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Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

Part 1:
Elementary applications

Détermination des limites caractéristiques (seuil de décision, limite de détection et extrémités de l'intervalle élargi) pour mesurages de rayonnements ionisants — Principes fondamentaux et applications —

Partie 1: Applications élémentaires

Reference number
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by This document was prepared by ISO/TC 85, *Nuclear energy, nuclear technologies, and radiological protection*, Subcommittee SC 2, *Radiological protection*.

This second edition of ISO 11929-1 together with ISO 11929-2 and ISO 11929-3, cancels and replaces ISO 11929:2010, which has been technically revised. The main change is specifically with reference to the type of statistical treatment of the data and extended with respect to the methodology of uncertainty assessment from ISO/IEC Guide 98-3:2009, to the ISO/IEC Guide 98-3-1:2008.

A list of all the parts in the ISO 11929 series can be found on the ISO website.

Introduction

Measurement uncertainties and characteristic values, such as the decision threshold, the detection limit and limits of the coverage interval for measurements, as well as the best estimate and its associated standard measurement uncertainty, are of importance in metrology in general and for radiological protection in particular. The quantification of the uncertainty associated with a measurement result provides a basis for the trust an individual can have in a measurement result. Conformity with regulatory limits, constraints or reference values can only be demonstrated by taking into account and quantifying all sources of uncertainty. Characteristic limits provide, at the end, the basis for deciding if uncertainties have to be taken into account.

This standard provides characteristic values of a non-negative measurand of ionizing radiation. It is also applicable for a wide range of measuring methods extending beyond measurements of ionizing radiation.

The limits to be provided according to ISO 11929 series for specified probabilities of wrong decisions allow detection possibilities to be assessed for a measurand and for the physical effect quantified by this measurand as follows:

— the “decision threshold” allows a decision to be made on whether or not the physical effect quantified by the measurand is present;

— the “detection limit” indicates the smallest true quantity value of the measurand that can still be detected with the applied measurement procedure; this gives a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose;

— the “limits of the coverage interval” enclose, in the case of the physical effect recognized as present, a coverage interval containing the true quantity value of the measurand with a specified probability.

Hereinafter, the limits mentioned are jointly called the “characteristic limits”.

NOTE According to ISO/IEC Guide 99:2007 updated by JCGM 200:2012 the term “coverage interval” is used here instead of “confidence interval” in order to distinguish the wording of Bayesian terminology from that of conventional statistics.

All the characteristic values are based on Bayesian statistics and on the ISO/IEC 98-3 as well as on the ISO/IEC Guide 98-3-1 and ISO/IEC 98-3-2. As explained in detail in ISO 11929-2, the characteristic values are mathematically defined by means of moments and quantiles of probability distributions of the possible measurand values.

Since measurement uncertainty plays an important part in ISO 11929, the evaluation of measurements and the treatment of measurement uncertainties are carried out by means of the general procedures according to the ISO/IEC Guide 98-3 and to the ISO/IEC Guide 98-3-1; see also References [13] to [17]. This enables the strict separation of the evaluation of the measurements, on the one hand, and the provision and calculation of the characteristic values, on the other hand. The ISO 11929 series makes use of a theory of uncertainty in measurement [18] to [20] based on Bayesian statistics (e.g. see References [21] to [26]) in order to take into account those uncertainties that cannot be derived from repeated or counting measurements. The latter uncertainties cannot be handled by frequentist statistics.

Because of developments in metrology concerning measurement uncertainty laid down in the ISO/IEC Guide 98-3, ISO 11929:2010 was drawn up on the basis of the ISO/IEC Guide 98-3, but using Bayesian statistics and the Bayesian theory of measurement uncertainty. This theory provides a Bayesian foundation for the ISO/IEC Guide 98-3. Moreover, ISO 11929:2010 was based on the definitions of the characteristic values[13], the standard proposal[14], and the explanatory article[15]. It unified and replaced all earlier parts of ISO 11929 and was applicable not only to a large variety of particular measurements of ionizing radiation but also, in analogy, to other measurement procedures.

In 2008 the ISO/IEC Guide 98-3-1 has been published, dealing comprehensively with a more general treatment of measurement uncertainty using the Monte Carlo method in complex measurement evaluations. This provided an incentive for writing a corresponding Monte Carlo supplement[16] to ISO 11929:2010 and to revise ISO 11929:2010. The revised ISO 11929 is also essentially founded on Bayesian statistics and can serve as a bridge between ISO 11929:2010 and the ISO/IEC Guide 98-3-1. Moreover, more general definitions of the characteristic values (ISO 11929-2) and the Monte Carlo computation of the characteristic values make it possible to go a step beyond the present state of standardization laid down in ISO 11929:2010 since probability distributions rather than uncertainties can be propagated. It is thus more comprehensive and extending the range of applications.

The revised ISO 11929, moreover, is more explicit on the calculation of the characteristic values. It corrects also a problem in ISO 11929:2010 regarding uncertain quantities and influences, which do not behave randomly in measurements repeated several times. Reference [17] gives a survey on the basis of the revision. Furthermore, in ISO 11929-3, it gives detailed advice how to calculate characteristic values in the case of multivariate measurements using unfolding methods. For such measurements, the ISO/IEC Guide 3-2 provides the basis of the uncertainty evaluation.

Formulas are provided for the calculation of the characteristic values of an ionizing radiation measurand via the "standard measurement uncertainty" of the measurand (hereinafter the "standard uncertainty") derived according to the ISO/IEC Guide 98-3 as well as via probability density functions (PDFs) of the measurand derived in accordance with the ISO/IEC Guide 98-3-1. The standard uncertainties or probability density functions take into account the uncertainties of the actual measurement as well as those of sample treatment, calibration of the measuring system and other influences. The latter uncertainties are assumed to be known from previous investigations.

Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

Part 1: Elementary applications

1 Scope

The ISO 11929 series specifies a procedure, in the field of ionizing radiation metrology, for the calculation of the “decision threshold”, the “detection limit” and the “limits of the coverage interval” for a non-negative ionizing radiation measurand when counting measurements with preselection of time or counts are carried out. The measurand results from a gross count rate and a background count rate as well as from further quantities on the basis of a model of the evaluation. In particular, the measurand can be the net count rate as the difference of the gross count rate and the background count rate, or the net activity of a sample. It can also be influenced by calibration of the measuring system, by sample treatment and by other factors.

ISO 11929 has been divided into four parts covering elementary applications in this document, advanced applications on the basis of the ISO/IEC Guide 3-1 in ISO 11929-2, applications to unfolding methods in ISO 11929-3, and guidance to the application in ISO 11929-4.

This document covers basic applications of counting measurements frequently used in the field of ionizing radiation metrology. It is restricted to applications for which the uncertainties can be evaluated on the basis of the ISO/IEC Guide 98-3 (JCGM 2008). In Annex A, the special case of repeated counting measurements with random influences is covered, while measurements with linear analogous ratemeters are covered in Annex B.

ISO 11929-2 extends the former ISO 11929:2010 to the evaluation of measurement uncertainties according to the ISO/IEC Guide 98-3-1. ISO 11929-2 also presents some explanatory notes regarding general aspects of counting measurements and on Bayesian statistics in measurements.

ISO 11929-3 deals with the evaluation of measurements using unfolding methods and counting spectrometric multi-channel measurements if evaluated by unfolding methods, in particular, for alpha- and gamma-spectrometric measurements. Further, it provides some advice on how to deal with correlations and covariances.

ISO 11929-4 gives guidance to the application of the ISO 11929 series, summarizes shortly the general procedure and then presents a wide range of numerical examples. Information on the statistical roots of ISO 11929 and on its current development may be found elsewhere[33][34].

The ISO 11929 series also applies analogously to other measurements of any kind especially if a similar model of the evaluation is involved. Further practical examples can be found, for example, in ISO 18589[1], ISO 9696[2], ISO 9697[3], ISO 9698[4], ISO 10703[5], ISO 7503[6], ISO 28218[7], and ISO 11665[8].

NOTE A code system, named UncertRadio, is available for calculations according to ISO 11929-1 to ISO 11929-3. UncertRadio[31][32] can be downloaded for free from https://www.thuenen.de/de/fi/arbeitsbereiche/meeresumwelt/leitstelle-umweltradioaktivitaet-in-fisch/uncertradio/. The download contains a setup installation file which copies all files and folders into a folder specified by the user. After installation one has to add information to the PATH of Windows as indicated by a pop-up window during installation. English language can be chosen and extensive “help” information is available.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 80000-1, *Quantities and units — Part 1: General*

ISO 80000-10, *Quantities and units — Part 10: Atomic and nuclear physics*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 1: Guide to the expression of uncertainty in measurement, JCGM 100:2008.*

ISO/IEC Guide 98-3-1, *Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" — a Propagation of distributions using a Monte Carlo method, JCGM 101:2008*

ISO/IEC Guide 98-3-2, *Evaluation of measurement data — Supplement 2 to the "Guide to the expression of uncertainty in measurement" — Models with any number of output quantities, JCGM 102:2011*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM), JCGM 200:2012*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 80000-1, ISO 80000-10, ISO/IEC Guide 98-3, ISO/IEC Guide 98-3-1, ISO/IEC 98-3-2, ISO/IEC Guide 99 and ISO 3534-1 and the following apply.

3.1
quantity value
value of a quantity
value
number and reference together expressing magnitude of a quantity

[SOURCE: JCGM 200:2012, 1.19]

3.2
measurement
process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity

[SOURCE: JCGM 200:2012, 2.1]

3.3
measurand
quantity intended to be measured

[SOURCE: JCGM 200:2012, 2.3]

3.4
coverage interval
interval containing the set of true quantity values of a measurand with a stated probability, based on the information available

[SOURCE: JCGM 200:2012, 2.36]

Note 1 to entry: A coverage interval does not need to be centred on the chosen measured quantity value (see JCGM 101:2008).

Note 2 to entry: A coverage interval should not be termed "confidence interval" to avoid confusion with the statistical concept.

3.5
measurement method
method of measurement
generic description of a logical organization of operations used in a measurement

[SOURCE: JCGM 200:2012, 2.4]

3.6
measurement procedure
detailed description of a measurement according to one or more measurement principles and to a given measurement method, based on a measurement model and including any calculation to obtain a measurement result

[SOURCE: JCGM 200:2012, 2.6]

3.7
measurement result
result of measurement
set of quantity values being attributed to a measurand together with any other available relevant information

[SOURCE: JCGM 200:2012, 2.9]

3.8
measured quantity value
value of a measured quantity
measured value
quantity value representing a measurement result

[SOURCE: JCGM 200:2012, 2.10]

3.9
true quantity value
true value of a quantity
true value
quantity value consistent with the definition of a quantity

[SOURCE: JCGM 200:2012, 2.11]

Note 1 to entry: In the Error Approach to describing measurement, a true quantity value is considered unique and, in practice, unknowable. The Uncertainty Approach is to recognize that, owing to the inherently incomplete amount of detail in the definition of a quantity, there is not a single true quantity value but rather a set of true quantity values consistent with the definition. However, this set of values is, in principle and in practice, unknowable. Other approaches dispense altogether with the concept of true quantity value and rely on the concept of metrological compatibility of measurement results for assessing their validity.

Note 2 to entry: When the definitional uncertainty associated with the measurand is considered to be negligible compared to the other components of the measurement uncertainty, the measurand may be considered to have an "essentially unique" true quantity value. This is the approach taken by the ISO/IEC Guide 98-3 and associated documents, where the word "true" is considered to be redundant.

3.10
measurement uncertainty
uncertainty of measurement
uncertainty
non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used

[SOURCE: JCGM 200:2012, 2.26]

Note 1 to entry: Measurement uncertainty includes components arising from systematic effects, such as components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty. Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.

Note 2 to entry: The parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.

Note 3 to entry: Measurement uncertainty comprises, in general, many components. Some of these may be evaluated by Type A evaluation of measurement uncertainty from the statistical distribution of the quantity values from series of measurements and can be characterized by standard deviations. The other components, which may be evaluated by Type B evaluation of measurement uncertainty, can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information.

Note 4 to entry: In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to the measurand. A modification of this value results in a modification of the associated uncertainty.

3.11
model of evaluation
set of mathematical relationships between all measured and other quantities involved in the evaluation of measurements

Note 1 to entry: The model of evaluation does not need to be an explicit function; it can also be an algorithm realized by a computer code.

3.12
decision threshold
value of the estimator of the measurand, which, when exceeded by the result of an actual measurement using a given measurement procedure of a measurand quantifying a physical effect, is used to decide that the physical effect is present

Note 1 to entry: The decision threshold is defined such that in cases where the measurement result, y, exceeds the decision threshold, y^*, the probability of a wrong decision, namely that the true value of the measurand is not zero if in fact it is zero, is less or equal to a chosen probability α.

Note 2 to entry: If the result, y, is below the decision threshold, y^*, it is decided to conclude that the result cannot be attributed to the physical effect; nevertheless, it cannot be concluded that it is absent.

3.13
detection limit
smallest true value of the measurand which ensures a specified probability of being detectable by the measurement procedure

Note 1 to entry: With the decision threshold according to 4.13, the detection limit is the smallest true value of the measurand for which the probability of wrongly deciding that the true value of the measurand is zero is equal to a specified value, β, when, in fact, the true value of the measurand is not zero. The probability of being detectable is consequently (1-β).

Note 2 to entry: The terms detection limit and decision threshold are used in an ambiguous way in different standards (e.g. standards related to chemical analysis or quality assurance). If these terms are referred to one has to state according to which standard they are used.

3.14
probabilistically symmetric coverage interval
coverage interval for a quantity such that the probability that the quantity is less than the smallest value in the interval is equal to the probability that the quantity is greater than the largest value in the interval

[SOURCE: JCGM 101:2008, 3.15]

3.15
shortest coverage interval
coverage interval for a quantity with the shortest length among all coverage intervals for that quantity having the same coverage probability

[SOURCE: JCGM 101:2008, 3.16]

3.16
limits of the coverage interval
values which define a coverage interval

Note 1 to entry: The limits are calculated in the ISO 11929 series to contain the true value of the measurand with a specified probability $(1-\gamma)$.

Note 2 to entry: The definition of a coverage interval is ambiguous without further stipulations. In this standard two alternatives, namely the probabilistically symmetric and the shortest coverage interval are used.

3.17
best estimate of the true quantity value of the measurand
expectation value of the probability distribution of the true quantity value of the measurand, given the experimental result and all prior information on the measurand

Note 1 to entry: The best estimate is the one among all possible estimates of the measurand on the basis of given information, which is associated with the minimum uncertainty.

3.18
guideline value
value which corresponds to scientific, legal or other requirements with regard to the detection capability and which is intended to be assessed by the measurement procedure by comparison with the detection limit

Note 1 to entry: The guideline value can be given, for example, as an activity, a specific activity or an activity concentration, a surface activity or a dose rate.

Note 2 to entry: The comparison of the detection limit with a guideline value allows a decision on whether or not the measurement procedure satisfies the requirements set forth by the guideline value and is therefore suitable for the intended measurement purpose. The measurement procedure satisfies the requirement if the detection limit is smaller than the guideline value.

Note 3 to entry: The guideline value shall not be confused with other values stipulated as conformity requests or as regulatory limits.

3.19
background effect
measurement effect caused by radiation other than that caused by the object of the measurement itself

Note 1 to entry: The background effect can be due to natural radiation sources or radioactive materials in or around the measuring instrumentation and also to the sample itself (for instance, from other lines in a spectrum).

3.20
background effect in spectrometric measurement
number of events of no interest in the region of a specific line in the spectrum

3.21
net effect
contribution of the possible radiation of a measurement object (for instance, of a radiation source or radiation field) to the measurement effect

3.22
gross effect
measurement effect caused by the background effect and the net effect

3.23
shielding factor
factor describing the reduction of the background count rate by the effect of shielding caused by the measurement object

3.24
relaxation time constant
duration in which the output signal of a linear-scale ratemeter decreases to 1/e times the starting value after stopping the sequence of the input pulses

4 Quantities and symbols

The symbols for auxiliary quantities and the symbols only used in the annexes are not listed. Physical quantities are denoted by upper-case letters but shall be carefully distinguished from their values, denoted by the corresponding lower-case letters.

NOTE In this document, a quantity is considered to have a true value which is unknown and unknowable. In some applications, one needs to assume a true value.

m	number of input quantities
X_i	input quantity $(i=1,...,m)$
x_i	estimate of the input quantity X_i
$u(x_i)$	standard uncertainty of the input quantity X_i associated with the estimate x_i
W	calibration factor
w	estimate of the calibration factor
$u_{rel}(w)$	relative standard uncertainty of a quantity W associated with the estimate w
G	model function
Y	non-negative measurand, which quantifies the physical effect of interest
Y_0	random variable serving as an estimator of Y without taking into account that Y is non-negative
Y_1	random variable serving as an estimator of Y taking into account that Y is non-negative
$\tilde{y}$	possible or assumed true quantity values of the measurand; if the physical effect of interest is not present, then $\tilde{y}=0$; otherwise, $\tilde{y}>0$
y	determined value of the estimator Y_0, estimate of the measurand, primary measurement result of the measurand; also used as a variable describing possible measurement results (estimates)

y_j	values y from different measurements $(j = 0, 1, 2, ...)$
$u(y)$	standard uncertainty of the measurand associated with the primary measurement result y
$\tilde{u}(\tilde{y})$	standard uncertainty of the estimator Y_0 as a function of an assumed true value $\tilde{y}$ of the measurand
c_i	sensitivity coefficient. $c_i = \left.\frac{\partial G}{\partial X_i}\right\rvert_{X_1 = x_1, ..., X_m = x_m}$
$\hat{y}$	best estimate of the measurand based on the estimator Y_1
$u(\hat{y})$	standard uncertainty of the measurand associated with the best estimate $\hat{y}$
$\omega, \kappa, \vartheta, \Psi, \Theta$	auxiliary quantities
y^*	decision threshold of the measurand
$y^\#$	detection limit of the measurand
$\tilde{y}_i$	approximations of the detection limit $y^\#$
y_r	guideline value of the measurand
y_low, y_up	lower and upper limit of an unspecified coverage interval, respectively, of the measurand
$y^\triangleleft$, $y^\triangleright$	lower and upper limit of the probabilistically symmetric coverage interval, respectively, of the measurand
$y^<$, $y^>$	lower and upper limit of the shortest coverage interval, respectively, of the measurand
n_i	number of counted pulses obtained from the measurement of the count rate R_i
n_g, n_0	number of counted pulses of the gross effect and of the background effect, respectively
t_i	measurement duration of the measurement of the count rate R_i
t_g, t_0	measurement duration of the measurement of the gross effect and of the background effect, respectively
R_g, R_0	gross count rate and background count rate, respectively
r_g, r_0	estimate of the gross count rate and of the background count rate, respectively
τ_g, τ_0	relaxation time constant of a ratemeter used for the measurement of the gross effect and of the background effect, respectively
α, β	probability of a false positive and false negative decision, respectively

$1-\gamma$ probability for the coverage interval of the measurand

k_p, k_q quantiles of the standardized normal distribution for the probabilities p and q, respectively (for instance $p=1-\alpha$, $1-\beta$ or $1-\gamma/2$)

$\Phi(t)$ distribution function of the standardized normal distribution; $\Phi(k_p)=p$ applies

5 Summary of procedures for evaluating a measurement and calculating the characteristic limits

5.1 General aspects

This clause gives in a concise form, the procedure to be followed for evaluating a measurement of a single measurand on the basis of the ISO/IEC Guide 98-3 and calculating the characteristic limits, i.e. the decision threshold, the detection limit and the limits of a coverage interval. This procedure is suitable in the majority of cases that can be encountered in measurements of ionizing radiation.

It is assumed that the measurand is non-negative. This information is only used when calculating a coverage interval and the best estimate of the measurand and its associated uncertainty. It is a further characteristic of measurements of ionizing radiation that they have to be performed in the presence of a radiation background that has to be subtracted from a gross measurement quantity. The procedures described in this Standard likewise are applicable to any measurements where a background or blank contribution has to be subtracted from a gross quantity.

However, important exceptions exist for which the procedures do not provide reliable results and other procedures need to be applied, such as those described in the ISO/IEC Guide 98-3-1. Such procedures are dealt with in ISO 11929-2. Both the aspects of the procedure and the tools to ascertain whether this document or ISO 11929-2 is suitable or not for the specific application are described in later clauses of this document.

The application of this document is structured into 9 consecutive steps that are explained in the following 5.2 to 5.10. A detailed workflow of this document is given in Figure 1.

These steps are as follows:

- Step 1: Modelling the measurement
- Step 2: General considerations about the applicability of the ISO/IEC Guide 98-3 and decision whether to proceed according to the ISO/IEC Guide 98-3 and this document or to apply the ISO/IEC Guide 98-3-1 and ISO 11929-2
- Step.3: Evaluating the input quantities, standard uncertainties and covariances, and the primary result and its associated standard uncertainty
- Step 4: Evaluating the standard uncertainty as a function of an assumed true value of the measurand
- Step 5: Calculating the decision threshold and decisions to be made
- Step 6: Calculating the detection limit and assessment of the measurement procedure
- Step 7: Calculating a coverage interval for the measurand
- Step 8: Calculating the best estimate of the measurand and its associated standard uncertainty
- Step 9: Reporting of the results

5.2 Modelling the measurement

The objective of the evaluation of a measurement is to obtain an estimate y of a measurand Y and the associated standard uncertainty $u(y)$. In most cases the measurand is not measured directly, but is determined from other quantities $X_1, ..., X_m$ through a functional relationship G such that

$$Y = G(X_1, ..., X_m) \quad (1)$$

and is known as the model of evaluation. In the model of evaluation, the measurand is the output quantity and the quantities on which it depends are the input quantities. The input quantity X_1, the gross counting rate, is a special one because it is characterized by exclusively carrying the information of the effect to be measured. The model of evaluation constitutes a mathematical expression or computer algorithm that gives an estimate y of the measurand Y corresponding to any plausible estimates of the input quantities $x_1, ..., x_m$ via the relationship $y = G(x_1, ..., x_m)$. y is an estimate of the true value $\tilde{y}$ of the measurand Y which itself is unknown and unknowable and consequently uncertain. The standard uncertainty $u(y)$ associated with the estimate y quantifies the uncertainty about $\tilde{y}$.

NOTE It is assumed that selecting a given value $\tilde{y}$ corresponds to modifying only the value of x_1. For a given value $\tilde{y}$, it is usually possible to derive the associated value of $\tilde{x}_1$. One may write this as $\tilde{x}_1 = G^{-1}(\tilde{y}, x_2, ..., x_m)$ keeping $x_2, ..., x_m$ constant. See 2.5 for an example.

5.3 General considerations about the applicability of the ISO/IEC Guide 98-3

The evaluation of the measurement is in this part of the standard strictly based on procedures recommended in the ISO/IEC Guide 98-3. The application of the ISO/IEC Guide 98-3 is limited to models of evaluation that can – at least locally – be linearized and it represents an approximation using a first order Taylor expansion. Independent of these limitations, the ISO/IEC Guide 98-3 is successful for a wide range of applications that frequently do exceed the limits of its declared requirements.

The ISO/IEC Guide 98-3 makes use of a Bayesian theory of measurement uncertainties[18-20]. A clear statement, however, of the statistical roots and statistical methodology as well as provisions for the lacking generality of the ISO/IEC Guide 98-3 were made only in the ISO/IEC Guide 98-3-1. The ISO/IEC Guide 98-3-1 states that its framework is exclusively based on Bayesian statistics (e.g. see References [21-26]) and that the distribution density functions that completely describe the uncertainties can be derived by Bayes' Theorem or by the Principle of Maximum Information Entropy (PME)[27] based on the available information. A particular aspect is that only Bayesian statistics can handle uncertainties (so-called type B uncertainties) which cannot be derived from repeated or counting measurements and which originate from other sources.

The original ISO/IEC Guide 98-3 is contained in the ISO/IEC Guide 98-3-1 as a special case, namely that the available information comprises only the estimates of the input quantities, x_i, and their associated standard uncertainties, $u(x_i)$. Details on the statistical foundation of ISO 11929 are given in Annex C of ISO 11929-2. There are also details given regarding the need of a decision theory (e.g. Reference [28]) for defining a decision threshold.

The user is responsible for assuring that the ISO/IEC Guide 98-3 is applicable, i.e. that the model of evaluation is linear or can at least locally be approximated by a linear function. If the ISO/IEC Guide 98-3 approach is used and the primary measurement result, y, and its associated standard uncertainty, $u(y)$, are calculated, and if during the subsequent calculation of the characteristic values it turns out that a detection limit does not exist, the user has to decide whether this lack of knowledge is acceptable. If this is not the case, the user has to use ISO 11929-2 in connection with the ISO/IEC Guide 98-3-1 approach that also allows handling large relative uncertainties.

NOTE 1 If the formula for the detection limit has not a solution it is usually due to a too large relative uncertainty of the calibration factor; see Formula (37).

NOTE 2 If there are doubts about the applicability of the ISO/IEC Guide 98-3, IEC/TR 62461[9] gives the recommendation “the results of both methods should be given in order to display their difference. When the 95 % coverage intervals of the Monte Carlo method and of the analytical method do not deviate by more than 10 %, then the analytical one may be used for the uncertainty determination in similar cases, i.e. a similar model function and similar or smaller values of the uncertainty of the input quantities.”

5.4 Evaluating the input quantities, standard uncertainties and covariances, and the primary result and its associated standard uncertainty

To obtain an estimate y of the measurand, it is first necessary to determine estimates x_i of the X_i. Accordingly, the standard uncertainty $u(y)$ is evaluated from the standard uncertainties $u(x_i)$ associated with the estimates x_i, and their covariances $u(x_i, x_j)$. Guidance on how to deal with covariances is given in ISO 11929-3:2019, Annex A.

NOTE The estimates x_i of the X_i are given data and are not uncertain. The standard uncertainties $u(x_i)$ are associated with the estimates x_i. They are parameters characterising the dispersion of the (true) quantity values which can be assigned to the quantity based on the available information.

The primary estimate y of Y is given by

$$y = G(x_1, ..., x_m) \tag{2}$$

To calculate the standard uncertainty $u(y)$, first determine the sensitivity coefficients $c_1, ..., c_m$. The ith sensitivity coefficient, c_i, is $\partial G / \partial X_i$, the first partial derivative of G with respect to X_i evaluated at $x_1, ..., x_m$.

The standard uncertainty $u(y)$ associated with y is given by the law of propagation of uncertainty. When the x_i are not correlated, this law takes the form

$$u^2(y) = \sum_{i=1}^{m} c_i^2 \cdot u^2(x_i) \tag{3}$$

Otherwise, covariances are also taken into consideration

$$u^2(y) = \sum_{i=1}^{m} c_i^2 \cdot u^2(x_i) + 2 \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} c_i \cdot c_j \cdot u(x_i, x_j) \tag{4}$$

In many cases of measurements of ionizing radiation, the input quantities X_i are not correlated and Formula (3) can be applied.

Formulas (2), (3) and (4) hold when Y can be expressed as a linear combination of the x_i or it is judged that the non-linearity of G is negligible in the neighbourhood of the estimates x_i. When it is judged that the model cannot adequately be linearized, or in any case of doubt, the Monte Carlo method given in the ISO/IEC Guide 98-3-1 and ISO 11929-2 should be applied to determine the probability density function (PDF) for Y and thence to obtain y and $u(y)$ and the characteristic limits.

5.5 Evaluating the standard uncertainty as a function of an assumed true value of the measurand

Deriving values of the decision threshold or of the detection limit requires performing calculations for an assumed true value $\tilde{y}$ of the measurand, which is considered as a given value. Such a given value corresponds to an assumed value of that single input quantity which carries the information of the measurement effect, usually the gross count rate quantity, taken to be X_1. All other input quantities are not affected. In the model type of radioactivity measurements considered here, the assumed true (given) value $\tilde{y}$ depends linearly on the associated value $\tilde{x}_1$, while x_2 through x_m are left unchanged.

This enables calculating $\tilde{x}_1$ from $\tilde{y}$ by a linear function. One may denote this as $\tilde{x}_1 = G^{-1}(\tilde{y}, x_2, ..., x_m)$. As $\tilde{x}_1$ denotes a gross count rate, its standard uncertainty in most cases is given by $\tilde{u}(\tilde{x}_1) = \sqrt{\tilde{x}_1 / t}$ with t being the measurement time. Replacing then the pair (x_1, $u(x_1)$) within the uncertainty propagation formula by $(\tilde{x}_1, \tilde{u}(\tilde{x}_1))$ finally leads to the standard uncertainty of the assumed true value $\tilde{y}$. The standard uncertainty of $\tilde{y}$ then is given as, based on $\tilde{y} = G(\tilde{x}_1, x_2, ..., x_m)$,

$$\tilde{u}^2(\tilde{y}) = \left(\left. \frac{\partial G}{\partial \tilde{x}_1} \right|_{X_1=\tilde{x}_1, X_2...X_m=x_2...x_m} \right)^2 \tilde{u}^2(\tilde{x}_1) + \sum_{i=2}^{m} \left(\left. \frac{\partial G}{\partial x_i} \right|_{X_1=\tilde{x}_1, X_2...X_m=x_2...x_m} \right)^2 u^2(x_i) \quad (5)$$

If the model is of the form $Y = (X_1 - X_2) \cdot W$ with X_1 being a gross counting rate, X_2 a background counting rate and W a calibration factor, then $\tilde{u}^2(\tilde{y})$ turns out to be a quadratic function of $\tilde{y}$ with a quadratic term $\tilde{y}^2 \cdot u_{\mathrm{rel}}^2(w)$. See Clause 7 for details.

In some cases, however, $\tilde{u}^2(\tilde{y})$ can only be approximated by interpolation. Such an interpolation has to be based on the evaluation of $\tilde{u}^2(0)$ and at least one measurement result $y > 0$ and its associated standard uncertainty $u(y)$ via $\tilde{u}^2(\tilde{y}) = \tilde{u}^2(0) \cdot (1 - \tilde{y}/y) + u^2(y) \cdot \tilde{y}/y$.

For a model of evaluation of the form $Y = (X_1 - X_2) \cdot W$ with X_1 being an arbitrary gross quantity, X_2 an arbitrary background quantity and W a calibration factor, $\tilde{u}^2(0)$ can be estimated by assuming that for $\tilde{y} = 0$ one expects $x_1 = x_2$ and hence $\tilde{u}^2(0) = 2 \cdot u^2(x_2) \cdot w^2$. A.2 and A.4 give detailed advice for such cases.

When extending the application of this Standard beyond counting measurements of ionizing radiation, either information about $\tilde{u}(\tilde{y})$ has to be obtained (e.g. by repeated measurements of different reference materials) or the interpolation approach has to be used.

5.6 Calculating the decision threshold and decisions to be made

The decision threshold allows a decision to be made on whether or not the physical effect quantified by the measurand is present. The models of evaluation dealt with in this Standard all have the characteristic that a background contribution has to be subtracted from a gross quantity. The background may be a result of the background radiation or of any blank affecting the measurement procedure. The existence of an uncertain background radiation or blank raises the question whether or not a contribution from the sample can be recognized. Since all estimates of the input quantities and consequently of the measurand are uncertain, this question can only be dealt with by decision theory (e.g. Reference [28]) allowing for a predefined probability α of a wrong decision. This leads to the definition of the decision threshold, y^*,

$$P(y > y^* | \tilde{y} = 0) = \alpha \quad (6)$$

i.e. the decision threshold, y^*, is defined by the condition that the probability to obtain a primary measurement result, y, that is larger than the decision threshold, y^*, is equal to α if in reality the true value, $\tilde{y}$, is zero.

NOTE The probability α is that for a false positive decision if in fact there is no effect of the sample.

Using a quadratic loss function (see 8.2) and applying the ISO/IEC Guide 98-3 for the evaluation of the uncertainties the decision threshold is given by,

$$y^* = k_{1-\alpha} \cdot \tilde{u}(0) \quad (7)$$

with $k_{1-\alpha}$ being the $(1-\alpha)-$ quantile of the standardized normal distribution; see Annex E.

If the primary measurement result, y, exceeds the decision threshold, y^*, it is decided to conclude that the physical effect provided by the measurand is present, i.e. that a contribution from the sample has been recognized.

If the result, y, is below the decision threshold, y^*, it is decided to conclude that the result cannot be attributed to the physical effect. Nevertheless, it cannot be concluded that it is absent. If the physical effect is really absent, the probability of taking the wrong decision, that the effect is present, is equal to the specified probability, α (probability of a wrong decision; see 8.1).

5.7 Calculating the detection limit and assessment of the measurement procedure

The detection limit indicates the smallest true value of the measurand which can still be detected with a specified probability by the applied measurement procedure; this gives a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose.

Any primary measurement result, y, bears the possibility that given the decision rule provided by the decision threshold, y^*, one decides wrongly that the physical effect is not present in spite of its presence in reality. In order to keep the probability of such a wrong decision beyond a predefined probability, β, the detection limit is defined as the smallest true value of the measurand fulfilling the condition

$$P(y < y^* \mid \tilde{y} = y^\#) = \beta \quad (8)$$

i.e. the detection limit, $y^\#$, is defined by the condition that the probability to obtain a primary measurement result, y, that is smaller than the decision threshold, y^*, is equal to β if in reality the true value, $\tilde{y}$, is equal to the detection limit, $y^\#$.

If the ISO/IEC Guide 98-3 is used for the evaluation of the uncertainties the detection limit is given by the implicit Formula

$$y^\# = y^* + k_{1-\beta} \cdot \tilde{u}(y^\#) \quad (9)$$

with $k_{1-\beta}$ being the $(1-\beta)-$ quantile of the standardized normal distribution; see Annex E.

For iterative solutions of the implicit Formula (9), see Annex C and for explicit solutions see Annex D.

A non-existence of the detection limit points to the fact that there are too large relative uncertainties of the input quantities and that the ISO/IEC Guide 98-3 approximation is not sufficient. In such a case, the solution is to obtain the necessary information to allow for an evaluation of the uncertainty based on distributions according to the ISO/IEC Guide 98-3-1 and to proceed according to ISO 11929-2.

5.8 Calculating a coverage interval for the measurand

If the primary measurement result, y, exceeds the decision threshold, y^*, lower and upper limits of a coverage interval, $y_{\text{low}}, y_{\text{up}}$, are calculated. A coverage interval $[y_{\text{low}}, y_{\text{up}}]$ is an interval containing the true value of a non-negative measurand with a stated coverage probability $(1-\gamma)$, often taken as 0,95,

$$P\left(\tilde{y} \in [y_{\text{low}}, y_{\text{up}}] \,\middle|\, y, u(y), Y \geq 0\right) = 1 - \gamma \quad (10)$$

The definition of a coverage interval is ambiguous without specification of additional conditions. In this standard, the limits of two frequently cited coverage intervals are calculated alternatively.

The limits of the probabilistically symmetric coverage interval, $\left[y^{\lhd}, y^{\rhd}\right]$, for the coverage probability $(1-\gamma)$ are unambiguously defined by

$$P\left(\tilde{y} < y^{\lhd} \middle| y, u(y), Y \geq 0\right) = P\left(\tilde{y} > y^{\rhd} \middle| y, u(y), Y \geq 0\right) = \gamma / 2 \tag{11}$$

If the ISO/IEC Guide 98-3 is used for the evaluation of the uncertainties the limits of the probabilistically symmetric coverage interval are calculated with the auxiliary quantity $\omega = \frac{1}{2\pi} \int_{-\infty}^{y/u(y)} \exp(-\frac{v^2}{2}) \mathrm{d}v = \Phi(y / u(y))$ by

$$y^{\lhd} = y - k_p u(y) \text{ with } p = \omega \cdot (1 - \gamma / 2) \text{ and } y^{\rhd} = y + k_q u(y) \text{ with } q = 1 - \omega\gamma / 2 \tag{12}$$

with k_p, k_q being the quantiles of the standardized normal distribution for the probabilities p and q; see Annex E.

The limits of the shortest coverage interval, $\left[y^{<}, y^{>}\right]$, for the coverage probability $(1-\gamma)$ are unambiguously defined by

$$P\left(\tilde{y} \in [y^{<}, y^{>}] \middle| y, u(y), Y \geq 0, (y^{>} - y^{<}) = \min\right) = 1 - \gamma \tag{13}$$

If the ISO/IEC Guide 98-3 is used for the evaluation of the uncertainties, the limits of the shortest coverage interval are calculated by

$$\begin{aligned} & y^{<,>} = y \pm k_p u(y) \text{ with } p = \omega \cdot (1 + \omega(1-\gamma)) / 2 \text{ and if} \\ & y^{<} < 0 : y^{<} = 0; y^{>} = y + k_q \cdot u(y) \text{ with } q = 1 - \omega\gamma \end{aligned} \tag{14}$$

with k_p, k_q being the quantiles of the standardized normal distribution for the probabilities p and q; see Annex E.

NOTE 1 While the probabilistically symmetric coverage interval never contains the true value zero, the shortest coverage interval will contain the true value zero if the relative uncertainty of the measurement result is large.

NOTE 2 The decision on whether to use the probabilistically symmetric coverage interval or the shortest coverage interval is due to the regulator or the customer.

5.9 Calculating the best estimate of the measurand and its associated standard uncertainty

If the primary measurement result, y, exceeds the decision threshold, y^*, also the best estimate, $\hat{y}$, and its associated uncertainty, $u(\hat{y})$, can be calculated. In contrast to the primary result, y, and its associated uncertainty, $u(y)$, the best estimate, $\hat{y}$, and its associated uncertainty, $u(\hat{y})$, take into account that the measurand is non-negative.

If the ISO/IEC Guide 98-3 is used for the evaluation of the uncertainties, the best estimate and its associated uncertainty are calculated by

$$\hat{y} = y + \frac{u(y) \cdot \exp\left\{-y^2 / \left[2u^2(y)\right]\right\}}{\omega\sqrt{2\pi}} \text{ and } u(\hat{y}) = \sqrt{u^2(y) - (\hat{y} - y)\hat{y}} \,. \tag{15}$$

For $y \geq 4u(y)$, the approximations $\hat{y} = y$; $u(\hat{y}) = u(y)$ are sufficient and a separate calculation of the best estimate, $\hat{y}$, and its associated uncertainty, $u(\hat{y})$, is not necessary.

5.10 Reporting the results

Advice how to document and report the results is given in Clause 11.

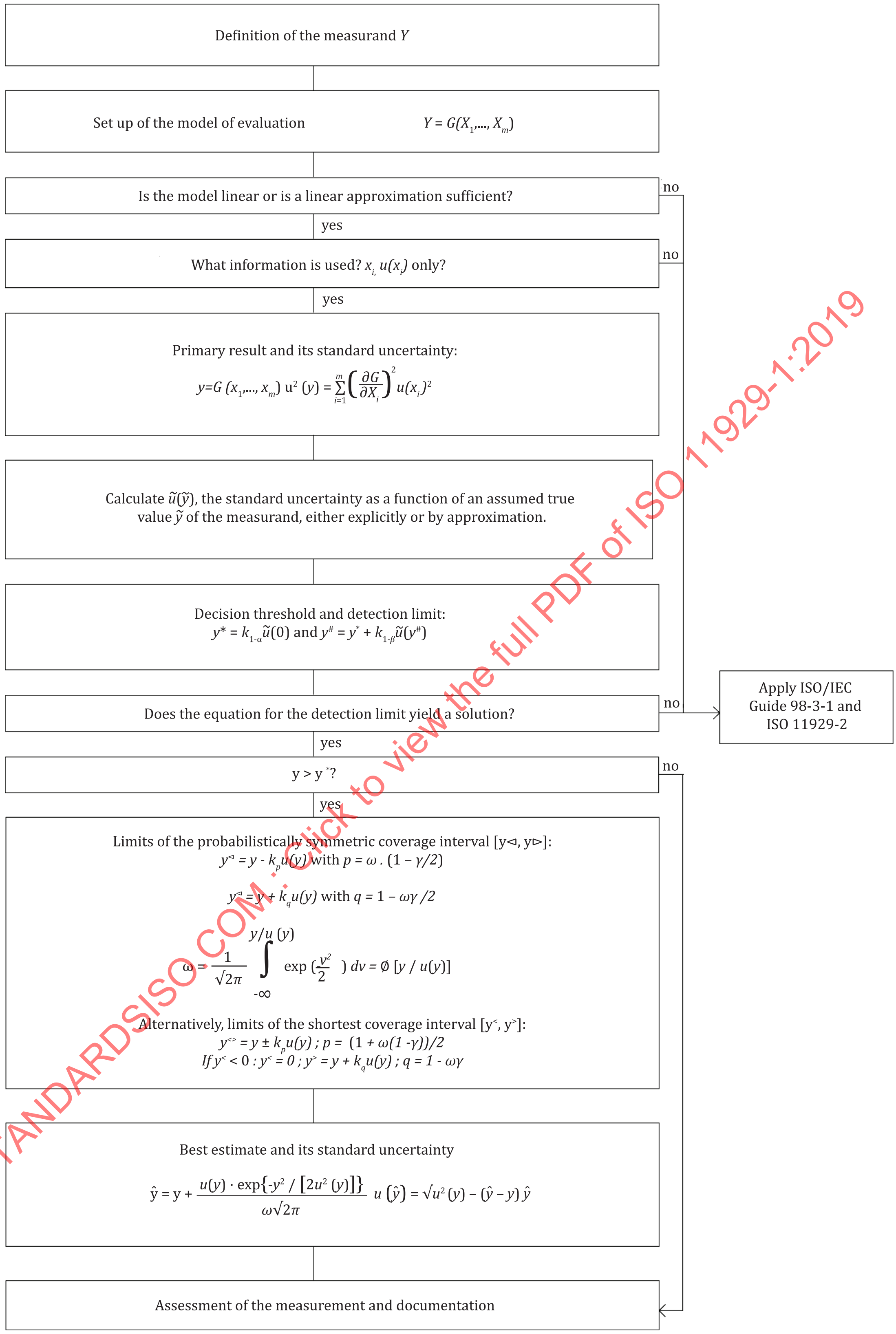

Figure 1 — Flow diagramm of ISO 11929-1 when applying the ISO/IEC Guide 98-3

6 Fundamentals and evaluation of the measurement

6.1 General aspects concerning the measurand

A non-negative measurand shall be assigned to the physical effect to be investigated according to a given measurement task. The measurand shall quantify the effect. It assumes the true value zero if the effect is not present in a particular case.

A random variable Y_0, an estimator, shall be assigned to the measurand. The symbol Y is used in this clause for the measurand itself. A value y of the estimator Y_0, determined from measurements, is an estimate of the measurand. It shall be calculated as the primary measurement result together with the primary standard uncertainty $u(y)$, of the measurand associated with y. These two values form the complete primary measurement result for the measurand and are obtained in accordance with the ISO/IEC Guide 98-3 or ISO/IEC Guide 98-3-1 (see also References [13] to [17]) by evaluation of the measurement data and other information by means of a model (of the evaluation), which mathematically connects all the quantities involved (see 6.2). In general, the fact that the measurand is non-negative is not explicitly taken into account in the evaluation. Therefore, y may be negative, especially when the measurand nearly assumes the true value zero. This is, however, without any consequence given the decision rule provided by the decision threshold (see 8.2). The primary measurement result, y, differs from the best estimate, $\hat{y}$, of the measurand calculated in Clause 10. With $\hat{y}$ and with the limits of the coverage interval according to Clause 9, the knowledge that the measurand is non-negative is taken into account. The standard uncertainty, $u(\hat{y})$, associated with $\hat{y}$ is smaller than $u(y)$.

Part 1 of the ISO 11929 series applies to cases in which the evaluation of measurement uncertainties can be performed according to the ISO/IEC Guide 98-3. In the ISO/IEC Guide 98-3 methodology only the measurement results as estimates x_i of the input quantities X_i and their associated standard uncertainties $u(x_i)$ comprise the available information. This information is used to derive an estimate y of the measurand Y and to propagate the standard uncertainties associated with the measurement results of the input quantities to calculate the standard uncertainty $u(y)$ associated with y.

The ISO/IEC Guide 98-3 uncertainty propagation is the result of a Taylor expansion truncated after the linear term and requires that the model of evaluation is linear or that it can be at least locally linearized. If this condition is not met, the methodology of the ISO/IEC Guide 98-3-1 has to be applied. This is for instance also the case if there are large and dominating relative uncertainties. See Clause 5 of this document for the decisions to be made regarding the application of the ISO/IEC Guide 98-3 or ISO/IEC Guide 98-3-1. If the ISO/IEC Guide 98-3 cannot be applied since its underlying assumptions are not met, the methodology of ISO/IEC Guide 98-3-1 has to be used. The latter case is dealt with in ISO 11929-2.

6.2 Model of evaluation

6.2.1 General model

In many cases, the measurand, Y, is a function of several input quantities, X_i, in the form of Formula (16),

$$Y = G(X_1, ..., X_m) \tag{16}$$

Formula (16) is the model of the evaluation. Substituting given estimates, x_i, of the input quantities, X_i, in the model function, G, Formula (16) yields the primary measurement result y of the measurand as

$$y = G(x_1, ..., x_m) \tag{17}$$

For the application of this standard measurement uncertainties have to be evaluated using all available information about the measurement procedure and the quantities and values involved taking into account all known sources of uncertainty. This evaluation shall be performed according to the ISO/IEC Guide 98-3 or ISO/IEC Guide 98-3-1.

If the model of evaluation can be at least locally linearized and if only the x_i and their associated standard uncertainties $u(x_i)$ are known, and if there are no large dominating uncertainties of individual input quantities, the ISO/IEC Guide 98-3 approximation can be used; see the ISO/IEC Guide 98-3 for details. Otherwise, the methods described in the ISO/IEC Guide 98-3-1 have to be used and the characteristic values have to be calculated as stipulated in ISO 11929-2.

For the general model according to Formula (16) the standard uncertainty, $u(y)$, of the measurand associated with the primary measurement result, y, is calculated according to Formula (18), if the input quantities, X_i, are independently measured and standard uncertainties, $u(x_i)$, associated with the estimates, x_i, are given, from the relation

$$u^2(y) = \sum_{i=1}^{m} c_i^2\, u^2(x_i) \text{ with the sensitivity coefficients } c_i = \left.\frac{\partial G}{\partial X_i}\right|_{X_1=x_1,\dots,X_m=x_m} \quad (18)$$

NOTE If the input quantities are correlated the standard uncertainty is calculated using the covariances. Advice to handle correlations and covariances is given in ISO 11929-3:2019, Annex A.

In Formula (18), the estimates, x_i, shall be substituted for the input quantities, X_i, in the partial derivatives of G. The determination of the estimates, x_i, and the associated standard uncertainties, $u(x_i)$, and also the numerical or experimental determination of the partial derivatives are in accordance with the ISO/IEC Guide 98-3 or References [13][14].

For a count rate, $X_i = R_i$, with the given counting result, n_i, recorded during the measurement of duration, t_i, the specifications $x_i = r_i = n_i / t_i$ and $u^2(x_i) = n_i / t_i^2 = r_i / t_i$ apply. If a n_i is equal to zero $x_i = r_i = 1/t_i$ and $u^2(x_i) = u^2(r_i) = 1/t_i^2$ shall be set (see ISO 11929-2:2019, Annex A).

If the input quantities are not independently measured and for more complicated measurement evaluations such as unfolding, see ISO 11929-3.

Stipulations for rate meter measurements and the special cases in which random influences have to be taken into account in addition to those of the Poisson process are laid down in Annexes B and A, respectively.

If the partial derivatives are not explicitly available (e.g. because the model of evaluation is only available in form of a computer code), it is sufficient to approximate them numerically by differential quotients according to Formula (19),

$$c_i = \frac{1}{u(x_i)} \cdot \left(G\left(x_1,\dots,x_i + u(x_i)/2,\dots,x_m\right) - G\left(x_1,\dots,x_i - u(x_i)/2,\dots,x_m\right) \right). \quad (19)$$

6.2.2 Model in ionizing radiation counting measurements

In this document, the measurand, Y relates to a sample of radioactive material and is determined from counting the gross effect and the background effect with preselection of time or counts. In particular, Y can be the net count rate, R_n, or the net activity, A, of the sample. The symbols belonging to the counting of the gross effect and of the background effect are marked in the following by the subscripts g and 0, respectively.

In this document, the model is specified as follows:

$$Y = G(X_1, ..., X_m) = (X_1 - X_2 X_3 - X_4) \cdot \frac{X_6 X_8 \cdots}{X_5 X_7 \cdots} = (X_1 - X_2 X_3 - X_4) \cdot W \tag{20}$$

with

$$W = \frac{X_6 X_8 \cdots}{X_5 X_7 \cdots}. \tag{21}$$

$X_1 = R_g$ is the gross count rate and $X_2 = R_0$ is the background count rate. The other input quantities, X_i, are calibration, correction or influence quantities, or conversion factors, for instance the emission or response probability or, in particular, X_3 is a shielding factor and X_4 an additional background correction quantity. If some of the input quantities are not involved, $x_i = 1 \quad (i = 3; i > 4)$, $x_4 = 0$ and $u(x_i) = 0$ shall be set for them.

NOTE If the model of evaluation is not of the general form of Formula (20) one has to proceed according to 6.2.1. Some examples for more complex models are given in ISO 11929-3.

6.3 Evaluation of the primary measurement result

By substituting the estimates, x_i, in Formula (20), the primary estimate, y, of the measurand, Y, gives the result

$$y = G(x_1, ..., x_m) = (x_1 - x_2 x_3 - x_4) \cdot w = (r_g - r_0 x_3 - x_4) \cdot w = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0} x_3 - x_4 \right) \cdot w \tag{22}$$

with

$$w = \frac{x_6 x_8 \cdots}{x_5 x_7 \cdots}. \tag{23}$$

NOTE 1 It is assumed that the estimates w and $x_5, x_6, ...$ of the calibration factor W and its components $X_5, X_6, ...$ and their standard uncertainties were evaluated in separate experiments and that these quantities are not correlated with the quantities X_1 to X_4.

NOTE 2 The determination of the primary measurement result, y, does not take into account that the measurand, Y, is non-negative. Thus, a primary measurement result, y, may be negative. The fact that the measurand, Y, is non-negative is only taking into account when calculating the limits of the coverage interval, $y^{\triangleleft}, y^{\triangleright}$ or $y^{<}, y^{>}$, and the best estimate, $\hat{y}$, and its associated standard uncertainty, $u(\hat{y})$.

6.4 Standard uncertainty associated with the primary measurement result

In case of the general model in ionizing radiation measurements according to Formula (20) and if the x_i are not correlated, the standard uncertainty $u(y)$ associated with y is calculated as follows.

For the count rates, $x_1 = r_g = n_g / t_g$ and $u^2(x_1) = n_g / t_g^2 = r_g / t_g$ as well as $x_2 = r_0 = n_0 / t_0$ and $u^2(x_2) = n_0 / t_0^2 = r_0 / t_0$ apply. If a n_i is equal to zero $x_i = r_i = 1/t_i$ and $u^2(x_i) = u^2(r_i) = 1/t_i^2$ shall be set (see ISO 11929-2:2019, Annex A).

With the partial derivatives

$$\frac{\partial G}{\partial X_1}=W\,;\ \frac{\partial G}{\partial X_2}=-X_3W\,;\ \frac{\partial G}{\partial X_3}=-X_2W\,;\ \frac{\partial G}{\partial X_4}=-W\,;\ \frac{\partial G}{\partial X_i}=\pm\frac{Y}{X_i}\quad(i\geq 5)\,;\ \frac{\partial G}{\partial W}=\frac{Y}{W} \tag{24}$$

and by substituting the estimates x_i, w and y, Formula (22) yields the standard uncertainty $u(y)$ of the measurand associated with y

$$\begin{aligned}u(y)&=\sqrt{w^2\cdot\left[u^2(x_1)+x_3^2u^2(x_2)+x_2^2u^2(x_3)+u^2(x_4)\right]+y^2u_{\mathrm{rel}}^2(w)}\\&=\sqrt{w^2\cdot\left[r_{\mathrm{g}}/t_{\mathrm{g}}+x_3^2r_0/t_0+r_0^2u^2(x_3)+u^2(x_4)\right]+y^2u_{\mathrm{rel}}^2(w)}\end{aligned} \tag{25}$$

where

$$u_{\mathrm{rel}}^2(w)=\sum_{i=5}^{m}\frac{u^2(x_i)}{x_i^2} \tag{26}$$

is the sum of the squared relative standard uncertainties of the quantities X_5 to X_m. For $m<5$, the values $w=1$ and $u_{\mathrm{rel}}^2(w)=0$ apply.

The estimates x_i and the standard uncertainties $u(x_i)$ of X_i $(i=3,...,m)$ are taken as known from previous investigations or as values of experience according to other information. In the previous investigations, x_i can be determined as an arithmetic mean value and $u^2(x_i)$ as an empirical variance.

7 Standard uncertainty as a function of an assumed value of the measurand

For the numerical calculation of the decision threshold in 8.2 and of the detection limit in 8.3, the standard uncertainty of the measurand is needed as a function $\tilde{u}(\tilde{y})$ of an assumed true value $\tilde{y}>0$ of the measurand. This function shall be determined in a way similar to $u(y)$ within the framework of the evaluation of the measurements by application of the ISO/IEC Guide 98-3 or ISO/IEC Guide 98-3-1; see also References [13] to [17]. In most cases, $\tilde{u}(\tilde{y})$ shall be formed as a positive square root of a variance function $\tilde{u}^2(\tilde{y})$ calculated first. This function shall be defined, unique and continuous for all $\tilde{y}>0$, and shall not assume negative values.

In most cases of models according to Formula (22), $\tilde{u}(\tilde{y})$ can be explicitly specified, provided that $u(x_1)$ is given as a function of x_1. In such cases, y shall be formally replaced by $\tilde{y}$ in Formula (17) and this Formula shall be solved for x_1 resulting in a function $x_1=G^{-1}(\tilde{y},x_2,...,x_m)$. For an assumed $\tilde{y}$, the value $\tilde{x}_1$ can also be calculated numerically from Formula (22); for instance, by means of an iteration procedure, which results again in a function $x_1=G^{-1}(\tilde{y},x_2,...,x_m)$. This function shall replace x_1 in Formula (25) and the calculation of $u(x_1)$, which finally yields with Formula (18) $\tilde{u}(\tilde{y})$ instead of $u(y)$.

In the case of the model according to Formula (22), the standard uncertainty, $u(x_1)$, of the gross count rate $X_1=R_{\mathrm{g}}$, is given as a function of the estimate, $x_1=r_{\mathrm{g}}$, either $\sqrt{x_1/t_{\mathrm{g}}}$ or $x_1/\sqrt{n_{\mathrm{g}}}$ applies if the measurement duration, t_{g} (time preselection), or, respectively, the number, n_{g}, of recorded pulses (preselection of counts) is specified.

The value y shall be formally replaced by $\tilde{y}$ in Formula (22). This allows the elimination of x_1 in the general case and, in particular, of n_{g} with time preselection and of t_{g} with preselection of counts in

Formula (25) by means of Formula (22). These values are not available if a true value of the measurand $\tilde{y}$ is assumed. This yields in the case according to Formula (22) to

$$x_1 = \tilde{y}/w + x_2 x_3 + x_4 \tag{27}$$

With time preselection

$$n_g = t_g \cdot (\tilde{y}/w + r_0 x_3 + x_4) \tag{28}$$

is obtained from Formula (22). Then, with $u^2(x_1) = x_1/t_g = n_g/t_g^2$ and by substituting n_g according to Formula (28) and with $u^2(x_2) = r_0/t_0$, Formula (25) leads in the case of **time preselection** to

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[(\tilde{y}/w + r_0 x_3 + x_4)/t_g + x_3^2 r_0 / t_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{rel}^2(w)}\,. \tag{29}$$

With preselection of counts

$$t_g = \frac{n_g}{\tilde{y}/w + r_0 x_3 + x_4} \tag{30}$$

is analogously obtained. Then, with $u^2(x_1) = x_1/t_g = n_g/t_g^2$ and by substituting t_g according to Formula (30) and with $u^2(x_2) = r_0^2/n_0$, Formula (25) leads in the case of **preselection of counts** to:

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[(\tilde{y}/w + r_0 x_3 + x_4)^2 / n_g + x_3^2 r_0^2 / n_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{rel}^2(w)} \tag{31}$$

If the standard uncertainty cannot be explicitly given as a function of an assumed true value of the measurand, an approximation by interpolation described in 5.5 has to be applied.

Preselection of counts is frequently chosen in order to economize the available time for measurements. If in addition a maximum measurement time, t_{max}, is set, it shall be ensured that $t_{max} \geq n_g / (n_0 \cdot x_3 / t_0 + x_4)$. Otherwise, the evaluation changes the chosen model of evaluation and the decision threshold and the detection limit are wrongly calculated.

If no explicit specification of $\tilde{u}(\tilde{y})$ is available, it is often sufficient to use the following approximations for the function $\tilde{u}(\tilde{y})$, in particular, if the standard uncertainty, $u(x_1)$, is not known as a function of x_1. A prerequisite is that measurement result, y_j, and associated standard uncertainties, $u(y_j)$, calculated from previous measurements of the same kind, are already available $(j = 0,1,2,...)$. The measurements shall be carried out on different samples with differing activities, but in other respects as far as possible under similar conditions. One of the measurements can be a background effect measurement or a blank measurement with $\tilde{y} = 0$ and, for instance, $j = 0$. Then, $y_0 = 0$ shall be set and $\tilde{u}(0) = u(y_0)$. The measurement currently carried out can be taken as a further measurement with j = 1. See A.2 for details.

$\tilde{u}(\tilde{y} = 0)$ is available as soon as a background or blank measurement is performed and the standard uncertainty $u(x_0)$ is evaluated. In the case of the simplest model, $y = w \cdot (x_1 - x_0)$, of a non-counting measurement, one expects $x_1 = x_0$ for $\tilde{y} = 0$ and consequently $\tilde{u}^2(\tilde{y} = 0) = w^2 \cdot 2 \cdot u^2(x_0)$.

8 Decision threshold, detection limit and assessments

8.1 Specifications

The probability, α, of a wrong decision in favour of the presence of the physical effect investigated, the probability, β, of a wrong decision in favour of the absence of the physical effect investigated and the probability, $1-\gamma$, for the coverage interval shall be specified. The choice depends on the application. A frequently cited choice is $\alpha=\beta$ and the value 0,05 for α and β. Then, $k_{1-\alpha}=k_{1-\beta}=1{,}65$. For the coverage interval the probability $\gamma=0{,}05$ is frequently chosen. If this is the case, then $k_{1-\gamma/2}=1{,}96$ (see Annex E).

If it is to be assessed whether or not a measurement procedure for the measurand satisfies the requirements to be fulfilled for scientific, legal or other reasons. To allow for such an assessment, a guideline value, y_r, shall also be specified as a value of the measurand.

8.2 Decision threshold

The decision threshold, y^*, of the non-negative measurand according to 6.1, quantifying the physical effect of interest, is the value of the estimator, Y_0, which allows the conclusion that the physical effect is present, if the primary measurement result, y, exceeds the decision threshold, y^*. If the result, y, is below the decision threshold, y^*, it is decided to conclude that the result cannot be attributed to the physical effect. Nevertheless, it cannot be concluded that it is absent. If the physical effect is really absent, the probability of taking the wrong decision, that the effect is present, is equal to the specified probability, α (probability of a wrong decision; see 8.1).

NOTE The decision rule represented by the decision threshold makes use of decision theory applying a quadratic loss function. See Reference [28] for an introduction to decision theory.

For a determined primary measurement result, y, for the non-negative measurand it is only decided to conclude that the true value of the measurand differs from zero ($\tilde{y}>0$), if the primary measurement result is larger than the decision threshold given by

$$y^*=k_{1-\alpha}\cdot\tilde{u}(0) \tag{32}$$

With Formulas (29) and (31) one obtains the decision threshold for the model in ionizing radiation measurement in both cases of time preselection and preselection of counts

$$y^*=k_{1-\alpha}\cdot\tilde{u}(0)=k_{1-\alpha}\cdot w\cdot\sqrt{(x_3\cdot n_0/t_0+x_4)/t_g+x_3^2\cdot n_0/t_0^2+u^2(x_3)\cdot n_0^2/t_0^2+u^2(x_4)}\,. \tag{33}$$

8.3 Detection limit

The detection limit, $y^\#$, is the smallest true value of the measurand, for which, by applying the decision rule according to 8.2, the probability of the wrong decision that the physical effect is absent does not exceed the specified probability, β (see 8.1). In order to find out whether a measurement procedure is suitable for the measurement purpose, the detection limit, $y^\#$, is compared with the specified guideline value, y_r, of the measurand.

The detection limit, $y^{\#}$, is obtained in the case of **time preselection** as the smallest solution of Formulas (34),

$$y^{\#} = y^{*} + k_{1-\beta} \cdot \tilde{u}(y^{\#}) = y^{*} + k_{1-\beta} \cdot \sqrt{w^2 \cdot \left((y^{\#} / w + x_3 \cdot n_0 / t_0 + x_4) / t_g + x_3^2 \cdot n_0 / t_0^2 + u^2(x_3) \cdot n_0^2 / t_0^2 + u^2(x_4) \right) + y^{\#2} \cdot u_{\mathrm{rel}}^2(w)} \quad (34)$$

Formula (34) has a solution, which is the detection limit, $y^{\#}$, if, with time preselection, the following condition is satisfied:

$$k_{1-\beta} u_{\mathrm{rel}}(w) < 1 . \quad (35)$$

The detection limit, $y^{\#}$, is obtained in the case of **preselection of counts** as the smallest solution of Formulas (36),

$$y^{\#} = y^{*} + k_{1-\beta} \cdot \tilde{u}(y^{\#}) = y^{*} + k_{1-\beta} \cdot \sqrt{w^2 \cdot \left[(y^{\#} / w + n_0 \cdot x_3 / t_0 + x_4)^2 / n_g + x_3^2 \cdot n_0 / t_0^2 + n_0^2 \cdot u^2(x_3) / t_0^2 + u^2(x_4) \right] + y^{\#2} \cdot u_{\mathrm{rel}}^2(w)} \quad (36)$$

Formula (36) has a solution, which is the detection limit, $y^{\#}$, if, with preselection of counts, the following condition is satisfied:

$$k_{1-\beta} \cdot \sqrt{\frac{1}{n_{\mathrm{g}}} + u_{\mathrm{rel}}^2(w)} < 1 \quad (37)$$

NOTE 1 For iterative solutions of Formulas (34) and (36) see Annex C and for an explicit solution of the detection limits in the case of preselection of time see Annex D.

NOTE 2 If the left-hand sides of Formulas (35) or (37) exceed 0,5, the detection limit will be significantly overestimated by Formulas (34) and (36), respectively. In this case, the application of ISO 11929-2 in connection with the application of the ISO/IEC Guide 98-3-1 leads to a better estimate of the detection limit.

A non-existence of the detection limit points to the fact that there are too large relative uncertainties of the quantities X_5 to X_m, summarily expressed by $u_{\mathrm{rel}}(w)$, and that the ISO/IEC Guide 98-3 approximation (see 6.2) is not sufficient. In such a case the solution is to obtain the necessary information to allow for an evaluation of the uncertainty on the basis of distributions according to the ISO/IEC Guide 98-3-1 and to proceed according to ISO 11929-2.

8.4 Assessments

The primary measurement result, y, has to be compared with the decision threshold, y^*. If the primary measurement result, y, exceeds the decision threshold, y^*, it is decided to conclude that the physical effect provided by the measurand is present, i.e. that a contribution from the sample has been recognized.

If the result, y, is below the decision threshold, y^*, it is decided to conclude that the result cannot be attributed to the physical effect. Nevertheless, it cannot be concluded that it is absent. If the physical effect is really absent, the probability of taking the wrong decision, that the effect is present, is equal to the specified probability, α.

The decision on whether or not a measurement procedure to be applied sufficiently satisfies the requirements regarding the detection of the physical effect quantified by the measurand is made by comparing the detection limit, $y^{\#}$, with the specified guideline value, y_{r}. If $y^{\#} > y_{\mathrm{r}}$, the measurement procedure is not suitable for the intended measurement purpose with respect to the requirements.

To improve the situation in the case of $y^{\#} > y_{\mathrm{r}}$, it can often be sufficient to choose longer measurement durations or to preselect more counts of the measurement procedure. This reduces the detection limit.

NOTE Occasionally, it can happen that a primary measurement result is larger than the decision threshold, i.e. $y > y^{*}$, is obtained and thus an effect of the sample is recognized, but the detection limit is larger than the specified guideline, i.e. $y^{\#} > y_{\mathrm{r}}$. This is, for instance, the case if due to particular circumstances the background counting rate is too high and at the same time the contribution from the sample is also high. If the primary measurement result, y, and its associated standard uncertainty, $u(y)$, conform with the measurement objective the result can be accepted though formally the criterion of the detection limit in comparison with the guideline value is not fulfilled.

If Formulas (34) or (36) have no solution $y^{\#}$ the ISO/IEC Guide 98-3 approximation, which provides a basis for ISO 11929-1, is not sufficient to evaluate the measurement uncertainties. In this case the Supplement 1 of the ISO/IEC Guide 98-3 has to be used for the evaluation of the uncertainties and the characteristic limits can be obtained as stipulated in ISO 11929-2.

9 Limits of the coverage interval

9.1 General aspects

The limits of the coverage interval are provided for a physical effect, recognized as present according to 8.2, limit the coverage interval in such a way that it contains the true value of the measurand with the specified probability $1-\gamma$ (see 8.1). The limits of the coverage interval take into account the fact that the measurand is non-negative by making use of a truncated and renormalized Gaussian distribution density (see Annex C in ISO 11929-2).

There is no unique definition for the coverage interval if only the condition $1-\gamma$ is given. Further conditions are required which lead among others to the definitions of the probabilistically symmetric coverage interval and the shortest coverage interval. For the calculation of the limits of both types of coverage intervals this document provides formulas for the case that uncertainties can be evaluated according to the ISO/IEC Guide 98-3.

NOTE For the purpose of radiation protection the regulator has to decide which type of coverage interval shall be used. When comparing upper limits of the two coverage intervals one has to take into account that the might have different probabilities.

9.2 The probabilistically symmetric coverage interval

With a primary measurement result, y, of the measurand and the standard uncertainty, $u(y)$, associated with y, the lower limit of the probabilistically symmetric coverage interval, $y^{\triangleleft}$, and the upper limit of the probabilistically symmetric coverage interval, $y^{\triangleright}$, are calculated by

$$y^{\triangleleft} = y - k_p \cdot u(y) \text{ with } p = \omega \cdot (1-\gamma/2) \tag{38}$$

$$y^{\triangleright} = y + k_q \cdot u(y) \text{ with } q = 1 - \omega \cdot \gamma/2 \tag{39}$$

where

$$\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y/u(y)} \exp(-\frac{v^2}{2}) \mathrm{d}v = \Phi[y/u(y)]. \tag{40}$$

For the distribution function, $\Phi(t)$, of the standardized normal distribution and for its inversion, $k_p = t$ for $\Phi(t) = p$, see Table E.1. For methods for its calculation, see Annex E or, for instance, Reference [30].

In general, the limits of the probabilistically symmetric coverage interval are located neither symmetrical to y, nor to the best estimate, $\hat{y}$, but the probabilities of the measurand being smaller than $y^{\triangleleft}$ or larger than $y^{\triangleright}$ both equal $\gamma/2$. The relations $0 < y^{\triangleleft} < y^{\triangleright}$ apply.

$\omega = 1$ may be set if $y > 4u(y)$. In this case, the following approximations symmetrical to y apply:

$$y^{\triangleleft} = y - k_{1-\gamma/2} \cdot u(y) \text{ and } y^{\triangleright} = y + k_{1-\gamma/2} \cdot u(y) \tag{41}$$

and the result may be expressed as $y \pm k_{1-\gamma/2} \cdot u(y)$.

9.3 The shortest coverage interval

As described in detail in Reference [17], the lower limit of the shortest coverage interval, $y^{<}$, and the upper limit of the shortest coverage interval, $y^{>}$, are calculated from a primary measurement result, y, of the measurand and the standard uncertainty, $u(y)$, associated with y, either by

$$y^{<}, y^{>} = y \pm k_p \cdot u(y)\ ; p = (1 + \omega \cdot (1 - \gamma))/2 \tag{42}$$

or if $y^{<} < 0$ were the result by

$$y^{<} = 0 \quad ; \quad y^{>} = y + k_q \cdot u(y)\ ; \quad q = 1 - \omega \cdot \gamma \tag{43}$$

with ω given by Formula (40). The relations $0 \le y^{<} < y^{>}$ apply and the approximation of Formula (41) is valid.

10 The best estimate and its associated standard uncertainty

The determined primary measurement result, y, of the measurand shall be compared with the decision threshold, y^*. If $y > y^*$, the physical effect quantified by the measurand is recognized as present. Otherwise, it is decided to conclude that the result cannot be attributed to the physical effect. Nevertheless it cannot be concluded that it is absent.

If $y \ge y^*$ and with ω according to Formula (40), the best estimate, $\hat{y}$, of the measurand is calculated by

$$\hat{y} = y + \frac{u(y) \cdot \exp\left\{-y^2 / \left[2u^2(y)\right]\right\}}{\omega\sqrt{2\pi}} \tag{44}$$

The standard uncertainty associated with $\hat{y}$ reads

$$u(\hat{y}) = \sqrt{u^2(y) - (\hat{y} - y)\hat{y}} \tag{45}$$

The relations $0 \le y^{<} < y^{\triangleleft} < y \le \hat{y} < y^{>} \le y^{\triangleright}$ as well as $u(\hat{y}) < u(y)$ and $u(\hat{y}) < \hat{y}$ apply. Moreover, for $y \ge 4u(y)$, the approximations

$$\hat{y} = y\ ; \ u(\hat{y}) = u(y) \tag{46}$$

are sufficient.

NOTE 1 If the best estimate, $\hat{y}$, and its associated standard uncertainty, $u(\hat{y})$, are calculated, the recording of the primary measurement result, y, and its associated standard uncertainty, $u(y)$, may be omitted.

NOTE 2 If the decision rule defined by the decision threshold is not used and if $y < y^*$, the best estimate, $\hat{y}$, and its standard uncertainty, $u(\hat{y})$, can also be calculated.

11 Documentation

The content of the test report depends on the specific application as well as on demands of the customer or regulator.

Independent of the requirements for the test report stipulated by the customer or regulator, information shall be retained in order to justify the data of the test report and to guarantee traceability. This applies in particular to:

a) a reference to this document, i.e. ISO 11929-1:2019;

b) the physical effect, Y, of interest, measurand and model of the evaluation;

c) the probabilities α and β of a false positive and a false negative decision, respectively, and, if necessary, the guideline value, y_r;

d) the primary measurement result, y, and the standard uncertainty, $u(y)$, associated with y;

e) the decision threshold, y^*;

f) detection limit, $y^\#$;

g) a statement, if necessary, as to whether or not the measurement procedure is suitable for the intended measurement purpose;

h) a statement as to whether or not the physical effect is recognized as being present;

NOTE 1 If the physical effect is not recognized as being present given the probability α, i.e. if $y < y^*$ (see 8.4), it is occasionally demanded by the regulator to document $< y^\#$ instead of the measured result, y. Such documentation can be meaningful since it allows, by comparison with the guideline value, to demonstrate that the measurement procedure is suitable for the intended measurement purpose. It is, however, misleading because the mathematical meaning is not correct.

NOTE 2 Occasionally, it is requested by the customer or regulator to compare the primary measurement result, y, with the detection limit, $y^\#$, in order to decide whether the physical effect is recognized or not. Such stipulations are not in accordance with the ISO 11929 series. They have the consequence that it is decided to conclude too frequently that the physical effect is absent when in fact it is not absent.

i) if the physical effect is recognized as being present, of the lower limit of the symmetric coverage interval, $y^{\triangleleft}$, and the upper limit of the symmetric coverage interval, $y^{\triangleright}$, with the probability, $1-\gamma$, for the coverage interval, the best estimate, $\hat{y}$, of the measurand, and the standard uncertainty, $u(\hat{y})$ associated with $\hat{y}$.

NOTE 3 Alternatively, the lower limit of the shortest coverage interval, $y^<$, and the upper limit of the shortest coverage interval, $y^>$, with the probability, $1-\gamma$, for the coverage interval, the best estimate, $\hat{y}$, of the measurand, and the standard uncertainty, $u(\hat{y})$ associated with $\hat{y}$ can be documented.

Annex A
(normative)

Repeated counting measurements with random influences

A.1 General aspects

Random influences due to, for instance, sample treatment and instruments cause measurement deviations, which can be different from sample to sample. In such cases, the counting results, n_i, of the counting measurements on several samples of a radioactive material to be examined, on several blanks of a radioactively labelled blank material, and on several reference samples of a standard reference material are therefore respectively averaged to obtain suitable estimates, x_1 and x_2, of the input quantities, X_1 and X_2, and the associated standard uncertainties, $u(x_1)$ and $u(x_2)$, respectively. Accordingly, X_1 shall be considered as the mean gross count rate and X_2 as the mean background count rate. Therefore, the measurand, Y, shall also be taken as an averaged quantity, for instance as the mean net count rate or mean activity of the samples. In this annex, all symbols belonging to the count values measured on the samples, blanks and reference samples are marked by the subscripts g, 0 and r, respectively. In each case, arithmetic averaging over m count values of the same kind carried out with the same preselected measurement duration, t (time preselection), is denoted by an overline. As shown in Reference [17], for m counting results, n_i $(i = 1, ..., m\,;\; m > 3)$, which are obtained in such a way and shall be averaged, the mean value, $\bar{n}$, and its uncertainty, $u^2(\bar{n})$, of the values, n_i, are given by

$$\bar{n} = \frac{1}{m}\sum_{i=1}^{m} n_i \;;\quad u^2(\bar{n}) = \frac{1}{m}\left(\bar{n} + \frac{m-1}{m-3}\bar{n} + \frac{1}{m-3}\sum_{i=1}^{m}(n_i - \bar{n})^2\right) \tag{A.1}$$

The procedures in A.2 and A.3 are approximations for sufficiently large counting results $n_i >> 1$ and $\bar{n} << u^2(\bar{n})$, which allow the random influences to be recognized in addition to those of the Poisson statistics [see Formula (A.10)].

A.2 Procedure with unknown influences

In the case of unknown influences, the following expressions are valid for the mean gross count rate, X_1, and the mean background count rate, X_2:

$$x_1 = \bar{n}_g / t_g \;;\quad x_2 = \bar{n}_0 / t_0 \tag{A.2}$$

$$u^2(x_1) = u^2(\bar{n}_g) / t_g^2 \;;\quad u^2(x_2) = u^2(\bar{n}_0) / t_0^2 \tag{A.3}$$

With the approaches according to Formulas (A.2) and (A.3), Formulas (20) and (25) yield

$$y = \left(\frac{\bar{n}_g}{t_g} - \frac{\bar{n}_0}{t_0} x_3 - x_4\right) \cdot w \tag{A.4}$$

$$u(y) = \sqrt{w^2 \cdot \left[u^2(\bar{n}_g / t_g^2 + x_3^2 u^2(\bar{n}_0) / t_0^2 + (\bar{n}_0 / t_0)^2 \cdot u^2(x_3) + u^2(x_4)\right] + y^2 u_{rel}^2(w)} \tag{A.5}$$

$u^2(x_1)$ is not given as a function of x_1. Therefore, $\tilde{u}^2(\tilde{y})$ shall be determined as an approximation, for instance, according to Formula (A.8), where the current result, y, can be used as y_1. For this purpose and for the calculation of $\tilde{u}^2(0)$, i.e. for $\tilde{y}=0$, the missing $\overline{n}_g^2 / t_g^2$ shall be replaced by $\overline{n}_0^2 / t_0^2$, since both these values are then variance estimates of the same distribution of count rate values, independent of t_g, t_0, m_g and m_0.

It is often sufficient to use the approximations of Formulas (A.8) and (A.9) for the function $\tilde{u}(\tilde{y})$, in particular, if the standard uncertainty, $u(x_1)$, is not known as a function of x_1. A prerequisite is that measurement result, y_j, and associated standard uncertainties, $u(y_j)$, calculated according to 6.3 from previous measurements of the same kind, are already available $(j=0,1,2,...)$. The measurements shall be carried out on different samples with differing activities, but in other respects as far as possible under similar conditions. One of the measurements can be a background effect measurement or a blank measurement with $\tilde{y}=0$ and, for instance, $j=0$. Then, $y_0=0$ shall be set and $\tilde{u}(0)=u(y_0)$. The measurement currently carried out can be taken as a further measurement with $j=1$.

For an assumed true value of the measurand $\tilde{y}=0$ one expects $\frac{\overline{n}_g}{t_g}=\frac{\overline{n}_0}{t_0}$ and obtains for the model according to Formula (A.4) with Formula (A.5)

$$\tilde{u}^2(0)=w^2\cdot\left[u^2(\overline{n}_0)/t_g^2+x_3^2\cdot u^2(\overline{n}_0)/t_0^2+(\overline{n}_0/t_0)^2\cdot u^2(x_3)+u^2(x_4)\right] \tag{A.6}$$

and the decision threshold

$$y^*=k_{1-\alpha}\cdot\tilde{u}(0)=k_{1-\alpha}\cdot w\cdot\sqrt{u^2(\overline{n}_0)/t_g^2+x_3^2\cdot u^2(\overline{n}_0)/t_0^2+(\overline{n}_0/t_0)^2\cdot u^2(x_3)+u^2(x_4)} \tag{A.7}$$

The function $\tilde{u}(\tilde{y})$ often shows a rather slow increase. Therefore, the approximation is sufficient in some of these cases, especially if the primary measurement result, y_1, of the measurand is not much larger than the associated standard uncertainty $u(y_1)$.

If only $\tilde{u}(0)=u(y_0)$, $y_1>0$ and $u(y_1)$ are known, the following linear interpolation often suffices:

$$\tilde{u}^2(\tilde{y})=\tilde{u}^2(0)(1-\tilde{y}/y_1)+u^2(y_1)\tilde{y}/y_1 \tag{A.8}$$

If the results y_0, y_1 and y_2 as well as the associated standard uncertainties $u(y_0)$, $u(y_1)$, and $u(y_2)$ from three measurements are available, the following bilinear interpolation can be used:

$$\tilde{u}^2(\tilde{y})=u^2(y_0)\cdot\frac{(\tilde{y}-y_1)(\tilde{y}-y_2)}{(y_0-y_1)(y_0-y_2)}+u^2(y_1)\cdot\frac{(\tilde{y}-y_0)(\tilde{y}-y_2)}{(y_1-y_0)(y_1-y_2)}+u^2(y_2)\cdot\frac{(\tilde{y}-y_0)(\tilde{y}-y_1)}{(y_2-y_0)(y_2-y_1)} \tag{A.9}$$

If results from many similar measurements are given, the parabolic shape of the function $\tilde{u}^2(\tilde{y})$ can also be determined by an adjustment calculation.

The linear interpolation according to Formula (A.8) leads to the approximation

$$y^\#=a+\sqrt{a^2+(k_{1-\beta}^2-k_{1-\alpha}^2)\tilde{u}^2(0)} \tag{A.10}$$

with

$$a=k_{1-\alpha}\,\tilde{u}(0)+\frac{1}{2}\left\{\left(k_{1-\beta}^2/y_1\right)\left[\left(u^2(y_1)-\tilde{u}^2(0)\right]\right\}\right. \tag{A.11}$$

A.3 Procedure with known influences

Another procedure, appropriate when small random influences are present, is based on the approach

$$u^2(\bar{n}) = \frac{\bar{n} + u^2}{m} = \frac{\bar{n} + \vartheta^2 \bar{n}^2}{m} \quad \text{or} \quad \vartheta = u / \bar{n} \tag{A.12}$$

where u denotes here the two terms with $m-3$ in Formula (A.1), i.e. $u^2 = \frac{m-1}{m-3}\bar{n} + \frac{1}{m-3}\sum_{i=1}^{m}(n_i - \bar{n})^2$.

The first term, $\bar{n}$, of Formula (A.12) corresponds to the numbers n_i of pulses according to the Poisson law in the absence of random influences. These influences are described by the second term, $\vartheta^2 \bar{n}^2$, assuming an empirical relative standard deviation, ϑ, valid for all samples and countings and caused by these influences. This influence parameter, ϑ, can be calculated from the data of counting measurements of the reference samples by combining Formula (A.12) with Formula (A.1) so that

$$\vartheta^2 = \left(m \cdot u^2(\bar{n}_\mathrm{r}) - \bar{n}_\mathrm{r}\right) / \bar{n}_\mathrm{r}^2 \tag{A.13}$$

Instead of the data from counting measurements of the reference samples, those for other samples can be used which were previously examined, not explicitly for reference purposes but under conditions similar to those of the reference samples.

If $\vartheta^2 < 0$ results, the approach and the data are not compatible. The number, m_r, of the reference samples should then be enlarged or $\vartheta = 0$ be set. Moreover, $\vartheta < 0{,}2$ should be obtained. Otherwise, one can proceed according to A.2.

NOTE The influence parameter, ϑ, is assumed to apply to both, the gross and the background measurements.

Instead of Formula (A.3), the expressions

$$u^2(x_1) = \left(\bar{n}_\mathrm{g} + \vartheta^2 \bar{n}_\mathrm{g}^2\right) / \left(m_\mathrm{g} t_\mathrm{g}^2\right) ; \quad u^2(x_2) = \left(\bar{n}_0 + \vartheta^2 \bar{n}_0^2\right) / \left(m_0 t_0^2\right) \tag{A.14}$$

now apply with Formula (A.12). The cases $m_\mathrm{g} = 1$ and $m_0 = 1$ are permitted here. Therefore, with $x_1 = \bar{n}_\mathrm{g} / t_\mathrm{g}$ and Formula (A.14), $u^2(x_1)$ is given as a function of x_1 by

$$u^2(x_1) = h_1^2(x_1) = \left(x_1 / t_\mathrm{g} + \vartheta^2 x_1^2\right) / m_\mathrm{g} \tag{A.15}$$

Formulas (A.2) and (A.4) remain valid for $x_3 = 1$ with $u(x_3) = 0$ and $x_4 = 0$ with $u(x_4) = 0$. Furthermore, according to Formula (25), it follows that

$$u(y) = \sqrt{w^2 \cdot \left[u^2(x_1) + u^2(x_2)\right] + y^2 u_\mathrm{rel}^2(w)} \tag{A.16}$$

$u^2(x_1)$ and $u^2(x_2)$ according to Formula (A.12) shall be inserted.

In order to calculate $\tilde{u}(\tilde{y})$, the result, y, is replaced by $\tilde{y}$ and Formula (A.10) is solved for $x_1 = \bar{n}_g / t_g$. This yields $x_1 = \tilde{y}/w + \bar{n}_0 / t_0$. The estimate, x_1, determined in this way in the current case, shall be substituted in Formula (A.13) and $u^2(x_1)$ obtained there from in Formula (A.14). This finally leads to

$$\tilde{u}(\tilde{y}) = \sqrt{\tilde{y}^2 \left[\frac{\vartheta^2}{m_g} + u_{rel}^2(w) \right] + \frac{\tilde{y} w}{m_g} \left(\frac{2\bar{n}_0 \vartheta^2}{t_0} + \frac{1}{t_g} \right) + w^2 \left(\frac{\bar{n}_0}{m_g t_0 t_g} + \frac{\bar{n}_0^2 \vartheta^2}{m_g t_0^2} + \frac{\bar{n}_0 + \vartheta^2 \bar{n}_0^2}{m_0 t_0^2} \right)} \quad \text{(A.17)}$$

and one obtains the decision threshold

$$y^* = k_{1-\alpha} \sqrt{w^2 \left(\frac{\bar{n}_0}{m_g t_0 t_g} + \frac{\bar{n}_0^2 \vartheta^2}{m_g t_0^2} + \frac{\bar{n}_0 + \vartheta^2 \bar{n}_0^2}{m_0 t_0^2} \right)} \quad \text{(A.18)}$$

and the detection limit

$$y^\# = y^* + k_{1-\beta} \sqrt{y^{\#2} \left[\frac{\vartheta^2}{m_g} + u_{rel}^2(w) \right] + \frac{y^\# w}{m_g} \left(\frac{2\bar{n}_0 \vartheta^2}{t_0} + \frac{1}{t_g} \right) + w^2 \left(\frac{\bar{n}_0}{m_g t_0 t_g} + \frac{\bar{n}_0^2 \vartheta^2}{m_g t_0^2} + \frac{\bar{n}_0 + \vartheta^2 \bar{n}_0^2}{m_0 t_0^2} \right)} \quad \text{(A.19)}$$

A.4 Black-box measurements with random influences

Black-box measurements are measurements in which primary results, y_i, of the measurand, Y, are directly indicated without information for the user about the measurement technique and the model of evaluation involved. There are just series of observations y_i. The measurement problem is to compare a series of indications, $y_{b,i}$ $(i = 1,...,n_b)$, which are judged by the user to represent a “normal”, “background” or “blank” scenario, with a series of indications, $y_{g,i}$ $(i = 1,...,n_g)$, for another scenario, called “gross” scenario. To this end, a net quantity is investigated as

$$Y = \bar{Y}_g - \bar{Y}_b \quad \text{(A.20)}$$

and the characteristic limits are evaluated for the measurand Y.

Primary estimates of $\bar{Y}_g$ and $\bar{Y}_b$ are obtained as the arithmetic means

$$\bar{y}_g = \frac{1}{n_g} \sum_{i=1}^{n_g} y_{g,i} \quad \text{and} \quad \bar{y}_b = \frac{1}{n_b} \sum_{i=1}^{n_b} y_{b,i} \quad \text{(A.21)}$$

respectively, with their respective sample standard deviations

$$s_g = \left(\frac{1}{n_g - 1} \sum_{i=1}^{n_g} (y_{g,i} - \bar{y}_g)^2 \right)^{1/2} \quad \text{and} \quad s_b = \left(\frac{1}{n_b - 1} \sum_{i=1}^{n_b} (y_{b,i} - \bar{y}_b)^2 \right)^{1/2} \quad \text{(A.22)}$$

Since no other information is available the $y_{b,i}$ $(i = 1,...,n_b)$ and $y_{g,i}$ $(i = 1,...,n_g)$ are assumed to be samples from Gaussian distributions with unknown expectations and variances. According to ISO/IEC Guide 98-3-1, 9.2.3 (JCGM 2008b) the arithmetic means $\bar{y}_g$ and $\bar{y}_b$ according to Formula (A.21) are the best estimates and the standard uncertainties associated with $\bar{y}_g$, $\bar{y}_b$ and y are

$$u^2(\bar{y}_g) = \frac{n_g - 1}{n_g - 3} \cdot \frac{s_g^2}{n_g}, \quad u^2(\bar{y}_b) = \frac{n_b - 1}{n_b - 3} \cdot \frac{s_b^2}{n_b} \quad \text{and} \quad u^2(y) = u^2(\bar{y}_g) + u^2(\bar{y}_b) \quad \text{(A.23)}$$

NOTE 1 In the context of the methods described in ISO 11929-2 the PME yields the scaled and shifted t-distributions $t_{n_g-1}(\bar{y}_g, s_g^2/n_g)$ and $t_{n_b-1}(\bar{y}_b, s_b^2/n_b)$ as the respective PDFs (ISO/IEC Guide 98-3-1, 9.2.3 (JCGM 2008b)).

NOTE 2 Evidently $n_b, n_g > 3$ are required. The choice of suitable and meaningful numbers of indications $n_b > 3$ and $n_g > 3$ depends on the judgement of the user, the measurement objective and the prevailing circumstances.

Applying the ISO/IEC Guide 98-3 (JCGM 2008a) and this document (ISO 2016a), one assumes in order to obtain the decision threshold that $\bar{y}_g = \bar{y}_b$ and $s_g = s_b$ will hold for a true value $\tilde{y} = 0$ of Y. This yields

$$\tilde{u}^2(\tilde{y}=0) = \left(\frac{n_g - 1}{n_g - 3} \cdot \frac{1}{n_g} + \frac{n_b - 1}{n_b - 3} \cdot \frac{1}{n_b} \right) \cdot s_b^2 \qquad \text{(A.24)}$$

and the decision threshold

$$y^* = k_{1-\alpha} \cdot \tilde{u}(\tilde{y}=0) = k_{1-\alpha} \cdot \sqrt{\frac{n_g - 1}{n_g - 3} \cdot \frac{1}{n_g} + \frac{n_b - 1}{n_b - 3} \cdot \frac{1}{n_b}} \cdot s_b \qquad \text{(A.25)}$$

The detection limit $y^\#$ can only be obtained by applying the interpolation formula according to Formula (A.8).

Annex B
(normative)

Measurements with ratemeters

A ratemeter records the rate of pulses arriving at the input of the meter. Herein, a ratemeter is understood as a linear, analogously working count rate measuring instrument where the output signal increases sharply (with a negligible rise time constant) upon the arrival of an input pulse and then decreases exponentially with a relaxation time constant, τ, until the next input pulse arrives. The signal increase shall be the same for all pulses and the relaxation time constant shall be independent of the count rate. A digitally working count rate measuring instrument simulating the one just described is also taken as a ratemeter having to be considered here.

Each particular measurement using a ratemeter shall be carried out in the stationary state of the ratemeter. This requires at least a sufficiently large fixed time span between the start of measurement and reading the ratemeter indication. This applies to each sample and to each background effect measurement. According to Reference [29], fixed time spans of 3τ or 7τ correspond to deviations of the indication by 5 % or 0,1 % of the magnitude of the difference between the indication at the start of measurement and that at the end of the time span. If further uncertain influences have to be taken into account, a time span of 7τ should be chosen, if possible.

The count rates, R_g and R_0, indicated by the output signals of the ratemeter in the cases of measuring the gross and background effects, respectively, are taken as the input quantities, X_1 and X_2, of the model in 6.2.2 for the calculation of the characteristic values: $X_1 = R_g$ and $X_2 = R_0$. With the values r_g and r_0 of the output signals determined at the respective moments of measurement, the following approaches result for the values of the input quantities and the associated standard uncertainties:

$$x_1 = r_g \; ; \; x_2 = r_0 \tag{B.1}$$

$$u^2(x_1) = \frac{r_g}{2\tau_g} \; ; \; u^2(x_2) = \frac{r_0}{2\tau_0} \tag{B.2}$$

In Formula (B.2), approximations with a maximum relative deviation of 5 % for $r_g \cdot \tau_g \geq 0{,}65$ and of 1 % for $r_g \cdot \tau_g \geq 1{,}32$ are specified according to Reference [29]. The same applies to $r_0 \cdot \tau_0$. The relaxation time constants, τ_g and τ_0, shall be adjusted to fulfil requirements regarding the maximum relative deviations.

The ratemeter measurement is equivalent to a counting measurement with time preselection according to 6.2.2 and with the measurement durations, $t_g = 2\tau_g$ and $t_0 = 2\tau_0$. The quotients n_g / t_g and n_0 / t_0 of the counting measurement shall be replaced here by the measured count rate values, r_g and r_0, respectively, of the ratemeter measurement.

The model shall be specified in the form of $Y = W \cdot (X_1 - X_2) = W \cdot (R_g - R_0)$, Formulas (B.1) and (B.2) lead to

$$y = (r_b - r_0) \cdot w \quad u^2(y) = w^2 \cdot \left(\frac{r_g}{2\tau_g} + \frac{r_0}{2\tau_0} \right) + (r_g - r_0)^2 \cdot u^2(w) \tag{B.3}$$

Replacing y by $\tilde{y}$ and eliminating $r_g = \tilde{y} + r_0$, because of $u^2(x_1) = x_1/(2\tau_g) = r_g/(2\tau_g)$, yields

$$\tilde{u}^2(\tilde{y}) = w^2 \cdot \left[\frac{\tilde{y}}{2\tau_g \cdot w} + r_0 \cdot \left(\frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right) \right] + \tilde{y}^2 \cdot u_{rel}^2(w) \tag{B.4}$$

and one obtains the decision threshold with

$$y^* = k_{1-\alpha} \cdot w \cdot \sqrt{r_0 \cdot \left(\frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right)} \tag{B.5}$$

and the detection limit with

$$y^\# = y^* + k_{1-\beta} \cdot \sqrt{w^2 \cdot \left[\frac{y^\#}{2\tau_g \cdot w} + r_0 \cdot \left(\frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right) \right] + y^{\#2} \cdot u_{rel}^2(w)} \tag{B.6}$$

If the background count rate is very small, as for instance in measurements of contaminations with alpha-emitting radionuclides, Formulas (B.3) to (B.6) can be modified. Then the model of evaluation and the standard uncertainty associated with the primary result reads

$$y = (r_b - r_0 - \frac{1}{2\tau_0}) \cdot w \quad u^2(y) = w^2 \cdot \left(\frac{r_g}{2\tau_g} + \frac{r_0}{2\tau_0} \right) + (r_g - r_0 - \frac{1}{2\tau_0})^2 \cdot u^2(w) \tag{B.7}$$

Replacing again y by $\tilde{y}$ and eliminating $r_g = \tilde{y} + r_0 + \frac{1}{2\tau_0}$, because of $u^2(x_1) = x_1/(2\tau_g) = r_g/(2\tau_g)$, yields

$$\tilde{u}^2(\tilde{y}) = w^2 \cdot \left[\frac{\tilde{y}}{2\tau_g \cdot w} + r_0 \cdot \left(\frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right) + \frac{1}{4\tau_g \tau_0} \right] + \tilde{y}^2 \cdot u_{rel}^2(w) \tag{B.8}$$

and one obtains the decision threshold with

$$y^* = k_{1-\alpha} \cdot w \cdot \sqrt{r_0 \cdot \left(\frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right) + \frac{1}{4\tau_g \tau_0}} \tag{B.9}$$

and the detection limit with

$$y^\# = y^* + k_{1-\beta} \cdot \sqrt{w^2 \cdot \left[\frac{y^\#}{2\tau_g \cdot w} + r_0 \cdot \left(\frac{1}{2\tau_g} + \frac{1}{2\tau_0} \right) + \frac{1}{4\tau_g \tau_0} \right] + y^{\#2} \cdot u_{rel}^2(w)} \tag{B.10}$$

