
**Determination of the detection limit and
decision threshold for ionizing radiation
measurements —**

Part 1:

Fundamentals and application to counting
measurements without the influence of sample
treatment

*Détermination de la limite de détection et du seuil de décision des
mesurages des rayonnements ionisants —*

*Partie 1: Principes fondamentaux et application aux mesures par comptage,
sans l'influence du traitement de l'échantillon*



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International Organization for Standardization
Case postale 56 • CH-1211 Genève 20 • Switzerland
Internet iso@iso.ch

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 11929-1 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, SC 2, *Radiation protection*, Working Group WG 17 (formerly WG 2), *Radioactivity measurements*.

ISO 11929 consists of the following parts, under the general title *Determination of the detection limit and decision threshold for ionizing radiation measurements*:

- *Part 1: Fundamentals and application to counting measurements without the influence of sample treatment*
- *Part 2: Fundamentals and application to counting measurements with the influence of sample treatment*
- *Part 3: Fundamentals and application to counting measurements by high resolution gamma spectrometry, without the influence of sample treatment*
- *Part 4: Fundamentals and application to measurements by use of linear scale analogue ratemeters, without the influence of sample treatment*

Annex A of this part of ISO 11929 is for information only.

Introduction

This part of ISO 11929 gives basic information on the statistical fundamentals for determination of the detection limit and decision threshold (and additional directives for specification of a confidence interval) for nuclear counting radiation measurements based on the principles defined by Altschuler and Pasternack [1], Nicholson [7] and Currie [3].

As an example, information is given for a gross (integral) counting measurement. Other parts of ISO 11929 deal with counting measurements which take sample treatment into consideration and with analogue pulse-rate measurements and specific problems which occur when this part of ISO 11929 is applied (e.g. in the case of spectrometric measurements or continuous monitoring of radioactive effluents).

This part of ISO 11929 concerns the field of nuclear radiation measurement in which events (in particular pulses) are counted (for example on samples). It considers exclusively the random character of radioactive decay and of pulse counting and ignores all other influences (e.g. arising from sample treatment, weighing, enrichment or the instability of the test setup). It also assumes that the duration of measurement is small in relation to the half-life of the radionuclides involved and that dead-time losses are negligible.

For information on the influences mentioned above, see ISO 11929-2.

Whenever activities or specific activities are to be determined, it is assumed that the factors for the conversion of pulse rates into activities or specific activities have been determined to such an accuracy that the influence of their uncertainty of measurement can be ignored.

This part of ISO 11929 was prepared in parallel with other International Standards prepared by WG 2 (now WG 17): ISO 11932:1996, *Activity measurements of solid materials considered for recycling, re-use or disposal as non-radioactive waste*, and ISO 11929-2, ISO 11929-3 and 11929-4 and is, consequently, complementary to these documents.

Determination of the detection limit and decision threshold for ionizing radiation measurements —

Part 1:

Fundamentals and application to counting measurements without the influence of sample treatment

1 Scope

This part of ISO 11929 specifies suitable statistical values which allow an assessment of the detection capabilities in ionizing radiation measurements without the influence of sample treatment. For this purpose, statistical methods are used to specify the following two statistical values characterizing given probabilities of error.

- The decision threshold, which allows a decision to be made for each measurement with a given probability of error as to whether the registered pulses include a contribution by the sample.
- The detection limit, which specifies the minimum sample contribution which can be detected with a given probability of error using the measuring procedure in question. This consequently allows a decision to be made as to whether a measuring method defined in this part of ISO 11929 satisfies certain requirements and is consequently suitable for the given purpose of measurement.

The difference between using the decision threshold and using the detection limit is that measured values are to be compared with the decision threshold while the detection limit is to be compared with the guideline value.

2 Normative reference

The following normative document contains provisions which, through reference in this text, constitute provisions of this part of ISO 11929. For dated references, subsequent amendments to, or revisions of, such publications do not apply. However, parties to agreements based on this part of ISO 11929 are encouraged to investigate the possibility of applying the most recent edition of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML Guide to the Expression of Uncertainty in Measurement. Geneva 1993.

3 Terms and definitions

For the purposes of this part of ISO 11929, the following terms and definitions apply.

3.1

measuring method

use of a measuring instrument for counting measurements under given conditions

3.2

decision threshold

critical value of a statistical test for the decision between the hypothesis $\rho_s = \rho_0$ and the alternative hypothesis $\rho_s > \rho_0$

NOTE In the case of time preselection, it shall be the value R_n^* which, when exceeded by the determined value R_n , is taken to indicate that the hypothesis should be rejected. In the case of pulse preselection, it should be the value $(R_s/R_0)^*$ which, when exceeded by the determined value R_s/R_0 , is taken to indicate that the hypothesis should be rejected. The statistical test should be designed such that the probability of wrongly rejecting the hypothesis (error of the first kind) is equal to a value α which is fixed prior to commencement of the measurement.

3.3 detection limit

(time preselection counting) smallest expectation value of the net counting rate that can be detected on given probabilities and, therefore, the smallest difference $\rho_n = \rho_s - \rho_0$ associated with the statistical test concerned for the decision between the hypothesis $\rho_s = \rho_0$ and the alternative hypothesis $\rho_s > \rho_0$ and having the following characteristic: if in reality $\rho_n \geq \rho$, the probability of wrongly not rejecting the hypothesis $\rho_s = \rho_0$ (error of second kind) shall be at most equal to a value β which is fixed prior to commencement of the measurement

3.4 detection limit

(pulse preselection counting) smallest quotient ρ_s/ρ_0 associated with the statistical test concerned for the decision between the hypothesis $\rho_s = \rho_0$ and the alternative hypothesis $\rho_s > \rho_0$ and having the following characteristic: if in reality $\rho_s/\rho_0 \geq (\rho_s/\rho_0)^*$, the probability of wrongly not rejecting the hypothesis $\rho_s = \rho_0$ (error of second kind) shall at most be equal to a value β which is fixed prior to commencement of the measurement

3.5 confidence interval

interval to be specified for the measured value obtained for R_n or R_s/R_0

NOTE This interval includes the true value ρ_n or ρ_s/ρ_0 in at least $(1-\gamma) \times 100\%$ of all cases.

3.6 sample

whole amount or an aliquot of a material, the content of radioactive nuclides of which has to be determined by ionizing radiation measurement

3.7 background effect

measured counting rate without a sample

NOTE This covers radiation caused by external sources and radionuclides in detector and shielding.

3.8 gross effect

measured counting rate from the sample (sample contribution) and the background radiation

3.9 net effect

(sample contribution) gross effect minus the background effect

3.10 guideline value

value constituted by requirements on measuring procedures arising for scientific, legal or other reasons which are specified, for example, as activity, specific activity, surface activity, dose rate, etc.

NOTE If necessary, a calibration factor can be determined using a radioactive reference standard.

4 Symbols

N_0	Number of pulses counted during measurement of the background effect
N_s	Number of pulses counted during measurement of the gross effect
t_0	Duration of the background effect measurement
t_s	Duration of the gross effect measurement
R_0	Background effect counting rate, quotient of the pulses N_0 counted during the preselected duration of measurement t_0 and the duration of measurement t_0 : $R_0 = N_0/t_0$
ρ_0	Expectation value of R_0
R_s	Gross effect counting rate, quotient of the number of pulses N_s counted during the preselected duration of measurement t_s and the duration of measurement t_s : $R_s = N_s/t_s$
ρ_s	Expectation value of R_s
R_n	Net effect counting rate, difference between gross and background effect counting rates, $R_n = R_s - R_0$
ρ_n	Expectation value of R_n
R_n^*	Decision threshold for the net counting rate R_n using the time preselection (see Table 1)
$(R_s/R_0)^*$	Decision threshold for the quotient R_s/R_0 using pulse preselection
ρ_n^*	Detection limit for the expectation value of the net counting rate R_n using time preselection
$(\rho_s/\rho_0)^*$	Detection limit for the quotient ρ_s/ρ_0 using pulse preselection (see Table 1)
α	Error of the first kind; the probability of rejecting the null hypothesis $\rho_s = \rho_0$ for the alternative hypothesis $\rho_s > \rho_0$ when the null hypothesis is true
β	Error of the second kind; the probability of accepting the null hypothesis $\rho_s = \rho_0$ against the alternative hypothesis $\rho_s > \rho_0$ when the null hypothesis is false
$1-\gamma$	Confidence level of the confidence interval for ρ_n or ρ_s/ρ_0
$k_{1-\alpha}, k_{1-\beta}, k_{1-\gamma/2}$	Quantiles of the standard normal distribution (see Table 2)
$F(N_s, N_0, 1-\alpha)$ $F(N_0, N_s, 1-\beta)$ $F(N_0, N_s, 1-\gamma/2)$ $F(N_s, N_0, 1-\gamma/2)$	 Quantiles of the F -distribution (see Tables 3 and 4)
q	Quotient of the measuring times t_s and t_0 with time preselection: $q = t_s/t_0$; or quotient of the numbers of pulses N_s and N_0 with pulse preselection: $q = N_s/N_0$

5 Statistical values and confidence interval

5.1 Principles

5.1.1 General aspects

The definition of the statistical values for decision threshold, detection limit and confidence interval are based on the variances of the measured results. They are dependent on the variations caused by counting statistics. Measurement equipment instabilities normally can be neglected because usually they are small in comparison to the other influences. The influence of counting statistics can be calculated by the Poisson formula.

5.1.2 Model

If device instabilities are neglected, the following model can be applied.

The number of pulses, N_0 , counted without a sample is given by background radiation (external sources and detector noise).

The number of pulses, N_s , counted with a sample is the sum of background and sample radiation (net counting):

$$N_s = N_0 + N_n \quad (1)$$

It is assumed that, for constant radioactive emission, both the numbers of pulses, N_s, N_0 , counted in a measurement of the gross effect during the time t_s and in an independent measurement of the background effect during the time t_0 are Poisson distributed with the expectations $\rho_s \cdot t_s$ and $\rho_0 \cdot t_0$, respectively. Therefore, the expectations of the counting rates R_0 and R_s are ρ_0 and $\rho_s = \rho_0 + \rho_n$. The variances of R_0 and R_s are ρ_0/t_0 and ρ_s/t_s , respectively.

The net counting rate $R_n = R_s - R_0$ has the expectation ρ_n and the variance

$$\text{var}(R_n) = \text{var}(R_0) + \text{var}(R_s) \quad (2)$$

For $\rho_n = 0$ this variance is

$$\text{var}(R_n) = \rho_0(1/t_0 + 1/t_s) \quad (3)$$

In this case, ρ_0 and ρ_s are unknown parameters.

The values of statistics characterizing the decision threshold and the detection limit shall be formulated in a manner depending on whether a time preselection or a pulse preselection has been made.

5.2 Decision threshold

5.2.1 Time preselection

In the case of time preselection, the decision threshold shall refer to the value R_n^* which, when exceeded by a measured net counting rate R_n , is taken to indicate that a sample contribution exists.

If this decision rule is observed, a wrong decision occurs with the probability α that there is a sample contribution when in fact only a background effect exists (error of the first kind).

The decision threshold is given by

$$R_n^* \approx k_{1-\alpha} \sqrt{\text{var}(R_n = 0)} \quad (4)$$

where $k_{1-\alpha}$ is a factor given in Table 2.

Formulae for calculation of the decision threshold under different conditions are given in Table 1.

5.2.2 Pulse preselection

In the case of measuring time while counting up to preselected numbers of pulses, N_s , N_0 , the decision threshold for the pulse rate is given by

$$\left(\frac{R_s}{R_0}\right)^* = F(N_s, N_0, 1 - \alpha) \quad (5)$$

where $F(N_s, N_0, 1 - \alpha)$ is a factor given in Table 3; $R_s = N_s/t_s$, $R_0 = N_0/t_0$.

If a quotient (R_s/R_0) of measured results exceeds $(R_s/R_0)^*$, it is assumed that a sample contribution really exists and the null hypothesis should be rejected.

5.3 Detection limit

5.3.1 Time preselection

In the case of time preselection, the detection limit shall refer to the smallest expectation of the net counting rate ρ for which a wrong decision occurs with the probability β (if the decision rule as specified in 5.2 is applied) that there is no sample contribution but only a background effect (error of the second kind).

To check whether a measuring procedure is suitable for the purpose of measurement, the detection limit shall be compared with a specified guideline value (e.g. specified requirements on the sensitivity of the measuring procedure for scientific, legal or other reasons).

So, with α and β , the detection limit is

$$\rho_n^* = R_n^* + k_{1-\beta} \sqrt{\text{var}(R_n = \rho_n^*)} \quad (6)$$

$$= k_{1-\alpha} \sqrt{\text{var}(R_n = 0)} + k_{1-\beta} \sqrt{\text{var}(R_n = \rho_n^*)} \quad (7)$$

and for $\text{var}(R_n = 0) \approx \text{var}(R_n > 0)$

$$\rho_n^* = (k_{1-\alpha} + k_{1-\beta}) \sqrt{\text{var}(R_n = 0)} \quad (8)$$

If $\alpha = \beta$, the detection limit is

$$\rho_n^* = 2 \cdot R_n^* \quad (9)$$

where $k_{1-\alpha}$, $k_{1-\beta}$ are factors given in Table 2.

Formulae for calculation of the detection limit under different conditions are given in Table 1.

5.3.2 Pulse preselection

In the case of pulse number preselection counting, the detection limit is the smallest quotient of the expectations of gross and background counting rates $(\rho_s/\rho_0)^*$ for which there is a probability β for wrongly accepting the null hypothesis $\rho_s = \rho_0$ that there is no sample contribution but only a background effect.

The detection limit is

$$\left(\frac{\rho_s}{\rho_0}\right)^* = F(N_0, N_s, 1 - \beta) \cdot F(N_s, N_0, 1 - \alpha) \quad (10)$$

5.4 Confidence interval

By way of completion, formulae are given which can be used with a specified confidence level $1-\gamma$ to assign a confidence interval to each measured value. The confidence interval for ρ_n or ρ_s/ρ_0 is an interval to be specified for the measured value obtained for R_n or R_s/R_0 using the formulae given in Table 1. It includes the true value of ρ_n or ρ_s/ρ_0 in at least $(1-\gamma) \times 100\%$ of all cases.

6 Application of this part of ISO 11929 (see annex A)

6.1 Specified values

The error probabilities α , β and the confidence level $1-\gamma$ shall be specified in advance. Frequently cited values are $\alpha = \beta = \gamma = 0,05$. The measuring times t_0 and t_s , in the case of time preselection, and the numbers of pulses N_0 and N_s , in the case of pulse preselection, shall be chosen such that the detection limit is below the guideline value.

NOTE If $\alpha = \beta = \gamma/2$ are chosen and if $\text{var}(R_n)$ varies just a little with R_n , one obtains for $R_n = R_n^*$ the confidence interval $R_n^* \pm k_{1-\alpha} \sqrt{\text{var}(R_n)}$, that is the interval $(0, \rho_n^*)$. This choice avoids a discontinuity in the expression of results.

6.2 Assessment of a measuring procedure

The decision as to whether a measuring method (3.1) satisfies certain requirements with respect to the detection limit shall be made by comparing the detection limit which has been determined with the specified guideline value (see 5.3.1).

This may be carried out either in advance, for the assessment of an intended measuring method on the basis of an empirically determined value for the background effect or a separate measurement, or else in retrospect, for the assessment of a measurement already carried out on the basis of the background effect value then available.

The detection limit may be either read off from Figure 1 or Figure 2 or calculated by means of the formulae in Table 1.

If the detection limit thus determined is greater than the guideline value, the measuring procedure is not suitable for the purpose of the measurement.

NOTE Under certain circumstances, a measuring procedure may be suitable for the purpose of measurement, for example, by preselecting a greater duration of measurement or a higher number of pulses, by reducing the background effect, or by increasing sample quantity or by sample enrichment.

6.3 Assessment of measured results

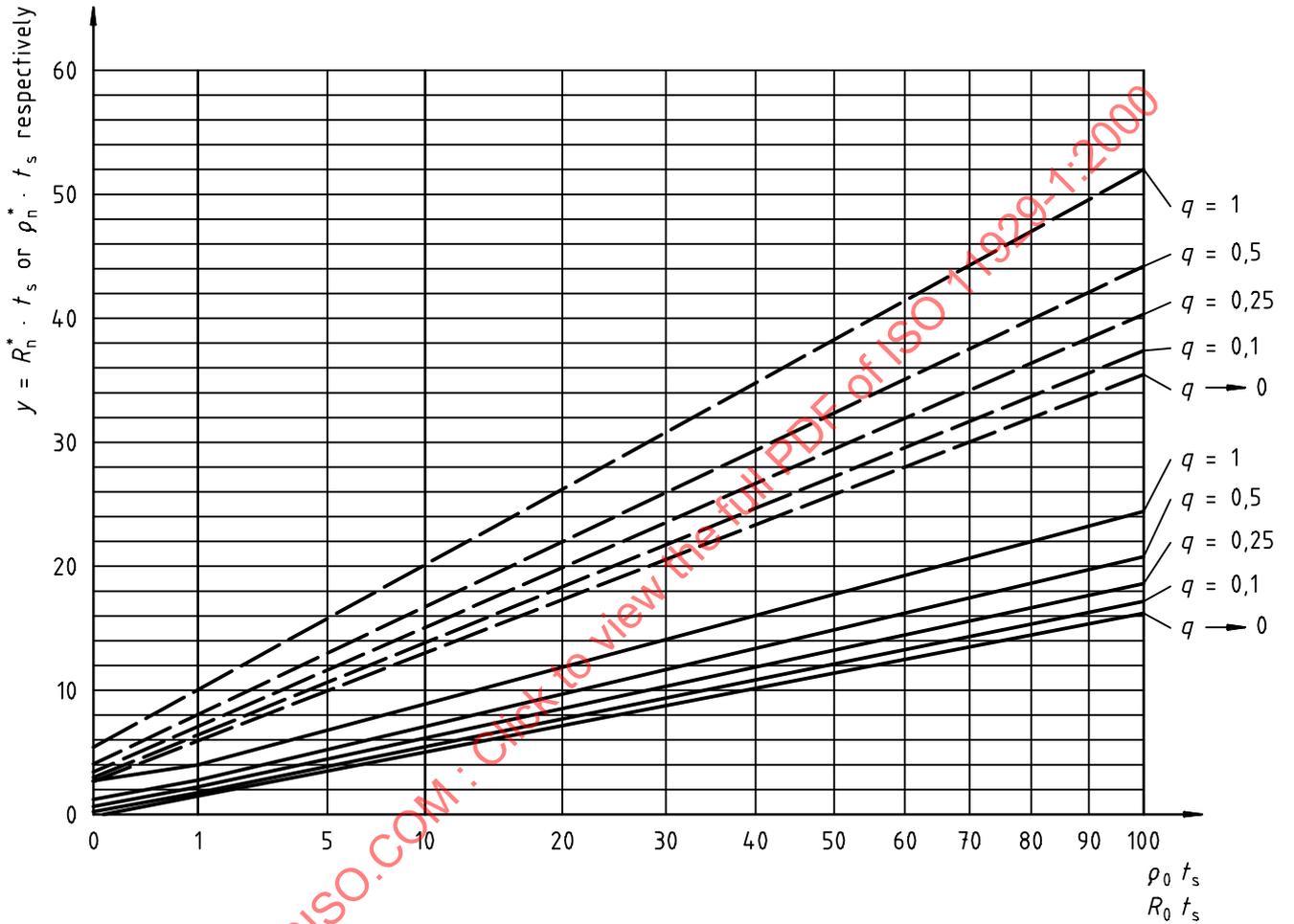
The decision threshold may be read off from Figure 1 or Figure 2 or calculated by means of the formulae in Table 1.

A measured result shall be compared with the decision threshold thus obtained (see 5.2). If a result is greater than the decision threshold, it is assumed to be a real sample contribution.

6.4 Documentation

A report on measurements in accordance with this part of ISO 11929 shall be accompanied by details on the probabilities of error, the decision threshold and the detection limit.

For established sample contributions, in addition to the measured value, confidence intervals determined in accordance with the equations in Table 1, and the confidence level, shall also be reported.



————— Decision threshold: $R_n^* = y/t_s$; (x-axis: $R_0 \cdot t_s$) (11)

----- Detection limit: $\rho_n^* = y/t_s$; (x-axis: $\rho_0 \cdot t_s$) (12)

NOTE Values for $q \rightarrow 0$ are calculated by the formulae in Table 1 for $t_0 \rightarrow \infty$

Figure 1 — Decision threshold and detection limit using preselected time as a function of $R_0 t_s$ or $\rho_0 t_s$ respectively for $\alpha = \beta = 0,05$; parameter $q = t_s/t_0$

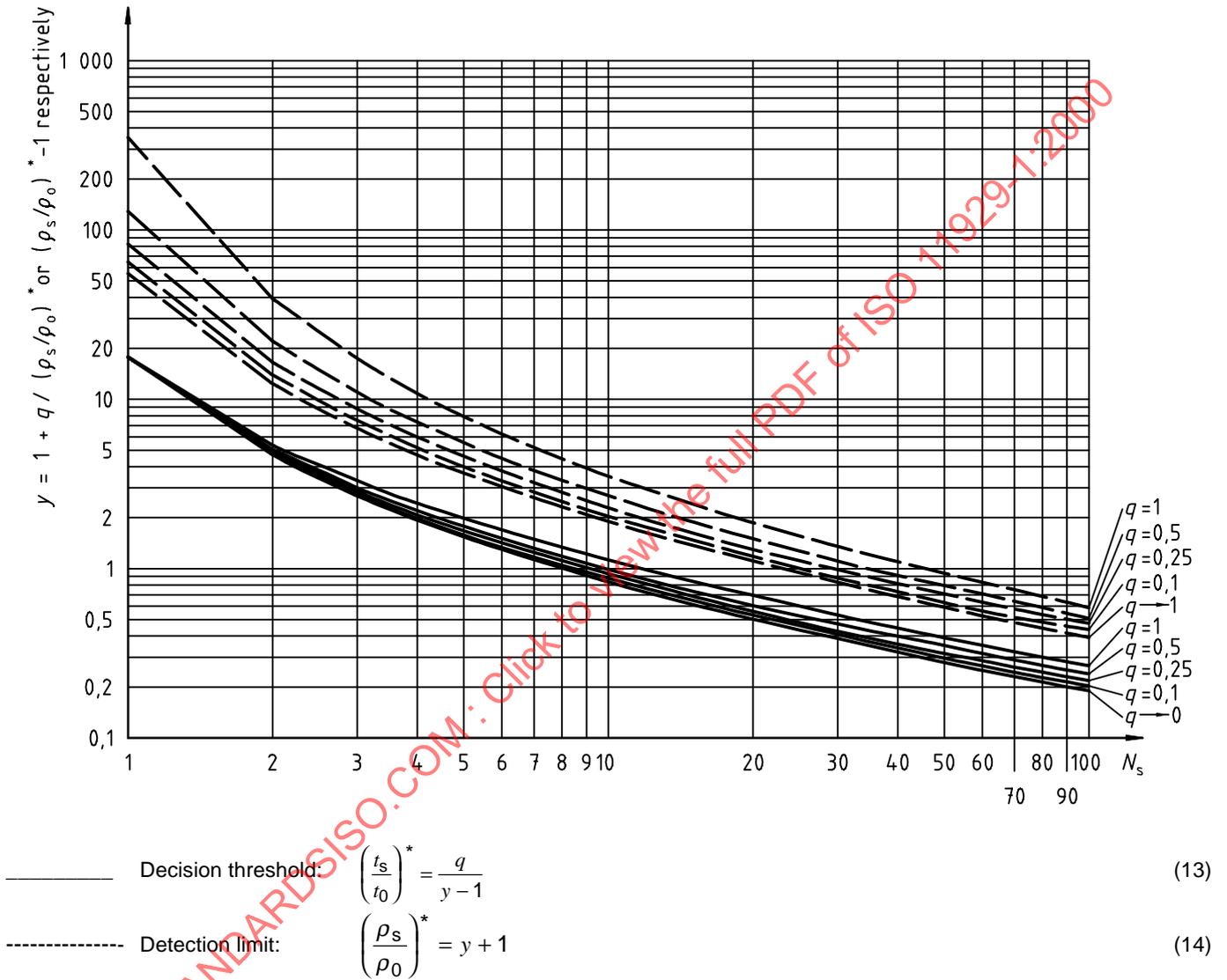


Figure 2 — Decision threshold and detection limit using preselected counts as a function of N_s for $\alpha = \beta = 0,05$; parameter $q = N_s/N_0$

Table 1 — Formulae for the calculation of the decision threshold, the detection limit and the confidence interval

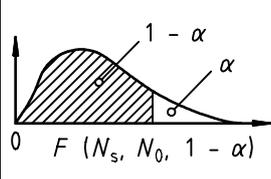
Time preselection	Remarks
<p>Decision threshold</p> $R_n^* = \frac{1}{2t_0} k_{1-\alpha}^2 \left(1 + \sqrt{1 + \frac{4R_0 t_0}{k_{1-\alpha}^2} \left(1 + \frac{t_0}{t_s} \right)} \right) \quad (15)$ <p>Detection limit</p> $\rho_n^* = (k_{1-\alpha} + k_{1-\beta}) \sqrt{\rho_0 \left(\frac{1}{t_0} + \frac{1}{t_s} \right)} + \frac{1}{4} (k_{1-\alpha} + k_{1-\beta})^2 \left(\frac{1}{t_0} + \frac{1}{t_s} \right) \quad (16)$ <p>Confidence interval</p> $R_n - k_{1-\gamma/2} \sqrt{\frac{R_0}{t_0} + \frac{R_s}{t_s}} \leq \rho_n \leq R_n + k_{1-\gamma/2} \sqrt{\frac{R_0}{t_0} + \frac{R_s}{t_s}} \quad (17)$	<p>The formulae (15) to (17) are based on approximations of the Poisson distribution by the normal distribution. With values of adequate size of $R_0 t_s$ and $\rho_0 t_s$, the following more simplified approximation formulae are obtained from the formulae for the decision threshold and the detection limit given on the left:</p> $R_n^* = k_{1-\alpha} \sqrt{R_0 \left(\frac{1}{t_0} + \frac{1}{t_s} \right)} \quad (15a)$ $\rho_n^* = (k_{1-\alpha} + k_{1-\beta}) \sqrt{\rho_0 \left(\frac{1}{t_0} + \frac{1}{t_s} \right)} \quad (16a)$
Pulse preselection	
<p>Decision threshold</p> $\left(\frac{R_s}{R_0} \right)^* = F(N_s, N_0, 1 - \alpha) \quad (18)$ <p>Detection limit</p> $\left(\frac{\rho_s}{\rho_0} \right)^* = F(N_0, N_s, 1 - \beta) \cdot F(N_s, N_0, 1 - \alpha) \quad (19)$ <p>Confidence interval</p> $\frac{1}{F(N_s, N_0, 1 - \gamma/2)} \cdot \frac{N_s}{N_0} \cdot \frac{t_0}{t_s} \leq \frac{\rho_s}{\rho_0} \leq F(N_0, N_s, 1 - \gamma/2) \cdot \frac{N_s}{N_0} \cdot \frac{t_0}{t_s} \quad (20)$	<p>The formulae are exact for all N_0 and N_s. With adequately large values of N_0 and N_s, the following approximations can be used:</p> <p>Decision threshold</p> $\left(\frac{R_s}{R_0} \right)^* = 1 + k_{1-\alpha} \sqrt{\frac{1}{N_0} + \frac{1}{N_s}} \quad (18a)$ <p>Detection limit</p> $\left(\frac{\rho_s}{\rho_0} \right)^* = 1 + (k_{1-\alpha} + k_{1-\beta}) \sqrt{\frac{1}{N_0} + \frac{1}{N_s}} \quad (19a)$ <p>Confidence interval</p> $\frac{N_s}{N_0} \left(1 - k_{1-\gamma/2} \sqrt{\frac{1}{N_0} + \frac{1}{N_s}} \right) \frac{t_0}{t_s} \leq \frac{\rho_s}{\rho_0} \leq \frac{N_s}{N_0} \left(1 + k_{1-\gamma/2} \sqrt{\frac{1}{N_0} + \frac{1}{N_s}} \right) \frac{t_0}{t_s} \quad (20a)$ <p>alternatively</p> $\frac{N_s}{t_s} - \frac{N_0}{t_0} - k_{1-\gamma/2} \cdot \sqrt{\frac{N_0}{t_0^2} + \frac{N_s}{t_s^2}} \leq \rho_n \leq \frac{N_s}{t_s} - \frac{N_0}{t_0} + k_{1-\gamma/2} \cdot \sqrt{\frac{N_0}{t_0^2} + \frac{N_s}{t_s^2}} \quad (20b)$

Table 2 — Values $k_{1-\alpha}$, $k_{1-\beta}$, $k_{1-(\gamma/2)}$ as a function of the error probabilities α and β and of the confidence level $1-\gamma$ (quantiles of normal distribution)

Error of probability α or β	$k_{1-\alpha}$, $k_{1-\beta}$	Confidence level $1-\gamma$	$k_{1-(\gamma/2)}$
0,158 5	1,000	0,682	1,000
0,100 0	1,282	0,800	1,282
0,050 0	1,645	0,900	1,645
0,025 0	1,960	0,950	1,960
0,022 8	2,000	0,955	2,000
0,010 0	2,326	0,980	2,326
0,005 0	2,576	0,990	2,576
0,001 4	3,000	0,997	3,000
0,001 0	3,090	0,998	3,090

Table 3 — Values for $F(N_s, N_0, 1-\alpha)^a$ and $F(N_0, N_s, 1-\beta)^b$ as a function of N_0 and $q = N_s/N_0$ for $\alpha = \beta = 0,05$

$N_0, N_s q$	$F(N_s, N_0, 1-\alpha)$					$F(N_0, N_s, 1-\beta)$				
	1,000	0,500 0	0,250 0	0,200 0	0,100 0	1,000 0	0,500 0	0,250 0	0,200 0	0,100 0
1	19,000					19,000				
2	6,388 2	19,247				6,388 2	6,944 3			
3	4,283 9					4,283 9				
4	3,438 1	6,041 0	19,371			3,438 1	3,837 9	4,459 0		
5	2,978 2			19,396		2,978 2			4,102 8	
6	2,686 6	3,999 9				2,686 6	2,996 1			
7	2,483 7					2,483 7				
8	2,333 5	3,201 6	5,844 1			2,333 5	2,591 1	3,006 9		
9	2,217 2					2,217 2				
10	2,124 2	2,774 0		5,802 5	19,446	2,124 2	2,347 9		2,866 1	3,492 8
25	1,599 6					1,599 6				
50	1,391 8	1,525 2				1,391 8	1,477 3			
100	1,262 6	1,341 8	1,484 0	1,551 4		1,262 6	1,320 7	1,414 8	1,455 5	
250	1,158 7	1,201 6		1,308 1	1,457 6	1,158 7	1,194 0		1,275 4	1,376 6
500	1,109 7	1,137 8	1,184 7	1,305 5	1,296 0	1,109 7	1,134 2	1,173 6	1,190 4	1,259 7
1 000	1,076 4	1,095 2	1,126 1	1,139 6	1,197 2	1,076 4	1,093 5	1,120 8	1,132 5	1,180 3
2 500	1,047 6	1,059 0	1,077 4	1,085 3	1,118 6	1,047 6	1,058 3	1,075 4	1,082 6	1,112 2
5 000	1,033 4	1,041 3	1,053 9	1,059 3	1,081 8	1,033 4	1,041 0	1,052 9	1,058 0	1,078 7
10 000	1,023 5	1,029 0	1,037 7	1,041 4	1,056 8	1,023 5	1,028 8	1,037 2	1,040 8	1,055 3



$\alpha = 5\%$ of the area.

a $(1-\alpha)$ quantile of F -distribution with $2N_0$ numerator degrees of freedom and $2N_s$ denominator degrees of freedom.

b $(1-\beta)$ quantile of F -distribution with $2N_s$ numerator degrees of freedom and $2N_0$ denominator degrees of freedom.

Table 4 — Values for $F(N_s, N_0, 1-\gamma/2)^a$ and $F(N_0, N_s, 1-\gamma/2)^b$ as a function of N_0 and $q = N_s/N_0$ for $1-\gamma = 0,95$

$N_0, N_s q$	$F(N_s, N_0, 1-\gamma/2)$					$F(N_0, N_s, 1-\gamma/2)$				
	1,000 0	0,500 0	0,250 0	0,200 0	0,100 0	1,000 0	0,500 0	0,250 0	0,200 0	0,100 0
1	39,000					39,000				
2	9,604 5	39,248				9,604 5	10,649			
3	5,819 8					5,819 8				
4	4,433 3	8,979 6	39,373			4,433 3	5,052 6	6,059 5		
5	3,716 8			39,389		3,716 8			5,456 4	
6	3,277 3	5,366 2				3,277 3	3,728 3			
7	2,978 6					2,978 6				
8	2,761 4	4,076 1	8,632 6			2,761 4	3,124 8	3,729 4		
9	2,595 6					2,595 6				
10	2,464 5	3,418 5		8,559 9	39,448	2,464 5	2,773 7		3,514 7	4,461 3
25	1,752 1					1,752 1				
50	1,483 3	1,656 2				1,483 3	1,591 7			
100	1,320 5	1,420 4	1,603 4	1,691 5		1,320 5	1,392 7	1,511 0	1,562 4	
250	1,191 9	1,244 9		1,378 2	1,569 7	1,191 9	1,235 2		1,335 6	1,461 9
500	1,132 1	1,166 4	1,224 1	1,249 9	1,363 2	1,132 1	1,161 8	1,209 9	1,230 5	1,315 9
1 000	1,091 6	1,114 5	1,152 2	1,168 7	1,239 7	1,091 6	1,112 3	1,145 4	1,159 6	1,218 1
2 500	1,057 0	1,070 7	1,092 9	1,102 5	1,143 0	1,057 0	1,069 9	1,090 4	1,099 1	1,135 0
5 000	1,040 0	1,049 4	1,064 6	1,071 1	1,098 3	1,040 0	1,049 0	1,063 3	1,069 4	1,094 4
10 000	1,028 1	1,034 6	1,045 1	1,049 6	1,068 1	1,028 1	1,034 4	1,044 5	1,048 8	1,066 2

a $(1-\gamma/2)$ quantile of F -distribution with $2N_0$ numerator degrees of freedom and $2N_s$ denominator degrees of freedom.
 b $(1-\gamma/2)$ quantile of F -distribution with $2N_s$ numerator degrees of freedom and $2N_0$ denominator degrees of freedom.

NOTE For Tables 3 and 4, see [4] for other values of α, β, γ . However, if $N_0, N_s \geq 15$, the quantiles of F -distribution can be calculated using the following formula:

$$F(r, s, v) = e^{2z} \tag{22}$$

with

$$z = \frac{k_v \sqrt{h + \lambda_v}}{h} + \left(\frac{1}{2r-1} - \frac{1}{2s-1} \right) \cdot \left[\frac{5}{6} + \lambda_v - \frac{1}{3} \left(\frac{1}{2r-1} + \frac{1}{2s-1} \right) \right] \tag{23}$$

$$h = \frac{2}{\frac{1}{2r-1} + \frac{1}{2s-1}} \tag{24}$$

$$\lambda_v = \frac{1}{6} [k_v^2 - 3] \tag{25}$$

The values for k_v shall be taken from Table 2.

The following assignment applies:

Quantile	r	s	v
$F(N_s, N_0, 1-\alpha)$	N_s	N_0	$1-\alpha$
$F(N_0, N_s, 1-\beta)$	N_0	N_s	$1-\beta$
$F(N_s, N_0, 1-\gamma/2)$	N_s	N_0	$1-\gamma/2$
$F(N_0, N_s, 1-\gamma/2)$	N_0	N_s	$1-\gamma/2$

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