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**Colorimetry —**

Part 4:

**CIE 1976 L\*a\*b\* Colour space**

*Colorimétrie —*

*Partie 4: Espace chromatique L\*a\*b\* CIE 1976*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

ISO 11664-4 was prepared as Standard CIE S 014-4/E by the International Commission on Illumination, which has been recognized by the ISO Council as an international standardizing body. It was adopted by ISO under a special procedure which requires approval by at least 75 % of the member bodies casting a vote, and is published as a joint ISO/CIE edition.

The International Commission on Illumination (abbreviated as CIE from its French title) is an organization devoted to international cooperation and exchange of information among its member countries on all matters relating to the science and art of lighting.

ISO 11664-4 was prepared by CIE Technical Committee 1-57 of Division 1, *Vision and colour*.

ISO 11664 consists of the following parts, under the general title *Colorimetry*:

- *Part 1: CIE standard colorimetric observers*
- *Part 2: CIE standard illuminants*
- *Part 4: CIE 1976 L\*a\*b\* Colour space*

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Standard

COMMISSION INTERNATIONALE DE L'ÉCLAIRAGE  
INTERNATIONAL COMMISSION ON ILLUMINATION  
INTERNATIONALE BELEUCHTUNGSKOMMISSION

CIE S 014-4/E:2007

# Colorimetry - Part 4: CIE 1976 L\*a\*b\* Colour Space

Colorimétrie - Partie 4: Espace chromatique L\*a\*b\* CIE 1976

Farbmessung - Teil 4: CIE 1976 L\*a\*b\* Farbenraum

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## FOREWORD

Standards produced by the Commission Internationale de l'Eclairage (CIE) are a concise documentation of data defining aspects of light and lighting, for which international harmony requires such unique definition. CIE Standards are therefore a primary source of internationally accepted and agreed data, which can be taken, essentially unaltered, into universal standard systems.

This CIE Standard has been prepared by the Technical Committee TC 1-57\* of Division 1 "Vision and Colour" and was approved by the National Committees of the CIE.

The following ISO and IEC committees and working groups co-operated in the preparation of this standard:

IEC TC100/TA2 (Audio, Video and Multimedia Systems)

ISO TC6 (Paper, Board and Pulps)

ISO TC35/SC9/WG22 (Paint and Varnishes)

ISO TC38/SC1/WG7 (Textiles)

ISO TC42 (Photography)

ISO TC130 (Graphic Technology)

ISO/IEC/JTC1/SC28 (Office Systems)

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## COLORIMETRY - PART 4: CIE 1976 L\*a\*b\* COLOUR SPACE

### INTRODUCTION

The three-dimensional colour space produced by plotting CIE tristimulus values (X,Y,Z) in rectangular coordinates is not visually uniform, nor is the (x,y,Y) space nor the two-dimensional CIE (x,y) chromaticity diagram. Equal distances in these spaces do not represent equally perceptible differences between colour stimuli. For this reason, in 1976, the CIE introduced and recommended two new spaces (known as CIELAB and CIELUV) whose coordinates are non-linear functions of X, Y and Z. The recommendation was put forward in an attempt to unify the then very diverse practice in uniform colour spaces and associated colour difference formulae (Robertson, 1990; CIE, 2004). Both these more-nearly uniform colour spaces have become well accepted and widely used. Numerical values representing approximately the magnitude of colour differences can be described by simple Euclidean distances in the spaces or by more sophisticated formulae that improve the correlation with the perceived size of differences. The purpose of this CIE Standard is to define procedures for calculating the coordinates of the CIE 1976 L\*a\*b\* (CIELAB) colour space and the Euclidean colour difference values based on these coordinates. The standard does not cover more sophisticated colour difference formulae based on CIELAB, such as the CMC formula (Clarke et al., 1984), the CIE94 formula (CIE, 1995), the DIN99 formula (DIN, 2001), and the CIEDE2000 formula (CIE, 2001), nor does it cover the alternative uniform colour space, CIELUV.

### 1. SCOPE

This CIE Standard specifies the method of calculating the coordinates of the CIE 1976 L\*a\*b\* colour space including correlates of lightness, chroma and hue. It includes two methods for calculating Euclidean distances in this space to represent the perceived magnitude of colour differences.

The Standard is applicable to tristimulus values calculated using colour-matching functions of the CIE 1931 standard colorimetric system or the CIE 1964 standard colorimetric system. The Standard may be used for the specification of colour stimuli perceived as belonging to a reflecting or transmitting object, where a three-dimensional space more uniform than tristimulus space is required. It does not apply to colour stimuli perceived as belonging to an area that appears to be emitting light as a primary light source, or that appears to be specularly reflecting such light. This Standard does apply to self-luminous displays, like cathode ray tubes, if they are being used to simulate reflecting or transmitting objects and if the stimuli are appropriately normalized.

### 2. NORMATIVE REFERENCES

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

CIE 17.4-1987. *International Lighting Vocabulary* (Joint publication IEC/CIE).

CIE S 014-1:2006. *Colorimetry Part 1. CIE Standard Colorimetric Observers*. [ISO 11664-1:2007]

CIE S 014-2:2006. *Colorimetry Part 2. CIE Standard Illuminants*. [ISO 11664-2:2007]

### 3. DEFINITIONS, SYMBOLS AND ABBREVIATIONS

For the purposes of this International Standard, the terms and definitions given in CIE 17.4-1987 (International Lighting Vocabulary), as amended by this standard and the following symbols and abbreviations apply.

X, Y, Z	tristimulus values of test stimulus calculated using the colour-matching functions of the CIE 1931 standard colorimetric system (also known as the CIE 2° standard colorimetric system)
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$X_n, Y_n, Z_n$	tristimulus values of a specific white colour stimulus calculated using the colour-matching functions of the CIE 1931 standard colorimetric system
$L^*$	CIELAB lightness
$a^*, b^*$	CIELAB $a^*, b^*$ coordinates
$C^*_{ab}$	CIELAB chroma
$h_{ab}$	CIELAB hue angle
$\Delta L^*$	CIELAB lightness difference
$\Delta a^*, \Delta b^*$	CIELAB $a^*, b^*$ difference
$\Delta C^*_{ab}$	CIELAB chroma difference
$\Delta h_{ab}$	CIELAB hue angle difference
$\Delta H^*_{ab}$	CIELAB hue difference
$\Delta E^*_{ab}$	CIELAB colour difference

If the character " $\Delta$ " is not available, it may be replaced by the character "D".

The phrase "CIE 1976  $L^*a^*b^*$ " and the term "CIELAB" may be used interchangeably.

Where tristimulus values are calculated using the colour-matching functions of the CIE 1964 standard colorimetric system (also known as the CIE 10° standard colorimetric system), a subscript 10 shall be added to all the above symbols.

## 4. CALCULATION METHOD

### 4.1 Basic coordinates

The CIE 1976  $L^*a^*b^*$  colour space is a three-dimensional, approximately uniform colour space produced by plotting in rectangular coordinates,  $L^*$ ,  $a^*$ ,  $b^*$ , quantities defined by the equations:

$$L^* = 116f(Y/Y_n) - 16 \quad (1)$$

$$a^* = 500 [f(X/X_n) - f(Y/Y_n)] \quad (2)$$

$$b^* = 200 [f(Y/Y_n) - f(Z/Z_n)] \quad (3)$$

where

$$f(X/X_n) = (X/X_n)^{1/3} \quad \text{if } (X/X_n) > (6/29)^3 \quad (4)$$

$$f(X/X_n) = (841/108)(X/X_n) + 4/29 \quad \text{if } (X/X_n) \leq (6/29)^3 \quad (5)$$

and

$$f(Y/Y_n) = (Y/Y_n)^{1/3} \quad \text{if } (Y/Y_n) > (6/29)^3 \quad (6)$$

$$f(Y/Y_n) = (841/108)(Y/Y_n) + 4/29 \quad \text{if } (Y/Y_n) \leq (6/29)^3 \quad (7)$$

and

$$f(Z/Z_n) = (Z/Z_n)^{1/3} \quad \text{if } (Z/Z_n) > (6/29)^3 \quad (8)$$

$$f(Z/Z_n) = (841/108)(Z/Z_n) + 4/29 \quad \text{if } (Z/Z_n) \leq (6/29)^3 \quad (9)$$

where  $X, Y, Z$  are the tristimulus values of the test colour stimulus based on the CIE 1931 standard colorimetric system defined in CIE S 014-1, and  $X_n, Y_n, Z_n$  are the corresponding tristimulus values of a specified white stimulus.

In the case of simulated reflecting or transmitting objects produced on a self-luminous display, all the tristimulus values shall be first normalized by the same factor so that Y would be equal to 100 for an object with 100% reflectance or transmittance.

If the angle subtended at the eye by the test stimulus is between about 1° and 4° the tristimulus values X, Y, Z calculated using the colour-matching functions of the CIE 1931 standard colorimetric system should be used. If this angular subtense is greater than 4° the tristimulus values X<sub>10</sub>, Y<sub>10</sub>, Z<sub>10</sub> calculated using the colour-matching functions of the CIE 1964 standard colorimetric system should be used. The same colour-matching functions and the same specified white stimulus shall be used for all stimuli to be compared with each other.

When tristimulus values based on the CIE 1964 standard colorimetric system defined in CIE S 014-1 are used, a subscript 10 shall be added to all the symbols in equations (1) to (9).

If the tristimulus values X, Y, Z are obtained by spectrophotometry, the tristimulus values X<sub>n</sub>, Y<sub>n</sub>, Z<sub>n</sub> of the specified white stimulus shall be calculated using the same method as used for the test stimulus (same colour-matching functions, same range and interval of wavelength, and same bandwidth). If the tristimulus values X, Y, Z are obtained by direct measurement using a tristimulus colorimeter, X<sub>n</sub>, Y<sub>n</sub>, Z<sub>n</sub> shall be measured using the same tristimulus colorimeter and a white reflectance standard calibrated relative to a perfect reflecting diffuser.

NOTE 1 For real object colours, the specified white stimulus normally chosen for X<sub>n</sub>, Y<sub>n</sub>, Z<sub>n</sub> is light reflected from a perfect reflecting diffuser illuminated by the same light source as the test object. In this case, X<sub>n</sub>, Y<sub>n</sub>, Z<sub>n</sub> are the tristimulus values of the light source normalized by a common factor so that Y<sub>n</sub> is equal to 100. For simulated object colours, the specified white stimulus normally chosen is one that has the appearance of a perfect reflecting diffuser, again normalized by a common factor so that Y<sub>n</sub> is equal to 100.

NOTE 2 Examples of values of X<sub>n</sub>, Y<sub>n</sub> and Z<sub>n</sub> for specific illuminants and specific calculation methods have been published (CIE, 2004).

NOTE 3 Equations (5), (7) and (9) are based on a suggestion by Pauli (1976).

NOTE 4 A value of 7,787 is approximately equal to the term (841/108) in equations (5), (7) and (9). The approximate value may be used in practice.

NOTE 5 A value of 0,008856 is approximately equal to the term (6/29)<sup>3</sup> in equations (4), (5), (6), (7), (8) and (9). The approximate value may be used in practice.

NOTE 6 The fractions 6/29 and 4/29 in equations (4) through (9) are exactly equal to the fractions 24/116 and 16/116 appearing in CIE 15:2004.

NOTE 7 The term (841/108) in equations (5), (7) and (9) is derived from and exactly equal to (1/3)(29/6)<sup>2</sup>.

NOTE 8 Equation (1) reduces to  $L^* \approx 903,3(Y/Y_n)$  when  $Y/Y_n \leq (6/29)^3$ .

When CIELAB values are reported, they should be accompanied by all relevant information relating to the measurement conditions and the procedures used to calculate the input tristimulus values.

#### 4.2 Correlates of lightness, chroma and hue

Approximate correlates of the perceived attributes lightness, chroma, and hue shall be calculated as follows:

CIE 1976 lightness:  $L^*$  as defined in section 4.1

CIE 1976 a,b chroma (CIELAB chroma):  $C^*_{ab} = \left[ (a^*)^2 + (b^*)^2 \right]^{1/2}$  (10)

CIE 1976 a,b hue angle (CIELAB hue angle):  $h_{ab} = \arctan(b^*/a^*)$  (11)

CIELAB hue angle,  $h_{ab}$  shall lie between  $0^\circ$  and  $90^\circ$  if  $a^*$  and  $b^*$  are both positive, between  $90^\circ$  and  $180^\circ$  if  $b^*$  is positive and  $a^*$  is negative, between  $180^\circ$  and  $270^\circ$  if  $b^*$  and  $a^*$  are both negative, and between  $270^\circ$  and  $360^\circ$  if  $b^*$  is negative and  $a^*$  is positive.

NOTE When the linear equations (5), (7) or (9) are used for  $X/X_n$ ,  $Y/Y_n$  or  $Z/Z_n$ , anomalous values of  $h_{ab}$  may be obtained (McLaren, 1980). Anomalous values are unlikely to occur for reflecting object colours but may occur for transparent object colours of low luminance factor lying close to the spectrum locus or purple line.

### 4.3 Colour differences

Euclidean distances in CIELAB colour space can be used to represent approximately the perceived magnitude of colour differences between object colour stimuli of approximately the same size, viewed in identical white to middle-grey surroundings, by an observer photopically adapted to a field with the chromaticity of CIE standard illuminant D65 defined in CIE S 014-2. The values given by this Standard may not correlate well with perceived colour differences in other viewing conditions.

Differences between two stimuli denoted by subscripts 0 (usually the reference) and 1 (usually the test) shall be calculated as follows:

$$\Delta L^* = L^*_1 - L^*_0 \quad (12)$$

$$\Delta a^* = a^*_1 - a^*_0 \quad (13)$$

$$\Delta b^* = b^*_1 - b^*_0 \quad (14)$$

$$\Delta C^*_{ab} = C^*_{ab,1} - C^*_{ab,0} \quad (15)$$

$$\Delta h_{ab} = h_{ab,1} - h_{ab,0} \quad (16)$$

$$\Delta H^*_{ab} = 2 \left( C^*_{ab,1} \cdot C^*_{ab,0} \right)^{1/2} \sin(\Delta h_{ab} / 2) \quad (17)$$

For small colour differences away from the achromatic axis  $C^*_{ab}=0$ , equation (17) reduces to

$$\Delta H^*_{ab} \approx \left( C^*_{ab,1} \cdot C^*_{ab,0} \right)^{1/2} \Delta h_{ab} \quad (18)$$

where the value of  $\Delta h_{ab}$  is in radians.

If the line joining the two colours crosses the positive  $a^*$  axis, equation (16) will give a value outside the range  $\pm 180^\circ$ . In this case, the value of  $\Delta h_{ab}$  must be corrected by adding or subtracting  $360^\circ$  to bring it within this range.

NOTE 1 The quantity  $\Delta H^*_{ab}$  is introduced to provide congruence with the perceptual understanding that a colour difference can be divided into a vector sum of a lightness difference, a chroma difference and a hue difference.

NOTE 2 The division of CIELAB colour differences into hue and chroma differences is progressively less useful as the absolute value of  $\Delta h_{ab}$  approaches  $180^\circ$ .

NOTE 3 In information technology and other fields the subscripts r (for reference) and t (for test) are sometimes used instead of 0 and 1, respectively. Similarly in industrial evaluation of small colour differences s (for standard) and b (for batch) are sometimes used. In other applications, std (for standard) and spl (for sample) are sometimes used.

The CIE 1976  $a, b$  colour difference,  $\Delta E^*_{ab}$  between two colour stimuli is calculated as the Euclidean distance between the points representing them in the space:

$$\Delta E^*_{ab} = \left[ (\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2 \right]^{1/2} \quad (19)$$

$$\text{or } \Delta E^*_{ab} = \left[ (\Delta L^*)^2 + (\Delta C^*_{ab})^2 + (\Delta H^*_{ab})^2 \right]^{1/2} \quad (20)$$

These two definitions of  $\Delta E^*_{ab}$  are equivalent.

Other ways of calculating  $\Delta H^*_{ab}$  are:

$$\Delta H^*_{ab} = \left[ (\Delta E^*_{ab})^2 - (\Delta L^*)^2 - (\Delta C^*_{ab})^2 \right]^{1/2} \quad (21)$$

where  $\Delta E^*_{ab}$  is calculated from equation (19) and  $\Delta H^*_{ab}$  has the same sign as  $\Delta h_{ab}$ ;

$$\Delta H^*_{ab} = k \left[ 2(C^*_{ab,1} \cdot C^*_{ab,0} - a^*_1 \cdot a^*_0 - b^*_1 \cdot b^*_0) \right]^{1/2} \quad (22)$$

where  $k=-1$  if  $a^*_1 \cdot b^*_0 \geq a^*_0 \cdot b^*_1$ , otherwise  $k=1$ ;

$$\text{and } \Delta H^*_{ab} = (a^*_0 \cdot b^*_1 - a^*_1 \cdot b^*_0) / \left[ 0,5(C^*_{ab,1} \cdot C^*_{ab,0} + a^*_1 \cdot a^*_0 + b^*_1 \cdot b^*_0) \right]^{1/2} \quad (23)$$

NOTE 4 Equation (23) cannot be used when either of the compared chromas is zero and is imprecise when either chroma is close to zero.

NOTE 5 More details on the various methods of calculating  $\Delta H^*_{ab}$  are given by Sève (1991) for Eq. 17, by Stokes and Brill (1992) for Eq. 22 and by Sève (1996) for Eq. 23.

NOTE 6 In different practical applications it may be necessary to modify equation (20) by using different weightings for  $\Delta L^*$ ,  $\Delta C^*_{ab}$  and  $\Delta H^*_{ab}$  to obtain better correlation with colour-difference perception. In 2001, the CIE recommended such weightings in a new formula for industrial evaluation of small colour differences (CIE, 2001), but this is outside the scope of this Standard. A colour difference obtained in this way is not a CIELAB colour difference and the symbol  $\Delta E^*_{ab}$  should not be used.

**ANNEX (INFORMATIVE): REVERSE TRANSFORMATION**

The following equations represent the reverse transformation, i.e. the calculation of X, Y, Z when L\*, a\*, b\* are given.

$$f(Y/Y_n) = (L^* + 16) / 116 \tag{A1}$$

$$f(X/X_n) = a^* / 500 + f(Y/Y_n) \tag{A2}$$

$$f(Z/Z_n) = f(Y/Y_n) - b^* / 200 \tag{A3}$$

$$X = X_n [f(X/X_n)]^3 \quad \text{if } f(X/X_n) > 6/29 \tag{A4}$$

$$X = (108/841) X_n [f(X/X_n) - 4/29] \quad \text{if } f(X/X_n) \leq 6/29 \tag{A5}$$

$$Y = Y_n [f(Y/Y_n)]^3 \quad \text{if } f(Y/Y_n) > 6/29 \tag{A6}$$

$$Y = (108/841) Y_n [f(Y/Y_n) - 4/29] \quad \text{if } f(Y/Y_n) \leq 6/29 \tag{A7}$$

$$Z = Z_n [f(Z/Z_n)]^3 \quad \text{if } f(Z/Z_n) > 6/29 \tag{A8}$$

$$Z = (108/841) Z_n [f(Z/Z_n) - 4/29] \quad \text{if } f(Z/Z_n) \leq 6/29 \tag{A9}$$

NOTE 1 The condition in equation (A6) is equivalent to L\* > 8.

NOTE 2 The condition in equation (A7) is equivalent to L\* ≤ 8.

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