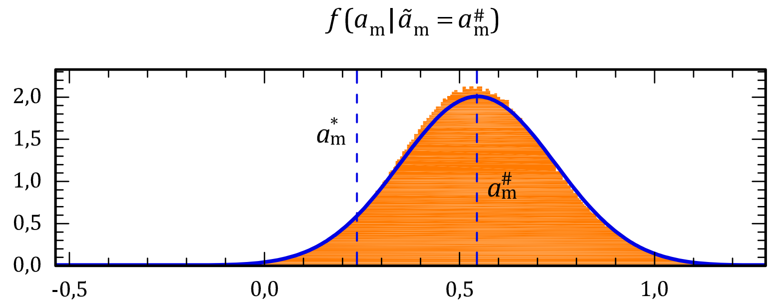
Table 20 — Result and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a_m	$\mathrm{Bq \cdot kg^{-1}}$	1,40	1,411

Table 20 *(continued)*

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Standard uncertainty associated with the primary result	$u(a_m)$	$Bq \cdot kg^{-1}$	0,261	0,265
Relative standard uncertainty associated with the primary result	$u_{rel}(a_m)$	1	0,186	0,188
Decision threshold	a_m^*	$Bq \cdot kg^{-1}$	0,234	0,237
Detection limit	$a_m^\#$	$Bq \cdot kg^{-1}$	0,546	0,545
Best estimate	$\hat{a}_m$	$Bq \cdot kg^{-1}$	1,40	1,411
Standard uncertainty associated with the best estimate	$u(\hat{a}_m)$	$Bq \cdot kg^{-1}$	0,261	0,264
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{a}_m)$	1	0,186	0,187
Lower limit of the probabilistically symmetric coverage interval	$a_m^\triangleleft$	$Bq \cdot kg^{-1}$	0,890	0,918
Upper limit of the probabilistically symmetric coverage interval	$a_m^\triangleright$	$Bq \cdot kg^{-1}$	1,91	1,96
Lower limit of the shortest coverage interval	$a_m^<$	$Bq \cdot kg^{-1}$	0,890	0,902
Upper limit of the shortest coverage interval	$a_m^>$	$Bq \cdot kg^{-1}$	1,91	1,94





NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 10 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

14.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a_m = 1{,}40\ \mathrm{Bq \cdot kg^{-1}}$ exceeds the decision threshold $a_m^* = 0{,}23\ \mathrm{Bq \cdot kg^{-1}}$. It is decided to conclude that an activity per unit mass of the sample was recognized.

— The detection limit $a_m^{\#} = 0{,}55\ \mathrm{Bq \cdot kg^{-1}}$ exceeds the guideline value $a_{m,r} = 0{,}5\ \mathrm{Bq \cdot kg^{-1}}$. It is decided to conclude that the measurement procedure is not suited for the measurement purpose in spite of the fact that a contribution of the sample was recognized.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a_m^{\lhd} = 0{,}89\ \mathrm{Bq \cdot kg^{-1}}$ and $a_m^{\rhd} = 1{,}91\ \mathrm{Bq \cdot kg^{-1}}$. These limits are identical with the limits of the shortest coverage interval.

— The best estimate is $\hat{a}_m = 1{,}40\ \mathrm{Bq \cdot kg^{-1}}$ with an associated standard uncertainty $u(\hat{a}_m) = 0{,}26\ \mathrm{Bq \cdot kg^{-1}}$.

The model of evaluation is linear, and there are no dominating large uncertainties so that the application of ISO 11929-1 is justified. This is a case with small or minor relative uncertainties. The results obtained by application of ISO 11929-2 are practically identical with those obtained by ISO 11929-1:2019, Table 20. The ISO/IEC Guide 98-3 approximations, which assume normal PDFs and make use of a Taylor expansion truncated after the linear term, hold.

Note that in spite of the fact that according to the comparison of the detection limit and the guideline value the measurement procedure is not suited for the measurement purpose, an effect of the sample has been recognized. The primary measurement result and its associated standard uncertainty can well be used to calculate limits of coverage intervals and the best estimate and its associated standard uncertainty.

15 Counting measurement with known influence of sample treatment

15.1 Definition of the task and general aspects

This is an example for the application ISO 11929-1:2019, A.3.

As in the example in Clause 14, a sample of solid material containing a radionuclide is examined by chemical separation of this nuclide and subsequent counting measurement of its radiation. The measurement is randomly influenced by sample treatment because of the chemical separation. In this example, the random influence of sample treatment was derived in form of the relative uncertainty of the sample treatment from measurements on reference samples or on other samples. The latter samples should be similar to the current samples and be measured under similar conditions, in order that they can be taken as reference samples although they need not be examined specifically for reference purposes. Since the influence of the sample treatment is now known one measurement, each of a sample and a blank is sufficient in the actual measurement. This is an example according to the general model of ISO 11929-1:2019, A.3.

This procedure, appropriate when small random influences are present, is based on the approach

$$u^2(\bar{n}) = \frac{\bar{n} + u^2}{m} = \frac{\bar{n} + \vartheta^2 \cdot \bar{n}^2}{m} \quad \text{with } \vartheta = u / \bar{n} \tag{175}$$

and with the auxiliary quantity $u^2 = \frac{m-1}{m-3}\bar{n} + \frac{1}{m-3}\sum_{i=1}^{m}(n_i - \bar{n})^2$ for any counts of the measurement procedure.

The influence parameter ϑ can be calculated from the data of counting measurements of m_r reference samples by

$$\vartheta^2 = \left(m_\mathrm{r} \cdot u^2(\bar{n}_\mathrm{r}) - \bar{n}_\mathrm{r}\right) / \bar{n}_\mathrm{r}^2 \quad \text{or } \vartheta = u_\mathrm{r} / \bar{n}_\mathrm{r} \tag{176}$$

If $\vartheta^2 < 0$ results, the approach and the data are not compatible. The number m_r of the reference samples should then be enlarged or $\vartheta = 0$ shall be set. Moreover, $\vartheta < 0{,}2$ should be obtained. Otherwise, one can proceed according to the example in Clause 14.

A guideline value $a_{\mathrm{m,r}} = 1\,\mathrm{Bq \cdot kg^{-1}}$ is assumed.

15.2 Model of evaluation and standard uncertainty

The measurand y is the specific activity a_m (activity of the sample divided by the total mass of the sample) for which the characteristic limits, the best estimate, and the associated standard uncertainty are calculated.

In the case of known influences, the following expressions are valid for the mean gross count rate r_g and the mean background count rate, r_0.

The model of the evaluation is given by:

$$a_{\mathrm{m}} = \left(\frac{n_{\mathrm{g}}}{t_{\mathrm{g}}} - \frac{n_0}{t_0}\right) \cdot w = \frac{r_{\mathrm{g}} - r_0}{m \cdot \kappa \cdot \varepsilon} = (r_{\mathrm{g}} - r_0) \cdot w \quad (177)$$

where

$$w = \frac{1}{m \cdot \kappa \cdot \varepsilon} \quad (178)$$

Then the standard uncertainties of the background and the gross count rate are given by:

$$u^2(r_{\mathrm{g}}) = (n_{\mathrm{g}} + \vartheta^2 \cdot n_{\mathrm{g}}^2) / t_{\mathrm{g}}^2 \text{ and } u^2(r_0) = (n_0 + \vartheta^2 \cdot n_0^2) / t_0^2 \quad (179)$$

The relative standard uncertainty of the calibration factor $u_{\mathrm{rel}}(w) = u(w)/w$ is calculated by:

$$u_{\mathrm{rel}}^2(w) = u_{\mathrm{rel}}^2(m) + u_{\mathrm{rel}}^2(\varepsilon) + u_{\mathrm{rel}}^2(\kappa) \quad (180)$$

The standard uncertainty of the measurand is calculated with $u^2(r_{\mathrm{g}})$ and $u^2(r_0)$ according to Formula (179) by

$$u(a_{\mathrm{m}}) = \sqrt{w^2 \cdot \left[u^2(r_{\mathrm{g}}) + u^2(r_0)\right] + a_{\mathrm{m}}^2 \cdot u_{\mathrm{rel}}^2(w)} \quad (181)$$

15.3 Available information, input data, and specifications

The efficiency of the detector κ and its associated standard uncertainty $u(\kappa)$ were independently determined to be $\kappa = 0{,}51\,\mathrm{s}^{-1} \cdot \mathrm{Bq}^{-1}$ and $u(\kappa) = 0{,}02\,\mathrm{s}^{-1} \cdot \mathrm{Bq}^{-1}$.

The mass of the sample was determined by a balance to be $m = 0{,}1\,\mathrm{kg}$. Its associated relative standard uncertainty was set to $u_{\mathrm{rel}}(m) = 1\ \%$ by expert guess.

The chemical yield ε was determined by separate experiments which resulted in $\varepsilon = 0{,}57$ and an associated standard uncertainty $u(\varepsilon) = 0{,}04$.

The input data and their associated uncertainties are given in Table 21. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 21.

Table 21 — Input data, intermediate values

Quantity	Symbol	Data		
Number of samples, blanks and reference samples	m_{g}, m_0, m_{r}	1, 1, 20		
Number of recorded events for the reference samples	$n_{\mathrm{r},i}$	74 349; 67 939; 88 449; 83 321; 66 657; 64 094, 74 348; 93 576; 56 402; 66 785; 78 194; 69 221; 63 965; 70 503; 74 220; 97 422; 74 476; 71 784; 68 235; 74 989		
Number of recorded events for the samples (gross effect)	n_{g}	2 040		
Number of recorded events for the blanks (background effect)	n_0	817		
Quantity	**Symbol**	**Value**	**Standard uncertainty**	**PDF**
measurement durations	t_{g}, t_0, t_{r}	30 000 s	—	
Sample mass	m with $u(m)$	0,100 kg	0,001 kg	$\mathrm{N}[\tilde{m}; m, u(m)]$

Table 21 *(continued)*

Quantity	Symbol	Data		
Detection efficiency	κ with $u(\kappa)$	0,51 s^{-1} Bq^{-1}	0,02 s^{-1} Bq^{-1}	$N[\tilde{\kappa};\kappa,u(\kappa)]$
Chemical yield	ε with $u(\varepsilon)$	0,57	0,04	$N[\tilde{\varepsilon};\varepsilon,u(\varepsilon)]$
Intermediate values				
Quantity	**Symbol**	**Value**		
Mean values	$\bar{n}_r$	73 946,5		
Auxiliary quantity	u_r	10 771,3		
Influence parameter	$\vartheta = u_r / \bar{n}_r$	0,146		

The PDFs of the calibration factor and of the count rates were chosen according to the ISO/IEC Guide 98-3:2008/Suppl.1; see also ISO 11929-2:2019, 6.3, for a detailed description and ISO 11929-1:2019, A.3 for the general methodology.

15.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

15.4.1 Determination of the relative uncertainty of the sample treatment

By analysing m_r reference samples, the relative uncertainty of the sample treatment was determined by

$$\vartheta = u_r / \bar{n}_r \tag{182}$$

with the auxiliary quantity, u

$$u_r^2 = \frac{m_r - 1}{m_r - 3}\bar{n}_r + \frac{1}{m_r - 3}\sum_{i=1}^{m_r}(n_{r,i} - \bar{n}_r)^2 \tag{183}$$

This yields the relative uncertainty of the sample treatment

$$\vartheta = \frac{1}{\bar{n}_r}\sqrt{\frac{m_r - 1}{m_r - 3}\bar{n}_r + \frac{1}{m_r - 3}\sum_{i=1}^{m_r}(n_{r,i} - \bar{n}_r)^2} = 0{,}146 \tag{184}$$

15.4.2 Background effect

The count rate of the background effect has been measured by an analysis of a blank sample. The number of counted pulses n_0 is given in Table 21. One calculates the background count rate by

$$r_0 = \frac{n_0}{t_0} = 0{,}027\,2\ s^{-1} \tag{185}$$

Its standard uncertainty associated with the background count rate is given by:

$$u(r_0) = \sqrt{\frac{n_0 + \vartheta^2 \cdot n_0^2}{t_0^2}} = 0{,}004\,08\ s^{-1} \tag{186}$$

15.4.3 Primary result and its associated standard uncertainty

The mass–related activity is calculated from Formula (177). This formula depends on the calibration factor the input quantities of which and their associated standard uncertainties were determined independently.

The calibration factor is calculated by:

$$w = \frac{1}{m \cdot k \cdot \varepsilon} = 34{,}40\ \text{s Bq} \cdot \text{kg}^{-1} \tag{187}$$

The relative standard uncertainty of the calibration factor $u_{\text{rel}}(w) = u(w)/w$ is calculated by:

$$u_{\text{rel}}(w) = \sqrt{u_{\text{rel}}^2(m) + u_{\text{rel}}^2(\varepsilon) + u_{\text{rel}}^2(\kappa)} = 0{,}081\,0 \tag{188}$$

which yields

$$u(w) = 2{,}79\ \text{s Bq} \cdot \text{kg}^{-1} \tag{189}$$

The primary result a_{m} of the measurement is given by:

$$a_{\text{m}} = \left(\frac{n_{\text{g}}}{t_{\text{g}}} - \frac{n_0}{t_0} \right) \cdot w = 1{,}40\ \text{Bq} \cdot \text{kg}^{-1} \tag{190}$$

The standard uncertainty $u(a_{\text{m}})$ associated with the primary measurement result a_{m} is calculated as

$$u(a_{\text{m}}) = \sqrt{w^2 \cdot \left[\left(n_{\text{g}} + \vartheta^2 \cdot n_{\text{g}}^2\right)/t_{\text{g}}^2 + \left(n_0 + \vartheta^2 \cdot n_0^2\right)/t_0^2 \right] + a_{\text{m}}^2 \cdot u_{\text{rel}}^2(w)} = 0{,}389\ \text{Bq} \cdot \text{kg}^{-1} \tag{191}$$

15.4.4 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}(\tilde{a}_{\text{m}})$ as a function of the true value $\tilde{a}_{\text{m}}$ of the measurand is needed to calculate the decision threshold and the detection limit.

For an assumed true value of the measurand $\tilde{a}_{\text{m}}$ one expects $\tilde{n}_{\text{g}} = \tilde{a}_{\text{m}}/w + n_0/t_0$ and one obtains with Formula (181) and $u^2(n_0)$ and $u^2(n_{\text{g}})$ from Formula (179)

$$\tilde{u}(\tilde{a}_{\text{m}}) = \sqrt{\tilde{a}_{\text{m}}^2 \left[\vartheta^2 + u_{\text{rel}}^2(w) \right] + \tilde{a}_{\text{m}} \cdot w \left(\frac{2 n_0 \cdot \vartheta^2}{t_0} + \frac{1}{t_{\text{g}}} \right) + w^2 \left(\frac{n_0}{t_0 \cdot t_{\text{g}}} + \frac{n_0^2 \cdot \vartheta^2}{t_0^2} + \frac{n_0 + \vartheta^2 \cdot n_0^2}{t_0^2} \right)} \tag{192}$$

15.4.5 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1 - \alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold of the activity is calculated by:

$$a_{\text{m}}^* = k_{1-\alpha} \sqrt{w^2 \left(\frac{n_0}{t_0 \cdot t_{\text{g}}} + \frac{n_0^2 \cdot \vartheta^2}{t_0^2} + \frac{n_0 + \vartheta^2 \cdot n_0^2}{t_0^2} \right)} = 0{,}327\ \text{Bq} \cdot \text{kg}^{-1} \tag{193}$$

The measured primary result a_{m} exceeds the decision threshold a_{m}^*.

15.4.6 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit is calculated by:

$$a_\mathrm{m}^\# = a_\mathrm{m}^* + k_{1-\beta}\sqrt{a_\mathrm{m}^{\#\,2}\left[\vartheta^2 + u_\mathrm{rel}^2(w)\right] + a_\mathrm{m}^\# \cdot w\left(\frac{2n_0\cdot\vartheta^2}{t_0} + \frac{1}{t_\mathrm{g}}\right) + w^2\left(\frac{n_0}{t_0\cdot t_\mathrm{g}} + \frac{n_0^2\cdot\vartheta^2}{t_0^2} + \frac{n_0+\vartheta^2\cdot n_0^2}{t_0^2}\right)} = 0{,}826\ \mathrm{Bq}\cdot\mathrm{kg}^{-1} \quad (194)$$

The guideline value $a_\mathrm{m,r}$ exceeds the detection limit $a_\mathrm{m}^\#$.

15.4.7 Limits of coverage intervals

The lower and upper limits of the probabilistically symmetric coverage interval are calculated since the measurement result a_m exceeds the decision threshold a_m^*. With $\omega = \Phi[a_\mathrm{m}/u(a_\mathrm{m})] = 1{,}00$ and $\gamma = 0{,}05$ one obtains the probabilities by $p = \omega\cdot(1-\gamma/2) = 0{,}975$ and $q = 1-\omega\cdot\gamma/2 = 0{,}975$, and hence the quantiles k_p and k_q are both equal to 1,96. Then, the lower and upper limits of the probabilistically symmetric coverage interval are given by:

$$a_\mathrm{m}^\triangleleft = a_\mathrm{m} - k_p\cdot u(a_\mathrm{m}) = 0{,}641\ \mathrm{Bq}\cdot\mathrm{kg}^{-1} \text{ and } a_\mathrm{m}^\triangleright = a_\mathrm{m} + k_q\cdot u(a_\mathrm{m}) = 2{,}165\ \mathrm{Bq}\cdot\mathrm{kg}^{-1} \quad (195)$$

Alternatively, the lower and upper limits of the shortest coverage interval are calculated. With $\omega = \Phi[a_\mathrm{m}/u(a_\mathrm{m})] = 1{,}00$, one obtains $p = [1+\omega\cdot(1-\gamma)]/2 = 0{,}975$, and hence the quantile k_p is equal to 1,96. With this, the limits of the shortest coverage interval according to Formula (196) are

$$a_\mathrm{m}^< = a_\mathrm{m} - k_p\cdot u(a_\mathrm{m}) = 0{,}640\ \mathrm{Bq}\cdot\mathrm{kg}^{-1} \text{ and } a_\mathrm{m}^> = a_\mathrm{m} + k_p\cdot u(a_\mathrm{m}) = 2{,}16\ \mathrm{Bq}\cdot\mathrm{kg}^{-1} \quad (196)$$

15.4.8 The best estimate and its associated standard uncertainty

The best estimate $\hat{a}_\mathrm{m}$ of the activity of the sample is given by:

$$\hat{a}_\mathrm{m} = a_\mathrm{m} + \frac{u(a_\mathrm{m})\cdot\exp\left\{-a_\mathrm{m}^2\Big/\left[2u^2(a_\mathrm{m})\right]\right\}}{\omega\sqrt{2\pi}} = 1{,}40\ \mathrm{Bq}\cdot\mathrm{kg}^{-1} \quad (197)$$

with its associated standard uncertainty $u(\hat{a}_\mathrm{s})$

$$u(\hat{a}_\mathrm{m}) = \sqrt{u^2(a_\mathrm{m}) - (\hat{a}_\mathrm{m} - a_\mathrm{m})\hat{a}_\mathrm{m}} = 0{,}389\ \mathrm{Bq}\cdot\mathrm{kg}^{-1} \quad (198)$$

15.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

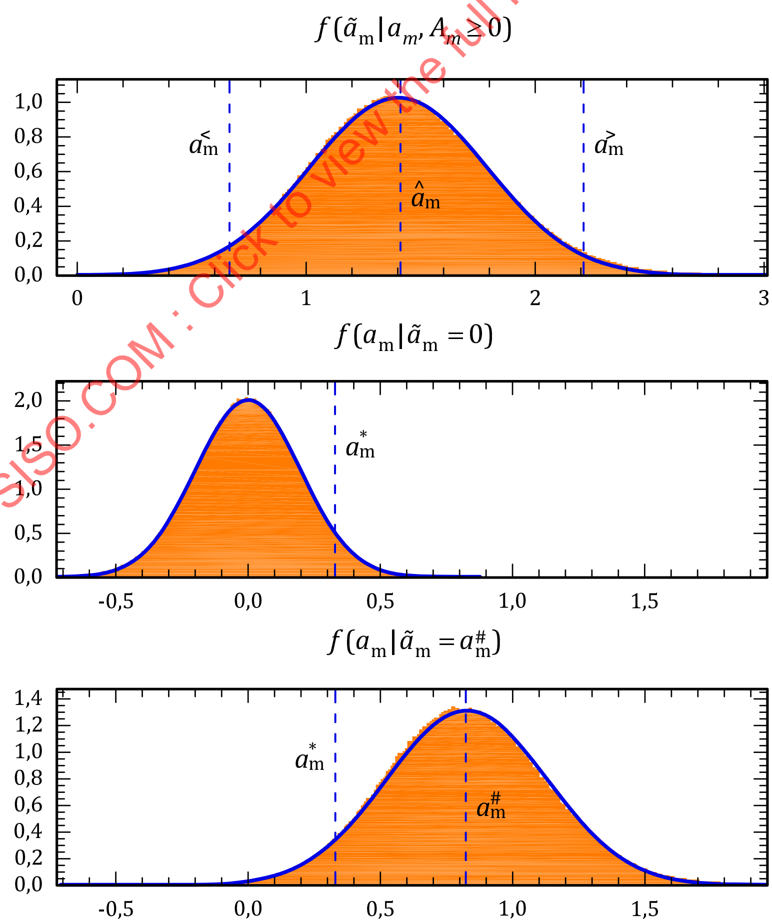
The results and characteristic limits are summarized in Table 22 and the PDFs obtained by the Monte Carlo simulations are presented in Figure 11.

Table 22 — Results and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result	a_m	$\mathrm{Bq}\cdot\mathrm{kg}^{-1}$	1,40	1,412
Standard uncertainty associated with the primary result	$u(a_\mathrm{m})$	$\mathrm{Bq}\cdot\mathrm{kg}^{-1}$	0,389	0,393

Table 22 *(continued)*

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Relative standard uncertainty associated with the primary result	$u_{rel}(a_m)$	1	0,277	0,278
Decision threshold	a_m^*	Bq·kg^{-1}	0,327	0,330
Detection limit	$a_m^\#$	Bq·kg^{-1}	0,826	0,823
Best estimate	$\hat{a}_m$	Bq·kg^{-1}	1,40	1,41
Standard uncertainty associated with the best estimate	$u(\hat{a}_m)$	Bq·kg^{-1}	0,389	0,393
Relative uncertainty associated with the best estimate	$u_{rel}(\hat{a}_m)$	1	0,277	0,278
Lower limit of the probabilistically symmetric coverage interval	$a_m^{\triangleleft}$	Bq·kg^{-1}	0,641	0,665
Upper limit of the probabilistically symmetric coverage interval	$a_m^{\triangleright}$	Bq·kg^{-1}	2,17	2,210
Lower limit of the shortest coverage interval	$a_m^<$	Bq·kg^{-1}	0,640	0,647
Upper limit of the shortest coverage interval	$a_m^>$	Bq·kg^{-1}	2,16	2,190



NOTE The blue PDFs are the respective Gaussian approximations. The blue vertical lines indicate the best estimate and the limits of the coverage interval (upper panel), the decision threshold (middle panel) and the decision threshold and the detection limit (lower panel).

Figure 11 — PDFs calculated by Monte Carlo methods according to ISO 11929-2 in red colour

15.6 Assessment and explanations

The assessment according to ISO 11929-1 concludes the following:

— The primary measurement result $a_{\mathrm{m}} = 1{,}40\ \mathrm{Bq \cdot kg^{-1}}$ exceeds the decision threshold $a_{\mathrm{m}}^{*} = 0{,}33\ \mathrm{Bq \cdot kg^{-1}}$. It was decided to conclude that an activity per unit mass of the sample was recognized.

— The detection limit $a_{\mathrm{m}}^{\#} = 0{,}83\ \mathrm{Bq \cdot kg^{-1}}$ is below the guideline value $a_{\mathrm{m,r}} = 1\ \mathrm{Bq \cdot kg^{-1}}$. It is decided to conclude that the measurement procedure is suited for the measurement purpose.

— The lower and upper limits of the probabilistically symmetric coverage interval are $a_{\mathrm{m}}^{\triangleleft} = 0{,}64\ \mathrm{Bq \cdot kg^{-1}}$ and $a_{\mathrm{m}}^{\triangleright} = 2{,}17\ \mathrm{Bq \cdot kg^{-1}}$. These limits are identical to the limits of the shortest coverage interval.

— The best estimate is $\hat{a}_{\mathrm{m}} = 1{,}40\ \mathrm{Bq \cdot kg^{-1}}$ with an associated standard uncertainty $u(\hat{a}_{\mathrm{m}}) = 0{,}39\ \mathrm{Bq \cdot kg^{-1}}$.

The model of evaluation is linear, and there are no dominating large uncertainties so that the application of ISO 11929-1 is justified. This is a case with small or minor relative uncertainties. The results obtained by application of ISO 11929-2 are practically identical with those obtained by ISO 11929-1:2019, Table 22. The ISO/IEC Guide 98-3 approximations, assuming normal PDFs and using a Taylor expansion truncated after the linear term, hold.

16 Dose measurement using an active personal dosemeter

16.1 Definition of the task and general aspects

As an illustration for the application of ISO 11929 (all parts) to others than counting measurements, a numerical example for the determination of a personal dose equivalent $H_{\mathrm{p}}(10)$ with an active dosemeter and the calculation of the characteristic limits is given in this Clause 16.

It is assumed that the dosemeter is used for an entire working day and afterwards the dose is read. It is further assumed that in the context of the calibration of the dosimetry system the correction and calibration factors and the background dose rate $\dot{M}_{\mathrm{b}}$ due to the ambient radiation during the exposure duration t_{e} and their associated standard uncertainties were independently determined.

A guideline value of $H_{\mathrm{p,r}}(10) = 10\ \mu\mathrm{Sv}$ is assumed in the example.

16.2 Model of evaluation and standard uncertainty

The model of the evaluation is given by:

$$H_{\mathrm{p}}(10) = (M_{\mathrm{g}} - \dot{M}_{\mathrm{b}} \cdot t_{\mathrm{e}}) \cdot \frac{N_0}{r_{\mathrm{rel}}} = (M_{\mathrm{g}} - \dot{M}_{\mathrm{b}} \cdot t_{\mathrm{e}}) \cdot w \qquad (199)$$

with

$H_{\mathrm{p}}(10)$	personal dose equivalent in µSv;
M_{g}	reading of the dosemeter in µSv;
$\dot{M}_{\mathrm{b}}$	background dose rate during the time of exposure in µSv per hour;
t_{e}	time of exposure in h;
N_0	calibration factor measured independently in a reference radiation field;

r_{rel} relative response as determined e.g. during type testing;

$w = \frac{N_0}{r_{rel}}$ composite calibration factor;

u constant standard uncertainty of M_g.

The standard uncertainty $u[H_p(10)]$ associated with the personal dose equivalent $H_p(10)$ is given by:

$$\begin{aligned} u^2(H_p(10)) &= u^2(M_g - \dot{M}_b \cdot t_e) \cdot w^2 + (M_g - \dot{M}_b \cdot t_e)^2 \cdot u^2(w) \\ &= u^2(M_g) \cdot w^2 + u^2(\dot{M}_b) \cdot t_e{}^2 \cdot w^2 + H_p(10)^2 \cdot u_{rel}^2(w) \end{aligned} \quad (200)$$

with

$$u_{rel}^2(w) = u_{rel}^2(N_0) + u_{rel}^2(r_{rel}) \quad (201)$$

16.3 Available information, input data, and specifications

The input data and their associated uncertainties are given in Table 23. The PDFs assigned to the input data and used in the application of ISO 11929-2 are also given in Table 23.

Table 23 — Input quantities and data for personal dose

Quantity	Symbol	x_i	$u(x_i)$	PDF	Unit
Reading of the dosemeter	M_g	20	0,288 7	Rectangular as the display has a resolution of 1 digit, half-width 0,5 digits	µSv
Background dose rate	$\dot{M}_b$	0,1	0,016 0	Gaussian as the value was determined by several repeated measurements	µSv h^{-1}
Time of exposure	t_e	8	—	—	h
Calibration factor	N_0	1,0	0,115 47	Rectangular as the official verification measurement only requires "result shall be within ±20 %", half-width 0,20	1
Relative response and correction factor	r_{rel}	1,0	0,230 94	Rectangular as the type approval requirement is "result shall be within ±40 %", half-width 0,40	1

With regard to the calculation of the decision threshold and the detection limit, the gross indication M_g is the analogous quantity to the gross count rate x_1 in the general model of evaluation given in ISO 11929-1. However, the actual example is not a counting measurement and demonstrates that the possible application of ISO 11929 (all parts) extends far beyond counting measurements. The particular feature of this example is that the gross indication M_g has a constant uncertainty u, which allows calculating the standard uncertainty of the measurand as a function of an assumed true value in 16.4.3.

16.4 Evaluation of the measurement and characteristic limits according to ISO 11929-1

16.4.1 Background effect

The background dose rate $\dot{M}_b$ is set by assuming an average ambient dose rate of 0,1 µSv per hour. The standard uncertainty was obtained from a Gaussian distribution, since the value was determined by several repeated measurements resulting in $u(\dot{M}_b) = 0{,}016\ 0\ \mu\text{Sv} \cdot \text{h}^{-1}$.

16.4.2 Primary result and its associated standard uncertainty

With the data of Table 23 the primary result of the personal dose equivalent is calculated to be

$$H_\mathrm{p}(10) = (M_\mathrm{g} - \dot{M}_\mathrm{b} \cdot t_\mathrm{e}) \cdot \frac{N_0}{r_\mathrm{rel}} = (M_\mathrm{g} - \dot{M}_\mathrm{b} \cdot t_\mathrm{e}) \cdot w = 19{,}20\,\mu\mathrm{Sv} \quad (202)$$

and its associated standard uncertainty

$$u[H_\mathrm{p}(10)] = \sqrt{u^2(M_\mathrm{g}) \cdot w^2 + u^2(\dot{M}_\mathrm{b}) \cdot {t_\mathrm{e}}^2 \cdot w^2 + H_\mathrm{p}(10)^2 \cdot u_\mathrm{rel}^2(w)} = 4{,}97\,\mu\mathrm{Sv} \quad (203)$$

16.4.3 Standard uncertainty as a function of an assumed true value

The standard uncertainty $\tilde{u}[\tilde{H}_\mathrm{p}(10)]$ as a function of the true value $\tilde{H}_\mathrm{p}(10)$ of the measurand is needed to calculate the decision threshold and the detection limit.

This function is available in this example. Since $u(M_\mathrm{g}) = u = \mathrm{const.}$, $\tilde{u}[\tilde{H}_\mathrm{p}(10)]$ is given by:

$$\tilde{u}^2[\tilde{H}_\mathrm{p}(10)] = u^2 \cdot w^2 + u^2(\dot{M}_\mathrm{b}) \cdot {t_\mathrm{e}}^2 \cdot w^2 + \tilde{H}_\mathrm{p}(10)^2 \cdot u_\mathrm{rel}^2(w) \quad (204)$$

16.4.4 Decision threshold

The value of the quantile $k_{1-\alpha}$ of the standardised normal distribution $\Phi(k_{1-\alpha}) = 1 - \alpha$ is 1,645 assuming a probability α of 5 %. The decision threshold is calculated from Formula (204). With the data from Table 23 the decision threshold is

$$H_\mathrm{p}(10)^* = k_{1-\alpha} \cdot \tilde{u}(0) = k_{1-\alpha} \cdot w \cdot \sqrt{u^2 + u^2(\dot{M}_\mathrm{b}) \cdot t_\mathrm{e}^2} = 0{,}519\,\mu\mathrm{Sv} \quad (205)$$

The measurement result $H_\mathrm{p}(10)$ exceeds the decision threshold $H_\mathrm{p}(10)^*$ and hence a personal dose equivalent greater than zero has been detected.

16.4.5 Detection limit

The probability β is set to 5 %. Then, the value of the corresponding quantile $k_{1-\beta}$ of the standardised normal distribution is 1,645. The detection limit follows from Formula (204) with the decision threshold as given in Formula (205).

$$\begin{aligned} H_\mathrm{p}(10)^{\#} &= H_\mathrm{p}(10)^{*} + k_{1-\beta} \cdot \tilde{u}[\tilde{H}_\mathrm{p}(10) = H_\mathrm{p}(10)^{\#}] \\ &= H_\mathrm{p}(10)^{*} + k_{1-\beta} \cdot \sqrt{u^2 \cdot w^2 + u^2(\dot{M}_\mathrm{b}) \cdot {t_\mathrm{e}}^2 \cdot w^2 + {H_\mathrm{p}(10)^{\#}}^2 \cdot u_\mathrm{rel}^2(w)} \end{aligned} \tag{206}$$

The implicit Formula (206) is solved by iteration. As the initial value for the iteration is selected $2 \cdot H_\mathrm{p}^{*}(10) \approx 1\,\mu\mathrm{Sv}$. This iteration is almost always convergent; otherwise the measurement procedure is completely unsuitable. Using the data from Table 23 the detection limit is

$$H_\mathrm{p}(10)^{\#} = H_\mathrm{p}(10)^{*} + k_{1-\beta} \cdot \tilde{u}[\tilde{H}_\mathrm{p}(10) = H_\mathrm{p}(10)^{\#}] = 1{,}27\,\mu\mathrm{Sv} \tag{207}$$

Since $\alpha = \beta$, $k_{1-\alpha} = k_{1-\beta} = k$. The detection limit can also be calculated by the explicit Formula (208):

$$H_\mathrm{p}(10)^{\#} = \frac{2 \cdot H_\mathrm{p}(10)^{*}}{1 - k^2 \cdot u_\mathrm{rel}^2(w)} = 1{,}27\,\mu\mathrm{Sv} \tag{208}$$

The detection limit is compared with the limit for legal or other requirements. If the detection limit is smaller than the limit, the method for determining the photon dose is useful in the personal dosimetry.

16.4.6 Limits of coverage intervals

The limits of the probabilistically symmetric coverage interval, $H_\mathrm{p}^{\triangleleft}(10)$ and $H_\mathrm{p}^{\triangleright}(10)$ and $\gamma = 0{,}05$ of the personal dose equivalent is calculated for the probability 1 - γ = 0,95 for the obtained value of the personal dose equivalent $H_\mathrm{p}(10) = 19{,}20\,\mu\mathrm{Sv}$ with the associated standard uncertainty $u[H_\mathrm{p}(10)] = 4{,}97\,\mu\mathrm{Sv}$. From the standard normal distribution results the parameter

$$\omega = \Phi(H_\mathrm{p}(10)/u(H_\mathrm{p}(10))) = \Phi(3{,}85) = 1{,}00 \tag{209}$$

and with

$$p = \omega \cdot (1 - \gamma/2) = 0{,}975 \text{ and } q = 1 - \omega \cdot \gamma/2 = 0{,}975 \tag{210}$$

and the quantiles of the standardised normal distribution $k_p = 1{,}96$ and $k_q = 1{,}96$, respectively, one obtains the limits of the probabilistically symmetric coverage interval

$$H_\mathrm{p}^{\triangleleft}(10) = H_\mathrm{p}(10) - k_p \cdot u[H_\mathrm{p}(10)] = 9{,}47\ \ \mu\mathrm{Sv} \tag{211}$$

and

$$H_\mathrm{p}^{\triangleright}(10) = H_\mathrm{p}(10) + k_q \cdot u[H_\mathrm{p}(10)] = 28{,}94\ \ \mu\mathrm{Sv} \tag{212}$$

The limits of the shortest coverage interval, $H_\mathrm{p}^{<}(10)$ and $H_\mathrm{p}^{>}(10)$, of the personal dose equivalent is calculated for the probability 1 - γ = 0,95 for the obtained value of the personal dose equivalent $H_\mathrm{p}(10) = 19{,}2\,\mu\mathrm{Sv}$ with the associated standard uncertainty $u[H_\mathrm{p}(10)] = 5{,}0\,\mu\mathrm{Sv}$.

With $\omega = \Phi((H_\mathrm{p}(10)/u(H_\mathrm{p}(10))) = \Phi(3{,}85) = 1{,}00$, the limits of the shortest coverage interval are identical to those of the probabilistically symmetric coverage interval.

16.4.7 The best estimate and its associated standard uncertainty

The calculation of the best estimate and its associated standard uncertainty can be omitted since $\omega = \Phi\left\{H_p(10)/u[H_p(10)]\right\} = \Phi(3{,}85) = 1{,}000$. Consequently, the best estimate and its associated standard uncertainty are equal to the primary measurement result and its associated standard uncertainty, respectively. The results are given in Table 24 anyway.

16.5 Documentation of the results obtained by ISO 11929-1 and ISO 11929-2

The results and characteristic limits are summarized in Table 24 and the PDFs obtained by the Monte Carlo approach are presented in Figure 12.

Table 24 — Results and characteristic values

Results	Symbol	Unit	ISO 11929-1	ISO 11929-2
Primary result of the personal dose equivalent	$H_p(10)$	µSv	19,2	20,3
Standard uncertainty associated with the primary result	$u[H_p(10)]$	µSv	5,0	5,6
Relative standard uncertainty associated with the primary result	$u_{rel}[H_p(10)]$	1	0,26	0,28
Decision threshold	$H_p(10)^*$	µSv	0,5	0,6
Detection limit	$H_p(10)^\#$	µSv	1,3	1,2
Best estimate	$\hat{H}_p(10)$	µSv	19,2	20,3
Standard uncertainty associated with the best estimate	$u[\hat{H}_p(10)]$	µSv	5,0	5,6
Relative uncertainty associated with the best estimate	$u_{rel}[\hat{H}_p(10)]$	1	0,26	0,28
Lower limit of the probabilistically symmetric coverage interval	$H_p^{\triangleleft}(10)$	µSv	9,5	12,3
Upper limit of the probabilistically symmetric coverage interval	$H_p^{\triangleright}(10)$	µSv	28,9	33,1
Lower limit of the shortest coverage interval	$H_p^{<}(10)$	µSv	9,5	11,5
Upper limit of the shortest coverage interval	$H_p^{>}(10)$	µSv	28,9	31,4