
**Lasers and laser-related equipment —
Test methods for laser beam
widths, divergence angles and beam
propagation ratios —**

**Part 2:
General astigmatic beams**

*Lasers et équipements associés aux lasers — Méthodes d'essai des
largeurs du faisceau, angles de divergence et facteurs de limite de
diffraction —*

Partie 2: Faisceaux astigmatiques généraux

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 9, *Laser and electro-optical systems*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 123, *Lasers and photonics*, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

This second edition cancels and replaces the first edition (ISO 11146-2:2005), which has been technically revised. The main changes compared to the previous edition are as follows:

- The terms and definitions were harmonized with the new ISO 11145.
- The "principal axes" were defined more thoroughly and named as x' and y' . Quantities related to the principal axes coordinate system refer to this definition and use x' and y' in their indices.
- The requirements for the integration range for the determination of the second order moments have been relaxed.

A list of all parts in the ISO 11146 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The propagation properties of laser beams can be characterized by ten independent parameters when applying the method of second order moments (see ISO/TR 11146-3). Most laser beams need few parameters for a complete description due to their higher symmetry. Lasers emit beams which are stigmatic or simple astigmatic due to their resonator design.

ISO 11146-1 describes the measurement methods for stigmatic and simple astigmatic beams while this document deals with the measurement procedures for general astigmatic beams. This document is applicable to beams of unknown type. Beam characterization, based on the method of second order moments as described in ISO 11146-1 and this document, is only valid within the paraxial approximation.

The theoretical description of beam characterization and propagation as well as the classification of laser beams is given in ISO/TR 11146-3, which is a Technical Report. The procedures for background subtraction and offset correction are also given in ISO/TR 11146-3.

In ISO 11146, the second order moments of the power (energy) density distribution function are used for the determination of beam widths. If problems are experienced in the direct measurements of these quantities, other indirect methods of measurement of second order moments may be used as long as comparable results are achievable.

In ISO/TR 11146-3, three alternative methods for beam width measurement and their correlation with the method used in this document are described. These methods are:

- variable aperture method;
- moving knife-edge method;
- moving slit method.

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Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

Part 2: General astigmatic beams

1 Scope

This document specifies methods for measuring beam widths (diameter), divergence angles and beam propagation ratios of laser beams. This document is applicable to general astigmatic beams or unknown types of beams. For stigmatic and simple astigmatic beams, ISO 11146-1 is applicable.

Within this document, the description of laser beams is accomplished by means of the second order moments of the Wigner distribution rather than physical quantities such as beam widths and divergence angles. However, these physical quantities are closely related to the second order moments of the Wigner distribution. In ISO/TR 11146-3, formulae are given to calculate all relevant physical quantities from the measured second order moments.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11145, *Optics and photonics — Lasers and laser-related equipment — Vocabulary and symbols*

ISO 11146-1, *Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios — Part 1: Stigmatic and simple astigmatic beams*

EN 61040:1992, *Power and energy measuring detectors, instruments and equipment for laser radiation*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 11145, ISO 11146-1, EN 61040 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

Note The x-, y- and z-axes in the following definitions refer to the laboratory system (as described in [Clause 4](#)). Here and throughout this document the term “power density distribution $E(x,y,z)$ ” refers to continuous wave sources. It might be replaced by “energy density distribution $H(x,y,z)$ ” in case of pulsed sources.

**3.1
generalized beam diameter**

d_g
measure of the extent of the power density distribution of a beam in a cross-section at an axial location z , derived from the second order moments by

$$d_g = 2\sqrt{2} \sqrt{\langle x^2 \rangle + \langle y^2 \rangle} \tag{1}$$

Note 1 to entry: This definition is similar to the beam diameter defined in ISO 11145 or ISO 11146-1. But in this context the definition is not restricted to circular power density distributions.

**3.2
generalized beam waist location**

$z_{0,g}$
position where the *generalized beam diameter* (3.1) reaches its minimum value along the axis of propagation

**3.3
generalized Rayleigh length**

$z_{R,g}$
distance along the beam axis from the generalized beam waist where the generalized beam diameter is a factor of $\sqrt{2}$ larger than the generalized beam waist diameter

**3.4
Wigner distribution**

phase space distribution representing a laser beam in a transverse plane at location z

Note 1 to entry: The Wigner distribution is a function of two spatial and two angular coordinates, giving the amount of beam power propagating through the point (x, y) in the direction (θ_x, θ_y) .

**3.5
spatial first order moments of the Wigner distribution**

$\langle x \rangle, \langle y \rangle$

subset of the first order moments, which can be directly obtained from measured power density distribution by

$$\langle x \rangle(z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) x \, dx \, dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) \, dx \, dy} \tag{2}$$

and

$$\langle y \rangle(z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) y \, dx \, dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) \, dx \, dy} \tag{3}$$

where $E(x,y,z)$ is the power density distribution at the specific plane $z = \text{constant}$

3.6

second order moments of the Wigner distribution

$$\langle x^2 \rangle, \langle y^2 \rangle, \langle xy \rangle, \langle \theta_x^2 \rangle, \langle \theta_y^2 \rangle, \langle \theta_x \theta_y \rangle, \langle x \theta_x \rangle, \langle x \theta_y \rangle, \langle y \theta_x \rangle, \langle y \theta_y \rangle$$

ten second order moments of the *Wigner distribution* (3.4) of the beam at location z

Note 1 to entry: The ten second order moments contain information on the following physical beam properties: beam size and orientation, divergence angles and their orientation, radii of curvature of the phase paraboloid and their orientation and the twist parameter. Details on these relations are given in ISO/TR 11146-3.

Note 2 to entry: In ISO 11146-1, the three spatial second order moments are defined as σ_x^2 , σ_y^2 and σ_{xy}^2 . In this document and ISO/TR 11146-3, the angular brackets are used to emphasize the coordinates of the moments. This means that $\sigma_x^2 = \langle x^2 \rangle$, $\sigma_y^2 = \langle y^2 \rangle$ and $\sigma_{xy}^2 = \langle xy \rangle$.

Note 3 to entry: Three angular moments $\langle \theta_x^2 \rangle$, $\langle \theta_y^2 \rangle$ and $\langle \theta_x \theta_y \rangle$ are independent of z . The other seven second order moments are, in general, functions of z .

3.7

spatial second order moments of the Wigner distribution

$$\langle x^2 \rangle, \langle y^2 \rangle, \langle xy \rangle$$

subset of the second order moments, which can be directly obtained from measured power density distribution by

$$\langle x^2 \rangle (z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) (x - \langle x \rangle)^2 dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy} \quad (4)$$

$$\langle y^2 \rangle (z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) (y - \langle y \rangle)^2 dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy} \quad (5)$$

and

$$\langle xy \rangle (z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) (x - \langle x \rangle)(y - \langle y \rangle) dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy} \quad (6)$$

3.8

beam matrix

P

symmetric and positive definite 4×4 matrix containing all ten *second order moments of the Wigner distribution* (3.6) and its elements and given by

$$P = \begin{bmatrix} \langle x^2 \rangle & \langle xy \rangle & \langle x \theta_x \rangle & \langle x \theta_y \rangle \\ \langle xy \rangle & \langle y^2 \rangle & \langle y \theta_x \rangle & \langle y \theta_y \rangle \\ \langle x \theta_x \rangle & \langle y \theta_x \rangle & \langle \theta_x^2 \rangle & \langle \theta_x \theta_y \rangle \\ \langle x \theta_y \rangle & \langle y \theta_y \rangle & \langle \theta_x \theta_y \rangle & \langle \theta_y^2 \rangle \end{bmatrix} \quad (7)$$

**3.9
effective beam propagation ratio**

M_{eff}^2
invariant quantity related to the focusability of a general astigmatic beam, defined as:

$$M_{\text{eff}}^2 = \frac{4\pi}{\lambda} [\det(\mathbf{P})]^{\frac{1}{4}} \tag{8}$$

where $\det(\mathbf{P})$ is the determinant of matrix \mathbf{P}

Note 1 to entry: The effective beam propagation ratio M_{eff}^2 is an invariant related to the volume that the beam occupies in the four-dimensional phase space (two lateral spatial and two lateral angular dimensions) and thus a measure for the focusability of the beam.

Note 2 to entry: For simple astigmatic beams, the effective beam propagation ratio is the geometric mean of the beam propagation ratios of the principal axes of the beam: $M_{\text{eff}}^2 = \sqrt{M_x^2 \times M_y^2}$. For stigmatic beams $M_{\text{eff}}^2 = M^2$.

**3.10
intrinsic astigmatism**

a
degree of how close to a stigmatic beam the general astigmatic beam can be transformed by using lenses and free space propagation

$$a = \frac{8\pi^2}{\lambda^2} \left[(\langle x^2 \rangle \langle \theta_x^2 \rangle - \langle x\theta_x \rangle^2) + (\langle y^2 \rangle \langle \theta_y^2 \rangle - \langle y\theta_y \rangle^2) + 2(\langle xy \rangle \langle \theta_x \theta_y \rangle - \langle x\theta_y \rangle \langle y\theta_x \rangle) \right] - (M_{\text{eff}}^2)^2 \geq 0 \tag{9}$$

Note 1 to entry: Beams are classified according to their intrinsic astigmatism, a , which is an invariant quantity. A beam with $a = 0$ is called intrinsic stigmatic, a beam with $a > 0$ is called intrinsic astigmatic. For simple astigmatic beams $a = (1/2)(M_x^2 - M_y^2)^2$. More details are given in ISO/TR 11146-3.

**3.11
twist parameter**

t
parameter related to the rotational properties of the phase front of a beam, and also to the orbital angular momentum carried by the beam

$$t = \langle x\theta_y \rangle - \langle y\theta_x \rangle \tag{10}$$

Note 1 to entry: The twist parameter is invariant under propagation through free space and spherical lenses. It might be altered under propagation through cylindrical lenses.

**3.12
principal axes**

x', y'
<power density distribution> axes of the maximum and minimum beam extent based on the second order moments of the power density distribution in a cross-section of the beam

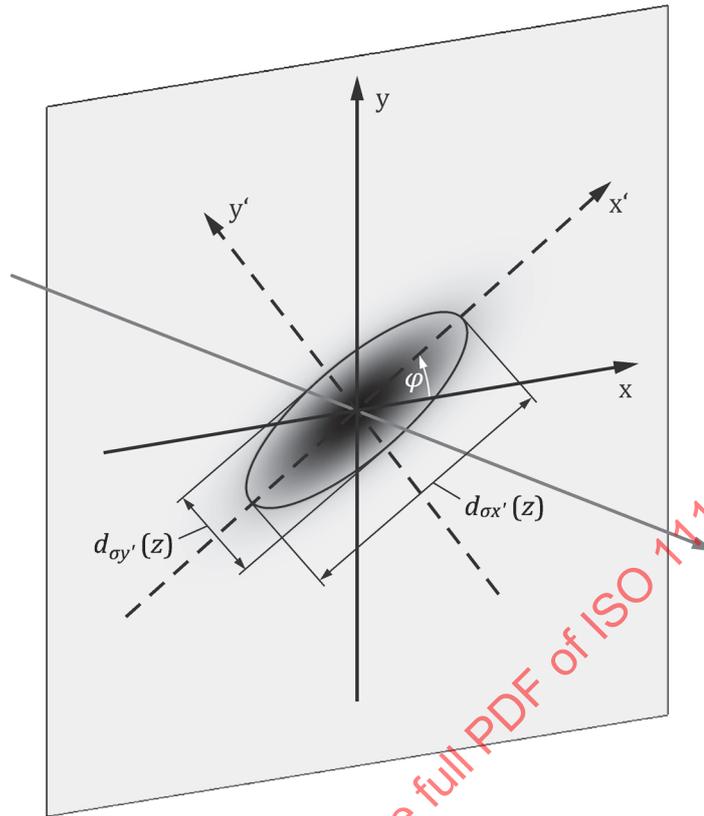


Figure 1 — Beam profile with the laboratory and principle axes coordinate systems

Note 1 to entry: The axes of maximum and minimum extent are always perpendicular to each other.

Note 2 to entry: Unless otherwise stated, in this document x' is the *principal axis* which is closer to the x -axis of the laboratory coordinate system, and y' is the principal axis which is closer to the y -axis of the laboratory coordinate system.

Note 3 to entry: If the *principal axes* make the angle $\pi/4$ with the x - and y -axes of the laboratory coordinate system, then the x -axis is by convention the direction of maximum extent.

Note 4 to entry: See [Figure 1](#).

[SOURCE: ISO 11146-1:2021, 3.3]

3.13 azimuthal orientation

φ

<power density distribution> azimuthal angle between the x -axis of the laboratory system and the principal axis x'

[SOURCE: ISO 11146-1:2021, 3.4]

3.14 beam widths

$d_{\sigma x'}(z), d_{\sigma y'}(z)$

extent of a power density distribution in a cross-section of the beam at an axial location z along the principal axes x' and y' , respectively, based on the second order moments of the power density distribution

Note 1 to entry: This definition differs from that given in ISO 11145:2018, 3.5.2, where the beam widths are defined only in the laboratory system, whereas for the purposes of this document the beam widths are defined in the *principal axes* ([3.12](#)) system of the beam.

Note 2 to entry: Formulae for calculation of the *beam widths* from the three second order moments are given in [7.2](#).

[SOURCE: ISO 11146-1:2021, 3.5]

4 Coordinate system

The x-, y- and z-axes define the orthogonal space directions in the laboratory axes system and shall be specified by the user. The z-axis shall approximately coincide with the direction of the beam. The x- and y- axes are transverse axes, usually horizontal and vertical, respectively. The origin of the z-axis is in a reference x-y plane defined by the manufacturer, e.g. the front of the laser enclosure.

5 Test principles

5.1 General

The following test principles are valid for general astigmatic beams. For stigmatic and simple astigmatic beams ISO 11146-1 may be applied.

5.2 Spatial second order moments of the Wigner distribution

Spatial second order moments are obtained by acquisition of power density distributions by means of spatially resolving detectors, correcting the measured profiles and calculating the first and second order moments.

5.3 Second order moments of the Wigner distribution

For the determination of all ten second order moments two different measurement set-ups are required.

Eight of the ten second order moments and the sum $(\langle x\theta_y \rangle + \langle y\theta_x \rangle)$ are obtained by acquisition of power density distributions along the propagation axis z in different planes near the generalized waist location, calculating the three spatial second order moments of each measured power density profile and fitting three independent parabolas to them.

The difference $(\langle x\theta_y \rangle - \langle y\theta_x \rangle)$ is obtained from the spatial moments of a power density distribution acquired behind a cylindrical lens, see [7.3](#).

5.4 Derived quantities

The effective beam propagation ratio M_{eff}^2 , the intrinsic astigmatism, a , and the twist parameter, t , are obtained from the second order moments of the Wigner distribution according to the [Formulae \(8\)](#) to [\(10\)](#).

6 Measurement arrangement and test equipment

6.1 General

The test is based on the measurement of the cross-sectional power density distribution at several axial locations along the beam propagation axis.

6.2 Preparation

The optical axis of the measuring system should be coaxial with the laser beam to be measured. Suitable optical alignment devices are available for this purpose (e.g. aligning lasers or steering mirrors).

The aperture of the optical system should accommodate the entire cross-section of the laser beam. Clipping shall be smaller than 1 % of the total beam power or energy.

The attenuators or beam-forming optics should be mounted so that the optical axis runs through the geometrical centres. Care shall be taken to avoid systematic errors. Reflections, interference effects, external ambient light, thermal radiation or air draughts are all potential sources of error.

An evaluation shall be made to determine if the entire laser beam reaches the detector surface. For testing this, apertures of different widths can be introduced into the beam path in front of each optical component. The aperture that reduces the output signal by 5 % should have a diameter less than 0,8 times the aperture of the optical component.

6.3 Control of environment

Suitable measures such as mechanical and acoustical isolation of the test set-up, shielding from extraneous radiation, temperature stabilization of the laboratory, choice of low noise amplifiers shall be taken to ensure that the contribution to the total probable error of the parameter to be measured is low.

Care should be taken to ensure that the atmospheric environment in high-power laser beam paths does not contain gases or vapours that can absorb the laser radiation and cause thermal distortion in the beam to be measured.

6.4 Detector system

Measurement of the cross-sectional power density distribution requires the use of a detection system with high spatial resolution and high signal-to-noise ratio.

The accuracy of the measurement is directly related to the spatial resolution of the detector system and its signal-to-noise ratio. The latter is important for laser beams with low power densities at larger diameters (e.g. for diffracted parts of the laser beams).

For pixel based detector systems the spatial resolution should be at least 1/20 of the smaller beam width.

In practice, noise in the wings of the power density distribution $E(x,y,z)$ may readily dominate the second order moment integral. Thus, it is usually necessary to apply background correction procedures. See ISO/TR 11146-3 for further details.

The radiation detector system shall be in accordance with EN 61040:1992, Clauses 3 and 4 in particular. Furthermore, the following points shall be noted.

- Care shall be taken to ascertain the damage thresholds of the detector surface so as to make sure that these thresholds are not exceeded by the laser beam.
- It shall be confirmed, from manufacturers' data or by measurement, that the output quantity of the detector system (e.g. the voltage) is linearly dependent on the input quantity (laser power). Any wavelength dependency, non-linearity or non-uniformity of the detector or the electronic device shall be minimized or corrected by use of a calibration procedure.
- When using a scanning device for determining the power density distribution function, care shall be taken to ensure that the laser output is temporally stable during the whole scanning period.
- When measuring pulsed laser beams, the trigger time delay of sampling as well as the measuring time interval play an important role because the beam parameters may change during the pulse. Therefore it is necessary to specify these parameters in the test report.

6.5 Beam-forming optics and optical attenuators

If the beam cross-sectional area is greater than the detector area, a suitable optical system shall be used to reduce the beam cross-sectional area on the detector surface. The change in magnification shall be taken into account during the evaluation procedure.

Optics shall be selected appropriate to wavelength.

An attenuator may be required to reduce the laser power density at the surface of the detector.

Optical attenuators shall be used when the laser output-power or power density exceeds the detector's working (linear) range or the damage threshold. Any wavelength, polarization and angular dependency, non-linearity or non-uniformity, including thermal effects of the optical attenuator, shall be minimized or corrected by use of a calibration procedure.

None of the optical elements used shall significantly influence the relative power (energy) density distribution.

7 Measurement of the second order moments

7.1 General

Before the measurements are started, the laser shall be warmed up to achieve thermal equilibrium. This would be the time stated by the manufacturer or otherwise for at least 1 h. The measurements shall be carried out at the operating conditions specified by the laser manufacturer for the type of laser being evaluated.

7.2 Measurement of the second order moments of power density distributions

Spatial second order moments are calculated from measured and corrected power density distributions. The corresponding integrations are carried out on a subset of the measured data, called the integration area, because otherwise crucial noise in the data may dominate the integrals. In many cases, a proper choice of the integration area is important for obtaining reliable results. The following procedure relates the size and position of the integration area to the size and position of the measured power density distribution, which are initially unknown. Hence, an iterative procedure is required.

All integrations according to [Formulae \(2\) to \(6\)](#) are performed on a rectangular integration area which is centred to the beam centroid, defined by the spatial first order moments of the Wigner distribution, orientated parallel to the principal axes of the power density distribution, and sized three times the beam widths $d_{ox'}$ and $d_{oy'}$ (see [Figure 2](#)). Since the beam's centroid, orientation and widths are initially unknown, the procedure starts with an approximation for the integration area. The approximation should include the beam's extent, orientation and position. Using this integration area, initial values for the beam position, size and orientation are obtained which are used to redefine the integration area. From the new integration area, new values for the beam size, orientation and position are calculated. This procedure shall be repeated until convergence of the results is obtained.

In particular situations the ideal integration area may differ. In any case, the amplitude of the beam should have fallen below the detection limit within the integration range.

The azimuthal orientation φ of the principal axes of a power density distribution can be obtained from the second order moments of the power density distribution by [Formula \(11\)](#):

$$\varphi(z) = \frac{1}{2} \arctan \left[\frac{2\langle xy \rangle}{(\langle x^2 \rangle - \langle y^2 \rangle)} \right] \quad (11)$$

for $\langle x^2 \rangle \neq \langle y^2 \rangle$;

for $\langle x^2 \rangle = \langle y^2 \rangle$, $\varphi(z)$ is obtained as given by [Formula \(12\)](#):

$$\varphi(z) = \text{sgn}(\langle xy \rangle) \times \frac{\pi}{4} \quad (12)$$

where

$$\text{sgn}(\langle xy \rangle) = \frac{\langle xy \rangle}{|\langle xy \rangle|} \quad (13)$$

The beam widths in the direction of its principal axes are given by [Formulae \(14\)](#) and [\(15\)](#):

$$d_{\sigma_{x'}}(z) = 2\sqrt{2} \left\{ \langle x^2 \rangle + \langle y^2 \rangle + \gamma \left[(\langle x^2 \rangle - \langle y^2 \rangle)^2 + 4(\langle xy \rangle)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \quad (14)$$

$$d_{\sigma_{y'}}(z) = 2\sqrt{2} \left\{ \langle x^2 \rangle + \langle y^2 \rangle - \gamma \left[(\langle x^2 \rangle - \langle y^2 \rangle)^2 + 4(\langle xy \rangle)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \quad (15)$$

where

$$\gamma = \text{sgn}(\langle x^2 \rangle - \langle y^2 \rangle) = \frac{\langle x^2 \rangle - \langle y^2 \rangle}{|\langle x^2 \rangle - \langle y^2 \rangle|} \quad (16)$$

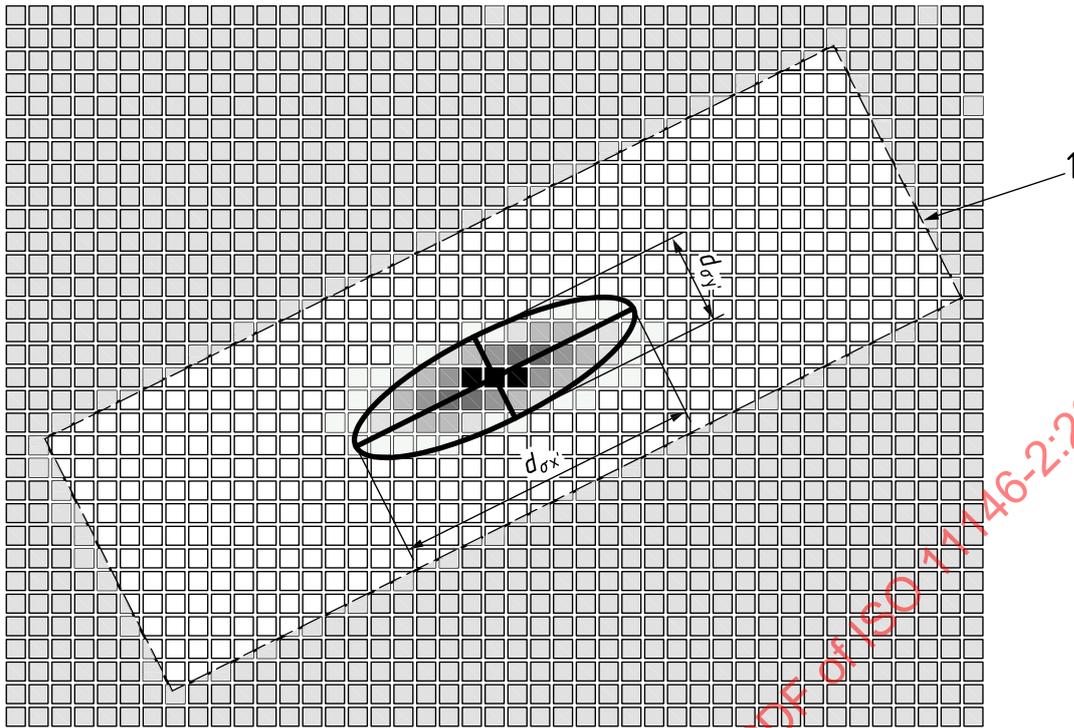
[Formulae \(14\)](#) and [\(15\)](#) are valid only for $\langle x^2 \rangle \neq \langle y^2 \rangle$.

For $\langle x^2 \rangle = \langle y^2 \rangle$ they shall be replaced by [Formulae \(17\)](#) and [\(18\)](#):

$$d_{\sigma_{x'}}(z) = 2\sqrt{2} \left(\langle x^2 \rangle + \langle y^2 \rangle + 2\langle xy \rangle \right)^{\frac{1}{2}} \quad (17)$$

$$d_{\sigma_{y'}}(z) = 2\sqrt{2} \left(\langle x^2 \rangle + \langle y^2 \rangle - 2\langle xy \rangle \right)^{\frac{1}{2}} \quad (18)$$

These calculations are performed for each measurement and the mean values and the standard deviations of the beam widths and the azimuthal orientation are calculated.



Key

- 1 outline of integration area

Figure 2 — Integration area

7.3 Measurement of all second order moments of the Wigner distribution

For sufficient and balanced accuracy of all ten second order moments, power density profiles at no less than twenty equally distributed z locations within a range of at least three generalized Rayleigh lengths before and after the generalized beam waist shall be acquired. If the generalized beam waist is not accessible for direct measurement an artificial waist shall be created by means of an aberration-free focusing lens or focusing system.

From the acquired power density distributions at distance Δz from the reference plane or the rear principal plane of the focusing system used, respectively, the spatial moments $\langle x^2 \rangle(\Delta z)$, $\langle y^2 \rangle(\Delta z)$ and $\langle xy \rangle(\Delta z)$ shall be calculated according to 7.2. The second order moments $\langle x^2 \rangle_0$, $\langle y^2 \rangle_0$, $\langle xy \rangle_0$, $\langle \theta_x^2 \rangle_0$, $\langle \theta_y^2 \rangle_0$, $\langle \theta_x \theta_y \rangle_0$, $\langle x \theta_x \rangle_0$, $\langle y \theta_y \rangle_0$, and the quantity

$$s_0 = \langle x \theta_y \rangle_0 + \langle y \theta_x \rangle_0 \tag{19}$$

in the reference plane or the rear principal plane of the focusing system used, respectively, are obtained by fitting three independent parabolas to the spatial moments in the measurement planes according to [Formulae \(20\)](#) and [\(22\)](#):

$$\langle x^2 \rangle(\Delta z) = \langle x^2 \rangle_0 + 2\Delta z \langle x \theta_x \rangle_0 + \Delta z^2 \langle \theta_x^2 \rangle_0 \tag{20}$$

$$\langle xy \rangle(\Delta z) = \langle xy \rangle_0 + \Delta z s_0 + \Delta z^2 \langle \theta_x \theta_y \rangle_0 \tag{21}$$

$$\langle y^2 \rangle (\Delta z) = \langle y^2 \rangle_0 + 2\Delta z \langle y\theta_y \rangle_0 + \Delta z^2 \langle \theta_y^2 \rangle_0 \quad (22)$$

The proper choice of measurement plane positions requires that the distance of the generalized beam waist location from the reference position or the rear plane of the focusing system $\Delta z_{0,g}$ and the generalized Rayleigh length $z_{R,g}$ are determined using [Formulae \(23\)](#) and [\(24\)](#):

$$\Delta z_{0,g} = -\frac{\langle x\theta_x \rangle_0 + \langle y\theta_y \rangle_0}{\langle \theta_x^2 \rangle_0 + \langle \theta_y^2 \rangle_0} \quad (23)$$

$$z_{R,g} = \sqrt{\frac{\langle x^2 \rangle + \langle y^2 \rangle}{\langle \theta_x^2 \rangle + \langle \theta_y^2 \rangle} - \frac{(\langle x\theta_x \rangle + \langle y\theta_y \rangle)^2}{(\langle \theta_x^2 \rangle + \langle \theta_y^2 \rangle)^2}} \quad (24)$$

To determine the second order moments $\langle x\theta_y \rangle_0$ and $\langle y\theta_x \rangle_0$, a cylindrical lens with a focal length f_c shall be placed at the distance $\Delta z = \Delta z_{0,g} + f_c$ from the reference plane or the rear plane of the focusing system in horizontally focusing orientation (see Figure 3).

The power density distribution in the rear focal plane of the cylindrical lens shall be acquired. The spatial second order moment $\langle xy \rangle$ calculated according to [7.2](#) and denoted as $\langle xy \rangle_h$. The cylindrical lens shall be rotated to vertical focusing orientation (see Figure 4). The power density distribution in the rear focal plane shall be acquired again, the spatial second order moment $\langle xy \rangle$ calculated according to [7.2](#) and denoted as $\langle xy \rangle_v$.

A focal length of $f_c \approx z_{R,g}$ of the cylindrical lens is recommended. This ensures a proper beam size in the rear focal plane of the cylindrical lens.

The second order moments $\langle x\theta_y \rangle_0$ and $\langle y\theta_x \rangle_0$ in the plane $z = 0$ shall then be calculated using [Formulae \(25\)](#) and [\(26\)](#):

$$\langle x\theta_y \rangle_0 = \frac{s_0}{2} + \frac{1}{2f_c} (\langle xy \rangle_v - \langle xy \rangle_h) \quad (25)$$

$$\langle y\theta_x \rangle_0 = \frac{s_0}{2} - \frac{1}{2f_c} (\langle xy \rangle_v - \langle xy \rangle_h) \quad (26)$$

If a focusing system has been used to create an artificial waist, the following procedure shall be performed to obtain the ten second order moments in the reference plane from the already obtained ten second order moments.

Arrange all the ten second moments in a beam matrix \mathbf{P}_0 (see [Clause 3](#)). Calculate the beam matrix \mathbf{P}_{ref} of the reference plane according to [Formula \(27\)](#):

$$\mathbf{P}_{\text{ref}} = \mathbf{S} \times \mathbf{P}_0 \times \mathbf{S}^T \quad (27)$$

where \mathbf{S} is the inverse geometric optical system matrix representing the ray propagation from the reference plane to the rear principal plane of the focusing system and \mathbf{S}^T is the transposed matrix \mathbf{S} .

Assuming that a thin spherical lens has been chosen and the origin of the z-axis has been placed in the centre of the lens, this system matrix is given by [Formula \(28\)](#):

$$S = \begin{pmatrix} 1 - \frac{L}{f} & 0 & -L & 0 \\ 0 & 1 - \frac{L}{f} & 0 & -L \\ \frac{1}{f} & 0 & 1 & 0 \\ 0 & \frac{1}{f} & 0 & 1 \end{pmatrix} \quad (28)$$

where

L is the distance between the reference plane and the lens;

f is the focal length.

If another focusing element has been used, e.g. a thick spherical lens or a system of lenses, see ISO/TR 11146-3 on how to obtain the system matrix, S .

The desired second order moments in the reference plane are the elements of the matrix, P_{ref} .

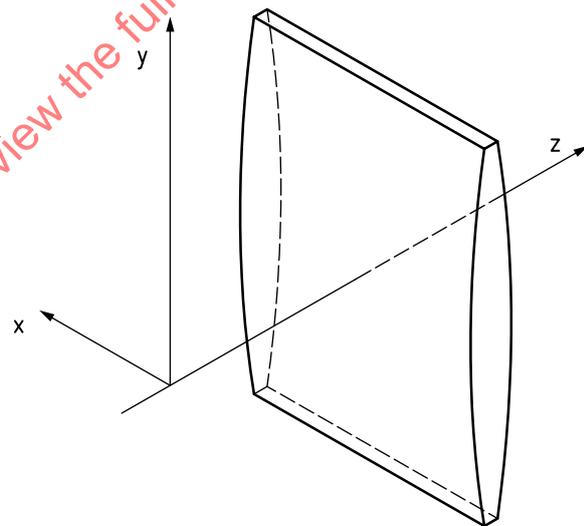
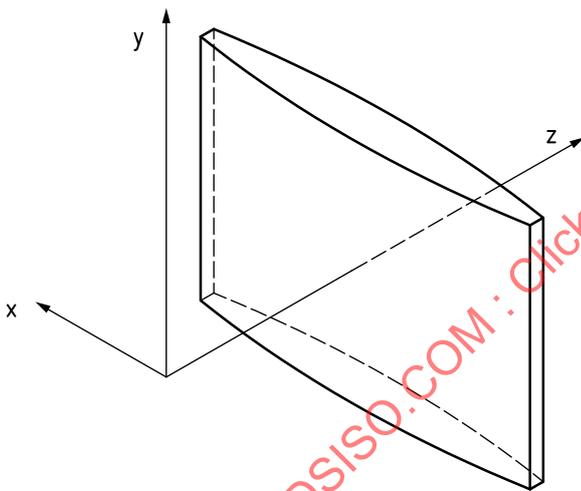


Figure 3 — Cylindrical lens in horizontally focusing orientation

Figure 4 — Cylindrical lens in vertically focusing orientation

8 Determination of effective beam propagation ratio

The effective beam propagation ratio is calculated according to [Formula \(8\)](#), where P is the so-called beam matrix which contains all second order moments of the Wigner distribution which are determined as given in [Clause 7](#). Further details are given in ISO/TR 11146-3.

9 Determination of intrinsic astigmatism

Another invariant during propagation is the intrinsic astigmatism a , which is defined in [Formula \(9\)](#). The beams are classified due to their intrinsic astigmatism a . A beam with $a = 0$ is called intrinsic stigmatic, a beam with $a > 0$ intrinsic astigmatic. For the determination of the intrinsic astigmatism it is

necessary to measure all ten second order moments of the Wigner distribution as described in [Clause 7](#). Further details are given in ISO/TR 11146-3.

10 Determination of the twist parameter

From the second order moments of the Wigner distribution determined according to [Clause 7](#) the twist parameter t is calculated using [Formula \(10\)](#).

11 Test report

The following information shall be included in the test report:

- a) General information:
 - 1) reference to this document, i.e. ISO 11146-2:2021;
 - 2) date of test;
 - 3) name and address of test organization;
 - 4) name of individual performing the test.
- b) Information concerning the tested laser:
 - 1) laser type;
 - 2) manufacturer;
 - 3) manufacturer's model designation;
 - 4) serial number.
- c) Test conditions:
 - 1) laser wavelength(s) at which tested;
 - 2) temperature in K (diode laser cooling fluid) (only applicable for diode lasers);
 - 3) operating mode [continuous wave (cw) or pulsed];
 - 4) laser parameter settings:
 - output power or energy;
 - current or energy input;
 - pulse energy;
 - pulse duration;
 - pulse repetition rate;
 - 5) mode structure;
 - 6) polarization;
 - 7) environmental conditions.
- d) Information concerning testing and evaluation:
 - 1) device used for the measurement of power distributions;