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Statistical interpretation of data — Tests and confidence intervals relating to proportions

Interprétation statistique des données — Tests et intervalles de confiance portant sur les proportions

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 11453 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 3, *Application of statistical methods in standardization*.

Annex A forms an integral part of this International Standard. Annexes B and C are for information only.

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Statistical interpretation of data — Tests and confidence intervals relating to proportions

1 Scope

This International Standard describes specific statistical methods for addressing the following questions.

a) Given a population of items from which a sample of n items has been drawn, x of the sample items are found to show a specified characteristic. What proportion of the population has that characteristic? (See A forms, subclause 8.1.)

b) Is the proportion estimated in a) different from a nominal (specified) value? (See B forms, subclause 8.2.)

c) Given two distinct populations, are the proportions with the characteristic in the two populations different? (See C forms, subclause 8.3.)

d) In b) and c) how many items must be sampled in the population(s) to be sufficiently sure that the result of the test is correct? (See 7.2.3 and 7.3.3.)

It is essential that the drawing of samples does not have any appreciable effect on the population. If the sample drawn at random is less than 10 % of the population this is usually satisfactory, but if the sample is greater than this, reliable results can be obtained only by replacing each item sampled before drawing the next item at random from the population.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

3 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1 and the following definition apply.

3.1 target item: One in which the specified characteristic is found.

4 Symbols

α	significance level chosen
α'	achieved significance level
$1-\alpha$	confidence level chosen
β	probability of the error of the second kind
n; n_1; n_2	sample size; sample size of sample 1; sample size of sample 2
X	number of target items in the sample (random variable)
x	value of X
p	proportion of target items in the population
$p_{u,o}$	upper limit of the one-sided confidence interval for p
$p_{l,o}$	lower limit of the one-sided confidence interval for p
$p_{u,t}$	upper limit of the two-sided confidence interval for p
$p_{l,t}$	lower limit of the two-sided confidence interval for p
T	value from table 2 for the determination of confidence limits for $n \leqslant 30$
$C_{l,o}$	critical value of the test of the null hypothesis H_0: $p \geqslant p_0$
$C_{u,o}$	critical value of the test of the null hypothesis H_0: $p \leqslant p_0$
$C_{l,t}$	lower critical value of the test of the null hypothesis H_0: $p = p_0$
$C_{u,t}$	upper critical value of the test of the null hypothesis H_0: $p = p_0$
p_0	given value for p
p'	value of p for which the probability of not rejecting the null hypothesis (P_a) is to be determined
P_a	probability of not rejecting the null hypothesis
f_1, f_2	number of degrees of freedom of the F-distribution
F_1, F_2	test statistics
$F_q(f_1, f_2)$	q-quantile of the F-distribution with f_1 and f_2 degrees of freedom
z_1, z_2	test statistics
u_q	q-quantile of the standard normal distribution
q, η, K	auxiliary values

5 Point estimator of the proportion p

The estimator of p from a sample of n items including x target items is

$$\hat{p} = x/n$$

This estimator is unbiased if the sample is drawn at random, whatever the sample size and population size may be, even if the sample forms an appreciable part of the population.

6 Confidence limits for the proportion p

The calculation of a confidence interval for p is described in forms A-1 to A-3.

The confidence limits will depend on sample size (n), the number of target items in the sample (x), and the desired confidence level ($1-\alpha$). It is not possible in general to achieve the desired confidence level exactly because the probability distribution governing the outcome x is discrete. Thus, the procedure yields the nearest confidence level greater than or equal to ($1-\alpha$).

The procedure used in this International Standard for determining the two-sided confidence limits for the desired confidence level ($1-\alpha$) uses the limits for the upper and lower one-sided limits each for the desired confidence level ($1-\alpha/2$). It thereby guarantees that the error probability is less than or equal to $\alpha/2$ on each side of the interval.

7 Significance tests on proportions p

7.1 General

For practical applications, forms B-1 to B-3 and C-1 to C-3 present the null hypotheses concerning proportions and schemes for carrying out tests. At the beginning of the procedures, the appropriate null hypothesis and the sample size n (the sample sizes n_1 and n_2) are to be determined and the significance level is to be chosen. Because the underlying sampling distributions are discrete, the procedures are designed to achieve the nearest significance level less than or equal to the desired (nominal) level. The alternative hypotheses are not indicated in the forms because in each application the alternative hypothesis is implicitly assumed to be complementary to the null hypothesis.

EXAMPLES

At the beginning of the B forms (procedure for the comparison of a proportion with a given value), one of the following three null hypotheses H_0 (with the complementary alternative hypothesis H_1) is to be chosen:

a) one-sided test with H_0: $p \geqslant p_0$ $\quad H_1$: $p < p_0$

b) one-sided test with H_0: $p \leqslant p_0$ $\quad H_1$: $p > p_0$

c) two-sided test with H_0: $p = p_0$ $\quad H_1$: $p \neq p_0$

where p_0 is the given value.

The result of each test is either to reject or not to reject the null hypothesis.

Rejecting the null hypothesis means adopting the alternative hypothesis. Not rejecting the null hypothesis does not necessarily mean accepting the null hypothesis (see 7.2.2).

7.2 Comparison of a proportion with a given value p_0

7.2.1 Test procedure

The test procedures for the null hypotheses

H_0: $p \geqslant p_0$

H_0: $p \leqslant p_0$

H_0: $p = p_0$

(where p_0 is the given value) are described in forms B-1 to B-3. They are especially easy to apply if the critical value(s) for the specified values of n, p and α is (are) known. The critical value(s) may have been determined through repeated execution of the test in accordance with the B forms. Otherwise the standard procedure to determine the critical values is the one given in the same forms.

7.2.2 Operating characteristics

The computation of the operating characteristics (including the probability of the error of the first kind, the achieved significance level and the probability of the error of the second kind) is described in annex A. For this purpose the critical value(s) need(s) to be known (see 7.2.1) and the alternative hypothesis, $p = p_1$, for which the probability of the error of the second kind is to be computed has to be chosen.

7.2.3 Determination of sample size n

If the sample size is not already specified (for example for economic or technical reasons), its minimum value shall be determined such that for a given null hypothesis H_0 (see 7.2.1), the achieved value of the significance level α is not greater than its chosen or given value. In addition, the achieved value of the type II error, probability β, shall be approximately equal to its chosen or given value if p equals a particular chosen or given value p'. For this purpose, p_0 and p' are to be marked on the p-scale and α, $(1-\alpha)$, $\alpha/2$, $(1-\alpha/2)$ on the P-scale and the straight lines 1 and 2 as defined through the procedure shown in table 1 are drawn in the Larson nomograph (figure 2).

Table 1 — Procedure for determining the sample size from the Larson nomograph (figure 2)

Case	Given value	Straight line 1 from p_0 to	Straight line 2 from p' to
H_0: $p \geqslant p_0$	$p' < p_0$	α	$1-\beta$
H_0: $p \leqslant p_0$	$p' > p_0$	$1-\alpha$	β
H_0: $p = p_0$	$p' > p_0$	$1-\alpha/2$	β
H_0: $p = p_0$	$p' < p_0$	$\alpha/2$	$1-\beta$

The point of intersection of the two lines leads to the values $C_{l,o}(C_{u,o})$ on the x-scale. If x is not an integer, round up or down to the next integer.

7.3 Comparison of two proportions

7.3.1 Test procedure

The test procedures for the null hypothesis

H_0: $p_1 \geqslant p_2$

H_0: $p_1 \leqslant p_2$

H_0: $p_1 = p_2$

(where p_1 is the proportion of target items in population 1 and p_2 is the proportion of target items in population 2) are described in forms C-1 to C-3. These procedures are also suitable for testing the independence of two attributes (dichotomous characteristics) of items in a population.

7.3.2 Operating characteristics

It is assumed:

a) that for a one-sided test of H_0: $p_1 \leqslant p_2$, the power $(1-\beta)$ must be determined for a given pair of proportions p_1 and p_2, with $p_1 > p_2$;

b) that the test is carried out with two samples of the same size, i.e. $n_1 = n_2 = n$.

The significance level is α. Then a very accurate approximate value of the power can be obtained by the arc sine transformation (proposed by Walters [1]) as follows:

$$1-\beta = \Phi(z - u_{1-\alpha})$$

where

Φ is the distribution function of the standard normal distribution,

$u_{1-\alpha}$ is the $(1-\alpha)$-quantile of that normal distribution, and

$$z = \sqrt{2n}\left[\arcsin\sqrt{p_1 - (1/2n)} - \arcsin\sqrt{p_2 - (1/2n)}\right]$$

This approximation can also be used for the two-sided case: H_0: $p_1 = p_2$ with the alternative hypothesis H_1: $p_1 > p_2$ if α is replaced by $\alpha/2$ in the formula.

7.3.3 Determination of sample size n

If the sample sizes n_1 and n_2 are not predetermined, their minimum values shall be determined such that the power of the test is at least $(1 - \beta)$ while the significance level is α.

It is assumed that the null hypothesis is H_0: $p_1 \leq p_2$. The following procedures, however, also apply in the two-sided case H_0: $p_1 = p_2$, with the restricted alternative hypothesis H_1: $p_1 > p_2$ if α is replaced by $\alpha/2$.

Exact values of the sample size are given in tables 5 and 6 (originally published by Haseman[2]) for selected values of α and β. These tables assume a common sample size $n = n_1 = n_2$.

For configurations of α, p_1, p_2 and $(1-\beta)$ not covered by these tables, the following approximation can be used which also allows for unequal sample sizes. It requires that the ratio r of sample sizes n_1/n_2 has been chosen in advance.

$$n_1 = \frac{n'}{4}\left[1 + \sqrt{1 + \frac{2(r+1)}{rn'(p_1 - p_2)}}\right]^2$$

$$n_2 = n_1/r$$

where

$$n' = \frac{\left\{u_{1-\alpha}\sqrt{(r+1)\bar{p}\,\bar{q}} + u_{1-\beta}\sqrt{\left[rp_1(1-p_1) + p_2(1-p_2)\right]}\right\}^2}{r(p_1 - p_2)^2}$$

$$\bar{p} = \frac{rp_1 + p_2}{r+1}$$

$$\bar{q} = 1 - \bar{p}$$

8 Forms

For ease of application, make a tick in the box representing the activated part of the form. (The horizontal position of the box symbolizes the position of the respective part in the hierarchy of the form, decreasing from the right to the left.) Then follow the procedure by entering the necessary data and carrying out the actions required.

8.1 A forms: Confidence interval for the proportion p

8.1.1 Form A-1: One-sided, with upper limit confidence interval for the proportion p

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Confidence level chosen: $1-\alpha =$
Sample size: $n =$
Number of target items in the sample: $x =$

Determination of confidence limits:

a) Procedure for $n \leqslant 30$ ☐

1) Case $x = n$ ☐
$p_{u,o} = 1$

2) Case $x < n$ ☐

Read the value from table 2 for the known values n, $X = x$ and $q = 1 - \alpha$
(this value is the confidence limit):
$T_{(1-\alpha)}(n, x) = p_{u,o} =$

b) Procedure for $n > 30$ ☐

1) Case $x = 0$ ☐
Computation:
$p_{u,o} = 1 - \alpha^{1/n} =$

2) Case $x = n$ ☐
$p_{u,o} = 1$

3) Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$
Read the value d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,411	0,677	1,353

Computation:

$$p_{u,o} = p_* + (1-2p_*)d/(n+1) + u_{1-\alpha}\sqrt{p_*(1-p_*)[1-d/(n+1)]/(n+1)} =$$

with $p_* = (x+1)/(n+1)$

Result:
$p \leqslant p_{u,o} =$

8.1.2 Form A-2: One-sided, with lower limit confidence interval for the proportion p

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Confidence level chosen: $1-\alpha =$
Sample size: $n =$
Number of target items in the sample: $x =$

Determination of confidence limits:

a) Procedure for $n \leq 30$ ☐

1) Case $x = 0$ ☐
$p_{l,o} = 0$

2) Case $x > 0$ ☐

Read the value from table 2 for the known values n, $X = n - x$ and $q = 1-\alpha$:

$T_{(1-\alpha)}(n, n-x) =$

Computation:
$p_{l,o} = 1 - T_{(1-\alpha)}(n, n-x) =$

b) Procedure for $n > 30$ ☐

1) Case $x = 0$ ☐
$p_{l,o} = 0$

2) Case $x = n$ ☐
Computation:
$p_{l,o} = \alpha^{1/n} =$

3) Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$
Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,411	0,677	1,353

Computation:
$p_{l,o} = p_* + (1-2p_*)d/(n+1) - u_{1-\alpha}\sqrt{p_*(1-p_*)[1-d/(n+1)]/(n+1)} =$
with $p_* = x/(n+1)$

Result:
$p_{l,o} = \qquad \leq p$

8.1.3 Form A-3: Two-sided confidence interval for the proportion p

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Confidence level chosen: $1-\alpha =$
Sample size: $n =$
Number of target items in the sample: $x =$

Determination of confidence limits:

a) Procedure for $n \leqslant 30$ ☐

1) Upper confidence limit

— Case $x = n$ ☐

$p_{u,t} = 1$

— Case $x < n$ ☐

Read the value from table 2 for the known values n, $X = x$ and $q = 1-\alpha/2$ (this value is the confidence limit):

$T_{(1-\alpha/2)}(n, x) = p_{u,t} =$

2) Lower confidence limit

— Case $x = 0$ ☐

— Case $x > 0$ ☐

Read the value from table 2 for the known values n, $X = n - x$ and $q = 1-\alpha/2$:

$T_{(1-\alpha/2)}(n, n-x) =$

Computation:

$p_{l,t} = 1 - T_{(1-\alpha/2)}(n, n-x) =$

b) Procedure for $n > 30$ ☐

1) Upper confidence limit

— Case $x = 0$ ☐

Computation:

$p_{u,t} = 1 - (\alpha/2)^{1/n} =$

— Case $x = n$ ☐

$p_{u,t} = 1$

— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1 - \alpha/2$: $u_{1-\alpha/2} =$

Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,677	0,960	1,659

Computation:

$$p_{u,t} = p_* + (1 - 2p_*)d/(n+1) + u_{1-\alpha/2}\sqrt{p_*(1-p_*)[1 - d/(n+1)]/(n+1)} =$$

with $p_* = (x+1)/(n+1)$

2) Lower confidence limit

— Case $x = 0$ ☐

$p_{l,t} = 0$

— Case $x = n$ ☐

Computation:

$$p_{l,t} = (\alpha/2)^{1/n} =$$

— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1 - \alpha/2$: $u_{1-\alpha/2} =$

Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,677	0,960	1,659

Computation:

$$p_{l,t} = p_* + (1 - 2p_*)d/(n+1) - u_{1-\alpha/2}\sqrt{p_*(1-p_*)[1 - d/(n+1)]/(n+1)} =$$

with $p_* = x/(n+1)$

Results:

$p_{l,t} =$; $p_{u,t} =$; $p_{l,t} \leqslant p \leqslant p_{u,t}$

8.2 B Forms: Comparison of the proportion p with a given value p_0

8.2.1 Form B-1: Comparison of the proportion p with a given value p_0 and with one-sided test with H_0: $p \geqslant p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value p_0 =
Significance level chosen: α =
Sample size: n =
Number of target items found in the sample: x =

Test procedure:

I The critical value(s) is (are) already known (see 7.2.1 and, if applicable, the determination of the critical values below): ☐

$C_{l,o}$ =

H_0 is rejected if $x < C_{l,o}$; otherwise it is not rejected.

II The critical value(s) is(are) not known: ☐

a) Case $x \geqslant p_0 n$ ☐

H_0 is not rejected

b) Case $x < p_0 n$ ☐

1) Procedure for $n \leqslant 30$ ☐

Determine according to form A-1 the one-sided upper confidence limit for n, x and the confidence level $(1-\alpha)$:

$p_{u,o}$ =

H_0 is rejected if $p_{u,o} < p_0$; otherwise it is not rejected.

2) Procedure for $n > 30$ ☐

— Case $x = 0$ ☐

Computation:

$p_{u,o} = 1 - \alpha^{1/n}$ = [see form A-1 b) 1)]

H_0 is rejected if $p_{u,o} < p_0$; otherwise it is not rejected.

— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1 - \alpha$: $u_{1-\alpha}$ =

Computation:

$$u_1 = 2\left[\sqrt{(n-x)p_0} - \sqrt{(x+1)(1-p_0)}\right] =$$

H_0 is rejected if $u_1 > u_{1-\alpha}$; otherwise it is not rejected.

Test result:

H_0 is rejected ☐ H_0 is not rejected ☐

Determination of the critical values: $C_{l,o}$ is the smallest non-negative integer x for which the test according to form B-1-II does not lead to the rejection of H_0. $C_{l,o}$ is to be determined iteratively through repeated application of form B-1-II with different values of x [1]. Thereby those values of x are to be determined which differ from each other by 1 and one of which leads to the rejection of the null hypothesis while the other does not. If desired a start value for x, x_{start} can be obtained as follows.
Computations: np_0, rounded to the next integer, is $x^* =$ $p_{l,o}\|_{x=x^*} =$ ($p_{l,o}\|_{x=x^*}$ from form A-2) $np_{l,o}\|_{x=x^*}$, rounded to the next integer, is $x_{start} =$
Interpretation of the test results from form B-1-II: for $x \leqslant C_{l,o} - 1 =$ H_0 is rejected for $x \geqslant C_{l,o} =$ H_0 is rejected
Result: $C_{l,o} =$
1) The critical value or one of the critical values, respectively, may not exist for extreme values of p_0 and/or for very small sample sizes n.

8.2.2 Form B-2: Comparison of the proportion p with a given value p_0 and with one-sided test with H_0: $p \leqslant p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value p_0 =
Significance level chosen: α =
Sample size: n =
Number of target items found in the sample: x =

Test procedure:

I The critical value(s) is (are) already known (see 7.2.1 and, if applicable, the determination of the critical values below): ☐

$C_{u,o}$ =

H_0 is rejected, if $x > C_{u,o}$; otherwise it is not rejected.

II The critical value(s) is (are) not known: ☐

a) Case $x \leqslant p_0 n$ ☐

H_0 is not rejected.

b) Case $x > p_0 n$ ☐

1) Procedure for $n \leqslant 30$ ☐

Determine according to form A-2 the one-sided lower confidence limit for n, x and the confidence level $(1-\alpha)$:

$p_{l,o}$ =

H_0 is rejected if $p_{l,o} > p_0$; otherwise it is not rejected.

2) Procedure for $n > 30$ ☐

— Case $x = n$ ☐

Computation :

$p_{l,o} = \alpha^{1/n}$ = [see form A-2 b) 2)]

H_0 is rejected if $p_{l,o} > p_0$; otherwise it is not rejected.

— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha}$ =

Computation:

$$u_2 = 2\left[\sqrt{x(1-p_0)} - \sqrt{(n-x+1)p_0}\right] =$$

H_0 is rejected if $u_2 > u_{1-\alpha}$; otherwise it is not rejected.

Test result:

H_0 is rejected ☐

H_0 is not rejected ☐

Determination of the critical values:

$C_{u,o}$ is the largest integer x for which the test according to form B-2-II does not lead to the rejection of the null hypothesis. $C_{u,o}$ is to be determined iteratively through repeated application of form B-2-II with different values of x [1]. Thereby those values of x are to be determined which differ from each other by 1 and one of which leads to the rejection of the null hypothesis while the other does not. If desired, a start value for x, x_{start} can be obtained as follows.

Computations:

np_0, rounded to the next integer, is $x^* =$

$p_{u,o}|_{x=x^*} =$ ($p_{u,o}|_{x=x^*}$ from form A-1)

$np_{u,o}|_{x=x^*}$, rounded to the next integer, is $x_{start} =$

Interpretation of the test results from form B-2-II:

for $x \leqslant C_{u,o} =$ H_0 is not rejected

for $x \geqslant C_{u,o}+1 =$ H_0 is rejected

Result:

$C_{u,o} =$

1) The critical value or one of the critical values, respectively, may not exist for extreme values of p_0 and/or for very small sample sizes n.

8.2.3 Form B-3: Comparison of the proportion p with a given value p_0 and with two-sided test with H_0: $p = p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value p_0 =
Significance level chosen: α =
Sample size: n =
Number of target items found in the sample: x =

Test procedure:

I The critical value(s) is (are) already known (see 7.2.1 and, if applicable, the determination of the critical values below): ☐

$C_{l,t}$ = $C_{u,t}$ =

H_0 is rejected if $x < C_{l,t}$ or $x > C_{u,t}$; otherwise, it is not rejected.

II The critical value(s) is (are) not known: ☐

a) Procedure for $n \leqslant 30$ ☐

Determine according to form A-3 the two-sided confidence limits for n, x, and the confidence level $(1-\alpha)$:
$p_{l,t}$ = and
$p_{u,t}$ =

H_0 is rejected if $p_{l,t} > p_0$ or $p_{u,t} < p_0$; otherwise, it is not rejected.

b) Procedure for $n > 30$ ☐

1) Case $x = 0$ ☐

Computation:
$p_{u,t} = 1-(\alpha/2)^{1/n}$ =
H_0 is rejected if $p_{u,t} < p_0$; otherwise, it is not rejected.

2) Case $x = n$ ☐

Computation:
$p_{l,t} = (\alpha/2)^{1/n}$ =
H_0 is rejected if $p_{l,t} > p_0$; otherwise, it is not rejected.

3) Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1 - \alpha/2$: $u_{1-\alpha/2} =$

Computations:

$u_1 = 2\left[\sqrt{(n-x)p_0} - \sqrt{(x+1)(1-p_0)}\right] =$

$u_2 = 2\left[\sqrt{x(1-p_0)} - \sqrt{(n-x+1)p_0}\right] =$

H_0 is rejected if $u_1 > u_{1-\alpha/2}$ or $u_2 > u_{1-\alpha/2}$; otherwise, it is not rejected.

Test result:

H_0 is rejected ☐

H_0 is not rejected ☐

Determination of the critical values:

$C_{l,t}$ is the smallest non-negative integer x and $C_{u,t}$ is the largest integer x for which the test according to form B-3-II does not lead to the rejection of H_0. $C_{l,t}$ and $C_{u,t}$ are to be determined iteratively through repeated application of form B-3-II with different values of x [1]. Thereby two pairs of values are to be determined such that in each pair the values differ from each other by 1 and one of the values leads to the rejection of the null hypothesis while the other does not. If desired, start values for x, x_{start} can be obtained as follows.

Computations:

np_0, rounded to the next integer, is $x^* =$

$p_{l,t}|_{x=x^*} =$ $\quad$ $p_{u,t}|_{x=x^*} =$

$p_{l,t}|_{x=x^*}$ and $p_{u,t}|_{x=x^*}$ from form A-3

$np_{l,t}|_{x=x^*}$, rounded to the next integer, is x_{start} (lower) =

$np_{u,t}|_{x=x^*}$, rounded to the next integer, is x_{start} (upper) =

Interpretation of the test results from form B-3-II:

for $x \leqslant C_{l,t} - 1 =$		H_0 is rejected
for $x = C_{l,t} =$	to $x = C_{u,t} =$	H_0 is not rejected
for $x \geqslant C_{u,t} + 1 =$		H_0 is rejected

Results:

$C_{l,t} =$ $\qquad$ $C_{u,t} =$

1) The critical value or one of the critical values, respectively, may not exist for extreme values of p_0 and/or for very small sample sizes n.

8.3 C forms: Comparison of two proportions

8.3.1 Form C-1: Comparison of two proportions with one-sided test with H_0: $p_1 \geqslant p_2$

Characteristic: Determination procedure: Items: Criterion for the identification of target items: Notes:
Significance level chosen: $\alpha =$ Sample size 1: $n_1 =$ Sample size 2: $n_2 =$ Number of target items in sample 1: $x_1 =$ Number of target items in sample 2: $x_2 =$
Check for trivial case: $\frac{x_1}{n_1} \geqslant \frac{x_2}{n_2}$ is true ☐ is not true ☐ In the "true" case, the null hypothesis is not rejected and the test result can be stated immediately. Otherwise the following procedure is to be followed which eventually may lead to rejecting or to not rejecting H_0.
Test procedure for the non-trivial cases: If at least one of the four values n_1, n_2, $(x_1 + x_2)$, $(n_1 + n_2 - x_1 - x_2)$ is smaller than or equal to $(n_1 + n_2)/4$, the binomial approximation, I, shall be applied; otherwise the normal approximation, II. However, even if the above condition is fulfilled, the normal approximation can be applied if the two following conditions are fulfilled: — while applying the binomial approximation, interpolation in the F-distribution table is necessary; — n_1 and n_2 are of the same order of magnitude or $(x_1 + x_2)$ and $(n_1 + n_2 - x_1 - x_2)$ are of the same order of magnitude. Decision: The binomial approximation is to be applied (proceed with I). ☐ The normal approximation is to be applied (proceed with II). ☐

I Binomial approximation

Definition of variables: K_1, K_2, η_1, η_2:

If either $[n_2 < n_1$ and $n_2 < (x_1 + x_2)]$
or $[(n_1 + n_2 - x_1 - x_2) < n_1$ and $(n_1 + n_2 - x_1 - x_2) < (x_1 + x_2)]$,

the variables are defined as follows:

$\eta_1 = n_2 =$
$\eta_2 = n_1 =$
$K_1 = n_2 - x_2 =$
$K_2 = n_1 - x_1 =$

Otherwise, they are:

$\eta_1 = n_1 =$
$\eta_2 = n_2 =$
$K_1 = x_1 =$
$K_2 = x_2 =$

Computation of test statistics and determination of values from tables:

I a) Case $\eta_1 \leqslant K_1 + K_2$ ☐

$$F_2 = \frac{(\eta_1 - K_1)(K_1 + 2K_2)}{(K_1 + 1)(\eta_1 + 2\eta_2 - K_1 - 2K_2 + 1)} =$$

Numbers of degrees of freedom of the F-distribution:

$f_1 = 2(K_1 + 1) =$
$f_2 = 2(\eta_1 - K_1) =$

Read the value from table 4 for $q = 1 - \alpha$, f_1 and f_2 (if necessary interpolate):
$F_{(1-\alpha)}(f_1, f_2) =$

I b) Case $\eta_1 > K_1 + K_2$ ☐

$$F_2 = \frac{K_2(2\eta_1 - K_1)}{(K_1 + 1)(2\eta_2 - K_2 + 1)} =$$

Number of degrees of freedom of the F-distribution:

$f_1 = 2(K_1 + 1) =$
$f_2 = 2K_2 =$

Read the value from table 4 for $q = 1-\alpha$, f_1 and f_2 (if necessary interpolate):

$$F_{(1-\alpha)}(f_1, f_2) =$$

Drawing the conclusion in the non-trivial case for the binomial approximation:

H_0 is rejected, if

$$F_2 \geqslant F_{(1-\alpha)}(f_1, f_2)$$

Otherwise H_0 is not rejected.

II Normal approximation

Computation of test statistics and determination of values from tables:

$$z_2 = \frac{n_1(x_1+x_2)-(x_1+1/2)(n_1+n_2)}{\sqrt{n_1 n_2 (x_1+x_2)(n_1+n_2-x_1-x_2)/(n_1+n_2)}} =$$

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$

Drawing the conclusion in the non-trivial case for the normal approximation:

H_0 is rejected if

$$z_2 \geqslant u_{1-\alpha}$$

Otherwise H_0 is not rejected.

Test result:

H_0 is rejected ☐

H_0 is not rejected ☐

8.3.2 Form C-2: Comparison of two proportions with one-sided test with H_0: $p_1 \leq p_2$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Significance level chosen: $\alpha =$
Sample size 1: $n_1 =$
Sample size 2: $n_2 =$
Number of target items in sample 1: $x_1 =$
Number of target items in sample 2: $x_2 =$

Check for trivial case:

$$\frac{x_1}{n_1} \leq \frac{x_2}{n_2}$$

is true ☐ is not true ☐

In the "true" case, the null hypothesis is not rejected and the test result can be stated immediately. Otherwise the following procedure is to be followed which eventually may lead to rejecting or to not rejecting H_0.

Test procedure for the non-trivial cases:

If at least one of the four values n_1, n_2, $(x_1 + x_2)$, $(n_1 + n_2 - x_1 - x_2)$ is smaller than or equal to $(n_1 + n_2)/4$, the binomial approximation, I, shall be applied; otherwise the normal approximation, II. However, even if the above condition is fulfilled, the normal approximation can be applied if the two following conditions are fulfilled:

— while applying the binomial approximation, interpolation in the F-distribution table is necessary;

— n_1 and n_2 are of the same order of magnitude or $(x_1 + x_2)$ and $(n_1 + n_2 - x_1 - x_2)$ are of the same order of magnitude.

Decision:
The binomial approximation is to be applied (proceed with I). ☐

The normal approximation is to be applied (proceed with II). ☐

I Binomial approximation

Definition of variables: K_1, K_2, η_1, η_2

If either $[n_2 < n_1$ and $n_2 < (x_1 + x_2)]$,
or $[(n_1 + n_2 - x_1 - x_2) < n_1$ and $(n_1 + n_2 - x_1 - x_2) < (x_1 + x_2)]$

the variables are defined as follows:

$\eta_1 = n_2 =$

$\eta_2 = n_1 =$

$K_1 = n_2 - x_2 =$

$K_2 = n_1 - x_1 =$

Otherwise they are:

$\eta_1 = n_1 =$

$\eta_2 = n_2 =$

$K_1 = x_1 =$

$K_2 = x_2 =$

Computation of test statistics and determination of values from tables:

I a) Case $\eta_1 \leq K_1 + K_2$ ☐

$$F_1 = \frac{K_1(\eta_1 + 2\eta_2 - K_1 - 2K_2)}{(\eta_1 - K_1 + 1)(K_1 + 2K_2 + 1)} =$$

Numbers of degrees of freedom of the F-distribution:

$f_1 = 2(\eta_1 - K_1 + 1) =$

$f_2 = 2K_1 =$

Read the value from table 4 for $q = 1 - \alpha$, f_1 and f_2 (if necessary interpolate): $F_{(1-\alpha)}(f_1, f_2) =$

I b) Case $\eta_1 > K_1 + K_2$ ☐

$$F_1 = \frac{K_1(2\eta_2 - K_2)}{(K_2 + 1)(2\eta_1 - K_1 + 1)} =$$

Numbers of degrees of freedom of the F-distribution:

$f_1 = 2(K_2 + 1) =$

$f_2 = 2K_1 =$

Read the value from table 4 for $q = 1 - \alpha$, f_1 and f_2 (if necessary interpolate): $F_{(1-\alpha)}(f_1, f_2) =$

Drawing the conclusion in the non-trivial case for the binomial approximation:

H_0 is rejected if:

$F_1 \geq F_{(1-\alpha)}(f_1, f_2)$

Otherwise H_0 is not rejected.

II Normal approximation

Computation of test statistics and determination of values from tables:

$$z_1 = \frac{(x_1 - 1/2)(n_1 + n_2) - n_1(x_1 + x_2)}{\sqrt{n_1 n_2 (x_1 + x_2)(n_1 + n_2 - x_1 - x_2)/(n_1 + n_2)}} =$$

Read the value from table 3 for $q = 1 - \alpha$:

$u_{1-\alpha} =$

Drawing the conclusion in the non-trivial case for the normal approximation:

H_0 is rejected, if

$z_1 \geqslant u_{1-\alpha}$

Otherwise H_0 is not rejected.

Test result:

H_0 is rejected ☐

H_0 is not rejected ☐

8.3.3 Form C-3: Comparison of two proportions with two-sided test with H_0: $p_1 = p_2$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Significance level chosen: $\alpha =$

Sample size 1: $n_1 =$

Sample size 2: $n_2 =$

Number of target items in sample 1: $x_1 =$

Number of target items in sample 2: $x_2 =$

Check for trivial case:

$$\frac{x_1}{n_1} = \frac{x_2}{n_2}$$

is true ☐ is not true ☐

In the "true" case, the null hypothesis is not rejected and the test result can be stated immediately. Otherwise the following procedure is to be followed which eventually may lead to rejecting or to not rejecting H_0.

Test procedure for the non-trivial cases:

If at least one of the four values n_1, n_2, $(x_1 + x_2)$, $(n_1 + n_2 - x_1 - x_2)$ is smaller than or equal $(n_1 + n_2)/4$, the binomial approximation, I, shall be applied; otherwise the normal approximation, II. However, even if the above condition is fulfilled, the normal approximation can be applied if the two following conditions are fulfilled:

— while applying the binomial approximation, interpolation in the F-distribution table is necessary;

— n_1 and n_2 are of the same order of magnitude or $(x_1 + x_2)$ and $(n_1 + n_2 - x_1 - x_2)$ are of the same order of magnitude.

Decision:

The binomial approximation is to be applied (proceed with I). ☐

The normal approximation is to be applied (proceed with II). ☐

I Binomial approximation

Definition of variables: K_1, K_2, η_1, η_2
If either $[n_2 < n_1$ and $n_2 < (x_1 + x_2)]$
or $[(n_1 + n_2 - x_1 - x_2) < n_1$ and $(n_1 + n_2 - x_1 - x_2) < (x_1 + x_2)]$

the variables are defined as follows:

$\eta_1 = n_2 =$
$\eta_2 = n_1 =$
$K_1 = n_2 - x_2 =$
$K_2 = n_1 - x_1 =$

Otherwise, they are:

$\eta_1 = n_1 =$
$\eta_2 = n_2 =$
$K_1 = x_1 =$
$K_2 = x_2 =$

Computation of test statistics and determination of values from tables:

I a) Case $\eta_1 \leqslant K_1 + K_2$ ☐

1) Case $\frac{K_1}{\eta_1} > \frac{K_2}{\eta_2}$ ☐

Determine F_1, f_1 and f_2 as in form C-2:

$F_1 =$ $f_1 =$ $f_2 =$

Read the value from table 4 for $q = 1 - \alpha/2, f_1$ and f_2 (if necessary interpolate):
$F_{(1-\alpha/2)}(f_1, f_2) =$

2) Case $\frac{K_1}{\eta_1} \leqslant \frac{K_2}{\eta_2}$ ☐

Determine F_2, f_1 and f_2 as in form C-1:

$F_2 =$ $f_1 =$ $f_2 =$

Read the value from table 4 for $q = 1 - \alpha/2, f_1$ and f_2 (if necessary interpolate):
$F_{(1-\alpha/2)}(f_1, f_2) =$

I b) Case $\eta_1 > K_1 + K_2$ ☐

1) Case $\frac{K_1}{\eta_1} > \frac{K_2}{\eta_2}$ ☐

Determine F_1, f_1 and f_2 as in form C-2:

$F_1 =$ $f_1 =$ $f_2 =$

Read the value from table 4 for $q = 1 - \alpha/2$, f_1 and f_2 (if necessary interpolate):

$F_{(1-\alpha/2)}(f_1, f_2) =$

2) Case $\frac{K_1}{\eta_1} \leqslant \frac{K_2}{\eta_2}$ ☐

Determine F_2, f_1 and f_2 as in form C-1:

$F_2 =$ $f_1 =$ $f_2 =$

Read the value from table 4 for $q = 1 - \alpha/2$, f_1 and f_2 (if necessary interpolate):

$F_{(1-\alpha/2)}(f_1, f_2) =$

Drawing the conclusion in the non-trivial case for the binomial approximation:

H_0 is rejected if:

in the case $\frac{K_1}{\eta_1} > \frac{K_2}{\eta_2}$: $F_1 \geqslant F_{(1-\alpha/2)}(f_1, f_2)$

in the case $\frac{K_1}{\eta_1} \leqslant \frac{K_2}{\eta_2}$: $F_2 \geqslant F_{(1-\alpha)}(f_1, f_2)$

Otherwise H_0 is not rejected.

II Normal approximation

Computation of test statistics and determination of values from tables:

a) Case $\frac{x_1}{n_1} > \frac{x_2}{n_2}$ ☐

Determine z_1 as in form C-2:

$z_1 =$

Read the value from table 3 for $q = 1 - \alpha/2$:

$u_{1-\alpha/2} =$

b) Case $\frac{x_1}{n_1} \leqslant \frac{x_2}{n_2}$ ☐

Determine z_2 as in form C-1:

$z_2 =$

Read the value from table 3 for $q = 1 - \alpha/2$:

$u_{1-\alpha/2} =$

Drawing the conclusion in the non-trivial case for the normal approximation:

H_0 is rejected if:

in the case $\frac{x_1}{n_1} > \frac{x_2}{n_2}$: $z_1 \geqslant u_{1-\alpha/2}$

in the case $\frac{x_1}{n_1} \leqslant \frac{x_2}{n_2}$: $z_2 \geqslant u_{1-\alpha/2}$

Otherwise H_0 is not rejected.

Test result:

H_0 is rejected ☐

H_0 is not rejected ☐

9 Tables and nomographs

9.1 Interpolation in table 4 of the quantiles of the *F*-distribution

Assume that $F_q(f_1, f_2) = F(f_1, f_2)$ is to be determined and that table 4 shows the adjacent values $F_{(f_{11}, f_2)}$ and $F_{(f_{12}, f_2)}$ with $F_{11} < f_1 < f_{12}$.

Then:

$$F_{(f_1,f_2)} = F_{(f_{11},f_2)} - \left[F_{(f_{11},f_2)} - F_{(f_{12},f_2)}\right]\frac{f_{12}}{f_1}\left(\frac{f_1 - f_{11}}{f_{12} - f_{11}}\right)$$

The interpolation with respect to f_2 is carried out in an analogous way if the adjacent values $F_{(f_1,f_{21})}$ and $F_{(f_1,f_{22})}$ with $f_{21} < f_2 < f_{22}$ are given in the table:

$$F_{(f_1,f_2)} = F_{(f_1,f_{21})} - \left[F_{(f_1,f_{21})} - F_{(f_2,f_{22})}\right]\frac{f_{22}}{f_2}\left(\frac{f_2 - f_{21}}{f_{22} - f_{21}}\right)$$

If the *F*-value to be determined is neither tabulated for f_1 nor for f_2, then three steps of interpolation are necessary: first two parallel steps with respect to one of the two numbers of degrees of freedom, and then another step with respect to the other number of degrees of freedom.

If $f_1 > 30$ and $f_2 > 30$, the quantile of the F-distribution is to be computed according to the appropriate one of the following equations:

$$\lg F_{(0,1)} = \frac{1{,}113\ 1}{\sqrt{h-0{,}77}} - 0{,}527g$$

$$\lg F_{(0,05)} = \frac{1{,}428\ 7}{\sqrt{h-0{,}95}} - 0{,}681g$$

$$\lg F_{(0,025)} = \frac{1{,}702\ 3}{\sqrt{h-1{,}14}} - 0{,}846g$$

$$\lg F_{(0,01)} = \frac{2{,}020\ 6}{\sqrt{h-1{,}40}} - 1{,}073g$$

$$\lg F_{(0,005)} = \frac{2{,}237\ 3}{\sqrt{h-1{,}61}} - 1{,}250g$$

$$\lg F_{(0,001)} = \frac{2{,}684\ 1}{\sqrt{h-2{,}09}} - 1{,}672g$$

where

$$g = \frac{1}{f_1} - \frac{1}{f_2}$$

$$h = 2/\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$$

$$F_q(f_1, f_2) = F_q$$

9.2 Example

An example for the determination of the critical value of the test of the null hypothesis H_0: $p \geqslant p_0$ is marked in the nomograph (figure 2) with a bold line (see 7.2.1). The given values are $p_0 = 0{,}15$, $\alpha = 0{,}05$ and $n = 35$. The nomograph yields a value of x between 1 and 2 and therefore $C_{\mathrm{l,o}} = 2$.

Suppose the sample size n is not already specified. If, in addition, it is given that $\beta = 0{,}10$ and $p' = 0{,}039$, then a second line is drawn from p' to $1-\beta$ for determining the sample size. The point of the intersection of the two lines leads to $n = 50$ in the nomograph and the value of x as 3; i.e. accept the null hypothesis when $x \leqslant 3$, otherwise reject the null hypothesis and accept the alternative hypothesis.

Table 2 — Upper one-sided confidence limits for the proportion p with $n \leqslant 30$

n	Value of x when q = 0,950																													
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0,950																													
2	0,777	0,975																												
3	0,632	0,865	0,984																											
4	0,528	0,752	0,903	0,988																										
5	0,451	0,658	0,811	0,924	0,990																									
6	0,394	0,582	0,729	0,847	0,938	0,992																								
7	0,349	0,521	0,659	0,775	0,872	0,947	0,993																							
8	0,313	0,471	0,600	0,711	0,808	0,889	0,954	0,994																						
9	0,284	0,430	0,550	0,656	0,749	0,832	0,903	0,959	0,995																					
10	0,259	0,395	0,507	0,607	0,697	0,778	0,850	0,913	0,964	0,995																				
11	0,239	0,365	0,471	0,565	0,651	0,729	0,801	0,865	0,922	0,967	0,996																			
12	0,221	0,339	0,439	0,828	0,610	0,685	0,755	0,819	0,878	0,929	0,970	0,996																		
13	0,206	0,317	0,411	0,495	0,573	0,646	0,713	0,777	0,835	0,888	0,934	0,972	0,997																	
14	0,193	0,297	0,386	0,466	0,541	0,610	0,675	0,737	0,794	0,848	0,896	0,939	0,975	0,997																
15	0,182	0,280	0,364	0,440	0,511	0,578	0,641	0,701	0,757	0,810	0,859	0,904	0,944	0,976	0,997															
16	0,171	0,264	0,344	0,417	0,485	0,549	0,609	0,667	0,722	0,774	0,823	0,868	0,910	0,947	0,978	0,997														
17	0,162	0,251	0,327	0,396	0,461	0,522	0,581	0,636	0,690	0,740	0,789	0,834	0,877	0,916	0,951	0,979	0,997													
18	0,154	0,238	0,311	0,377	0,439	0,498	0,555	0,608	0,660	0,709	0,757	0,802	0,844	0,884	0,921	0,953	0,980	0,998												
19	0,146	0,227	0,296	0,360	0,420	0,476	0,530	0,582	0,632	0,680	0,727	0,771	0,813	0,853	0,891	0,925	0,956	0,981	0,998											
20	0,140	0,217	0,283	0,344	0,402	0,456	0,508	0,559	0,607	0,654	0,699	0,742	0,783	0,823	0,861	0,896	0,929	0,958	0,982	0,998										
21	0,133	0,207	0,271	0,330	0,385	0,437	0,488	0,536	0,583	0,629	0,672	0,715	0,756	0,795	0,832	0,868	0,902	0,933	0,960	0,983	0,998									
22	0,128	0,199	0,260	0,316	0,370	0,420	0,469	0,516	0,561	0,605	0,648	0,689	0,729	0,768	0,805	0,841	0,874	0,906	0,936	0,962	0,984	0,998								
23	0,123	0,191	0,250	0,304	0,355	0,404	0,451	0,497	0,541	0,584	0,625	0,665	0,704	0,742	0,779	0,814	0,848	0,880	0,911	0,939	0,964	0,985	0,998							
24	0,118	0,183	0,240	0,293	0,342	0,390	0,435	0,479	0,522	0,563	0,604	0,643	0,681	0,718	0,754	0,789	0,823	0,855	0,886	0,915	0,944	0,966	0,985	0,998						
25	0,113	0,177	0,232	0,282	0,330	0,376	0,420	0,463	0,504	0,544	0,584	0,622	0,659	0,695	0,731	0,765	0,798	0,830	0,881	0,890	0,918	0,944	0,967	0,986	0,998					
26	0,109	0,170	0,223	0,272	0,319	0,363	0,406	0,447	0,487	0,527	0,565	0,602	0,638	0,674	0,708	0,742	0,775	0,807	0,837	0,867	0,895	0,922	0,946	0,968	0,987	0,999				
27	0,106	0,164	0,210	0,263	0,308	0,351	0,393	0,433	0,472	0,510	0,547	0,583	0,619	0,654	0,687	0,720	0,753	0,784	0,814	0,844	0,872	0,899	0,925	0,948	0,970	0,987	0,999			
28	0,102	0,159	0,209	0,255	0,298	0,340	0,380	0,419	0,457	0,494	0,530	0,566	0,600	0,634	0,667	0,700	0,731	0,762	0,792	0,821	0,850	0,877	0,903	0,927	0,950	0,971	0,988	0,999		
29	0,099	0,154	0,202	0,247	0,289	0,329	0,368	0,406	0,443	0,480	0,515	0,540	0,583	0,616	0,648	0,680	0,711	0,742	0,771	0,800	0,828	0,855	0,881	0,906	0,930	0,952	0,972	0,988	0,999	
30	0,096	0,149	0,196	0,239	0,280	0,319	0,358	0,394	0,430	0,466	0,500	0,534	0,567	0,599	0,631	0,662	0,692	0,722	0,751	0,779	0,807	0,834	0,860	0,886	0,910	0,932	0,954	0,973	0,989	0,999

Table 2 — Upper one-sided confidence limits for the proportion p with $n \leqslant 30$ *(continued)*

n	Value of x when q = 0,975																													
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0,975																													
2	0,842	0,988																												
3	0,708	0,906	0,992																											
4	0,603	0,806	0,933	0,994																										
5	0,522	0,717	0,854	0,948	0,995																									
6	0,460	0,642	0,778	0,882	0,957	0,996																								
7	0,410	0,579	0,710	0,816	0,902	0,964	0,997																							
8	0,370	0,527	0,651	0,756	0,843	0,915	0,969	0,997																						
9	0,337	0,483	0,601	0,701	0,788	0,864	0,926	0,972	0,998																					
10	0,309	0,446	0,557	0,653	0,738	0,813	0,879	0,934	0,975	0,998																				
11	0,285	0,413	0,518	0,610	0,693	0,767	0,833	0,891	0,940	0,978	0,998																			
12	0,265	0,385	0,485	0,572	0,652	0,724	0,790	0,849	0,901	0,946	0,980	0,998																		
13	0,248	0,361	0,455	0,539	0,615	0,685	0,749	0,808	0,862	0,910	0,950	0,981	0,999																	
14	0,232	0,339	0,429	0,508	0,582	0,649	0,712	0,770	0,824	0,873	0,917	0,954	0,983	0,999																
15	0,219	0,320	0,405	0,481	0,552	0,617	0,678	0,735	0,788	0,837	0,882	0,923	0,957	0,984	0,999															
16	0,206	0,303	0,384	0,457	0,524	0,587	0,646	0,702	0,754	0,803	0,849	0,890	0,928	0,960	0,985	0,999														
17	0,196	0,287	0,365	0,435	0,499	0,560	0,617	0,671	0,722	0,771	0,816	0,858	0,897	0,932	0,963	0,986	0,999													
18	0,186	0,273	0,348	0,415	0,477	0,535	0,591	0,643	0,693	0,740	0,785	0,828	0,867	0,904	0,936	0,965	0,987	0,999												
19	0,177	0,261	0,332	0,396	0,456	0,513	0,566	0,617	0,666	0,712	0,756	0,798	0,838	0,875	0,909	0,940	0,967	0,987	0,999											
20	0,169	0,249	0,317	0,379	0,437	0,492	0,543	0,593	0,640	0,685	0,729	0,770	0,809	0,847	0,882	0,914	0,943	0,968	0,988	0,999										
21	0,162	0,239	0,304	0,364	0,420	0,472	0,522	0,570	0,616	0,660	0,703	0,743	0,782	0,819	0,855	0,888	0,918	0,946	0,970	0,989	0,999									
22	0,155	0,229	0,292	0,350	0,403	0,454	0,503	0,549	0,594	0,637	0,678	0,718	0,757	0,793	0,829	0,862	0,893	0,922	0,949	0,971	0,989	0,999								
23	0,149	0,220	0,281	0,336	0,388	0,438	0,485	0,530	0,573	0,615	0,656	0,695	0,732	0,769	0,803	0,837	0,868	0,898	0,926	0,951	0,973	0,990	0,999							
24	0,143	0,212	0,270	0,324	0,374	0,422	0,468	0,511	0,554	0,595	0,634	0,672	0,709	0,745	0,779	0,813	0,844	0,874	0,903	0,929	0,953	0,974	0,990	0,999						
25	0,138	0,204	0,261	0,313	0,361	0,408	0,452	0,494	0,536	0,575	0,614	0,651	0,687	0,723	0,756	0,789	0,821	0,851	0,880	0,907	0,932	0,955	0,975	0,991	0,999					
26	0,133	0,197	0,252	0,302	0,349	0,394	0,437	0,478	0,518	0,557	0,595	0,631	0,667	0,701	0,735	0,767	0,798	0,828	0,857	0,885	0,911	0,935	0,957	0,976	0,991	1				
27	0,128	0,190	0,243	0,292	0,338	0,381	0,423	0,463	0,502	0,540	0,577	0,613	0,647	0,681	0,714	0,746	0,777	0,806	0,835	0,863	0,889	0,914	0,937	0,959	0,977	0,991	1			
28	0,124	0,184	0,236	0,283	0,327	0,369	0,410	0,449	0,487	0,524	0,560	0,595	0,629	0,662	0,694	0,725	0,756	0,785	0,814	0,842	0,868	0,894	0,918	0,940	0,960	0,978	0,992	1		
29	0,120	0,178	0,228	0,274	0,317	0,358	0,398	0,436	0,473	0,509	0,544	0,578	0,611	0,644	0,675	0,706	0,736	0,765	0,794	0,821	0,848	0,873	0,898	0,921	0,942	0,962	0,979	0,992	1	
30	0,116	0,173	0,221	0,266	0,308	0,348	0,386	0,423	0,459	0,494	0,529	0,562	0,594	0,626	0,657	0,688	0,717	0,746	0,774	0,801	0,828	0,853	0,878	0,901	0,923	0,944	0,963	0,979	0,992	1

Table 2 — Upper one-sided confidence limits for the proportion p with $n \leqslant 30$ *(continued)*

n	Value of x when q = 0,990																													
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0,990																													
2	0,900	0,995																												
3	0,785	0,942	0,997																											
4	0,684	0,860	0,959	0,998																										
5	0,602	0,778	0,895	0,968	0,998																									
6	0,536	0,706	0,827	0,916	0,974	0,999																								
7	0,483	0,644	0,764	0,858	0,930	0,978	0,999																							
8	0,438	0,590	0,707	0,802	0,880	0,940	0,981	0,999																						
9	0,401	0,545	0,657	0,750	0,830	0,895	0,947	0,983	0,999																					
10	0,370	0,505	0,612	0,703	0,782	0,850	0,907	0,953	0,985	0,999																				
11	0,343	0,470	0,573	0,661	0,738	0,807	0,866	0,917	0,958	0,986	1																			
12	0,319	0,440	0,538	0,623	0,698	0,766	0,826	0,879	0,925	0,962	0,988	1																		
13	0,299	0,413	0,507	0,588	0,661	0,728	0,788	0,842	0,890	0,931	0,965	0,989	1																	
14	0,281	0,390	0,479	0,557	0,628	0,693	0,752	0,806	0,855	0,899	0,936	0,967	0,990	1																
15	0,265	0,368	0,454	0,529	0,597	0,660	0,718	0,772	0,821	0,866	0,906	0,941	0,970	0,990	1															
16	0,251	0,349	0,431	0,503	0,569	0,630	0,687	0,740	0,789	0,834	0,875	0,913	0,945	0,972	0,991	1														
17	0,238	0,332	0,410	0,480	0,544	0,603	0,658	0,710	0,758	0,803	0,845	0,884	0,918	0,949	0,974	0,992	1													
18	0,226	0,317	0,392	0,459	0,520	0,578	0,631	0,682	0,729	0,774	0,816	0,855	0,891	0,923	0,952	0,975	0,992	1												
19	0,216	0,302	0,375	0,439	0,499	0,554	0,607	0,656	0,702	0,747	0,788	0,827	0,864	0,897	0,928	0,954	0,977	0,992	1											
20	0,206	0,289	0,359	0,421	0,479	0,533	0,583	0,631	0,677	0,720	0,762	0,800	0,837	0,871	0,903	0,932	0,957	0,978	0,993	1										
21	0,197	0,277	0,344	0,405	0,460	0,512	0,562	0,609	0,653	0,696	0,736	0,775	0,811	0,846	0,878	0,908	0,935	0,959	0,979	0,993	1									
22	0,189	0,266	0,331	0,389	0,443	0,494	0,542	0,587	0,631	0,673	0,712	0,750	0,787	0,821	0,854	0,884	0,913	0,938	0,961	0,980	0,994	1								
23	0,182	0,256	0,319	0,375	0,427	0,476	0,523	0,567	0,610	0,651	0,690	0,727	0,763	0,797	0,830	0,861	0,890	0,917	0,941	0,963	0,981	0,994	1							
24	0,175	0,247	0,307	0,362	0,412	0,460	0,505	0,549	0,590	0,630	0,668	0,705	0,741	0,775	0,807	0,838	0,867	0,895	0,921	0,944	0,965	0,982	0,994	1						
25	0,169	0,238	0,296	0,349	0,398	0,445	0,489	0,531	0,572	0,611	0,648	0,684	0,719	0,753	0,785	0,816	0,845	0,873	0,899	0,924	0946	0,966	0,982	0,994	1					
26	0,163	0,230	0,286	0,338	0,385	0,430	0,473	0,515	0,554	0,592	0,629	0,664	0,699	0,732	0,764	0,794	0,824	0,852	0,879	0,904	0,927	0,948	0,967	0,983	0,995	1				
27	0,157	0,222	0,277	0,327	0,373	0,417	0,459	0,499	0,538	0,575	0,611	0,646	0,679	0,712	0,743	0,774	0,803	0,831	0,858	0,883	0,908	0,930	0,951	0,969	0,984	0,995	1			
28	0,152	0,215	0,268	0,317	0,362	0,404	0,445	0,484	0,522	0,558	0,594	0,628	0,661	0,693	0,724	0,754	0,783	0,811	0,838	0,864	0,888	0,911	0,933	0,952	0,970	0,984	0,995	1		
29	0,147	0,208	0,260	0,307	0,351	0,393	0,432	0,470	0,507	0,543	0,577	0,611	0,643	0,675	0,705	0,735	0,764	0,791	0,818	0,844	0,869	0,892	0,914	0,935	0,954	0,971	0,985	0,995	1	
30	0,143	0,202	0,252	0,298	0,341	0,381	0,420	0,457	0,493	0,528	0,562	0,594	0,626	0,657	0,687	0,717	0,745	0,773	0,799	0,825	0,850	0,874	0,896	0,918	0,937	0,956	0,972	0,986	0,995	1

Table 2 — Upper one-sided confidence limits for the proportion p with $n \leqslant 30$ *(concluded)*

n	Value of x when q = 0,995: 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0,995																													
2	0,930	0,998																												
3	0,830	0,959	0,999																											
4	0,735	0,890	0,971	0999																										
5	0,654	0,815	0,918	0,978	0,999																									
6	0,587	0,747	0,857	0,934	0,982	1																								
7	0,531	0,685	0,798	0,883	0,945	0,985	1																							
8	0,485	0,632	0,743	0,831	0,901	0,953	0,987	1																						
9	0,445	0,585	0,693	0,781	0,854	0,914	0,959	0,988	1																					
10	0,412	0,545	0,649	0,736	0,810	0,872	0,924	0,963	0,990	1																				
11	0,383	0,509	0,609	0,694	0,767	0,831	0,886	0,932	0,967	0,991	1																			
12	0,357	0,478	0,573	0,656	0,728	0,792	0,848	0,897	0,938	0,970	0,992	1																		
13	0,335	0,450	0,542	0,621	0,692	0,755	0,812	0,862	0,906	0,943	0,973	0,992	1																	
14	0,316	0,425	0,513	0,590	0,658	0,721	0,777	0,828	0,874	0,914	0,948	0,975	0,930	1																
15	0,298	0,402	0,487	0,561	0,628	0,689	0,744	0,795	0,842	0,884	0,920	0,952	0,977	0,993	1															
16	0,282	0,382	0,463	0,535	0,600	0,659	0,714	0,764	0,811	0,853	0,892	0,926	0,955	0,978	0,994	1														
17	0,268	0,364	0,442	0,511	0,574	0,631	0,685	0,735	0,781	0,824	0,863	0,899	0,931	0,958	0,980	0,994	1													
18	0,255	0,347	0,422	0,489	0,550	0,606	0,658	0,707	0,753	0,796	0,836	0,872	0,905	0,935	0,960	0,981	0,995	1												
19	0,244	0,332	0,404	0,469	0,528	0,582	0,633	0,681	0,727	0,769	0,809	0,846	0,880	0,911	0,939	0,963	0,982	0,995	1											
20	0,233	0,318	0,388	0,450	0,507	0,560	0,610	0,657	0,701	0,743	0,783	0,820	0,855	0,887	0,916	0,942	0,965	0,983	0,995	1										
21	0,223	0,305	0,372	0,433	0,488	0,540	0,588	0,634	0,678	0,719	0,758	0,795	0,830	0,862	0,893	0,920	0,945	0,967	0,984	0,995	1									
22	0,215	0,293	0,358	0,417	0,470	0,521	0,568	0,613	0,655	0,696	0,735	0,771	0,806	0,839	0,870	0,898	0,924	0,948	0,968	0,985	0,996	1								
23	0,206	0,282	0,345	0,402	0,454	0,503	0,549	0,593	0,634	0,674	0,712	0,748	0,783	0,816	0,847	0,876	0,903	0,928	0,950	0,970	0,985	0,996	1							
24	0,199	0,272	0,333	0,388	0,438	0,486	0,531	0,574	0,614	0,654	0,691	0,727	0,761	0,794	0,825	0,854	0,882	0,908	0,931	0,953	0,971	0,986	0,996	1						
25	0,191	0,262	0,322	0,375	0,424	0,470	0,514	0,556	0,596	0,634	0,671	0,706	0,740	0,772	0,803	0,833	0,861	0,887	0,912	0,934	0,955	0,972	0,987	0,996	1					
26	0,185	0,253	0,311	0,363	0,410	0,456	0,498	0,539	0,578	0,615	0,652	0,686	0,720	0,752	0,782	0,812	0,840	0,867	0,892	0,915	0,937	0,956	0,973	0,987	0,996	1				
27	0,179	0,245	0,301	0,351	0,398	0,442	0,483	0,523	0,561	0,598	0,633	0,667	0,700	0,732	0,762	0,792	0,820	0,847	0,872	0,896	0,919	0,940	0,958	0,974	0,988	0,997	1			
28	0,173	0,237	0,292	0,340	0,386	0,429	0,469	0,508	0,545	0,581	0,616	0,650	0,682	0,713	0,743	0,772	0,800	0,827	0,853	0,877	0,900	0,922	0,942	0,960	0,975	0,988	0,997	1		
29	0,167	0,230	0,283	0,330	0,375	0,416	0,456	0,494	0,530	0,566	0,600	0,632	0,664	0,695	0,725	0,754	0,781	0,808	0,834	0,859	0,882	0,904	0,925	0,944	0,961	0,976	0,989	0,997	1	
30	0,162	0,223	0,275	0,321	0,364	0,405	0,443	0,480	0,516	0,551	0,584	0,616	0,647	0,678	0,707	0,736	0,763	0,790	0,815	0,840	0,864	0,886	0,908	0,928	0,946	0,963	0,977	0,989	0,997	1

Table 3 — Quantiles of the standard normal distribution, u_q

$q = \Phi(u)$	u_q
0,950	1,645
0,975	1,960
0,990	2,326
0,995	2,576

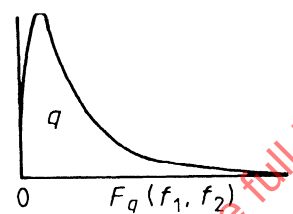


Figure 1 — Quantiles of the *F*-distribution

Table 4 — Quantiles of the F-distribution (see figure 1)

f_2	q	f_1 = 1	2	3	4	5	6	7	8	9	10	12	15	20	30	50	∞
1	0,9	39,9	49,5	53,6	55,8	57,2	58,2	58,9	59,4	59,9	60,2	60,7	61,2	61,7	62,3	62,7	63,3
	0,95	161	200	216	225	230	234	237	239	241	242	244	246	248	250	252	254
	0,975	648	800	864	900	922	937	948	957	963	969	977	985	993	1001	1008	1018
	0,990	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6106	6157	6209	6261	6303	6366
	0,995	16210	20000	21610	22500	23060	23440	23710	23930	24090	24220	24430	24630	24840	25040	25210	25460
	0,999	405300	500000	540400	562500	576400	585900	592900	598100	602300	605600	610700	615800	620900	626100	630300	636600
2	0,9	8,53	9,0	9,16	9,24	9,29	9,33	9,35	9,37	9,38	9,39	9,41	9,42	9,44	9,46	9,47	9,49
	0,95	18,5	19,0	19,2	19,2	19,3	19,3	19,4	19,4	19,4	19,4	19,4	19,4	19,4	19,5	19,5	19,5
	0,975	38,5	39,0	39,2	39,2	39,3	39,3	39,4	39,4	39,4	39,4	39,4	39,4	39,4	39,5	39,5	39,5
	0,990	98,5	99,0	99,2	99,2	99,3	99,3	99,4	99,4	99,4	99,4	99,4	99,4	99,4	99,5	99,5	99,5
	0,995	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199
	0,999	999	999	999	999	999	999	999	999	999	999	999	999	999	999	999	999
3	0,9	5,54	5,46	5,39	5,34	5,31	5,28	5,27	5,25	5,24	5,23	5,22	5,20	5,18	5,17	5,15	5,13
	0,95	10,1	9,55	9,28	9,12	9,01	8,94	8,89	8,85	8,81	8,79	8,74	8,70	8,66	8,62	8,58	8,53
	0,975	17,4	16,0	15,4	15,1	14,9	14,7	14,6	14,5	14,5	14,4	14,3	14,3	14,2	14,1	14,0	13,9
	0,990	34,1	30,8	29,5	28,7	28,2	27,9	27,7	27,5	27,3	27,2	27,1	26,9	26,7	26,5	26,4	26,1
	0,995	55,6	49,8	47,5	46,2	45,4	44,8	44,4	44,1	43,9	43,7	43,4	43,1	42,8	42,5	42,2	41,8
	0,999	167	149	141	137	135	133	132	131	130	129	128	127	126	125	125	123
4	0,9	4,54	4,32	4,19	4,11	4,05	4,01	3,98	3,95	3,94	3,92	3,90	3,87	3,84	3,82	3,80	3,76
	0,95	7,71	6,94	6,59	6,39	6,26	6,16	6,09	6,04	6,00	5,96	5,91	5,86	5,80	5,75	5,70	5,63
	0,975	12,2	10,6	9,98	9,60	9,36	9,20	3,07	8,96	8,90	8,84	8,75	8,66	8,56	8,46	8,38	8,26
	0,990	21,2	18,0	16,7	16,04	15,5	15,2	15,0	14,8	14,7	14,5	14,4	14,2	14,0	13,8	13,7	13,5
	0,995	31,3	26,3	24,3	23,2	22,5	22,0	21,6	21,4	21,1	21,0	20,7	20,4	20,2	19,9	19,7	19,3
	0,999	74,1	61,2	56,2	53,4	51,7	50,5	49,7	49,0	48,5	48,1	47,4	46,8	46,1	45,4	44,9	44,1
5	0,9	4,06	3,78	3,62	3,52	3,45	3,40	3,37	3,34	3,32	3,30	3,27	3,24	3,21	3,17	3,15	3,10
	0,95	6,61	5,79	5,41	5,19	5,05	4,95	4,88	4,82	4,77	4,74	4,68	4,62	4,56	4,50	4,44	4,36
	0,975	10,0	8,43	7,76	7,39	7,15	6,98	6,85	6,76	6,68	6,62	6,52	6,43	6,33	6,23	6,14	6,02
	0,990	16,3	13,3	12,1	11,4	11,0	10,7	10,5	10,3	10,2	10,1	9,89	9,72	9,55	9,38	9,24	9,02
	0,995	22,8	18,3	16,5	15,6	14,9	14,5	14,2	14,0	13,8	13,6	13,4	13,1	12,9	12,7	12,5	12,1
	0,999	47,2	37,1	33,2	31,1	29,8	28,8	28,2	27,6	27,2	26,9	26,4	25,9	25,4	24,9	24,4	23,8
6	0,9	3,78	3,46	3,29	3,18	3,11	3,05	3,01	2,98	2,96	2,94	2,90	2,87	2,84	2,80	2,77	2,72
	0,95	5,99	5,14	4,76	4,53	4,39	4,28	4,21	4,15	4,10	4,06	4,00	3,94	3,87	3,81	3,75	3,67
	0,975	8,81	7,26	6,60	6,23	5,99	5,82	5,70	5,60	5,52	5,46	5,37	5,27	5,17	5,07	4,98	4,85
	0,990	13,7	10,9	9,78	9,15	8,75	8,47	8,26	8,10	7,98	7,87	7,72	7,56	7,40	7,23	7,09	6,88
	0,995	18,6	14,5	12,9	12,0	11,5	11,1	10,8	10,6	10,4	10,3	10,0	9,81	9,59	9,36	9,17	8,88
	0,999	35,5	27,0	23,7	21,9	20,8	20,0	19,5	19,0	18,7	18,4	18,0	17,6	17,1	16,7	16,3	15,7
7	0,9	3,59	3,26	3,07	2,96	2,88	2,83	2,78	2,75	2,72	2,70	2,67	2,63	2,59	2,56	2,52	2,47
	0,95	5,59	4,74	4,35	4,12	3,97	3,87	3,79	3,73	3,68	3,64	3,57	3,51	3,44	3,38	3,32	3,23
	0,975	8,07	6,54	5,89	5,52	5,29	5,12	4,99	4,90	4,82	4,76	4,67	4,57	4,47	4,36	4,28	4,14
	0,990	12,2	9,55	8,45	7,85	7,46	7,19	6,99	6,84	6,72	6,62	6,47	6,31	6,16	5,99	5,86	5,65
	0,995	16,2	12,4	10,9	10,1	9,52	9,16	8,89	8,68	8,51	8,38	8,18	7,97	7,75	7,53	7,35	7,08
	0,999	29,2	21,7	18,8	17,2	16,2	15,5	15,0	14,6	14,3	14,1	13,7	13,3	12,9	12,5	12,2	11,7

Table 4 — Quantiles of the F-distribution *(continued)*

f_2	q	f_1 = 1	2	3	4	5	6	7	8	9	10	12	15	20	30	50	∞
8	0,9	3,46	3,11	2,92	2,81	2,73	2,67	2,62	2,59	2,56	2,54	2,50	2,46	2,42	2,38	2,35	2,29
	0,95	5,32	4,46	4,07	3,84	3,69	3,58	3,50	3,44	3,39	3,35	3,28	3,22	3,15	3,08	3,02	2,93
	0,975	7,57	6,06	5,42	5,05	4,82	4,65	4,53	4,43	4,36	4,30	4,20	4,10	4,00	3,89	3,81	3,67
	0,990	11,3	8,65	7,59	7,01	6,63	6,37	6,18	6,03	5,91	5,81	5,67	5,52	5,36	5,20	5,07	4,86
	0,995	14,7	11,0	9,60	8,81	8,30	7,95	7,69	7,50	7,34	7,21	7,01	6,81	6,61	6,40	6,22	5,95
	0,999	25,4	18,5	15,8	14,4	13,5	12,9	12,4	12,0	11,8	11,5	11,2	10,8	10,5	10,1	9,80	9,33
9	0,9	3,36	3,01	2,81	2,69	2,61	2,55	2,51	2,47	2,44	2,42	2,38	2,34	2,30	2,25	2,22	2,16
	0,95	5,12	4,26	3,86	3,63	3,48	3,37	3,29	3,23	3,18	3,14	3,07	3,01	2,94	2,86	2,80	2,71
	0,975	7,21	5,71	5,08	4,72	4,48	4,32	4,20	4,10	4,03	3,96	3,87	3,77	3,67	3,56	3,47	3,33
	0,990	10,6	8,02	6,99	6,42	6,06	5,80	5,61	5,47	5,35	5,26	5,11	4,96	4,81	4,65	4,52	4,31
	0,995	13,6	10,1	8,72	7,96	7,47	7,14	6,88	6,69	6,54	6,42	6,23	6,03	5,83	5,62	5,45	5,19
	0,999	22,9	16,4	13,9	12,6	11,7	11,1	10,7	10,4	10,1	9,89	9,57	9,24	8,90	8,55	8,26	7,81
10	0,9	3,29	2,92	2,73	2,61	2,52	2,46	2,41	2,38	2,35	2,32	2,28	2,24	2,20	2,16	2,12	2,06
	0,95	4,96	4,10	3,71	3,48	3,33	3,22	3,14	3,07	3,02	2,98	2,91	2,85	2,77	2,70	2,64	2,54
	0,975	6,94	5,46	4,83	4,47	4,24	4,07	3,95	3,85	3,78	3,72	3,62	3,52	3,42	3,31	3,22	3,08
	0,990	10,0	7,56	6,55	5,99	5,64	5,39	5,20	5,06	4,94	4,85	4,71	4,56	4,41	4,25	4,12	3,91
	0,995	12,8	9,43	8,08	7,34	6,87	6,54	6,30	6,12	5,97	5,85	5,66	5,47	5,27	5,07	4,90	4,64
	0,999	21,0	14,9	12,6	11,3	10,5	9,93	9,52	9,20	8,96	8,75	8,45	8,13	7,80	7,47	7,19	6,76
11	0,9	3,23	2,86	2,66	2,54	2,45	2,39	2,34	2,30	2,27	2,25	2,21	2,17	2,12	2,08	2,04	1,97
	0,95	4,84	3,98	3,59	3,36	3,20	3,09	3,01	2,95	2,90	2,85	2,79	2,72	2,65	2,57	2,51	2,40
	0,975	6,72	5,26	4,63	4,28	4,04	3,88	3,76	3,66	3,59	3,53	3,43	3,33	3,23	3,12	3,03	2,88
	0,990	9,65	7,21	6,22	5,67	5,32	5,07	4,89	4,74	4,63	4,54	4,40	4,25	4,10	3,94	3,81	3,60
	0,995	12,2	8,91	7,60	6,88	6,42	6,10	5,86	5,68	5,54	5,42	5,24	5,05	4,86	4,65	4,49	4,23
	0,999	19,7	13,8	11,6	10,3	9,58	9,05	8,66	8,35	8,12	7,92	7,63	7,32	7,01	6,68	6,42	6,00
12	0,9	3,18	2,81	2,61	2,48	2,39	2,33	2,28	2,24	2,21	2,19	2,15	2,10	2,06	2,01	1,97	1,90
	0,95	4,75	3,89	3,49	3,26	3,11	3,00	2,91	2,85	2,80	2,75	2,69	2,62	2,54	2,47	2,40	2,30
	0,975	6,55	5,10	4,47	4,12	3,89	3,73	3,61	3,51	3,44	3,37	3,28	3,18	3,07	2,96	2,87	2,72
	0,990	9,33	6,93	5,95	5,41	5,06	4,82	4,64	4,50	4,39	4,30	4,16	4,01	3,86	3,70	3,57	3,36
	0,995	11,8	8,51	7,23	6,52	6,07	5,76	5,52	5,35	5,20	5,09	4,91	4,72	4,53	4,33	4,17	3,90
	0,999	18,6	13,0	10,8	9,63	8,89	8,38	8,00	7,71	7,48	7,29	7,00	6,71	6,40	6,09	5,83	5,42
13	0,9	3,14	2,76	2,56	2,43	2,35	2,28	2,23	2,20	2,16	2,14	2,10	2,05	2,01	1,96	1,92	1,85
	0,95	4,67	3,81	3,41	3,18	3,03	2,92	2,83	2,77	2,71	2,67	2,60	2,53	2,46	2,38	2,31	2,21
	0,975	6,41	4,97	4,35	4,00	3,77	3,60	3,48	3,39	3,31	3,25	3,15	3,05	2,95	2,84	2,74	2,60
	0,990	9,07	6,70	5,74	5,21	4,86	4,62	4,44	4,30	4,19	4,10	3,96	3,82	3,66	3,51	3,38	3,17
	0,995	11,4	8,19	6,93	6,23	5,79	5,48	5,25	5,08	4,94	4,82	4,64	4,46	4,27	4,07	3,91	3,65
	0,999	17,8	12,3	10,2	9,07	8,35	7,86	7,49	7,21	6,98	6,80	6,52	6,23	5,93	5,63	5,37	4,97
14	0,9	3,10	2,73	2,52	2,39	2,31	2,24	2,19	2,15	2,12	2,10	2,05	2,01	1,96	1,91	1,87	1,80
	0,95	4,60	3,74	3,34	3,11	2,96	2,85	2,76	2,70	2,65	2,60	2,53	2,46	2,39	2,31	2,24	2,13
	0,975	6,30	4,86	4,24	3,89	3,66	3,50	3,38	3,29	3,21	3,15	3,05	2,95	2,84	2,73	2,64	2,49
	0,990	8,86	6,51	5,56	5,04	4,69	4,46	4,28	4,14	4,03	3,94	3,80	3,66	3,51	3,35	3,22	3,00
	0,995	11,1	7,92	6,68	6,00	5,56	5,26	5,03	4,86	4,72	4,60	4,43	4,25	4,06	3,86	3,70	3,44
	0,999	17,1	11,8	9,73	8,62	7,92	7,44	7,08	6,80	6,58	6,40	6,13	5,85	5,56	5,25	5,00	4,60

Table 4 — Quantiles of the *F*-distribution *(continued)*

f_2	q	f_1 = 1	2	3	4	5	6	7	8	9	10	12	15	20	30	50	∞
15	0,9	3,07	2,70	2,49	2,36	2,27	2,21	2,16	2,12	2,09	2,06	2,02	1,97	1,92	1,87	1,83	1,76
	0,95	4,54	3,68	3,29	3,06	2,90	2,79	2,71	2,64	2,59	2,54	2,48	2,40	2,33	2,25	2,18	2,07
	0,975	6,20	4,77	4,15	3,80	3,58	3,41	3,29	3,20	3,12	3,06	2,96	2,86	2,76	2,64	2,55	2,40
	0,990	8,68	6,36	5,42	4,89	4,56	4,32	4,14	4,00	3,89	3,80	3,67	3,52	3,37	3,21	3,08	2,87
	0,995	10,8	7,70	6,48	5,80	5,37	5,07	4,85	4,67	4,54	4,42	4,25	4,07	3,88	3,69	3,52	3,26
	0,999	16,6	11,3	9,34	8,25	7,57	7,09	6,74	6,47	6,26	6,08	5,81	5,54	5,25	4,95	4,70	4,31
16	0,9	3,05	2,67	2,46	2,33	2,24	2,18	2,13	2,09	2,06	2,03	1,99	1,94	1,89	1,84	1,79	1,72
	0,95	4,49	3,63	3,24	3,01	2,85	2,74	2,66	2,59	2,54	2,49	2,42	2,35	2,28	2,19	2,12	2,01
	0,975	6,12	4,69	4,03	3,73	3,50	3,34	3,22	3,12	3,05	2,99	2,89	2,79	2,68	2,57	2,47	2,32
	0,990	8,53	6,23	5,29	4,77	4,44	4,20	4,03	3,89	3,78	3,69	3,55	3,41	3,26	3,10	2,97	2,75
	0,995	10,6	7,51	6,30	5,64	5,21	4,91	4,69	4,52	4,38	4,27	4,10	3,92	3,73	3,54	3,37	3,11
	0,999	16,1	11,0	9,01	7,94	7,27	6,80	6,46	6,19	5,98	5,81	5,55	5,27	4,99	4,70	4,45	4,06
17	0,9	3,03	2,64	2,44	2,31	2,22	2,15	2,10	2,06	2,03	2,00	1,96	1,91	1,86	1,81	1,76	1,69
	0,95	4,45	3,59	3,20	2,96	2,81	2,70	2,61	2,55	2,49	2,45	2,38	2,31	2,23	2,15	2,08	1,96
	0,975	6,04	4,62	4,01	3,66	3,44	3,28	3,16	3,06	2,98	2,92	2,82	2,72	2,62	2,50	2,41	2,25
	0,990	8,40	6,11	5,18	4,67	4,34	4,10	3,93	3,79	3,68	3,59	3,46	3,31	3,16	3,00	2,87	2,65
	0,995	10,4	7,35	6,16	5,50	5,07	4,78	4,56	4,39	4,25	4,14	3,97	3,79	3,61	3,41	3,25	2,98
	0,999	15,7	10,7	8,73	7,68	7,02	6,56	6,22	5,96	5,75	5,58	5,32	5,05	4,78	4,48	4,24	3,85
18	0,9	3,01	2,62	2,42	2,29	2,20	2,13	2,08	2,04	2,00	1,98	1,93	1,89	1,84	1,78	1,74	1,66
	0,95	4,41	3,55	3,16	2,93	2,77	2,66	2,58	2,51	2,46	2,41	2,34	2,27	2,19	2,11	2,04	1,92
	0,975	5,98	4,56	3,95	3,61	3,38	3,22	3,10	3,01	2,93	2,87	2,77	2,67	2,56	2,44	2,35	2,19
	0,990	8,29	6,01	5,09	4,58	4,25	4,01	3,84	3,71	3,60	3,51	3,37	3,23	3,08	2,92	2,78	2,57
	0,995	10,2	7,21	6,03	5,37	4,96	4,66	4,44	4,28	4,14	4,03	3,86	3,68	3,50	3,30	3,14	2,87
	0,999	15,4	10,4	8,49	7,46	6,81	6,35	6,02	5,76	5,56	5,39	5,13	4,87	4,59	4,30	4,06	3,67
19	0,9	2,99	2,61	2,40	2,27	2,18	2,11	2,06	2,02	1,98	1,96	1,91	1,86	1,81	1,76	1,71	1,63
	0,95	4,38	3,52	3,13	2,90	2,74	2,63	2,54	2,48	2,42	2,38	2,31	2,23	2,16	2,07	2,00	1,88
	0,975	5,92	4,51	3,90	3,56	3,33	3,17	3,05	2,96	2,88	2,82	2,72	2,62	2,51	2,39	2,30	2,13
	0,990	8,18	5,93	5,01	4,50	4,17	3,94	3,77	3,63	3,52	3,43	3,30	3,15	3,00	2,84	2,71	2,49
	0,995	10,1	7,09	5,92	5,27	4,85	4,56	4,34	4,18	4,04	3,93	3,76	3,59	3,40	3,21	3,04	2,78
	0,999	15,1	10,2	8,28	7,27	6,62	6,18	5,85	5,59	5,39	5,22	4,97	4,70	4,43	4,14	3,90	3,51
20	0,9	2,97	2,59	2,38	2,25	2,16	2,09	2,04	2,00	1,96	1,94	1,89	1,84	1,79	1,74	1,69	1,61
	0,95	4,35	3,49	3,10	2,87	2,71	2,60	2,51	2,45	2,39	2,35	2,28	2,20	2,12	2,04	1,97	1,84
	0,975	5,87	4,46	3,86	3,51	3,29	3,13	3,01	2,91	2,84	2,77	2,68	2,57	2,46	2,35	2,25	2,09
	0,990	8,10	5,85	4,94	4,43	4,10	3,87	3,70	3,56	3,46	3,37	3,23	3,09	2,94	2,78	2,64	2,42
	0,995	9,94	6,99	5,82	5,17	4,76	4,47	4,26	4,09	3,96	3,85	3,68	3,50	3,32	3,12	2,96	2,69
	0,999	14,8	9,95	8,10	7,10	6,46	6,02	5,69	5,44	5,24	5,08	4,82	4,56	4,29	4,00	3,77	3,38
21	0,9	2,96	2,57	2,36	2,23	2,14	2,08	2,02	1,98	1,95	1,92	1,87	1,83	1,78	1,72	1,67	1,59
	0,95	4,32	3,47	3,07	2,84	2,68	2,57	2,49	2,42	2,37	2,32	2,25	2,18	2,10	2,01	1,94	1,81
	0,975	5,83	4,42	3,82	3,48	3,25	3,09	2,97	2,87	2,80	2,73	2,64	2,53	2,42	2,31	2,21	2,04
	0,990	8,02	5,78	4,87	4,37	4,04	3,81	3,64	3,51	3,40	3,31	3,17	3,03	2,88	2,72	2,58	2,36
	0,995	9,83	6,89	5,73	5,09	4,68	4,39	4,18	4,01	3,88	3,77	3,60	3,43	3,24	3,05	2,88	2,61
	0,999	14,6	9,77	7,94	6,95	6,32	5,88	5,56	5,31	5,11	4,95	4,70	4,44	4,17	3,88	3,64	3,26

Table 4 — Quantiles of the F-distribution *(continued)*

f_2	q	f_1 = 1	2	3	4	5	6	7	8	9	10	12	15	20	30	50	∞
22	0,9	2,95	2,56	2,35	2,22	2,13	2,06	2,01	1,97	1,93	1,90	1,86	1,81	1,76	1,70	1,65	1,57
	0,95	4,30	3,44	3,05	2,82	2,66	2,55	2,46	2,40	2,34	2,30	2,23	2,15	2,07	1,98	1,91	1,78
	0,975	5,79	4,38	3,78	3,44	3,22	3,05	2,93	2,84	2,76	2,70	2,60	2,50	2,39	2,27	2,17	2,00
	0,990	7,95	5,72	4,82	4,31	3,99	3,76	3,59	3,45	3,35	3,26	3,12	2,98	2,83	2,67	2,53	2,31
	0,995	9,73	6,81	5,65	5,02	4,61	4,32	4,11	3,94	3,81	3,70	3,54	3,36	3,18	2,98	2,82	2,55
	0,999	14,4	9,61	7,80	6,81	6,19	5,76	5,44	5,19	4,99	4,83	4,58	4,33	4,06	3,78	3,54	3,15
23	0,9	2,94	2,55	2,34	2,21	2,11	2,05	1,99	1,95	1,92	1,89	1,84	1,80	1,74	1,69	1,64	1,55
	0,95	4,28	3,42	3,03	2,80	2,64	2,53	2,44	2,37	2,32	2,27	2,20	2,13	2,05	1,96	1,88	1,76
	0,975	5,75	4,35	3,75	3,41	3,18	3,02	2,90	2,81	2,73	2,67	2,57	2,47	2,36	2,24	2,14	1,97
	0,990	7,88	5,66	4,76	4,26	3,94	3,71	3,54	3,41	3,30	3,21	3,07	2,93	2,78	2,62	2,48	2,26
	0,995	9,63	6,73	5,58	4,95	4,54	4,26	4,05	3,88	3,75	3,64	3,47	3,30	3,12	2,92	2,76	2,48
	0,999	14,2	9,47	7,64	6,70	6,08	5,65	5,33	5,09	4,89	4,73	4,48	4,23	3,96	3,68	3,44	3,05
24	0,9	2,93	2,54	2,33	2,19	2,10	2,04	1,98	1,94	1,91	1,88	1,83	1,78	1,73	1,67	1,62	1,53
	0,95	4,26	3,40	3,01	2,78	2,62	2,51	2,42	2,36	2,30	2,25	2,18	2,11	2,03	1,94	1,86	1,73
	0,975	5,72	4,32	3,72	3,38	3,15	2,99	2,87	2,78	2,70	2,64	2,54	2,44	2,33	2,21	2,11	1,94
	0,990	7,82	5,61	4,72	4,22	3,90	3,67	3,50	3,36	3,26	3,17	3,03	2,89	2,74	2,58	2,44	2,21
	0,995	9,55	6,66	5,52	4,89	4,49	4,20	3,99	3,83	3,69	3,59	3,42	3,25	3,06	2,87	2,70	2,43
	0,999	14,0	9,34	7,55	6,59	5,98	5,55	5,23	4,99	4,80	4,64	4,39	4,14	3,87	3,59	3,36	2,97
25	0,9	2,92	2,53	2,32	2,18	2,09	2,02	1,97	1,93	1,89	1,87	1,82	1,77	1,72	1,66	1,61	1,52
	0,95	4,24	3,39	2,99	2,76	2,60	2,49	2,40	2,34	2,28	2,24	2,16	2,09	2,01	1,92	1,84	1,71
	0,975	5,69	4,29	3,69	3,35	3,13	2,97	2,85	2,75	2,68	2,61	2,51	2,41	2,30	2,18	2,08	1,91
	0,990	7,77	5,57	4,68	4,18	3,85	3,63	3,46	3,32	3,22	3,13	2,99	2,85	2,70	2,54	2,40	2,17
	0,995	9,48	6,60	5,46	4,84	4,43	4,15	3,94	3,78	3,64	3,54	3,37	3,20	3,01	2,82	2,65	2,38
	0,999	13,9	9,22	7,45	6,49	5,89	5,46	5,15	4,91	4,71	4,56	4,31	4,06	3,79	3,52	3,28	2,89
30	0,9	2,88	2,49	2,28	2,14	2,05	1,98	1,93	1,88	1,85	1,82	1,77	1,72	1,67	1,61	1,55	1,46
	0,95	4,17	3,32	2,92	2,69	2,53	2,42	2,33	2,27	2,21	2,16	2,09	2,01	1,93	1,84	1,76	1,62
	0,975	5,57	4,18	3,59	3,25	3,03	2,87	2,75	2,65	2,57	2,51	2,41	2,31	2,20	2,07	1,97	1,79
	0,990	7,56	5,39	4,51	4,02	3,70	3,47	3,30	3,17	3,07	2,98	2,84	2,70	2,55	2,39	2,25	2,01
	0,995	9,18	6,35	5,24	4,62	4,23	3,95	3,74	3,58	3,45	3,34	3,18	3,01	2,82	2,63	2,46	2,18
	0,999	13,3	8,77	7,05	6,12	5,53	5,12	4,82	4,58	4,39	4,24	4,00	3,75	3,49	3,22	2,98	2,59
35	0,9	2,85	2,46	2,25	2,11	2,02	1,95	1,90	1,85	1,82	1,79	1,74	1,69	1,63	1,57	1,51	1,41
	0,95	4,12	3,27	2,87	2,64	2,49	2,37	2,29	2,22	2,16	2,11	2,04	1,96	1,88	1,79	1,70	1,56
	0,975	5,48	4,11	3,52	3,18	2,96	2,80	2,68	2,58	2,50	2,44	2,34	2,23	2,12	2,00	1,89	1,70
	0,990	7,42	5,27	4,40	3,91	3,59	3,37	3,20	3,07	2,96	2,88	2,74	2,60	2,44	2,28	2,14	1,89
	0,995	8,98	6,19	5,39	4,48	4,09	3,81	3,61	3,45	3,32	3,21	3,05	2,88	2,69	2,50	2,33	2,04
	0,999	10,9	8,47	6,79	5,88	5,30	4,89	4,59	4,36	4,18	4,03	3,79	3,55	3,29	3,02	2,78	2,38
40	0,9	2,84	2,44	2,23	2,09	2,00	1,93	1,87	1,83	1,79	1,76	1,71	1,66	1,61	1,54	1,48	1,38
	0,95	4,08	3,23	2,84	2,61	2,45	2,34	2,25	2,18	2,12	2,08	2,00	1,92	1,84	1,74	1,66	1,51
	0,975	5,42	4,05	3,46	3,13	2,90	2,74	2,62	2,53	2,45	2,39	2,29	2,18	2,07	1,94	1,83	1,64
	0,990	7,31	5,18	4,31	3,83	3,51	3,29	3,12	2,99	2,89	2,80	2,66	2,52	2,37	2,20	2,06	1,80
	0,995	8,83	6,07	4,98	4,37	3,99	3,71	3,51	3,35	3,22	3,12	2,95	2,78	2,60	2,40	2,23	1,93
	0,999	12,6	8,25	6,59	5,70	5,13	4,73	4,44	4,21	4,02	3,87	3,64	3,40	3,14	2,87	2,64	2,23

Table 4 — Quantiles of the F-distribution *(concluded)*

f_2	q	f_1 = 1	2	3	4	5	6	7	8	9	10	12	15	20	30	50	∞
45	0,9	2,82	2,42	2,21	2,07	1,98	1,91	1,85	1,81	1,77	1,74	1,70	1,64	1,58	1,52	1,46	1,35
	0,95	4,06	3,20	2,81	2,58	2,42	2,31	2,22	2,15	2,10	2,05	1,97	1,89	1,81	1,71	1,63	1,47
	0,975	5,38	4,01	3,42	3,09	2,86	2,70	2,58	2,49	2,41	2,35	2,25	2,14	2,03	1,90	1,79	1,59
	0,990	7,23	5,11	4,25	3,77	3,45	3,23	3,07	2,94	2,83	2,74	2,61	2,46	2,31	2,14	2,00	1,74
	0,995	8,71	5,97	4,89	4,29	3,91	3,64	3,43	3,28	3,15	3,04	2,88	2,71	2,53	2,33	2,16	1,85
	0,999	12,4	8,09	6,45	5,56	5,00	4,61	4,32	4,09	3,91	3,76	3,53	3,29	3,04	2,76	2,53	2,12
50	0,9	2,81	2,41	2,20	2,06	1,97	1,90	1,84	1,80	1,76	1,73	1,68	1,63	1,57	1,50	1,44	1,33
	0,95	4,03	3,18	2,79	2,56	2,40	2,29	2,20	2,13	2,07	2,03	1,95	1,87	1,78	1,69	1,60	1,44
	0,975	5,34	3,97	3,39	3,05	2,83	2,67	2,55	2,46	2,38	2,32	2,22	2,11	1,99	1,87	1,75	1,55
	0,990	7,17	5,06	4,20	3,72	3,41	3,19	3,02	2,89	2,78	2,70	2,56	2,42	2,27	2,10	1,95	1,68
	0,995	8,63	5,90	4,83	4,23	3,85	3,58	3,38	3,22	3,09	2,99	2,82	2,65	2,47	2,27	2,10	1,79
	0,999	12,2	7,96	6,34	5,46	4,90	4,51	4,22	4,00	3,82	3,67	3,44	3,20	2,95	2,68	2,44	2,03
60	0,9	2,79	2,39	2,18	2,04	1,95	1,87	1,82	1,77	1,74	1,71	1,66	1,60	1,54	1,48	1,41	1,29
	0,95	4,00	3,15	2,76	2,53	2,37	2,25	2,17	2,10	2,04	1,99	1,92	1,84	1,75	1,65	1,56	1,39
	0,975	5,29	3,93	3,34	3,01	2,79	2,63	2,51	2,41	2,33	2,27	2,17	2,06	1,94	1,82	1,70	1,48
	0,990	7,08	4,98	4,13	3,65	3,34	3,12	2,95	2,82	2,72	2,63	2,50	2,35	2,20	2,03	1,88	1,60
	0,995	8,49	5,79	4,73	4,14	3,76	3,49	3,29	3,13	3,01	2,90	2,74	2,57	2,39	2,19	2,01	1,69
	0,999	12,0	7,77	6,17	5,31	4,76	4,37	4,09	3,86	3,69	3,54	3,32	3,08	2,83	2,55	2,32	1,89
80	0,9	2,77	2,37	2,15	2,02	1,92	1,85	1,79	1,75	1,71	1,68	1,63	1,57	1,51	1,44	1,38	1,24
	0,95	3,96	3,11	2,72	2,49	2,33	2,21	2,13	2,06	2,00	1,95	1,88	1,79	1,70	1,60	1,51	1,32
	0,975	5,22	3,86	3,28	2,95	2,73	2,57	2,45	2,35	2,28	2,21	2,11	2,00	1,88	1,75	1,63	1,40
	0,990	6,96	4,88	4,04	3,56	3,26	3,04	2,87	2,74	2,64	2,55	2,42	2,27	2,12	1,94	1,79	1,49
	0,995	8,33	5,67	4,61	4,03	3,65	3,39	3,19	3,03	2,91	2,80	2,64	2,47	2,29	2,08	1,90	1,56
	0,999	11,7	7,54	5,97	5,12	4,58	4,20	3,92	3,70	3,53	3,39	3,16	2,93	2,68	2,41	2,16	1,72
100	0,9	2,76	2,36	2,14	2,00	1,91	1,83	1,78	1,73	1,69	1,66	1,61	1,56	1,49	1,42	1,35	1,21
	0,95	3,94	3,09	2,70	2,46	2,31	2,19	2,10	2,03	1,97	1,93	1,85	1,77	1,68	1,57	1,48	1,28
	0,975	5,18	3,83	3,25	2,92	2,70	2,54	2,42	2,32	2,24	2,18	2,08	1,97	1,85	1,71	1,59	1,35
	0,990	6,90	4,82	3,98	3,51	3,21	2,99	2,82	2,69	2,59	2,50	2,37	2,22	2,07	1,89	1,74	1,43
	0,995	8,24	5,59	4,54	3,96	3,59	3,33	3,13	2,97	2,85	2,74	2,58	2,41	2,23	2,02	1,84	1,49
	0,999	11,5	7,41	5,86	5,02	4,48	4,11	3,83	3,61	3,44	3,30	3,07	2,84	2,59	2,32	2,08	1,62
120	0,9	2,75	2,35	2,13	1,99	1,90	1,82	1,77	1,72	1,68	1,65	1,60	1,55	1,48	1,41	1,34	1,19
	0,95	3,92	3,07	2,68	2,45	2,29	2,18	2,09	2,02	1,96	1,91	1,83	1,75	1,66	1,55	1,46	1,25
	0,975	5,15	3,80	3,23	2,89	2,67	2,52	2,39	2,30	2,22	2,16	2,05	1,94	1,82	1,69	1,56	1,31
	0,990	6,85	4,79	3,95	3,48	3,17	2,96	2,79	2,66	2,56	2,47	2,34	2,19	2,03	1,86	1,70	1,38
	0,995	8,18	5,54	4,50	3,92	3,55	3,28	3,09	2,93	2,81	2,71	2,54	2,37	2,19	1,98	1,80	1,43
	0,999	11,4	7,32	5,78	4,95	4,42	4,04	3,77	3,55	3,38	3,24	3,02	2,78	2,53	2,26	2,02	1,54
∞	0,9	2,71	2,30	2,08	1,94	1,85	1,77	1,72	1,67	1,63	1,60	1,55	1,49	1,42	1,34	1,26	1,00
	0,95	3,84	3,00	2,60	2,37	2,21	2,10	2,01	1,94	1,88	1,83	1,75	1,67	1,57	1,46	1,35	1,00
	0,975	5,02	3,69	3,12	2,79	2,57	2,41	2,29	2,19	2,11	2,05	1,94	1,83	1,71	1,57	1,43	1,00
	0,990	6,63	4,61	3,78	3,32	3,02	2,80	2,64	2,51	2,41	2,32	2,18	2,04	1,88	1,70	1,52	1,00
	0,995	7,88	5,30	4,28	3,72	3,35	3,09	2,90	2,74	2,62	2,52	2,36	2,19	2,00	1,79	1,59	1,00
	0,999	10,8	6,91	5,42	4,62	4,10	3,74	3,47	3,27	3,10	2,96	2,74	2,51	2,27	1,99	1,73	1,00

NOTE — $F_\alpha(f_1, f_2) = 1/F_{(1-\alpha)}(f_2, f_1)$

Table 5 — Common size of the two samples, $n_1 = n_2$, to obtain a given power, $1-\beta$ (= 0,9; 0,8 or 0,5), the tests being one-sided with H_0: $p_1 \leqslant p_2$ and $\alpha = 0{,}05$ for various pairs of p_1 and p_2 with $p_1 > p_2$

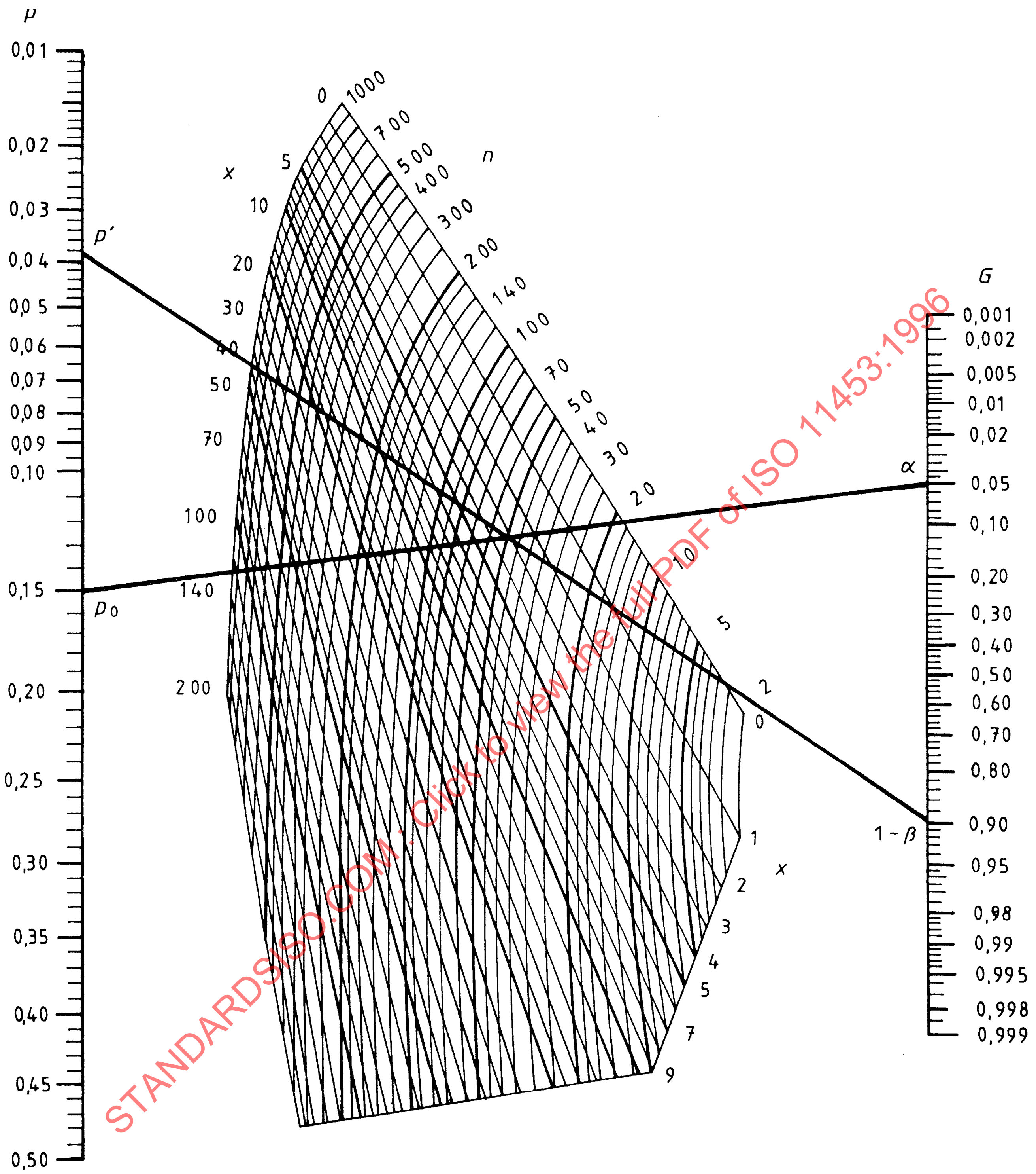
p_2	p_1									
	0,95	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1
0,9	503 371 184									
0,8	89 67 38	232 173 87								
0,7	42 34 19	74 56 31	338 249 121							
0,6	25 20 12	39 30 17	97 73 37	408 302 143						
0,5	18 14 9	25 19 11	47 36 19	111 84 43	445 321 155					
0,4	13 11 7	17 13 9	30 23 12	53 41 22	116 85 43	445 321 155				
0,3	10 9 6	12 10 6	18 15 9	31 23 12	53 41 22	111 84 43	408 302 143			
0,2	8 6 5	10 8 5	12 10 6	18 15 9	30 23 12	47 36 19	97 73 37	338 249 121		
0,1	6 5 3	8 6 3	10 8 5	12 10 6	17 13 9	25 19 11	39 30 17	74 56 31	232 173 87	
0,05	5 5 3	6 5 3	8 6 5	10 9 6	13 11 7	18 14 9	25 20 12	42 34 19	89 67 38	503 371 184

NOTE — In each cell of the table the upper figure is the common sample size, $n_1 = n_2$, giving $1-\beta = 0{,}9$, the middle figure and the lower figure giving $1-\beta = 0{,}8$ and 0,5. For instance, if p_1 = 0,9 and p_2 = 0,8, one must take $n_1 = n_2$ = 232 units to have $1-\beta = 0{,}9$, $n_1 = n_2 = 173$ to have $1-\beta = 0{,}8$ and only $n_1 = n_2 = 87$ to have $1-\beta = 0{,}5$.

Table 6 — Common size of the two samples, $n_1 = n_2$, to obtain a given power, $1-\beta$ (= 0,9; 0,8 or 0,5), the tests being one-sided with H_0: $p_1 \leqslant p_2$ and $\alpha = 0{,}01$ for various pairs of p_1 and p_2 with $p_1 > p_2$

p_2	p_1									
	0,95	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1
0,9	745 583 333									
0,8	130 101 61	344 269 155								
0,7	60 49 32	108 86 52	503 393 221							
0,6	37 31 18	56 46 27	143 113 66	609 475 265						
0,5	25 20 14	35 29 18	69 55 34	163 129 73	667 519 285					
0,4	18 16 10	24 20 13	42 34 21	77 60 35	171 137 78	667 519 285				
0,3	14 12 9	18 15 10	28 22 13	43 35 22	77 60 35	163 129 73	609 475 265			
0,2	12 9 6	13 12 8	18 16 9	28 22 13	42 34 21	69 55 34	143 113 66	503 393 221		
0,1	9 8 6	9 9 6	13 12 8	18 15 10	24 20 13	35 29 18	56 46 27	108 86 52	344 269 155	
0,05	8 6 5	9 8 6	12 9 6	14 12 9	18 16 10	25 20 14	37 31 18	60 49 32	130 101 61	745 583 333

NOTE — In each cell of the table the upper figure is the common sample size, $n_1 = n_2$, giving $1-\beta = 0{,}9$, the middle figure and the lower figure giving $1-\beta = 0{,}8$ and 0,5. For instance, if $p_1 = 0{,}9$ and $p_2 = 0{,}8$, one must take $n_1 = n_2 = 344$ units to have $1-\beta = 0{,}9$, and only $n_1 = n_2 = 155$ to have $1-\beta = 0{,}5$.



NOTE — If $p < 0{,}01$ mark λp instead of p on the p-scale and multiply the values on the n-scale by λ. Determine λ from $0{,}01/p$ rounded to the next larger integer.

Figure 2 — Larson nomogragh of binomial distribution

Annex A
(normative)

Computation of the operating characteristic of the test according to the B forms

A.1 One-sided test with H_0: $p \geqslant p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value $p_0 =$
Significance level chosen: $\alpha =$
Sample size: $n =$
Proportion for which the probability of not rejecting H_0 is to be computed: $p' =$

If the critical value(s) corresponding to n and p_0 for the specified significance level α is (are) not known, it (they) is (are) to be computed according to the B forms:

$C_{\mathrm{l,o}} =$

Determination of the probability of not rejecting H_0, P_a and resulting values:

If H_0 is true, the probability of the error of the first kind is $1 - P_a$. The achieved significance level, α', is equal to the probability of the error of the first kind when $p' = p_0$.

If the alternative hypothesis is true, the probability of the error of the second kind is P_a.

Computation:

$$u' = 2\left[\sqrt{(C_{\mathrm{l,o}} + 1)(1 - p')} - \sqrt{(n - C_{\mathrm{l,o}})p'}\right] =$$

Read from table 3: $\Phi(u') =$

Results:

$$P_a = 1 - \Phi(u') =$$

if $p' = p_0$: $\alpha' = \Phi(u') =$

if $p' < p_0$: $\beta = 1 - \Phi(u') =$

A.2 One-sided test with H_0: $p \leqslant p_0$

Characteristic: Determination procedure: Items: Criterion for the identification of target items: Notes:
Given value p_0 = Significance level chosen: α = Sample size: n = Proportion for which the probability of not rejecting H_0 is to be computed: p' = If the critical value(s) corresponding to n and p_0 for the specified significance level α is (are) not known, it (they) is (are) to be computed according to the B forms: $C_{u,o}$ =
Determination of the probability of not rejecting H_0, P_a and resulting values: If H_0 is true, the probability of the error of the first kind is $1-P_a$. The achieved significance level, α', is equal to the probability of the error of the first kind when $p'=p_0$. If the alternative hypothesis is true, the probability of the error of the second kind is P_a. Computation: $u''=2\left[\sqrt{C_{u,o}(1-p')}-\sqrt{(n-C_{u,o}+1)p'}\right]=$ Read from table 3: $\Phi(u'')=$ Results: $P_a=\Phi(u'')=$ if $p'=p_0$: $\alpha'=1-\Phi(u'')=$ if $p'>p_0$: $\beta=\Phi(u'')=$

A.3 Two-sided test with H_0: $p = p_0$

Characteristic:
Determination procedure:
Items:
Criterion for the identification of target items:
Notes:

Given value $p_0 =$
Significance level chosen: $\alpha =$
Sample size: $n =$
Proportion for which the probability of not rejecting H_0 is to be computed: $p' =$

If the critical value(s) corresponding to n and p_0 for the specified significance level α is (are) not known it (they) is (are) to be computed according to the B forms:

$C_{l,t} =$ $\qquad\qquad$ $C_{u,t} =$

Determination of the probability of not rejecting H_0, P_a and resulting values:

If H_0 is true, the probability of the error of the first kind is $1 - P_a$. The achieved significance level, α', is equal to the probability of the error of the first kind when $p' = p_0$.

If the alternative hypothesis is true, the probability of the error of the second kind is P_a.

Computation:

$$u' = 2\left[\sqrt{(C_{l,t}+1)(1-p')} - \sqrt{(n-C_{l,t})p'}\right] =$$

$$u'' = 2\left[\sqrt{C_{u,t}(1-p')} - \sqrt{(n-C_{u,t}+1)p'}\right] =$$

Read from table 3: $\Phi(u') =$

$\Phi(u'') =$

Results:

$P_a = \Phi(u'') - \Phi(u') =$

if $p' = p_0$: $\alpha' = 1 - \Phi(u'') + \Phi(u') =$

if $p' \neq p_0$: $\beta = \Phi(u'') - \Phi(u') =$

Annex B
(informative)

Examples of completed forms

B.1 A forms

B.1.1 Example 1: Form A-2 — One-sided, with lower limit confidence interval for the proportion p

Characteristic: Existence of video recorders in homes
Determination procedure: Interviews
Items: Homes in a defined area
Criterion for the identification of target items: At least one video recorder existing
Notes:

Confidence level chosen: $1-\alpha = 0{,}95$
Sample size: $n = 20$
Number of target items in the sample: $x = 14$

Determination of confidence limits:

a) Procedure for $n \leqslant 30$ ☒

1) Case $x = 0$ ☐

$p_{l,o} = 0$

2) Case $x > 0$ ☒

Read the value from table 2 for the known n, $X = n - x$ and $q = 1-\alpha$:

$T_{(1-\alpha)}(n, n-x) = 0{,}508$

Computation:

$p_{l,o} = 1 - T_{(1-\alpha)}(n, n-x) = 0{,}492$

b) Procedure for $n > 30$ ☐

1) Case $x = 0$ ☐

$p_{l,o} = 0$

2) Case $x = n$ ☐

Computation:

$p_{l,o} = \alpha^{1/n} =$

3) Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$

Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,411	0,677	1,353

Computation:

$p_{l,o} = p_* + (1-2p_*)d/(n+1) - u_{1-\alpha}\sqrt{p_*(1-p_*)[1-d/(n+1)]/(n+1)} =$

with $p_* = x/(n+1)$

Result:

$p_{l,o} = 0{,}492 \quad \leqslant p$

B.1.2 Example 2: Form A-3 — Two-sided confidence interval for the proportion p

Characteristic: Existence of video recorders in homes
Determination procedure: Interviews
Items: Homes in a defined area
Criterion for the identification of target items: At least one video recorder existing
Notes:

Confidence level chosen: $1-\alpha = 0{,}99$
Sample size: $n = 90$
Number of target items in the sample: $x = 19$

Determination of confidence limits:

a) Procedure for $n \leqslant 30$ ☐

1) Upper confidence limit

— Case $x = n$ ☐
$p_{u,t} = 1$

— Case $x < n$ ☐

Read the value from table 2 for the known n, $X = x$ and $q = 1-\alpha/2$ (this value is the confidence limit):
$T_{(1-\alpha/2)}(n, x) = p_{u,t} =$

2) Lower confidence limit

— Case $x = 0$ ☐

— Case $x > 0$ ☐

Read the value from table 2 for the known n, $X = n - x$, and $q = 1-\alpha/2$:
$T_{(1-\alpha/2)}(n, n-x) =$
Computation:
$p_{l,t} = 1 - T_{(1-\alpha/2)}(n, n-x) =$

b) Procedure for $n > 30$ ☒

1) Upper confidence limit ☐

— Case $x = 0$ ☐
Computation
$p_{u,t} = 1-(\alpha/2)^{1/n} =$

— Case $x = n$ ☐
$p_{u,t} = 1$

— Case $0 < x < n$ ☒

Read the value from table 3 for $q = 1-\alpha/2$: $u_{1-\alpha/2} = 2{,}576$

Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,677	0,960	1,659

Computation:

$$p_{u,t} = p_* + (1-2p_*)d/(n+1) + u_{1-\alpha/2}\sqrt{p_*(1-p_*)[1-d/(n+1)]/(n+1)} = 0{,}341$$

with $p_* = (x+1)/(n+1)$

2) Lower confidence limit

— Case $x = 0$ ☐

$p_{l,t} = 0$

— Case $x = n$ ☐

Computation

$p_{l,t} = (\alpha/2)^{1/n} =$

— Case $0 < x < n$ ☒

Read the value from table 3 for $q = 1-\alpha/2$: $u_{1-\alpha/2} = 2{,}576$

Read the value of d corresponding to the confidence level chosen:

$1-\alpha$	0,90	0,95	0,99
d	0,677	0,960	1,659

Computation:

$$p_{l,t} = p_* + (1-2p_*)d/(n+1) - u_{1-\alpha/2}\sqrt{p_*[(1-p_*)(1-d)/(n+1)]/(n+1)} = 0{,}111$$

with $p_* = x/(n+1)$

Results:

$p_{l,t} = 0{,}111$; $p_{u,t} = 0{,}341$; $p_{l,t} \leq p \leq p_{u,t}$

B.2 B forms

B.2.1 Example 1: Form B-2 — Comparison of proportion p with a given value p_0 and with one-sided test with H_0: $p \leqslant p_0$

Characteristic: Existence of video recorders in homes
Determination procedure: Interviews
Items: Homes in a defined area
Criterion for the identification of target items: At least one video recorder existing
Notes:

Given value $p_0 = 0{,}48$
Significance level chosen: $\alpha = 0{,}05$
Sample size: $n = 20$
Number of target items found in the sample: $x = 14$

Test procedure:

I The critical value(s) is (are) already known (see 7.2.1 and, if applicable, the determination of critical values below): ☐

$C_{u,o} =$

H_0 is rejected if $x > C_{u,o}$; otherwise it is not rejected.

II The critical value(s) is (are) not known: ☒

a) Case $x \leqslant p_0 n$ ☐

H_0 is not rejected.

b) Case $x > p_0 n$ ☒

1) Procedure for $n \leqslant 30$ ☒

Determine according to form A-2 the one-sided lower confidence limit for n, x and the confidence level $(1-\alpha)$:

$p_{l,o} = 0{,}492$

H_0 is rejected if $p_{l,o} > p_0$; otherwise it is not rejected.

2) Procedure for $n > 30$ ☐

— Case $x = n$ ☐

Computation:

$p_{l,o} = \alpha^{1/n} =$ [see form A-2 b) 2)]

H_0 is rejected if $p_{l,o} > p_0$; otherwise it is not rejected.

— Case $0 < x < n$ ☐

Read the value from table 3 for $q = 1-\alpha$: $u_{1-\alpha} =$

Computation:

$$u_2 = 2\left[\sqrt{x(1-p_0)} - \sqrt{(n-x+1)p_0}\right] =$$

H_0 is rejected if $u_2 > u_{1-\alpha}$; otherwise it is not rejected.

Test result:

H_0 is rejected ☒

H_0 is not rejected ☐

Determination of the critical values:

$C_{u,o}$ is the largest integer x for which the test according to form B-2-II does not lead to the rejection of the null hypothesis. $C_{u,o}$ is to be determined iteratively through repeated application of form B-2-II with different values of x [1)]. Thereby those values of x are to be determined which differ from each other by 1 and one of which leads to the rejection of the null hypothesis while the other does not. If desired, a start value for x, x_{start} can be obtained as follows.

Computations:

np_0, rounded to the next integer, is $x^* = 10$

$p_{u,o}|_{x=x^*} = 0{,}699$ ($p_{u,o}|_{x=x^*}$ from form A-1)

$np_{u,o}|_{x=x^*}$, rounded to the next integer, is $x_{start} = 14$

Interpretation of the tests results from form B-2-II respectively:

for $x \leqslant C_{u,o} = 13$ H_0 is not rejected

for $x \geqslant C_{u,o} + 1 = 14$ H_0 is rejected

Result:

$C_{u,o} = 13$

1) The critical value or one of the critical values, respectively, may not exist for extreme values of p_0 and/or for very small sample sizes n.