International Standard

ISO 10928

Fourth edition
2024-03

Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes and fittings — Methods for regression analysis and their use

Systèmes de canalisations en matières plastiques — Tubes et raccords plastiques thermodurcissables renforcés de verre (PRV) — Méthodes pour une analyse de régression et leurs utilisations

Reference number
ISO 10928:2024(en)

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Published in Switzerland

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 138, *Plastics pipes, fittings and valves for the transport of fluids*, Subcommittee SC 6, *Reinforced plastics pipes and fittings for all applications.*

This fourth edition cancels and replaces the third edition (ISO 10928:2016), which has been technically revised.

The main changes are as follows:

— Annex B, "Non-linear relationships", has been removed due to its complexity and highly specialized and limited application;

— Formula (B.3) [Formula (C.3) in ISO 10928:2016] has been corrected to include a factor 2 before Bx_{L}.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document describes the procedures intended for analysing the regression of test data, usually with respect to time, and the use of the results in the design and assessment of conformity with performance requirements. Its applicability is limited to use with data obtained from tests carried out on samples. Referring standards require estimates to be made of the long-term properties of the pipe for such parameters as circumferential tensile strength, long-term ring deflection, strain corrosion and creep or relaxation stiffness.

A range of statistical techniques that can be used to analyse the test data produced by destructive tests were investigated in the preparation of this document. Many of these simple techniques require the logarithms of the data to:

a) be normally distributed;

b) produce a regression line having a negative slope; and

c) have a sufficiently high regression correlation (see Table 1).

Analysis of data from several tests showed that in the destructive test context, while conditions b) and c) can be satisfied, there is often a skew to the distribution and hence condition a) is not satisfied. Further investigation into techniques that can handle skewed distributions resulted in the adoption of the covariance method (method A, see 5.2) for the analysis of such data within this document.

The results from non-destructive tests, such as long-term creep or relaxation stiffness, often satisfy all three conditions. Therefore, a simpler procedure, using time as the independent variable (method B, see 5.3), can also be used in accordance with this document.

These two analysis procedures (method A and method B) are limited to analysis methods specified in ISO product standards or test methods. Other analysis procedures can be useful for the extrapolation and prediction of long-term behaviour of some properties of glass-reinforced thermosetting plastics (GRP) piping products. For example, a second-order polynomial analysis is sometimes useful in the extrapolation of creep and relaxation data. This is particularly the case for analysing shorter term data, where the shape of the creep or relaxation curve can deviate considerably from linear. A second-order polynomial analysis is included in Annex A.

Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes and fittings — Methods for regression analysis and their use

1 Scope

This document specifies procedures suitable for the analysis of data which, when converted into logarithms of the values, have either a normal or a skewed distribution. It is intended for use with test methods and referring standards for glass-reinforced thermosetting plastics (GRP) pipes or fittings for the analysis of properties as a function of time. However, it can also be used for the analysis of other data.

Two methods are specified, which are used depending on the nature of the data. Extrapolation using these techniques typically extends a trend from data gathered over a period of approximately 10 000 h to a prediction of the property at 50 years, which is the typical maximum extrapolation time.

This document only addresses the analysis of data. The test procedures for collecting the data, the number of samples required and the time period over which data are collected are covered by the referring standards and/or test methods. Clause 6 discusses how the data analysis methods are applied to product testing and design.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

4 Principle

Data are analysed for regression using methods based on least squares analysis which can accommodate the incidence of a skew or a normal distribution or both. The two methods of analysis used are the following:

— method A: covariance using a first-order relationship;

— method B: least squares, with time as the independent variable using a first-order relationship.

The methods include statistical tests for the correlation of the data and the suitability for extrapolation.

5 Procedures for determining the linear relationships — Methods A and B

5.1 Procedures common to methods A and B

Use method A (see 5.2) or method B (see 5.3) to fit a straight line of the form given in Formula (1):

$$y = a + b \times x \quad (1)$$

where

y is the logarithm, lg, of the property being investigated;

a is the intercept on the y-axis;

b is the slope;

x is the logarithm, lg, of the time, in hours.

5.2 Method A — Covariance method

5.2.1 General

For method A, calculate the following variables in accordance with 5.2.2 to 5.2.5, using Formulae (2), (3) and (4):

$$Q_{\mathrm{y}} = \frac{\sum (y_i - Y)^2}{n} \tag{2}$$

$$Q_{\mathrm{x}} = \frac{\sum (x_i - X)^2}{n} \tag{3}$$

$$Q_{\mathrm{xy}} = \frac{\sum [(x_i - X) \times (y_i - Y)]}{n} \tag{4}$$

where

Q_{y} is the sum of the squared residuals parallel to the y-axis, divided by n;

Q_{x} is the sum of the squared residuals parallel to the x-axis, divided by n;

Q_{xy} is the sum of the squared residuals perpendicular to the line, divided by n;

Y is the arithmetic mean of the y data, i.e. given as Formula (5):

$$Y = \frac{\sum y_i}{n} \tag{5}$$

X is the arithmetic mean of the x data, i.e. given as Formula (6):

$$X = \frac{\sum x_i}{n} \tag{6}$$

x_i, y_i are individual values;

n is the total number of results (pairs of readings for x_i, y_i).

NOTE If the value of Q_{xy} is greater than zero, the slope of the line is positive and if the value of Q_{xy} is less than zero, then the slope is negative.

5.2.2 Suitability of data

Calculate the linear coefficient of correlation, r, using Formulae (7) and (8):

$$r^2 = \frac{Q_{xy}^2}{Q_x \times Q_y} \tag{7}$$

$$r = \left| \left(r^2 \right)^{0,5} \right| \tag{8}$$

Table 1 gives the minimum acceptable values of the correlation coefficient, r, as a function of the number of variables, n, and Student´s t-distribution t_ν, where t_ν is based on a two-sided 0,01 level of significance.

Table 1 — Minimum values of the correlation coefficient, *r*, for acceptable data from *n* pairs of data

Number of variables n	Degrees of freedom $n - 2$	Student's t-distribution t_ν (0,01)	Minimum r
13	11	3,106	0,683 5
14	12	3,055	0,661 4
15	13	3,012	0,641 1
16	14	2,977	0,622 6
17	15	2,947	0,605 5
18	16	2,921	0,589 7
19	17	2,898	0,575 1
20	18	2,878	0,561 4
21	19	2,861	0,548 7
22	20	2,845	0,536 8
23	21	2,831	0,525 6
24	22	2,819	0,515 1
25	23	2,807	0,505 2

Number of variables n	Degrees of freedom $n - 2$	Student's t-distribution t_ν (0,01)	Minimum r
26	24	2,797	0,495 8
27	25	2,787	0,486 9
32	30	2,750	0,448 7
37	35	2,724	0,418 2
42	40	2,704	0,393 2
47	45	2,690	0,372 1
52	50	2,678	0,354 2
62	60	2,660	0,324 8
72	70	2,648	0,301 7
82	80	2,639	0,283 0
92	90	2,632	0,267 3
102	100	2,626	0,254 0

5.2.3 Functional relationships

Find a and b for the functional relationship line using Formula (1).

First, set the gamma function Γ as given in Formula (9):

$$\Gamma = \frac{Q_y}{Q_x} \tag{9}$$

then calculate a and b using Formulae (10) and (11):

$$b = -(\Gamma)^{0,5} \tag{10}$$

$$a = Y - b \times X \tag{11}$$

5.2.4 Calculation of variances

If t_u is the applicable time to failure, then set x_u, the logarithm of t_u, as given in Formula (12):

$$x_u = \lg t_u \tag{12}$$

Using Formulae (13), (14) and (15) respectively, calculate for i = 1 to n, the following sequence of statistics:

— the best fit x_i' for true x_i;

— the best fit y_i' for true y_i;

— the error variance, σ_δ^2, for x.

$$x_i' = \frac{\Gamma \times x_i + b \times (y_i - a)}{2 \times \Gamma} \tag{13}$$

$$y_i' = a + b \times x_i' \tag{14}$$

$$\sigma_\delta^2 = \frac{\left[\sum (y_i - y_i')^2 + \Gamma \times \sum (x_i - x_i')^2\right]}{(n-2) \times \Gamma} \tag{15}$$

Calculate quantities E and D using Formulae (16) and (17):

$$E = \frac{b \times \sigma_\delta^2}{2 \times Q_{xy}} \tag{16}$$

$$D = \frac{2 \times \Gamma \times b \times \sigma_\delta^2}{n \times Q_{xy}} \tag{17}$$

Calculate the variance, C, of the slope b, using Formula (18):

$$C = D \times (1 + E) \tag{18}$$

5.2.5 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate the parameter T using Formula (19):

$$T = \frac{b}{(\text{var } b)^{0,5}} = \frac{b}{C^{0,5}} \tag{19}$$

If the absolute value, $|T|$ (i.e. ignoring signs), of T is equal to or greater than the applicable value for Student's t-distribution, t_v, shown in Table 2 for (n − 2) degrees of freedom, then consider the data suitable for extrapolation.

Calculation of confidence limits is not required by the test methods or referring standards. However, the calculation of the lower confidence limit (LCL), and lower prediction limit (LPL) shall be in accordance with Annex B.

Table 2 — Percentage points of Student's *t*-distribution (upper 2,5 % points; two-sided 5 % level of confidence; t_v for 97,5 %)

Degree of freedom $(n-2)$	Student's *t* value t_v	Degree of freedom $(n-2)$	Student's *t* value t_v	Degree of freedom $(n-2)$	Student's *t* value t_v
1	12,706 2	36	2,028 1	71	1,993 9
2	4,302 7	37	2,026 2	72	1,993 5
3	3,182 4	38	2,024 4	73	1,993 0
4	2,776 4	39	2,022 7	74	1,992 5
5	2,570 6	40	2,021 1	75	1,992 1
6	2,446 9	41	2,019 5	76	1,991 7
7	2,364 6	42	2,018 1	77	1,991 3
8	2,306 0	43	2,016 7	78	1,990 8
9	2,262 2	44	2,015 4	79	1,990 5
10	2,228 1	45	2,014 1	80	1,990 1
11	2,201 0	46	2,012 9	81	1,989 7
12	2,178 8	47	2,011 2	82	1,989 3
13	2,160 4	48	2,010 6	83	1,989 0
14	2,144 8	49	2,009 6	84	1,988 6
15	2,131 5	50	2,008 6	85	1,988 3
16	2,119 9	51	2,007 6	86	1,987 9
17	2,109 8	52	2,006 6	87	1,987 6
18	2,100 9	53	2,005 7	88	1,987 3
19	2,093 0	54	2,004 9	89	1,987 0
20	2,086 0	55	2,004 0	90	1,986 7
21	2,079 6	56	2,003 2	91	1,986 4
22	2,073 9	57	2,002 5	92	1,986 1
23	2,068 7	58	2,001 7	93	1,985 8
24	2,063 9	59	2,001 0	94	1,985 5
25	2,059 5	60	2,000 3	95	1,985 3
26	2,055 5	61	1,999 6	96	1,985 0
27	2,051 8	62	1,999 0	97	1,984 7
28	2,048 4	63	1,998 3	98	1,984 5
29	2,045 2	64	1,997 7	99	1,984 2
30	2,042 3	65	1,997 1	100	1,984 0
31	2,039 5	66	1,996 6		
32	2,036 9	67	1,996 0		
33	2,034 5	68	1,995 5		
34	2,032 2	69	1,994 9		
35	2,030 1	70	1,994 4		

5.2.6 Validation of statistical procedures by an example calculation

The data given in Table 3 are used in the following example to aid in verifying that statistical procedures, as well as computer programs and spreadsheets adopted by users, will produce results similar to those obtained from the formulae given in this document. For the purposes of the example, the property in question is represented by *V*, the values for which are of a typical magnitude and in no particular units. Due to rounding errors, it is unlikely that the results will agree exactly, so for a calculation procedure to be acceptable, the results obtained for r, r^2, b, a, and the mean value of V, and V_m, shall agree to within ±0,1 % of the values given in this example. The values of other statistics are provided to assist the checking of the procedure.

Sums of squares:

$Q_x = 0{,}798\ 12$;

$Q_y = 0{,}000\ 88$;

$Q_{xy} = -0{,}024\ 84$.

Coefficient of correlation:

$r^2 = 0{,}879\ 99$;

$r = 0{,}938\ 08$.

Functional relationships:

$\Gamma = 0{,}001\ 10$;

$b = -0{,}033\ 17$;

$a = 1{,}627\ 31$.

Table 3 — Basic data for example calculation and statistical analysis validation

n	*V*	*Y* lg *V*	Time, *T* h	*X* lg h
1	30,8	1,488 6	5 184	3,714 7
2	30,8	1,488 6	2 230	3,348 3
3	31,5	1,498 3	2 220	3,346 4
4	31,5	1,498 3	12 340	4,091 3
5	31,5	1,498 3	10 900	4,037 4
6	31,5	1,498 3	12 340	4,091 3
7	31,5	1,498 3	10 920	4,038 2
8	32,2	1,507 9	8 900	3,949 4
9	32,2	1,507 9	4 173	3,620 4
10	32,2	1,507 9	8 900	3,949 4
11	32,2	1,507 9	878	2,943 5
12	32,9	1,517 2	4 110	3,613 8
13	32,9	1,517 2	1 301	3,114 3
14	32,9	1,517 2	3 816	3,581 6
15	32,9	1,517 2	669	2,825 4
16	33,6	1,526 3	1 430	3,155 3
17	33,6	1,526 3	2 103	3,322 8
18	33,6	1,526 3	589	2,770 1
19	33,6	1,526 3	1 710	3,233 0
20	33,6	1,526 3	1 299	3,113 6
21	35,0	1,544 1	272	2,434 6
22	35,0	1,544 1	446	2,649 3
23	35,0	1,544 1	466	2,668 4
24	35,0	1,544 1	684	2,835 1

Table 3 *(continued)*

n	*V*	*Y* lg *V*	Time, *T* h	*X* lg h
25	36,4	1,561 1	104	2,017 0
26	36,4	1,561 1	142	2,152 3
27	36,4	1,561 1	204	2,309 6
28	36,4	1,561 1	209	2,320 1
29	38,5	1,585 5	9	0,954 2
30	38,5	1,585 5	13	1,113 9
31	38,5	1,585 5	17	1,230 4
32	38,5	1,585 5	17	1,230 4
Means:		*Y* = 1,530 1		*X* = 2, 930 5

Calculated variances (see 5.2.4):

$E = 3{,}520\ 2 \times 10^{-2}$;

$D = 4{,}842\ 2 \times 10^{-6}$;

$C = 5{,}012\ 7 \times 10^{-6}$ (the variance of *b*);

$\sigma_\delta{}^2 = 5{,}271\ 1 \times 10^{-2}$ (the error variance of *x*).

Check for the suitability for extrapolation (see 5.2.5):

$n = 32$;

$t_v = 2{,}042\ 3$;

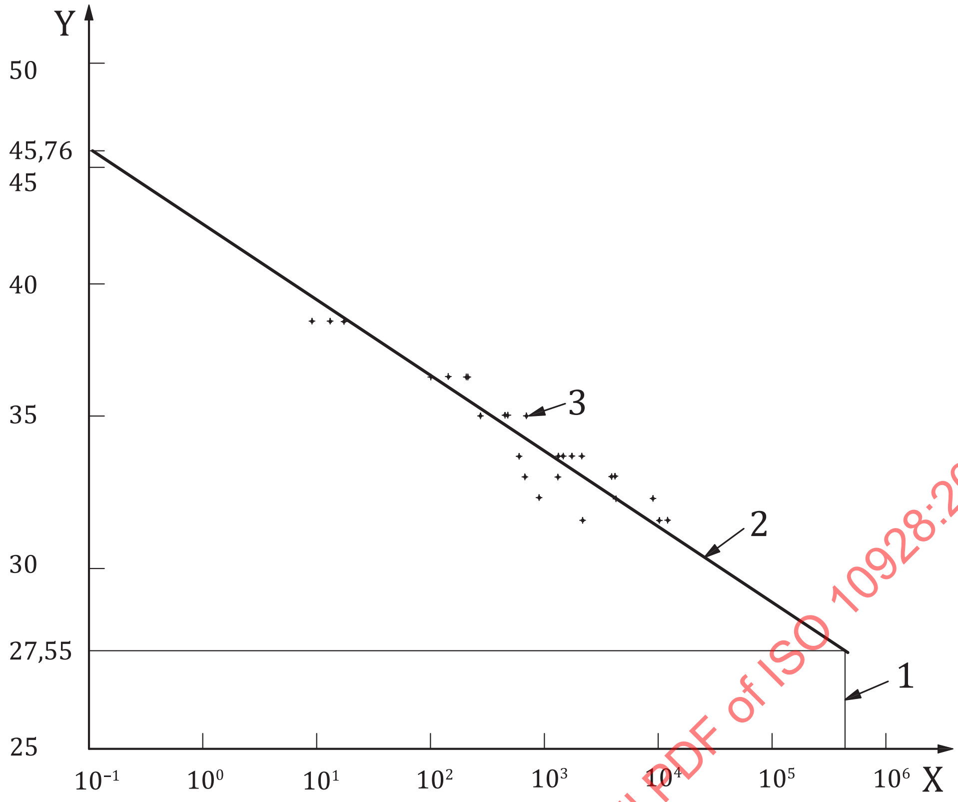
$T = -0{,}033\ 17 / (5{,}012\ 7 \times 10^{-6})^{0{,}5} = -14{,}816\ 7$;

$|T| = 14{,}816\ 7 > 2{,}042\ 3$.

The estimated mean values for *V* at various times are given in Table 4 and shown in Figure 1.

Table 4 — Estimated mean values, V_m, for *V*

Time h	V_m
0,1	45,76
1	42,39
10	39,28
100	36,39
1 000	33,71
10 000	31,23
100 000	28,94
438 000	27,55



Key

X lg scale of time, in hours
Y lg scale of property
1 438 000 h (50 years)
2 regression line from Table 4
3 data point

Figure 1 — Regression line from the results in Table 4

5.3 Method B — Regression with time as the independent variable

5.3.1 General

For method B, calculate the sum of the squared residuals parallel to the Y-axis, S_y, using Formula (20):

$$S_{\mathrm{y}} = \sum (y_i - Y)^2 \qquad (20)$$

Calculate the sum of the squared residuals parallel to the X-axis, S_x, using Formula (21):

$$S_{\mathrm{x}} = \sum (x_i - X)^2 \qquad (21)$$

Calculate the sum of the squared residuals perpendicular to the line, S_{xy}, using Formula (22):

$$S_{\mathrm{xy}} = \sum [(x_i - X) \times (y_i - Y)] \qquad (22)$$

where

Y is the arithmetic mean of the y data, i.e.

$$Y = \frac{\sum y_i}{n};$$

X is the arithmetic mean of the x data, i.e.

$$X = \frac{\sum x_i}{n};$$

x_i, y_i are individual values;

n is the total number of results (pairs of readings for x_i, y_i).

NOTE If the value of S_{xy} is greater than zero, the slope of the line is positive. If the value of S_{xy} is less than zero, then the slope is negative.

5.3.2 Suitability of data

Calculate the squared, r^2, and the linear coefficient of correlation, r, using Formulae (23) and (24):

$$r^2 = \frac{S_{xy}^2}{S_x \times S_y} \tag{23}$$

$$r = \left|\left(r^2\right)^{0,5}\right| \tag{24}$$

If the value of r^2 or r is less than the applicable minimum value given in Table 1 as a function of n, consider the data unsuitable for analysis.

5.3.3 Functional relationships

Calculate a and b for the functional relationship line [see Formula (1)], using Formulae (25) and (26):

$$b = \frac{S_{xy}}{S_x} \tag{25}$$

$$a = Y - b \times X \tag{26}$$

5.3.4 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate the parameter M using Formula (27):

$$M = \frac{S_x{}^2}{S_{xy}{}^2} - \frac{t_v{}^2 \times \left(S_x \times S_y - S_{xy}{}^2\right)}{(n-2) \times S_y{}^2} \tag{27}$$

where t_v is the applicable value for Student's t determined from Table 2.

If M is equal to or less than zero, consider the data unsuitable for extrapolation.

5.3.5 Validation of statistical procedures by an example calculation

The data given in Table 5 are used in the following example to aid in verifying that statistical procedures, as well as computer programs and spreadsheets adopted by users, will produce results similar to those obtained from the formulae given in this document. Use the data given in Table 5 for the calculation procedures described in 5.3.2 to 5.3.4 to ensure that the statistical procedures to be used in conjunction with this method will give results for r, r^2, a, b and V_m (the estimated mean value) to within ±0,1 % of the values given in this example.

Table 5 — Basic data for example calculation and statistical validation

n	**Time, *T*** h	***X*** lg T	***V***	***Y*** lg *V*
1	0,10	−1,000 0	7 114	3,852 1
2	0,27	−0,568 6	6 935	3,841 0
3	0,50	−0,301 0	6 824	3,834 1
4	1,00	0	6 698	3,825 9
5	3,28	0,515 9	6 533	3,815 1
6	7,28	0,862 1	6 453	3,809 8
7	20,0	1,301 0	6 307	3,799 9
8	45,9	1,661 8	6 199	3,792 3
9	72,0	1,857 3	6 133	3,787 7
10	166	2,220 1	5 692	3,755 2
11	219	2,340 4	5 508	3,741 0
12	384	2,584 3	5 393	3,731 8
13	504	2,702 4	5 364	3,729 5
14	3 000	3,477 1	5 200	3,716 0
15	10 520	4,022 0	4 975	3,696 8
Means:		X = 1,445 0		Y = 3,781 9

Sums of squares:

S_x = 31,681 1;

S_y = 0,034 7;

S_{xy} = −1,024 2.

Coefficient of correlation:

r^2 = 0,955 6;

r = 0,977 5.

Functional relationships (see 5.3.3):

a = 3,828 6;

b = −0,032 3.

Check for the suitability for extrapolation (see 5.3.4):

t_v = 2,160 4;

M = 942,21.

The estimated mean values, V_m, for V at various times are given in Table 6.

Table 6 — Estimated mean values, V_m, for V

Time h	V_m
0,1	7 259
1	6 739
10	6 256
100	5 808
1 000	5 391
10 000	5 005
100 000	4 646
438 000	4 428

6 Application of methods to product design and testing

6.1 General

The referring standards specify limiting requirements for the long-term properties and performance of a product. Some of these are based on destructive tests, for example, hoop tensile strength, while others are based on actual or derived physical properties, such as creep stiffness.

These properties require an extrapolated long-term (e.g. 50 years) value for the establishment of a product design or comparison with the requirement. This extrapolated value is determined by inserting, as necessary, the values for a and b, determined in accordance with 5.2 and 5.3 as appropriate, into Formula (28).

$$\lg y = a + b \times t_L \qquad (28)$$

where t_L is the logarithm, lg, of the long-term period, in hours [for 50 years (438 000 h), t_L = 5,641 47].

Solving Formula (28) for y gives the extrapolated value.

The use of the data and the specification of requirements in the product standards are separated into three distinct categories.

6.2 Product design

In the first category, the data are used for design or calculation of a product line. This is the case for long-term circumferential strength testing (ISO 7509). The long-term destructive test data are analysed using method A.

6.3 Comparison to a specified value

The second category is where the long-term extrapolated value is compared to a minimum requirement given in the product standard. This is the case for long-term ring bending (ISO 10471) and strain corrosion (ISO 10952). The long-term destructive test data are analysed using method A and can be used to establish a value to compare to the product standard requirement. As the analysis of the data provides a long-term stress or strain value, this value may also be utilized in analysis of product suitability for a range of installation and application conditions.

6.4 Declaration of a long-term value

The third category is when the long-term extrapolated value is used to calculate a long-term property and this value is then declared by the manufacturer. This is the case for long-term creep (ISO 10468) stiffness. The long-term non-destructive test data are analysed using method B.

Annex A
(informative)

Second-order polynomial relationships

A.1 General

This method fits a curved line of the form given in Formula (A.1):

$$y = c + d \times x + e \times x^2 \tag{A.1}$$

where

y is the logarithm, lg, of the property being investigated;

c is the intercept on the y-axis;

d, e are the coefficients to the two orders of x;

x is the logarithm, lg, of the time, in hours.

A.2 Variables

Calculate the following variables:

$\sum x_i$, the sum of all individual x data;

$\sum x_i^2$, the sum of all squared x data;

$\sum x_i^3$, the sum of all x data to the third power;

$\sum x_i^4$, the sum of all x data to the fourth power;

$\sum y_i$, the sum of all individual y data;

$\left(\sum y_i\right)^2$, the squared sum of all individual y data;

$\sum y_i^2$, the sum of all squared y data;

$\sum (x_i \times y_i)$, the sum of all products x_i, y_i;

$\sum \left(x_i^2 \times y_i\right)$, the sum of all products $x_i^2 y_i$;

$S_x = \sum (x_i - X)^2$, the sum of the squared residuals parallel to the X-axis for the linear part;

$S_{xx} = \sum \left(x_i^2 - X^2\right)^2$, the sum of the squared residuals parallel to the X-axis for the quadratic part;

$S_y = \sum (y_i - Y)^2$, the sum of the squared residuals parallel to the Y-axis;

$S_{xy} = \sum [(x_i - X) \times (y_i - Y)]$, the sum of the squared residuals perpendicular to the line for the linear part;

$S_{xxy} = \sum\left[\left(x_i^2 - X^2\right)\times\left(y_i - Y\right)\right]$, the sum of the squared residuals perpendicular to the line for the quadratic part

where

Y is the arithmetic mean of the y data, i.e. $Y = \frac{\sum y_i}{n}$;

X is the arithmetic mean of the x data, i.e. $X = \frac{\sum x_i}{n}$.

A.3 Solution system

Determine c, d and e using the matrix shown in Formulae (A.2), (A.3) and (A.4):

$$\sum y_i = c\times n + d\times\sum x_i + e\times\sum x_i^2 \tag{A.2}$$

$$\sum\left(x_i\times y_i\right) = c\times\sum x_i + d\times\sum x_i^2 + e\times\sum x_i^3 \tag{A.3}$$

$$\sum\left(x_i^2\times y_i\right) = c\times\sum x_i^2 + d\times\sum x_i^3 + e\times\sum x_i^4 \tag{A.4}$$

A.4 Suitability of data

Calculate the squared, r^2, and the linear coefficient of correlation, r, using Formulae (A.5) and (A.6):

$$r^2 = \frac{c\times\sum y_i + d\times\sum\left(x_i\times y_i\right) + e\times\sum\left(x_i^2\times y_i\right) - \left[\left(\sum y_i\right)^2/n\right]}{\sum y_i^2 - \left[\left(\sum y_i\right)^2/n\right]} \tag{A.5}$$

$$r = \left|\left(r^2\right)^{0,5}\right| \tag{A.6}$$

If the value of r^2 or r is less than the applicable minimum value given in Table 1 as a function of n, consider the data unsuitable for analysis.

A.5 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate M using Formula (A.7):

$$M = \frac{S_x^2}{S_{xy}^2} + \frac{S_{xx}^2}{S_{xxy}^2} - \frac{t_v^2\times\left(S_x\times S_y - S_{xy}^2 + S_{xx}\times S_y - S_{xxy}^2\right)}{(n-2)\times S_y^2} \tag{A.7}$$

If M is equal to or less than zero, consider the data unsuitable for extrapolation.

A.6 Validation of statistical procedures by an example calculation

Use the data given in Table 5 for the calculation procedures described in A.1 to A.5 to ensure that the statistical procedures to be used in conjunction with this method will give results for r, r^2, a, b and V_m to within ±0,1 % of the values given in this example (n = 15).

$\sum x_i$ = 21,671;

$\sum x_i^2$	= 62,989;
$\sum x_i^3$	= 180,623;
$\sum x_i^4$	= 584,233;
$\sum y_i$	= 56,728;
$\left(\sum y_i\right)^2$	= 3 218,09;
$\sum y_i^2$	= 214,574;
$\sum (x_i \times y_i)$	= 80,932;
$\sum \left(x_i^2 \times y_i\right)$	= 235,175;
$S_x = \sum (x_i - X)^2$	= 31,681;
$S_{xx} = \sum \left(x_i^2 - X^2\right)^2$	= 386,638;
$S_y = \sum (y_i - Y)^2$	= 0,034 7;
$S_{xy} = \sum [(x_i - X) \times (y_i - Y)]$	= −1,024 2;
$S_{xxy} = \sum \left[\left(x_i^2 - X^2\right) \times (y_i - Y)\right]$	= −3,041 8.

Solution system:

c = 3,828 8;

d = −0,026 2;

e = −0,002 2.

Coefficient of correlation:

r^2 = 0,964 7;

r = 0,982 2.

Check for the suitability for extrapolation:

t_v = 2,160 4;

M = 15 859,6.

The estimated mean values, V_m, for V at various times are given in Table A.1 and shown in Figure A.1.

Table A.1 — Estimated mean values, V_m, for V

Time h	V_m
0,1	7 125
1	6 742
10	6 315
100	5 856
1 000	5 375