INTERNATIONAL STANDARD

ISO 1151-3

Second edition
1989-04-01

Flight dynamics — Concepts, quantities and symbols —

Part 3 :
Derivatives of forces, moments and their coefficients

Mécanique du vol — Concepts, grandeurs et symboles —

Partie 3 : Dérivées des forces, des moments et de leurs coefficients

Reference number
ISO 1151-3 : 1989 (E)

Contents

International Organization for Standardization
Case postale 56 • CH-1211 Genève 20 • Switzerland
Printed in Switzerland

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council. They are approved in accordance with ISO procedures requiring at least 75 % approval by the member bodies voting.

International Standard ISO 1151-3 was prepared by Technical Committee ISO/TC 20, *Aircraft and space vehicles.*

This second edition cancels and replaces the first edition (ISO 1151-3 : 1972), of which it constitutes a technical revision.

Users should note that all International Standards undergo revision from time to time and that any reference made herein to any other International Standard implies its latest edition, unless otherwise stated.

ISO 1151, *Flight dynamics — Concepts, quantities and symbols,* comprises, at present, seven parts:

Part 1: Aircraft motion relative to the air.

Part 2: Motions of the aircraft and the atmosphere relative to the Earth.

Part 3: Derivatives of forces, moments and their coefficients.

Part 4: Parameters used in the study of aircraft stability and control.

Part 5: Quantities used in measurements.

Part 6: Aircraft geometry.

Part 7: Flight points and flight envelopes.

ISO 1151 is intended to introduce the main concepts, to include the more important terms used in theoretical and experimental studies and, as far as possible, to give corresponding symbols.

In all the parts comprising ISO 1151, the term "aircraft" denotes a vehicle intended for atmosphere or space flight. Usually, it has an essentially port and starboard symmetry with respect to a plane. That plane is determined by the geometric characteristics of the aircraft. In that plane, two orthogonal directions are defined: fore-and-aft and dorsal-ventral. The transverse direction, on the perpendicular to that plane, follows.

When there is a single plane of symmetry, it is the reference plane of the aircraft. When there is more than one plane of symmetry, or when there is none, it is necessary to choose a reference plane. In the former case, the reference plane is one of the planes of symmetry. In the latter case, the reference plane is arbitrary. In all cases, it is necessary to specify the choice made.

Angles of rotation, angular velocities and moments about any axis are positive clockwise when viewed in the positive direction of that axis.

All the axis systems are three-dimensional, orthogonal and right-handed, which implies that a positive rotation through $\pi/2$ around the x-axis brings the y-axis into the position previously occupied by the z-axis.

The centre of gravity coincides with the centre of mass if the field of gravity is homogeneous. If this is not the case, the centre of gravity can be replaced by the centre of mass in the definitions of ISO 1151; in this case, this should be indicated.

Numbering of sections and clauses

With the aim of easing the indication of references from a section or a clause, a decimal numbering system has been adopted such that the first figure is the number of the part of ISO 1151 considered.

Flight dynamics — Concepts, quantities and symbols —

Part 3:
Derivatives of forces, moments and their coefficients

3.0 Introduction

This part of ISO 1151 deals with derivatives of forces, moments and of other quantities characterizing such forces and moments.

The term "derivative" designates the partial derivative of a function with respect to an independent variable.

These derivatives appear in the terms of the Taylor series representing the variations of functions with the independent variables. This part of ISO 1151 is restricted to first-order terms. Terms of higher order would require additional definitions for derivatives of higher order.

The aircraft is assumed to be rigid. However, most of the definitions can be applied to the case of flexible aircraft. Aerolastic effects would require the introduction of further quantities.

3.1 Functions and independent variables

A set of derivatives is characterized by the set of the functions and the set of the independent variables, with respect to which differentiation takes place.

3.1.1 Functions and classes of derivatives

Different classes of derivatives are used in flight dynamics studies.

This part of ISO 1151 includes the following classes of derivatives:

Clause	Class	Distinguishing mark
3.2	Direct derivatives	
3.3	Specific derivatives	~
3.4	Normalized derivatives	^
3.5	Coefficient derivatives	

The distinguishing marks may be omitted if no confusion is likely.

In each class, the specific term for a particular derivative shall refer to the function and to the independent variable.

The functions used in a given problem refer to only one axis system.

In the chosen axis system, the components are numbered as follows:

1 Component with respect to the x-axis

2 Component with respect to the y-axis

3 Component with respect to the z-axis

3.1.2 Independent variables

The independent variables considered are

- variables representing the aircraft motion relative to the air (1.2 and 1.3);
- variables representing the motivator deflections (1.8.3).

NOTE — It may be necessary to introduce additional types of independent variables, for example parameters relating to the aircraft propulsive system.

It is necessary to specify the set of independent variables used. The value of the derivative of a given function with respect to a given independent variable depends, generally, in fact, on the choice of the other independent variables.

If different sets of independent variables are used simultaneously, each set of derivatives corresponding to a given set of independent variables shall be characterized by an appropriate distinguishing mark.

3.2 Direct derivatives

A direct derivative is the partial derivative of a component of a force or a moment with respect to a variable included in a given set of independent variables.

A direct derivative has the dimension of the ratio of the function to the independent variable.

The symbol for a direct derivative is the symbol of the function to which the symbol of the independent variable is added as a subscript[1].

EXAMPLE

$$\frac{\partial X}{\partial u} = X_u$$

The symbols of direct derivatives do not contain a distinguishing mark.

The direct derivatives of the components of the resultant force $\vec{R}$ (1.5.2) and of the components of the resultant moment $\vec{Q}$ (1.5.5) are the elements of matrix $\boldsymbol{R}$ (3.2.1) and matrix $\boldsymbol{Q}$ (3.2.2).

The symbols of the matrixes shall, preferably, be printed in bold type.

No.	Term	Definition	Symbol
3.2.1	(Direct) resultant force derivative matrix	The matrix consisting of the direct derivatives of the components of the resultant force (1.5.2). The rows of the matrix are ordered according to the convention given in 3.1.1. The ith row contains the derivatives of the ith function. The jth element in a row of the matrix is the direct derivative, with respect to the jth variable in the set of independent variables (3.1.2). The matrix has the following structure: $\boldsymbol{R} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \dots R_{1n} \\ R_{21} & R_{22} & R_{23} \dots R_{2n} \\ R_{31} & R_{32} & R_{33} \dots R_{3n} \end{pmatrix}$ with, for example $R_1 = X$ $R_2 = Y$ $R_3 = Z$ NOTE — An analogous matrix $\boldsymbol{R}^{A}$ can be defined with regard to the components of the airframe aerodynamic force (1.6.2.2).	$\boldsymbol{R}$

1) The independant variable is sometimes indicated in the symbol by a superscript, for example

$$\frac{\partial X}{\partial u} = X^u$$

No.	Term	Definition	Symbol
3.2.2	(Direct) resultant moment derivative matrix	The matrix consisting of the direct derivatives of the components of the resultant moment (1.5.5). The rows of the matrix are ordered according to the convention given in 3.1.1. The *i*th row contains the derivatives of the *i*th function. The *j*th element in a row of the matrix is the direct derivative, with respect to the *j*th variable in the set of independent variables (3.1.2). The matrix has the following structure: $\boldsymbol{Q} = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \cdots Q_{1n} \\ Q_{21} & Q_{22} & Q_{23} \cdots Q_{2n} \\ Q_{31} & Q_{32} & Q_{33} \cdots Q_{3n} \end{pmatrix}$ with, for example $Q_1 = L$ $Q_2 = M$ $Q_3 = N$ NOTE — An analogous matrix $\boldsymbol{Q}^{\mathrm{A}}$ may be defined with regard to the components of the airframe aerodynamic moment (1.6.2.10).	$\boldsymbol{Q}$

3.3 Specific derivatives

A specific derivative is the derivative of a component of the specific resultant (1.5.10) or of the specific resultant moment (1.5.12) with respect to a variable contained in a given set of independent variables.

The inertial characteristics of the aircraft,

— mass (1.4.1), and

— moments of inertia (1.4.2) and products of inertia (1.4.3) with respect to the body axis system,

are assumed to be constant.

If the inertial characteristics of the aircraft cannot be assumed to be constant, the parameters required for their definition shall be included in the set of independent variables.

A specific derivative has the dimension

— of the quotient of a linear acceleration by the independent variable, in the case of a specific force derivative, or

— of the quotient of an angular acceleration by the independent variable, in the case of a specific moment derivative.

The symbol of a specific derivative consists of

— the basic alphabetical symbol used for the corresponding resultant force component (1.5.2) or resultant moment component (1.5.5),

— the symbol of the independent variable as a subscript, and

— the distinguishing mark ~ above the basic alphabetical symbol.

Subclauses 3.3.1 and 3.3.2 give general definitions illustrated, for each type of specific derivative, by a particular example. Specific derivatives of other forces or other moments, or with respect to other independent variables, can be defined in an analogous manner.

3.3.1 Specific force derivatives

A specific force derivative is the product of the reciprocal of the aircraft mass (1.4.1) ($1/m$) by the corresponding direct force derivative (3.2). The matrix notation of the specific force derivative matrix, $\tilde{\boldsymbol{R}}$, is

$$\tilde{\boldsymbol{R}} = \frac{1}{m} \boldsymbol{R}$$

where

m is the aircraft mass (1.4.1);

$\boldsymbol{R}$ is the direct force derivative matrix (3.2.1).

The elements of matrix $\tilde{\boldsymbol{R}}$ are

$$\tilde{R}_{ij} = \frac{1}{m} R_{ij}$$

where

$\tilde{R}_{ij}$ is the derivative of the ith component of the specific resultant with respect to the jth independent variable;

R_{ij} is the direct derivative of the ith component of the resultant force with respect to the jth independent variable;

m is the aircraft mass (1.4.1).

No.	Term	Definition	Symbol
3.3.1.1	Specific force derivative with respect to an aircraft velocity component	The partial derivative of a component of the specific resultant (1.5.11) with respect to an aircraft velocity component (1.3.4). EXAMPLE $\tilde{Y}_w = \frac{1}{m} \frac{\partial Y}{\partial w}$	$\tilde{Y}_w$
3.3.1.2	Specific force derivative with respect to an angular velocity component	The partial derivative of a component of the specific resultant (1.5.11) with respect to an angular velocity component (1.3.6). EXAMPLE $\tilde{Y}_r = \frac{1}{m} \frac{\partial Y}{\partial r}$	$\tilde{Y}_r$
3.3.1.3	Specific force derivative with respect to a linear acceleration component	The partial derivative of a component of the specific resultant (1.5.11) with respect to the derivative of an aircraft velocity component (1.3.4) with respect to time. EXAMPLE $\tilde{Y}_{\dot{w}} = \frac{1}{m} \frac{\partial Y}{\partial \dot{w}}$ where $\dot{w} = \frac{\mathrm{d}w}{\mathrm{d}t}$	$\tilde{Y}_{\dot{w}}$
3.3.1.4	Specific force derivative with respect to a motivator deflection	The partial derivative of a component of the specific resultant (1.5.11) with respect to a motivator deflection (1.8.3.11 to 1.8.3.13). EXAMPLE $\tilde{Y}_{\delta \mathrm{n}} = \frac{1}{m} \frac{\partial Y}{\partial \delta_{\mathrm{n}}}$	$\tilde{Y}_{\delta \mathrm{n}}$

3.3.2 Specific moment derivatives

A specific moment derivative is the derivative of a component of the specific resultant moment (1.5.13) with respect to a variable contained in a given set of independent variables.

The matrix of the specific moment derivatives $\tilde{\boldsymbol{Q}}$ is the product of the inverse inertia matrix $\boldsymbol{J}$ (1.4.11) by the direct moment derivative matrix $\boldsymbol{Q}$ (3.2.2):

$$\tilde{\boldsymbol{Q}} = \boldsymbol{J}\,\boldsymbol{Q}$$

The elements of matrix $\tilde{\boldsymbol{Q}}$ are

$$\tilde{Q}_{ij} = \sum_{k=1}^{3} J_{ik}\,Q_{kj}$$

where

$\tilde{Q}_{ij}$ is the derivative of the ith component of the specific moment with respect to the jth independent variable;

J_{ik} is the kth element of the ith line in the inverse inertia matrix;

Q_{kj} is the derivative of the kth component of the resultant moment with respect to the jth independent variable.

No.	Term	Definition	Symbol
3.3.2.1	Specific moment derivative with respect to an aircraft velocity component	The partial derivative of a component of the specific resultant moment (1.5.13) with respect to an aircraft velocity component (1.3.4). EXAMPLE $\tilde{M}_w = \tilde{Q}_{2w} = J_{21}\frac{\partial L}{\partial w} + J_{22}\frac{\partial M}{\partial w} + J_{23}\frac{\partial N}{\partial w}$	$\tilde{M}_w$
3.3.2.2	Specific moment derivative with respect to an angular velocity component	The partial derivative of a component of the specific resultant moment (1.5.13) with respect to an angular velocity component (1.3.6). EXAMPLE $\tilde{M}_r = \tilde{Q}_{2r} = J_{21}\frac{\partial L}{\partial r} + J_{22}\frac{\partial M}{\partial r} + J_{23}\frac{\partial N}{\partial r}$	$\tilde{M}_r$
3.3.2.3	Specific moment derivative with respect to a linear acceleration component	The partial derivative of a component of the specific resultant moment (1.5.13) with respect to the derivative of an aircraft velocity component (1.3.4) with respect to time. EXAMPLE $\tilde{M}_{\dot{w}} = \tilde{Q}_{2\dot{w}} = J_{21}\frac{\partial L}{\partial \dot{w}} + J_{22}\frac{\partial M}{\partial \dot{w}} + J_{23}\frac{\partial N}{\partial \dot{w}}$ where $\dot{w} = \frac{\mathrm{d}w}{\mathrm{d}t}$	$\tilde{M}_{\dot{w}}$
3.3.2.4	Specific moment derivative with respect to a motivator deflection	The partial derivative of a component of the specific resultant moment (1.5.13) with respect to a motivator deflection (1.8.3.11 to 1.8.3.13). EXAMPLE $\tilde{M}_{\delta\mathrm{n}} = \tilde{Q}_{2\delta\mathrm{n}} = J_{11}\frac{\partial L}{\partial \delta_\mathrm{n}} + J_{12}\frac{\partial M}{\partial \delta_\mathrm{n}} + J_{13}\frac{\partial N}{\partial \delta_\mathrm{n}}$	$\tilde{M}_{\delta\mathrm{n}}$

3.4 Normalized derivatives

By convention, a normalized derivative is the quotient of a direct derivative divided by a normalization constant of the same dimension contained in a suitably chosen set. In this part of ISO 1151, a normalization set, based on aerodynamical quantities, is considered1).

The symbol for a normalized derivative consists of

- the basic alphabetical symbol used for the corresponding resultant force component (1.5.2) or resultant moment component (1.5.5),
- the symbol of the independent variable as a subscript, and
- the distinguishing mark ^ above the basic alphabetical symbol.

Normalization constants are quotients. Their numerators are a reference force or a reference moment. Their denominators have the same dimension as the independent variables of the direct derivatives to be normalized.

Reference quantities	Values
Force	$E_R = \frac{1}{2}\varrho_R V_R^2 S$
Moment	$Q_R = \frac{1}{2}\varrho_R V_R^2 S l$
Speed	V_R
Angular speed	V_R / l
Linear acceleration	V_R^2 / l

NOTE

S is the reference area (1.4.5);

l is the reference length (1.4.6);

ϱ_R is the reference value of the air density (5.1.3);

V_R is the reference value of the airspeed (1.3.1).

Subclauses 3.4.1 to 3.4.8 give general definitions illustrated, for each type of normalized derivative, by a particular example. Normalized derivatives of other forces or of other moments, or with respect to other independent variables, can be defined in an analogous manner.

No.	Term	Definition	Symbol
3.4.1	Normalized force derivative with respect to an aircraft velocity component	The direct derivative of a resultant force component (1.5.2) with respect to an aircraft velocity component (1.3.4), divided by E_R / V_R. EXAMPLE $\hat{Y}_w = \frac{\partial Y / \partial w}{E_R / V_R}$ $\hat{Y}_w = \frac{\partial Y / \partial w}{\frac{1}{2}\varrho_R V_R S}$	$\hat{Y}_w$
3.4.2	Normalized force derivative with respect to an angular velocity component	The direct derivative of a resultant force component (1.5.2) with respect to an angular velocity component (1.3.6), divided by $E_R / (V_R / l)$. EXAMPLE $\hat{Y}_r = \frac{\partial Y / \partial r}{E_R / (V_R / l)}$ $\hat{Y}_r = \frac{\partial Y / \partial r}{\frac{1}{2}\varrho_R V_R S l}$	$\hat{Y}_r$

1) It may be convenient to consider other normalization sets, based, for example, on dynamical quantities.

No.	Term	Definition	Symbol
3.4.3	Normalized force derivative with respect to a linear acceleration component	The direct derivative of a resultant force component (1.5.2) with respect to the derivative of an aircraft velocity component (1.3.4) with respect to time, divided by $E_R/(V_R^2/l)$. EXAMPLE $\hat{Y}_{\dot{w}} = \frac{\partial Y/\partial \dot{w}}{E_R/(V_R^2/l)}$ $\hat{Y}_{\dot{w}} = \frac{\partial Y/\partial \dot{w}}{\frac{1}{2}\varrho_R S l}$ where $\dot{w} = \frac{dw}{dt}$	$\hat{Y}_{\dot{w}}$
3.4.4	Normalized force derivative with respect to a motivator deflection	The direct derivative of a resultant force component (1.5.2) with respect to a normalized motivator deflection (1.8.3.11 to 1.8.3.13), divided by E_R. The non-angular motivator deflections shall be normalized. EXAMPLE $\hat{Y}_{\delta n} = \frac{\partial Y/\partial \delta_n}{E_R}$ $\hat{Y}_{\delta n} = \frac{\partial Y/\partial \delta_n}{\frac{1}{2}\varrho_R V_R^2 S}$	$\hat{Y}_{\delta n}$
3.4.5	Normalized moment derivative with respect to an aircraft velocity component	The direct derivative of a resultant moment component (1.5.5) with respect to an aircraft velocity component (1.3.4), divided by Q_R/V_R. EXAMPLE $\hat{M}_w = \frac{\partial M/\partial w}{Q_R/V_R}$ $\hat{M}_w = \frac{\partial M/\partial w}{\frac{1}{2}\varrho_R V_R S l}$	$\hat{M}_w$
3.4.6	Normalized moment derivative with respect to an angular velocity component	The direct derivative of a resultant moment component (1.5.5) with respect to an angular velocity component (1.3.6), divided by $Q_R/(V_R/l)$. EXAMPLE $\hat{M}_r = \frac{\partial M/\partial r}{Q_R/(V_R/l)}$ $\hat{M}_r = \frac{\partial M/\partial r}{\frac{1}{2}\varrho_R V_R S l^2}$	$\hat{M}_r$
3.4.7	Normalized moment derivative with respect to a linear acceleration component	The direct derivative of a resultant moment component (1.5.5) with respect to the derivative of an aircraft velocity component (1.3.4) with respect to time, divided by $Q_R/(V_R^2/l)$. EXAMPLE $\hat{M}_{\dot{w}} = \frac{\partial M/\partial \dot{w}}{Q_R/(V_R^2/l)}$ $\hat{M}_{\dot{w}} = \frac{\partial M/\partial \dot{w}}{\frac{1}{2}\varrho_R S l^2}$ where $\dot{w} = \frac{dw}{dt}$	$\hat{M}_{\dot{w}}$

No.	Term	Definition	Symbol
3.4.8	Normalized moment derivative with respect to a motivator deflector	The direct derivative of a resultant moment component (1.5.5) with respect to a motivator deflection (1.8.3.11 to 1.8.3.13), divided by Q_R. The non-angular motivator deflections shall be normalized. EXAMPLE $\widehat{M}_{\delta n} = \frac{\partial M / \partial \delta_n}{Q_R}$ $\widehat{M}_{\delta n} = \frac{\partial M / \partial \delta_n}{\frac{1}{2} \varrho_R V_R^2 S l}$	$\widehat{M}_{\delta n}$

3.5 Coefficient derivatives

A coefficient derivative is the partial derivative of a force coefficient (1.5.3) or of a moment coefficient (1.5.6) with respect to a variable contained in a given set of independent non-dimensional variables.

By convention, a standard set of independent variables is used for the coefficient derivatives:

— the angle of attack α (1.2.1.2) and the angle of sideslip β (1.2.1.1);

— the normalized angular velocities p^*, q^* and r^* (1.3.7);

— the Mach number M (1.3.3);

— the normalized derivatives of the angle of attack and the angle of sideslip with respect to time:

$$\dot{\alpha}^* = \frac{\dot{\alpha} l}{V}$$

$$\dot{\beta}^* = \frac{\dot{\beta} l}{V}$$

where

l is the reference length (1.4.6),

V is the airspeed (1.3.1);

— the normalized tangential acceleration [normalized derivative of the airspeed (1.3.1) with respect to time]:

$$\dot{V}^* = \frac{\dot{V} l}{V^2}$$

— the motivator deflections (1.8.3.7 to 1.8.3.13), normalized if necessary.

This standard set may be expanded according to need. A non-standard set shall be specified.

The symbol for a coefficient derivative comprises the symbol of the differentiated coefficient to which the symbol of the independent variable is added as a second subscript[1)].

NOTE — Analogous coefficient derivatives can be defined with regard to the coefficients of the components of the airframe aerodynamic force (1.7.2.1) and of the airframe aerodynamic moment (1.7.2.8) or with regard to the coefficients of the components of the thrust (1.7.1.1) and of the resultant moment of propulsive forces (1.7.1.2).

1) The independent variable is sometimes indicated in the symbol by a superscript.