INTERNATIONAL STANDARD

ISO 10928

Second edition
2009-09-15

Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes and fittings — Methods for regression analysis and their use

Systèmes de canalisation en matières plastiques — Tubes et raccords plastiques thermodurcissables renforcés de verre (PRV) — Méthodes pour une analyse de régression et leurs utilisations

Reference number
ISO 10928:2009(E)

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 10928 was prepared by Technical Committee ISO/TC 138, *Plastics pipes, fittings and valves for the transport of fluids*, Subcommittee SC 6, *Reinforced plastics pipes and fittings for all applications*.

This second edition cancels and replaces the first edition (ISO 10928:1997), which has been technically revised.

Introduction

This International Standard describes the procedures intended for analysing the regression of test data, usually with respect to time and the use of the results in design and assessment of conformity with performance requirements. Its applicability is limited to use with data obtained from tests carried out on samples. The referring standards require estimates to be made of the long-term properties of the pipe for such parameters as circumferential tensile strength, long-term ring deflection, strain-corrosion and creep or relaxation stiffness.

A range of statistical techniques that could be used to analyse the test data produced by destructive tests was investigated. Many of these simple techniques require the logarithms of the data to

a) be normally distributed,

b) produce a regression line having a negative slope, and

c) have a sufficiently high regression correlation (see Table 1).

Whilst the last two conditions can be satisfied, analysis shows that there is a skew to the distribution and hence this primary condition is not satisfied. Further investigation into techniques that can handle skewed distributions resulted in the adoption of the covariance method of analysis of such data for this International Standard.

However, the results from non-destructive tests, such as long-term creep or relaxation stiffness, often satisfy all three conditions and hence a simpler procedure, using time as the independent variable, can also be used in accordance with this International Standard.

These data analysis procedures are limited to analysis methods specified in ISO product standards or test methods. However, other analysis procedures can be useful for the extrapolation and prediction of long-term behaviour of some properties of glass-reinforced thermosetting plastics (GRP) piping products. For example, a second-order polynomial analysis is sometimes useful in the extrapolation of creep and relaxation data. This is particularly the case for analysing shorter term data, where the shape of the creep or relaxation curve can deviate considerably from linear. A second-order polynomial analysis is included in Annex B. In Annex C, there is an alternative non-linear analysis method. These non-linear methods are provided only for information and the possible use in investigating the behaviour of a particular piping product or material, therefore they might not be generally applicable to other piping products.

Plastics piping systems — Glass-reinforced thermosetting plastics (GRP) pipes and fittings — Methods for regression analysis and their use

1 Scope

This International Standard specifies procedures suitable for the analysis of data which, when converted into logarithms of the values, have either a normal or a skewed distribution. It is intended for use with the test methods and referring standards for glass-reinforced thermosetting plastics (GRP) pipes or fittings for the analysis of properties as a function of time. However, it can be used for the analysis of other data.

Depending upon the nature of the data, two methods are specified. The extrapolation using these techniques typically extends the trend from data gathered over a period of approximately 10 000 h, to a prediction of the property at 50 years, which is the typical maximum extrapolation time.

This International Standard only addresses the analysis of data. The test procedures to collect the data, the number of samples required and the time period over which data is collected, are covered by the referring standards and/or test methods. Clause 4 discusses how the data analysis methods are applied to product testing and design.

2 Principle

Data are analysed for regression using methods based on least squares analysis which can accommodate the incidence of a skew and/or a normal distribution. The two methods of analysis used are the following:

— method A: covariance using a first-order relationship;

— method B: least squares, with time as the independent variable using a first-order relationship.

The methods include statistical tests for the correlation of the data and the suitability for extrapolation.

3 Procedures for determining the linear relationships – Methods A and B

3.1 Procedures common to methods A and B

Use method A (see 3.2) or method B (see 3.3) to fit a straight line of the form given in Equation (1):

$$y = a + b \times x \tag{1}$$

where

y is the logarithm, lg, of the property being investigated;

a is the intercept on the Y-axis;

b is the slope;

x is the logarithm, lg, of the time, in hours.

3.2 Method A – Covariance method

3.2.1 General

For method A, calculate the following variables in accordance with 3.2.2 to 3.2.5, using Equations (2), (3) and (4):

$$Q_{\text{y}} = \frac{\sum (y_i - Y)^2}{n} \tag{2}$$

$$Q_{\text{x}} = \frac{\sum (x_i - X)^2}{n} \tag{3}$$

$$Q_{\text{xy}} = \frac{\sum \left[(x_i - X) \times (y_i - Y) \right]}{n} \tag{4}$$

where

Q_{y} is the sum of the squared residuals parallel to the Y-axis, divided by n;

Q_{x} is the sum of the squared residuals parallel to the X-axis, divided by n;

Q_{xy} is the sum of the squared residuals perpendicular to the line, divided by n;

Y is the arithmetic mean of the y data, i.e. given as Equation (5):

$$Y = \frac{\sum y_i}{n} \tag{5}$$

X is the arithmetic mean of the x data, i.e. given as Equation (6):

$$X = \frac{\sum x_i}{n} \tag{6}$$

x_i, y_i are individual values;

n is the total number of results (pairs of readings for x_i, y_i).

NOTE If the value of Q_{xy} is greater than zero, the slope of the line is positive and if the value of Q_{xy} is less than zero, then the slope is negative.

3.2.2 Suitability of data

Calculate the linear coefficient of correlation, r, using Equations (7) and (8):

$$r^2 = \frac{Q_{\text{xy}}^2}{Q_{\text{x}} \times Q_{\text{y}}} \tag{7}$$

$$r = \left| \left(r^2 \right)^{0,5} \right| \tag{8}$$

If the value of r is less than: $\dfrac{\text{Student's } t(f)}{\sqrt{n-2+\left[\text{Student's } t(f)\right]^2}}$

then the data are unsuitable for analysis.

Table 1 gives the minimum acceptable values of the correlation coefficient, r, as a function of the number of variables, n. The Student's t value is based on a two-sided 0,01 level of significance.

Table 1 — Minimum values of the correlation coefficient, r, for acceptable data from n pairs of data

Number of variables n	Degrees of freedom $n-2$	Student's $t(0{,}01)$	Minimum r
13	11	3,106	0,683 5
14	12	3,055	0,661 4
15	13	3,012	0,641 1
16	14	2,977	0,622 6
17	15	2,947	0,605 5
18	16	2,921	0,589 7
19	17	2,898	0,575 1
20	18	2,878	0,561 4
21	19	2,861	0,548 7
22	20	2,845	0,536 8
23	21	2,831	0,525 6
24	22	2,819	0,515 1
25	23	2,807	0,505 2

Number of variables n	Degrees of freedom $n-2$	Student's $t(0{,}01)$	Minimum r
26	24	2,797	0,495 8
27	25	2,787	0,486 9
32	30	2,750	0,448 7
37	35	2,724	0,418 2
42	40	2,704	0,393 2
47	45	2,690	0,372 1
52	50	2,678	0,354 2
62	60	2,660	0,324 8
72	70	2,648	0,301 7
82	80	2,639	0,283 0
92	90	2,632	0,267 3
102	100	2,626	0,254 0

3.2.3 Functional relationships

To find a and b for the functional relationship line:

$$y = a + b \times x \quad (1)$$

First set Γ as given in Equation (9):

$$\Gamma = \frac{Q_\text{y}}{Q_\text{x}} \quad (9)$$

then calculate a and b using Equations (10) and (11):

$$b = -(\Gamma)^{0{,}5} \quad (10)$$

$$a = Y - b \times X \quad (11)$$

3.2.4 Calculation of variances

If t_u is the applicable time to failure, then set x_u as given in Equation (12):

$$x_u = \lg t_u \tag{12}$$

Using Equations (13), (14) and (15) respectively, calculate for $i = 1$ to n, the following sequence of statistics:

— the best fit x_i' for true x_i;

— the best fit y_i' for true y_i;

— the error variance, σ_δ^2 for x.

$$x_i' = \frac{\Gamma \times x_i + b \times (y_i - a)}{2 \times \Gamma} \tag{13}$$

$$y_i' = a + b \times x_i' \tag{14}$$

$$\sigma_\delta^2 = \frac{\left[\sum (y_i - y_i')^2 + \Gamma \times \sum (x_i - x_i')^2\right]}{(n-2) \times \Gamma} \tag{15}$$

Calculate quantities E and D using Equations (16) and (17):

$$E = \frac{b \times \sigma_\delta^2}{2 \times Q_{xy}} \tag{16}$$

$$D = \frac{2 \times \Gamma \times b \times \sigma_\delta^2}{n \times Q_{xy}} \tag{17}$$

Calculate the variance, C, of the slope b, using Equation (18):

$$C = D \times (1 + E) \tag{18}$$

3.2.5 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate T using Equation (19):

$$T = \frac{b}{(\text{var } b)^{0,5}} = \frac{b}{C^{0,5}} \tag{19}$$

If the absolute value, $|T|$ (i.e. ignoring signs), of T is equal to or greater than the applicable value for Student's t, t_v, shown in Table 2 for $(n - 2)$ degrees of freedom, then consider the data suitable for extrapolation.

NOTE Calculation of confidence limits is not required by the test methods or referring standards, however, the calculation of lower confidence limit, LCL, and lower prediction limit, LPL, are given in Annex D.

Table 2 — Percentage points of Student's t distribution (upper 2,5 % points; two-sided 5 % level of confidence; t_v for 97,5 %)

Degree of freedom ($n-2$)	Student's t value t_v	Degree of freedom ($n-2$)	Student's t value t_v	Degree of freedom ($n-2$)	Student's t value t_v
1	12,706 2	36	2,028 1	71	1,993 9
2	4,302 7	37	2,026 2	72	1,993 5
3	3,182 4	38	2,024 4	73	1,993 0
4	2,776 4	39	2,022 7	74	1,992 5
5	2,570 6	40	2,021 1	75	1,992 1
6	2,446 9	41	2,019 5	76	1,991 7
7	2,364 6	42	2,018 1	77	1,991 3
8	2,306 0	43	2,016 7	78	1,990 8
9	2,262 2	44	2,015 4	79	1,990 5
10	2,228 1	45	2,014 1	80	1,990 1
11	2,201 0	46	2,012 9	81	1,989 7
12	2,178 8	47	2,011 2	82	1,989 3
13	2,160 4	48	2,010 6	83	1,989 0
14	2,144 8	49	2,009 6	84	1,988 6
15	2,131 5	50	2,008 6	85	1,988 3
16	2,119 9	51	2,007 6	86	1,987 9
17	2,109 8	52	2,006 6	87	1,987 6
18	2,100 9	53	2,005 7	88	1,987 3
19	2,093 0	54	2,004 9	89	1,987 0
20	2,086 0	55	2,004 0	90	1,986 7
21	2,079 6	56	2,003 2	91	1,986 4
22	2,073 9	57	2,002 5	92	1,986 1
23	2,068 7	58	2,001 7	93	1,985 8
24	2,063 9	59	2,001 0	94	1,985 5
25	2,059 5	60	2,000 3	95	1,985 3
26	2,055 5	61	1,999 6	96	1,985 0
27	2,051 8	62	1,999 0	97	1,984 7
28	2,048 4	63	1,998 3	98	1,984 5
29	2,045 2	64	1,997 7	99	1,984 2
30	2,042 3	65	1,997 1	100	1,984 0
31	2,039 5	66	1,996 6		
32	2,036 9	67	1,996 0		
33	2,034 5	68	1,995 5		
34	2,032 2	69	1,994 9		
35	2,030 1	70	1,994 4		

3.2.6 Validation of statistical procedures by an example calculation

The data given in Table 3 are used in the following example to aid in verifying that statistical procedures, as well as computer programs and spreadsheets adopted by users, will produce results similar to those obtained from the equations given in this International Standard. For the purposes of the example, the property in question is represented by V, the values for which are of a typical magnitude and in no particular units. Because of rounding errors, it is unlikely that the results will agree exactly, so for a calculation procedure to be acceptable, the results obtained for r, r^2, b, a, and the mean value of V, and V_m, shall agree to within ± 0,1 % of the values given in this example. The values of other statistics are provided to assist the checking of the procedure.

Sums of squares:

$Q_x = 0{,}798\ 12$;

$Q_y = 0{,}000\ 88$;

$Q_{xy} = -0{,}024\ 84$.

Coefficient of correlation:

$r^2 = 0{,}879\ 99$;

$r = 0{,}938\ 08$.

Functional relationships:

$\Gamma = 0{,}001\ 10$;

$b = -0{,}033\ 17$;

$a = 1{,}627\ 31$.

Table 3 — Basic data for example calculation and statistical analysis validation

n	V	Y lg V	**Time** h	X lg h
1	30,8	1,488 6	5 184	3,714 7
2	30,8	1,488 6	2 230	3,348 3
3	31,5	1,498 3	2 220	3,346 4
4	31,5	1,498 3	12 340	4,091 3
5	31,5	1,498 3	10 900	4,037 4
6	31,5	1,498 3	12 340	4,091 3
7	31,5	1,498 3	10 920	4,038 2
8	32,2	1,507 9	8 900	3,949 4
9	32,2	1,507 9	4 173	3,620 4
10	32,2	1,507 9	8 900	3,949 4
11	32,2	1,507 9	878	2,943 5
12	32,9	1,517 2	4 110	3,613 8
13	32,9	1,517 2	1 301	3,114 3
14	32,9	1,517 2	3 816	3,581 6
15	32,9	1,517 2	669	2,825 4
16	33,6	1,526 3	1 430	3,155 3
17	33,6	1,526 3	2 103	3,322 8
18	33,6	1,526 3	589	2,770 1
19	33,6	1,526 3	1 710	3,233 0
20	33,6	1,526 3	1 299	3,113 6
21	35,0	1,544 1	272	2,434 6
22	35,0	1,544 1	446	2,649 3
23	35,0	1,544 1	466	2,668 4
24	35,0	1,544 1	684	2,835 1
25	36,4	1,561 1	104	2,017 0
26	36,4	1,561 1	142	2,152 3
27	36,4	1,561 1	204	2,309 6
28	36,4	1,561 1	209	2,320 1
29	38,5	1,585 5	9	0,954 2
30	38,5	1,585 5	13	1,113 9
31	38,5	1,585 5	17	1,230 4
32	38,5	1,585 5	17	1,230 4
Means:		$\bar{Y} = 1{,}530\ 1$		$\bar{X} = 2{,}\ 930\ 5$

Calculated variances (see 3.2.4):

$E = 3{,}520\ 2 \times 10^{-2}$;

$D = 4{,}842\ 2 \times 10^{-6}$;

$C = 5{,}012\ 7 \times 10^{-6}$ (the variance of b);

$\sigma_\delta^2 = 5{,}271\ 1 \times 10^{-2}$ (the error variance of x).

Check for the suitability for extrapolation (see 3.2.5):

$n = 32$;

$t_v = 2{,}042\ 3$;

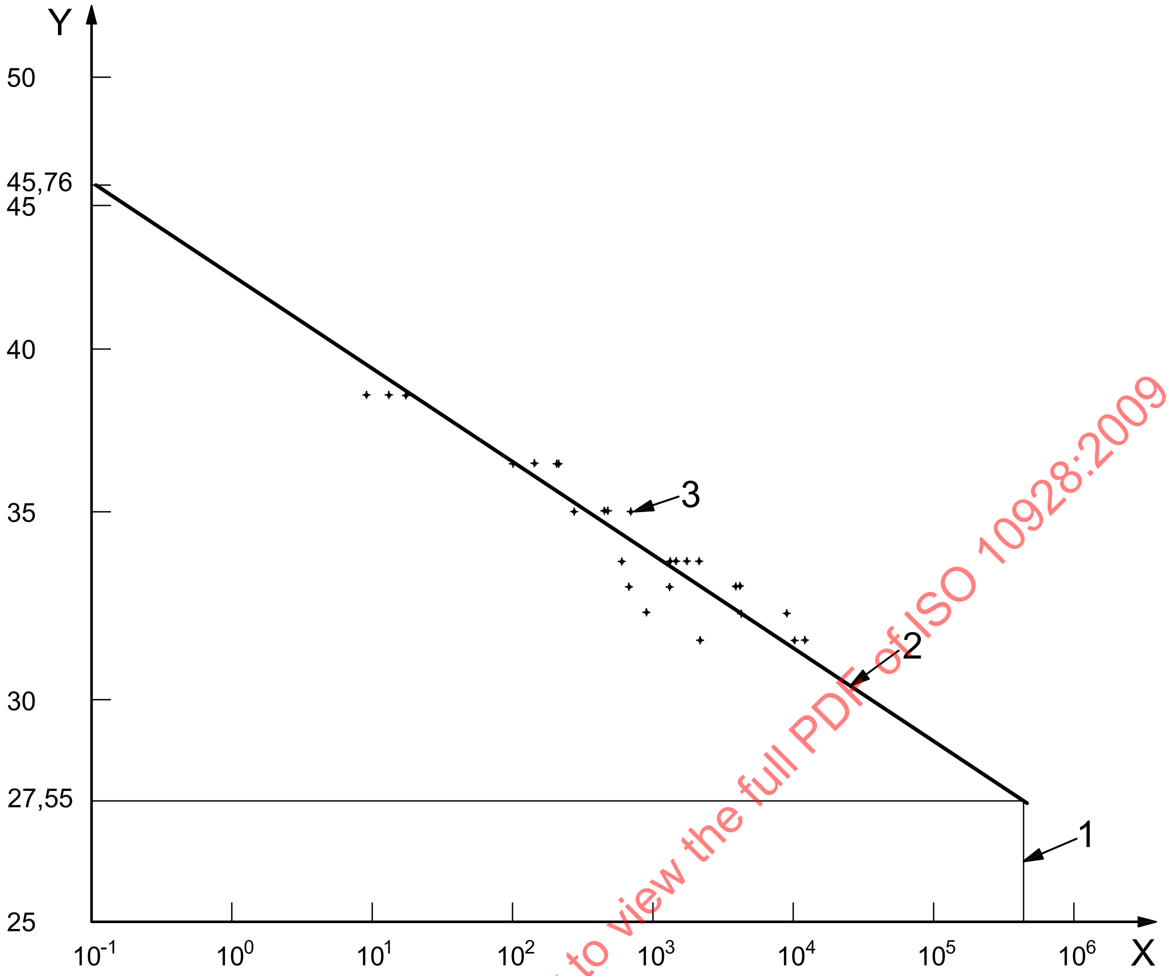
$T = -0{,}033\ 17 / (5{,}012\ 7 \times 10^{-6})^{0{,}5} = -14{,}816\ 7$;

$|T| = 14{,}816\ 7 > 2{,}042\ 3$.

The estimated mean values for V at various times are given in Table 4 and shown in Figure 1.

Table 4 — Estimated mean values, V_m, for V

Time h	V_m
0,1	45,76
1	42,39
10	39,28
100	36,39
1 000	33,71
10 000	31,23
100 000	28,94
438 000	27,55



Key

X-axis lg scale of time, in hours
Y-axis lg scale of property

1 438 000 h (50 years)
2 regression line from Table 4
3 data point

Figure 1 — Regression line from the results in Table 4

3.3 Method B – Regression with time as the independent variable

3.3.1 General

For method B, calculate the sum of the squared residuals parallel to the Y-axis, S_y, using Equation (20):

$$S_\mathrm{y} = \sum \left(y_i - Y \right)^2 \qquad (20)$$

Calculate the sum of the squared residuals parallel to the X-axis, S_x, using Equation (21):

$$S_\mathrm{x} = \sum \left(x_i - X \right)^2 \qquad (21)$$

Calculate the sum of the squared residuals perpendicular to the line, S_{xy}, using Equation (22):

$$S_{xy} = \sum\left[(x_i - X)\times(y_i - Y)\right] \tag{22}$$

where

Y is the arithmetic mean of the y data, i.e.

$$Y = \frac{\sum y_i}{n};$$

X is the arithmetic mean of the x data, i.e.

$$X = \frac{\sum x_i}{n};$$

x_i, y_i are individual values;

n is the total number of results (pairs of readings for x_i, y_i).

NOTE If the value of S_{xy} is greater than zero, the slope of the line is positive and if the value of S_{xy} is less than zero, then the slope is negative.

3.3.2 Suitability of data

Calculate the squared, r^2, and the linear coefficient of correlation, r, using Equations (23) and (24):

$$r^2 = \frac{S_{xy}^2}{S_x \times S_y} \tag{23}$$

$$r = \left|\left(r^2\right)^{0,5}\right| \tag{24}$$

If the value of r^2, or r, is less than the applicable minimum value given in Table 1 as a function of n, consider the data unsuitable for analysis.

3.3.3 Functional relationships

Calculate a and b for the functional relationship line [see Equation (1)], using Equations (25) and (26):

$$b = \frac{S_{xy}}{S_x} \tag{25}$$

$$a = Y - b \times X \tag{26}$$

3.3.4 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate M using Equation (27):

$$M = \frac{S_x{}^2}{S_{xy}{}^2} - \frac{t_v{}^2 \times \left(S_x \times S_y - S_{xy}{}^2\right)}{(n-2)\times S_y{}^2} \tag{27}$$

where

t_v is the applicable value for Student's t determined from Table 2.

If M is equal to or less than zero, consider the data unsuitable for extrapolation.

3.3.5 Validation of statistical procedures by an example calculation

The data given in Table 5 are used in the following example to aid in verifying that statistical procedures, as well as computer programs and spreadsheets, adopted by users will produce results similar to those obtained from the equations given in this International Standard. Use the data given in Table 5 for the calculation procedures described in 3.3.2 to 3.3.4 to ensure that the statistical procedures to be used in conjunction with this method will give results for r, r^2, a, b and V_m to within ± 0,1 % of the values given in this example.

Table 5 — Basic data for example calculation and statistical validation

n	Time T in h	X lg T	V	Y lg V
1	0,10	−1,000 0	7 114	3,852 1
2	0,27	−0,568 6	6 935	3,841 0
3	0,50	−0,301 0	6 824	3,834 1
4	1,00	0	6 698	3,825 9
5	3,28	0,515 9	6 533	3,815 1
6	7,28	0,862 1	6 453	3,809 8
7	20,0	1,301 0	6 307	3,799 9
8	45,9	1,661 8	6 199	3,792 3
9	72,0	1,857 3	6 133	3,787 7
10	166	2,220 1	5 692	3,755 2
11	219	2,340 4	5 508	3,741 0
12	384	2,584 3	5 393	3,731 8
13	504	2,702 4	5 364	3,729 5
14	3 000	3,477 1	5 200	3,716 0
15	10 520	4,022 0	4 975	3,696 8
Means:		X = 1,445 0		Y = 3,781 9

Sums of squares:

S_x = 31,681 1;

S_y = 0,034 7;

S_{xy} = −1,024 2.

Coefficient of correlation:

r^2 = 0,955 6;

r = 0,977 5.

Functional relationships (see 3.3.3):

a = 3,828 6;

b = −0,032 3.

Check for the suitability for extrapolation (see 3.3.4):

t_v = 2,160 4;

M = 942,21.

The estimated mean values, V_m, for V at various times are given in Table 6.

Table 6 — Estimated mean values, V_m, for V

Time h	V_m
0,1	7 259
1	6 739
10	6 256
100	5 808
1 000	5 391
10 000	5 005
100 000	4 646
438 000	4 428

4 Application of methods to product design and testing

4.1 General

The referring standards specify limiting requirements for the long-term properties and performance of a product. Some of these are based on destructive tests, for example hoop tensile strength, whilst others are based on actual or derived physical properties, such as creep or relaxation stiffness.

These properties require an extrapolated long-term (e.g. 50 years) value for the establishment of a product design or comparison with the requirement. This extrapolated value is determined by inserting, as necessary, the values for a and b determined in accordance with 3.1 or 3.2 as appropriate, into Equation (28).

$$\lg y = a + b \times t_L \quad (28)$$

where

t_L is the logarithm, lg, of the long-term period, in hours, [for 50 years (438 000 h), t_L = 5,641 47].

Solving Equation (28), for y gives the extrapolated value.

The use of the data, and the specification of requirements in the product standards, is in three distinct categories.

4.2 Product design

In the first category, the data is used for design or calculation of a product line. This is the case for long-term circumferential strength testing (ISO 7509)[1].The long-term destructive test data is analysed using method A. Short-term test data (ISO 8521) [2] is also required to carry out the design. Annex A describes the procedure for establishing the pressure design of a GRP pipe.

4.3 Comparison to a specified value

The second category is where the long-term extrapolated value is compared to a minimum requirement given in the product standard. This is the case for long-term ring bending (ISO 10471) [6] and strain corrosion (ISO 10952) [8]. The long-term destructive test data are analysed using method A to establish a value to compare to the product standard requirement.

4.4 Declaration of a long-term value

The third category is when the long-term extrapolated value is used to calculate a long-term property and this value is then declared by the manufacturer. This is the case for long-term creep (ISO 10468) [5] or relaxation (ISO 14828) [9] stiffness. These long-term non-destructive test data are analysed using method B.

Annex A
(normative)

GRP pressure pipe design procedure

A.1 Introduction

The design procedure described in this annex is used to formulate the minimum pressure performance requirements of GRP pipes made in accordance with the ISO system standards 10639 [7] and 10467 [4]. The recommended minimum factors of safety relative to the product's performance are given in these ISO system standards and are repeated in this annex.

NOTE The same procedures for pressure pipe design are used in CEN system standards, EN 1796 [10] and EN 14364 [11].

Like all plastics materials, GRP is subject to creep under applied loads. GRP pipe products are tested to establish the regression characteristics because they are influenced by the manufacturing method and the raw materials used.

This design procedure is based upon the principle that pipe products manufactured using a particular manufacturing process, product design and identified materials, when tested in accordance with a specified regression test method, e.g. ISO 7509 [1], will exhibit similar regression characteristics. Test data derived from this test is analysed using method A of this International Standard. The slope of the mean regression line derived from this analysis represents the general regression characteristics of products made with similar materials and processes. For products made with similar materials and processes, the regression behaviour is essentially not dimension-sensitive, i.e. testing products of different diameters and thicknesses will give similar results.

The properties of GRP products, like all manufactured materials, are recognized as having an inherent variability, but it is assumed that the manufacturing facility will be operating a quality control system which will permit the determination of the coefficient of variation and AQL for the initial circumferential tensile strength.

A.2 Minimum factors of safety for long-term pressure requirements

Most GRP pressure pipes are installed underground and are subjected to stress due not only to internal pressure, but also to ring bending resulting from soil and traffic loads. Consideration of these combined loadings and examination of the effects of varying the values for the probability of failure at 50 years has indicated that the factor of safety for the combined loadings, η_{hat}, shall be not less than 1,5.

Minimum ring deflection requirements are defined with respect to the pipe stiffness, which in effect defines the limits of the strain due to bending. Knowing the minimum acceptable value for η_{hat} and the bending conditions, the minimum acceptable value for the factor of safety in tension η_{t} is calculated. Using these concepts, the η_{t} values relating to the 97,5 % LCL and mean values have been calculated and are shown in Table A.1.

Table A.1 — Minimum long-term factors of safety, ($\eta_{\text{t, PN, 97,5\%LCL}}$) and ($\eta_{\text{t, PN, mean}}$)

Property to which factor of safety is to be applied	PN32	PN25	PN16	PN10	PN6	PN4	PN2,5
Minimum factor of safety to be applied to long-term 97,5 % LCL ($\eta_{\text{t, PN, 97,5\%LCL}}$)	1,3	1,3	1,45	1,55	1,6	1,65	1,7
Minimum factor of safety to be applied to long-term mean ($\eta_{\text{t, PN, mean}}$)	1,6	1,6	1,8	1,9	2,0	2,05	2,1
NOTE $\eta_{\text{t, PN, mean}}$ is based on a constant safety factor on combined loading (from pressure and bending) of 1,5. See ISO/TR 10465-3 [3] for a fuller explanation.							

The factors of safety given in Table A.1 shall be used when the coefficient of variation, Y, for the initial failure pressure, P_0 is 9 % or less. If the coefficient of variation is more than 9 % then the applicable factor of safety $(\eta_{t, PN, 97,5\ \%\ LCL})_{new}$ or $(\eta_{t, PN, mean})_{new}$ shall be determined using either Equation (A.1) or (A.2), as applicable:

$$\left(\eta_{t, PN, 97,5\,\%\,LCL}\right)_{Table\ A.1} \times \frac{1 - 9 \times 0,01 \times 1,96}{1 - Y \times 0,01 \times 1,96} = \left(\eta_{t, PN, 97,5\,\%\,LCL}\right)_{new} \quad (A.1)$$

$$\left(\eta_{t, PN, mean}\right)_{Table\ A.1} \times \frac{1 - 9 \times 0,01 \times 1,96}{1 - Y \times 0,01 \times 1,96} = \left(\eta_{t, PN, mean}\right)_{new} \quad (A.2)$$

where

$(\eta_{t, PN, 97,5\,\%\,LCL})_{Table\ A.1}$ is the applicable factor of safety given in Table A.1;

$(\eta_{t, PN, 97,5\,\%\,LCL})_{new}$ is the new applicable minimum factor of safety;

$(\eta_{t, PN, mean})_{Table\ A.1}$ is the applicable factor of safety given in Table A.1;

$(\eta_{t, PN, mean})_{new}$ is the new applicable minimum factor of safety;

Y is the coefficient of variation for the initial failure pressure, in per cent (%).

A.3 Establishment of pressure regression ratio

The regression characteristics of the pipes are established using test pieces selected at random from pipes of the same pressure and stiffness class, for a series of initial failure tests and pressure regression tests. Part of the test pieces are used for a long-term pressure regression test in accordance with ISO 7509 [1] and part of the test pieces are used to determine the mean short-term failure pressure ($P_{0, mean}$) of the test sample in accordance with ISO 8521 [2].

From the results of the long-term pressure test, determine a regression line using method A of this International Standard. From projected values of this line at 0,1 h (6 min) and at 438 000 h (50 years), determine the P_6 and P_{50} failure pressure values (see Figure A.1).

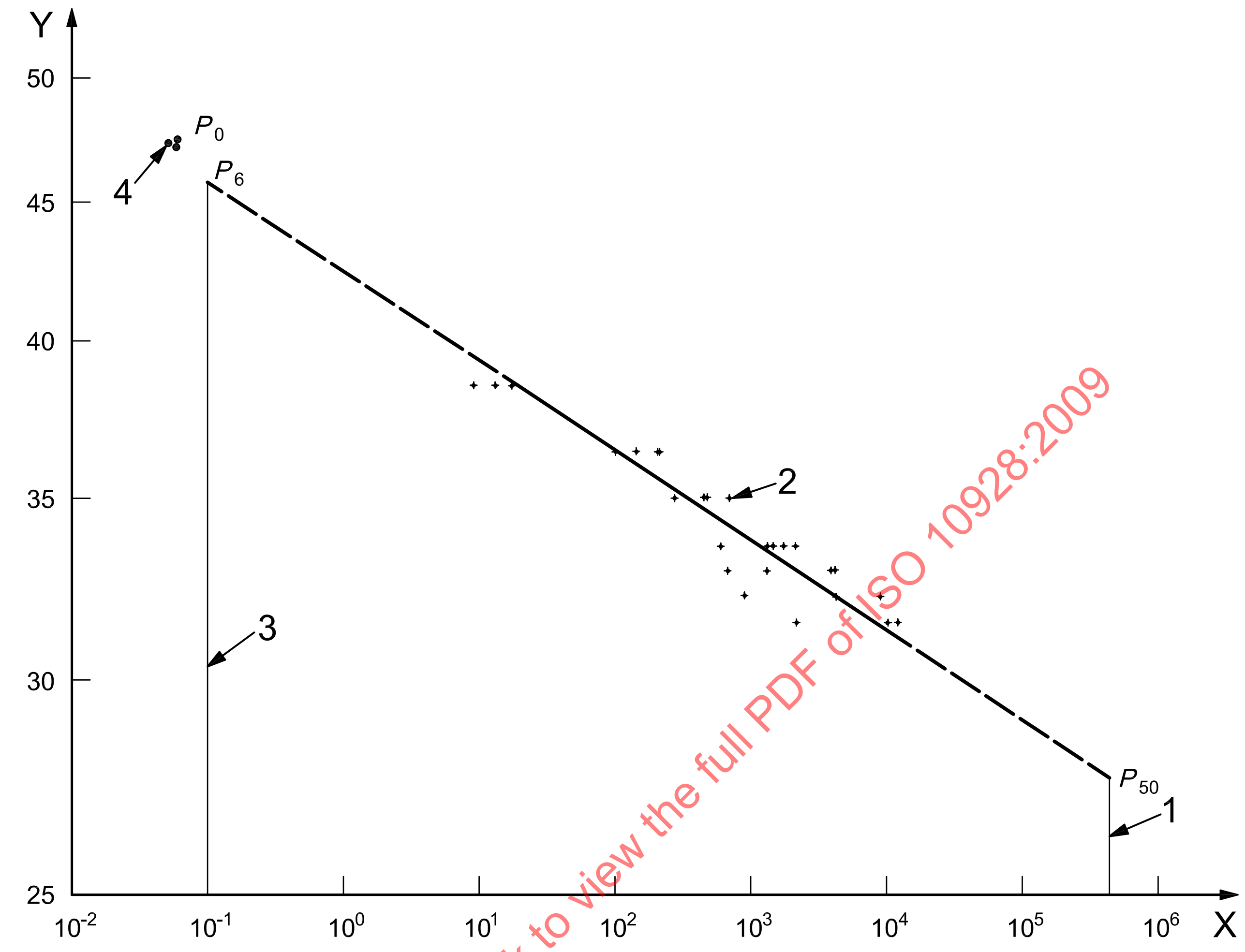
Derive the value of the pressure regression ratio, R_{RP}, for the products from Equation (A.3):

$$R_{RP} = P_{50} / P_6 \quad (A.3)$$

A.4 Determination of the design pressure value

A.4.1 Establishment of $P_{6, min}$

The purpose of the long-term pressure test in relation to this design procedure is only to establish the regression characteristics. The long-term failure pressure value obtained from the test is only directly relevant to the pipes tested. If the projected long-term failure value did not satisfy the defined minimum long-term design requirements for the pressure class of the pipes tested, the regression characteristic can still be used for product design purposes. However, in terms of validating the specific pressure class design of the pipes that were tested, it would show that the pipe, as manufactured, did not satisfy the procedure.



Key

X-axis lg scale of time, in hours

Y-axis lg scale of pressure, in bars[1)]

1 438 000 h (50 years)

2 results from long-term pressure tests

3 6 minutes

4 results from short-term pressure tests

P_0 mean initial failure pressure

P_6 projected failure pressure at 6 minutes

P_{50} projected failure pressure at 50 years

R_{RP} pressure regression ratio, $R_{RP} = P_{50}/P_6$

C correction factor for initial failure pressure, $C = P_0/P_6$

Figure A.1 — Derivation of pressure regression ratio, R_{RP}, and correction factor, C

1) 1 bar = 1 MPa = 0,1 N/mm^2 = 10^6·N/m^2.

Using the regression characteristics for the product, determined in accordance with A.3, calculate the design pressure values of the various pipe classes as follows.

Using the required value of PN, determine the appropriate value of $\eta_{t,\,PN,\,97,5\%\,LCL}$ from Table A.1 or Equation (A.1), as applicable, then calculate the minimum failure pressure at 50 years, $P_{50,\,min.}$, using Equation (A.4) where the nominal pressure PN is expressed in bars. Calculate the minimum failure pressure at 6 min, $P_{6,\,min.}$, using the value of the regression ratio R_{RP} derived from Equation (A.3) and the value of $P_{50,\,min.}$ from Equation (A.5).

$$P_{50,\,min.} = PN \times \eta_{t,\,PN,\,97,5\,\%\,LCL} \tag{A.4}$$

$$P_{6,\,min.} = \frac{PN \times \eta_{t,\,PN,\,97,5\%\,LCL}}{R_{RP}} = \frac{P_{50,\,min}}{R_{RP}} \tag{A.5}$$

A.4.2 Establishment of design pressure $P_{0,\,d}$

Initial failure pressure test results are influenced by the rate of pressurization and generally, the higher the pressurization rate, the higher the initial failure pressure. To allow for this, the design procedure includes a correction factor, C, which is the ratio of the mean of initial failure pressure tests performed as part of the regression test described in A.3, P_0, to the projected 6 minute failure pressure $P_{6,\,min}$ [see Equation (A.6)].

Calculate the correction factor for initial failure, C, using Equation (A.6):

$$C = P_0 / P_6 \tag{A.6}$$

Determine, from Equation (A.7), the lower 97,5 % confidence limit of the initial failure pressure, $P_{0,\,min}$, using $P_{6,\,min.}$ from Equation (A.5) and C from Equation (A.6):

$$P_{0,\,min} = C \times P_{6,\,min} \tag{A.7}$$

Determine from Equation (A.8) the minimum design pressure $P_{0,\,d}$ using $P_{0,\,min}$ from Equation (A.7) and the coefficient of variation, Y, which is established from the quality system results obtained from the facilities routine product testing. A graphical representation of this procedure is given in Figure A.2.

$$P_{0,\,d} = P_{0,\,min} \times \frac{1}{(1 - Y \times 0{,}01 \times 1{,}96)} \tag{A.8}$$

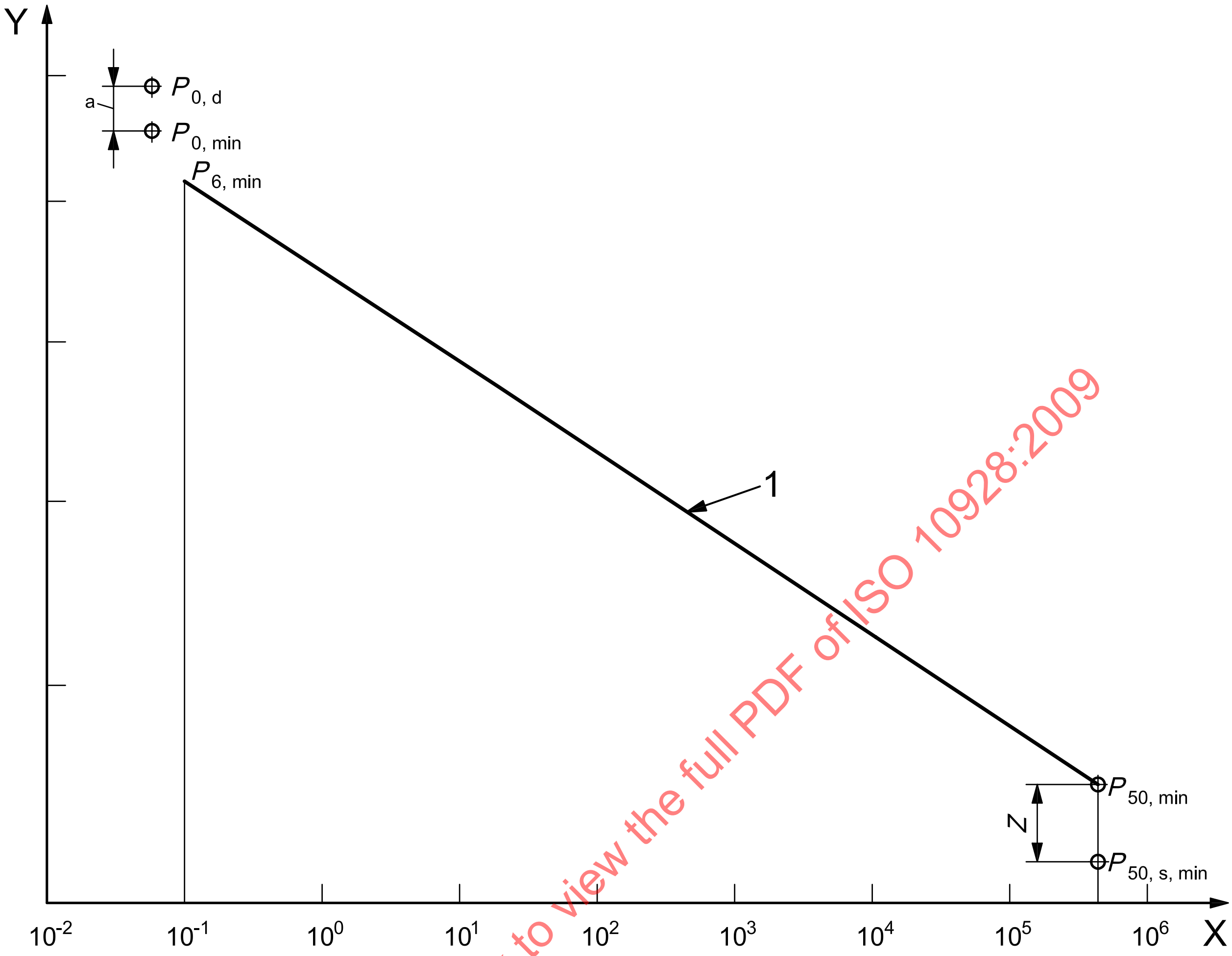
where

Y is the coefficient of variation in per cent (%) related to the mean initial failure pressure of the pipe of a particular pressure class manufactured at the facility;

1,96 is the multiplier for the 97,5 % confidence level.

A.5 Control procedures for $P_{0,\,d}$

A.5.1 To ensure that the factor of safety related to the 97,5 % LCL at 50 years, $\eta_{t,\,PN,\,97,5\%\,LCL}$ and the factor of safety, $\eta_{t,\,PN,\,mean}$ related to the minimum mean value at 50 years, $P_{50,\,mean,\,min}$.are met, perform the following procedures to determine the minimum design initial failure pressure value, $P_{0,\,d}$ (see Figure A.3).



Key

X-axis	lg scale of time, in hours
Y-axis	lg scale of pressure, in bars
1	97,5 % LCL line
C	correction factor, $C = P_0 / P_6$
$P_{0,\text{ d}}$	design minimum initial failure pressure
$P_{0,\text{ min}}$	minimum initial failure pressure
$P_{0,\text{ min}}$	$= C \times P_{6,\text{ min}}$
$P_{6,\text{ min}}$	minimum failure pressure at 6 minutes under test
$P_{50,\text{ min}}$	minimum failure pressure at 50 years
$P_{50,\text{ s, min}}$	specified minimum failure pressure at 50 years
Z	minimum long-term margin of safety, Z = Factor of safety – 1
σ	standard deviation of short-term pressure test data, derived from quality system
a	$1{,}96 \times \sigma$.

Figure A.2 — Derivation of the minimum design initial failure pressure, $P_{0,\text{ d}}$

A.5.2 Calculate the following.

a) Calculate the value for $P_{6,\ \text{mean}}$ using Equation (A.9):

$$\frac{P_{0,\text{d}}}{C} = P_{6,\ \text{mean}} \tag{A.9}$$

b) Calculate the value for $P_{50,\ \text{mean}}$ using Equation (A.10):

$$P_{6,\ \text{mean}} \times R_{\text{RP}} = P_{50,\ \text{mean}} \tag{A.10}$$

c) Calculate, using Equation (A.11), the minimum value for $P_{50,\ \text{mean}}$:

$$P_{50,\ \text{mean, min}} = PN \times \eta_{\text{t, PN, mean}} \tag{A.11}$$

where

$\eta_{\text{t, PN, mean}}$ is the appropriate value from Table A.1 or, if applicable, calculated from Equation (A.2).

d) If $P_{50,\ \text{mean}}$ from Equation (A.10) is equal to or greater than $P_{50,\ \text{mean, min.}}$ found from Equation (A.11), then $P_{0,\ \text{d}}$ is sufficient to satisfy the long-term factor of safety relative to the mean. If it is not, then $P_{0,\ \text{d}}$ has to be increased until this requirement is satisfied. Satisfying this requirement also ensures that the factor of safety related to the 97,5 % LCL at 50 years is also met, as $P_{50,\ \text{mean, min.}}$ includes the value of $1{,}96 \times \sigma$.

A.6 Pressure products assessment

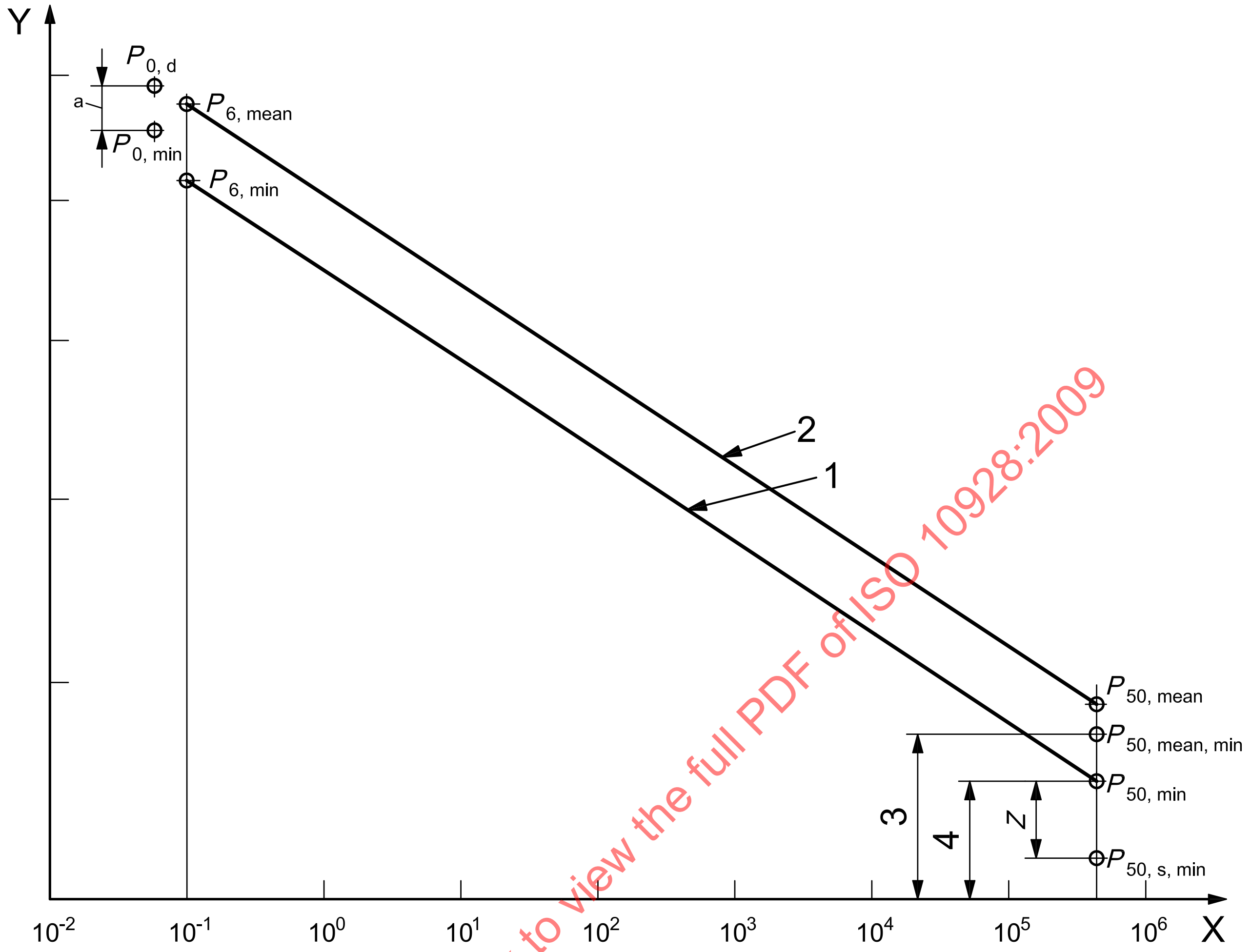
Using the results from initial failure tests performed over a period of time on a particular pressure class of pipe, determine the mean value, $P_{0,\ \text{mean}}$, and the standard deviation, σ, of the initial failure pressure. Calculate the coefficient of variation of this property, Y, in per cent (%) using Equation (A.12):

$$Y = \frac{\sigma}{P_{0,\ \text{mean}}} \tag{A.12}$$

where

Y is the standard deviation × 100, divided by the mean.

To assess a product designed using the foregoing design method, the quality system is required to be able to confirm that the product passes the minimum pressure requirements and also needs to establish whether or not the average failure pressure of the product, $P_{0,\ \text{mean}}$, is equal to or greater than the minimum design value $P_{0,\ \text{d}}$.



Key

X-axis	lg scale of time, in hours
Y-axis	lg scale of pressure, in bars
1	97,5 % LCL line
2	mean line
3	$PN \times \eta_{t,\ PN,\ mean}$
4	$PN \times \eta_{t,\ 97,5\%\ LCL}$
C	correction factor, $C = P_0 / P_6$
$P_{50,\ mean,\ min.}$	minimum mean failure pressure at 50 years
$P_{0,\ d}$	minimum design initial failure pressure
$P_{0,\ min}$	$= C \times P_{6,\ min}$
$P_{0,\ min.}$	minimum initial failure pressure
$P_{6,\ mean}$	mean failure pressure at 6 minutes
$P_{6,\ min.}$	minimum failure pressure at 6 minutes
$P_{50,\ mean}$	mean failure pressure at 50 years
$P_{50,\ min.}$	minimum failure pressure at 50 years
$P_{50,\ s,\ min.}$	specified minimum failure pressure at 50 years
Z	minimum long-term margin of safety, Z = Factor of safety – 1
σ	standard deviation from quality system
a	1,96 × σ.

Figure A.3 — Derivation of long-term mean failure pressures

Annex B
(informative)

Second-order polynomial relationships

B.1 General

This method fits a curved line of the form given in Equation (B.1):

$$y = c + d \times x + e \times x^2 \tag{B.1}$$

where

y is the logarithm, lg, of the property being investigated;

c is the intercept on the Y-axis;

d, e are the coefficients to the two orders of x;

x is the logarithm, lg, of the time, in hours.

B.2 Variables

Calculate the following variables:

$\sum x_i$, the sum of all individual x data;

$\sum x_i^2$, the sum of all squared x data;

$\sum x_i^3$, the sum of all x data to the third power;

$\sum x_i^4$, the sum of all x data to the fourth power;

$\sum y_i$, the sum of all individual y data;

$(\sum y_i)^2$, the squared sum of all individual y data;

$\sum y_i^2$, the sum of all squared y data;

$\sum (x_i \times y_i)$, the sum of all products $x_i y_i$;

$\sum \left(x_i^2 \times y_i\right)$, the sum of all products $x_i^2 y_i$;

$S_\mathrm{x} = \sum (x_i - X)^2$, the sum of the squared residuals parallel to the X-axis for the linear part;

$S_{xx} = \sum\left(x_i^2 - X^2\right)^2$, the sum of the squared residuals parallel to the X-axis for the quadratic part;

$S_y = \sum\left(y_i - Y\right)^2$, the sum of the squared residuals parallel to the Y-axis;

$S_{xy} = \sum\left[\left(x_i - X\right)\times\left(y_i - Y\right)\right]$, the sum of the squared residuals perpendicular to the line for the linear part;

$S_{xxy} = \sum\left[\left(x_i^2 - X^2\right)\times\left(y_i - Y\right)\right]$, the sum of the squared residuals perpendicular to the line for the quadratic part.

where

Y is the arithmetic mean of the y data, i.e. $Y = \frac{\sum y_i}{n}$;

X is the arithmetic mean of the x data, i.e. $X = \frac{\sum x_i}{n}$.

B.3 Solution system

Determine c, d and e using the following matrix:

$$\sum y_i = c\times n + d\times\sum x_i + e\times\sum x_i^2 \tag{B.2}$$

$$\sum\left(x_i\times y_i\right) = c\times\sum x_i + d\times\sum x_i^2 + e\times\sum x_i^3 \tag{B.3}$$

$$\sum\left(x_i^2\times y_i\right) = c\times\sum x_i^2 + d\times\sum x_i^3 + e\times\sum x_i^4 \tag{B.4}$$

B.4 Suitability of data

Calculate the squared, r^2, and the linear coefficient of correlation, r, using Equations (B.5) and (B.6):

$$r^2 = \frac{c\times\sum y_i + d\times\sum\left(x_i\times y_i\right) + e\times\sum\left(x_i^2\times y_i\right) - \left[\left(\sum y_i\right)^2 / n\right]}{\sum y_i^2 - \left[\left(\sum y_i\right)^2 / n\right]} \tag{B.5}$$

$$r = \left|\left(r^2\right)^{0,5}\right| \tag{B.6}$$

If the value of r^2 or r is less than the applicable minimum value given in Table 1 as a function of n, consider the data unsuitable for analysis.

B.5 Check for the suitability of data for extrapolation

If it is intended to extrapolate the line, calculate M using Equation (B.7):

$$M = \frac{S_x{}^2}{S_{xy}{}^2} + \frac{S_{xx}{}^2}{S_{xxy}{}^2} - \frac{t_v{}^2 \times \left(S_x \times S_y - S_{xy}{}^2 + S_{xx} \times S_y - S_{xxy}{}^2\right)}{(n-2) \times S_y{}^2} \qquad \text{(B.7)}$$

If M is equal to or less than zero, consider the data unsuitable for extrapolation.

B.6 Validation of statistical procedures by an example calculation

Use the data given in Table 5 for the calculation procedures described in B.1 to B.5 to ensure that the statistical procedures used in conjunction with this method gives results for r, r^2, a, b and V_m to within ± 0,1 % of the values given in this example (n = 15).

$\sum x_i$ = 21,671;

$\sum x_i{}^2$ = 62,989;

$\sum x_i{}^3$ = 180,623;

$\sum x_i{}^4$ = 584,233;

$\sum y_i$ = 56,728;

$(\sum y_i)^2$ = 3 218,09;

$\sum y_i{}^2$ = 214,574;

$\sum (x_i \times y_i)$ = 80,932;

$\sum \left(x_i{}^2 \times y_i\right)$ = 235,175;

$S_x = \sum (x_i - X)^2$ = 31,681;

$S_{xx} = \sum \left(x_i{}^2 - X^2\right)^2$ = 386,638;

$S_y = \sum (y_i - Y)^2$ = 0,034 7;

$S_{xy} = \sum \left[(x_i - X) \times (y_i - Y)\right]$ = −1,024 2;

$S_{xxy} = \sum \left[\left(x_i{}^2 - X^2\right) \times (y_i - Y)\right]$ = −3,041 8.

Solution system:

$c = 3{,}828\ 8$;

$d = -0{,}026\ 2$;

$e = -0{,}002\ 2$.

Coefficient of correlation:

$r^2 = 0{,}964\ 7$;

$r = 0{,}982\ 2$.

Check for the suitability for extrapolation:

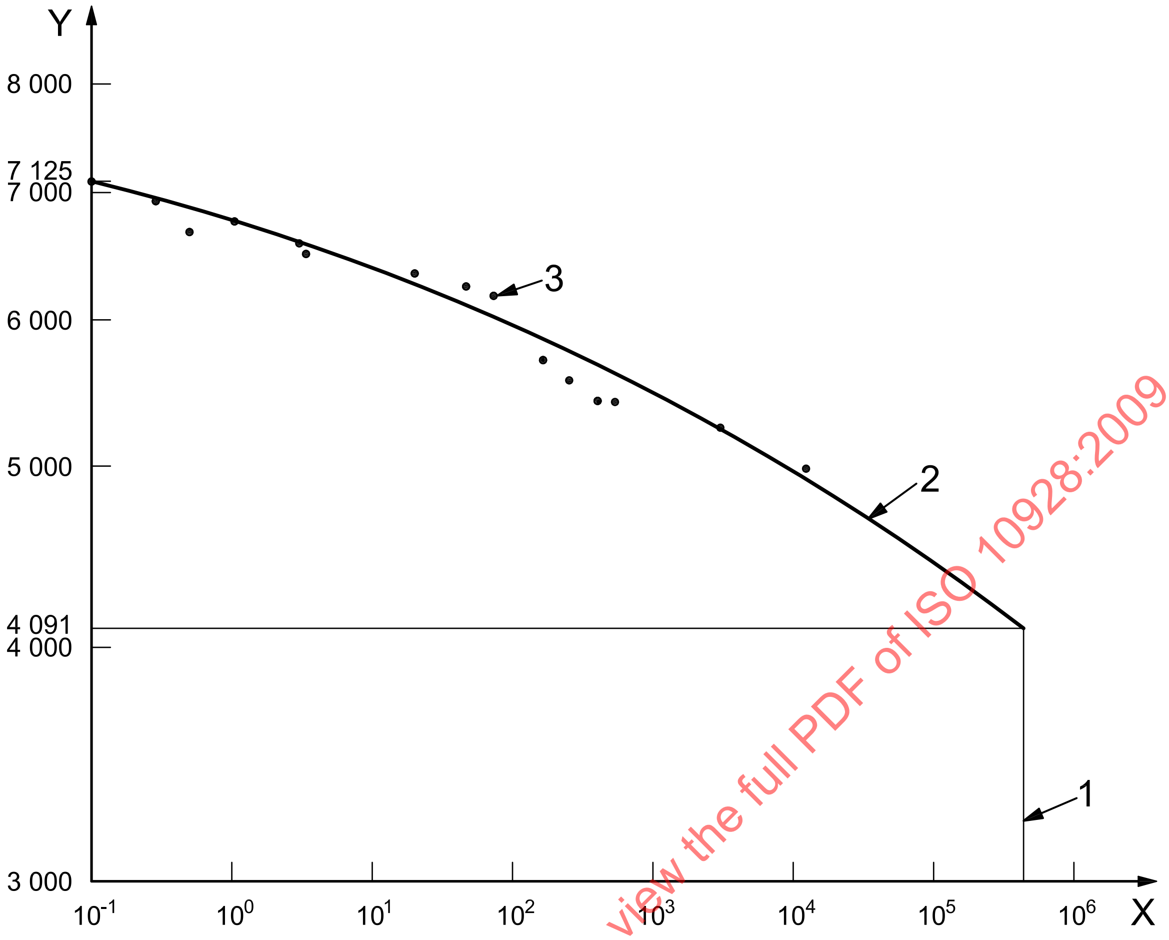
$t_v = 2{,}160\ 4$;

$M = 15\ 859{,}6$.

The estimated mean values, V_m, for V at various times are given in Table B.1 and shown in Figure B.1.

Table B.1 — Estimated mean values, V_m, for V

Time h	V_m
0,1	7 125
1	6 742
10	6 315
100	5 856
1 000	5 375
10 000	4 884
100 000	4 393
438 000	4 091



Key

X-axis lg scale of time, in hours

Y-axis lg scale of property

1 438 000 h (50 years)

2 regression line from Table B.1

3 data point

Figure B.1 — Regression line from the results in Table B.1

Annex C
(informative)

Non-linear relationships

C.1 General

Given a model for a set of data obtained from a long-term stiffness test on GRP test pieces, the objective of this annex is to produce explicit formulae for:

a) estimating all four parameters in the model, i.e. a, b, c and d;

b) calculating confidence and prediction intervals about the curve.

These formulae are presented in this annex, along with associated graphical displays for a typical data set.

NOTE While the data and procedures refer to a long-term stiffness test, the method can also be applied to data which fits the mathematical model and requires extrapolation to 50 years.

C.2 Model

As described in C.4, the model shown can be re-expressed as two linked straight-line regression models called Line 1 and Line 2. Answers obtained from the procedures described for Line 1 are then applied in the Line 2 procedures to obtain the four parameters in the model, which is then used to obtain the long-term value for the property under investigation.

C.2.1 Procedure for Line 1

The equations given in the following subclauses are used to develop the models for Lines 1 and 2.

NOTE Many of the equations make reference to a subscript i, which is the count value shown in the tables. For the data used in the tables, i runs from 0 to 16, where data indexed 1 to 15 are the experimental measured values, and those indexed 0 or 16 are the calculated values. When using the procedures described in this annex, the amount of data that can be analysed is not limited to 15 data sets but can be any number.

C.2.1.1 Determination of derived values for Y_i, x_i and y_i

Calculate Y_i, x_i, y_i, $\bar{x}$ and $\bar{y}$ using Equations (C.1), (C.2), (C.3), (C.4) and (C.5):

$$Y_i = \lg_{10}(S_i) \quad \text{(C.1)}$$

$$x_i = \lg_{10}(\text{minutes} + 1) \quad \text{(C.2)}$$

$$y_i = \ln\left[(a + b - Y_i)/(Y_i - a)\right] \quad \text{(C.3)}$$

$$\bar{x} = \left(\sum x_i\right)/n \quad \text{(C.4)}$$

$$\bar{y} = \left(\sum y_i\right)/n \quad \text{(C.5)}$$

Equations (C.1), (C.2), (C.3), (C.4) and (C.5) relate to values obtained for the property, S_i, after various periods of time under test, x_i.

C.2.1.2 Determination of parameters a and b

Calculate the following using Equations (C.6), (C.7) and (C.8):

$$a_0 = 0{,}995\ \text{min.}(Y_i) \tag{C.6}$$

$$a_0 + b_0 = 1{,}005\ \text{max.}(Y_i) \tag{C.7}$$

$$b_0 = (a_0 + b_0) - a_0 \tag{C.8}$$

Equations (C.6), (C.7) and (C.8) produce the initial estimates for two of the parameters in the model, a_0 and b_0.

C.2.1.3 Determination of the least squares estimates for A and B and unbiased estimate for $\tilde{\sigma}_1^2$

Calculate the following using Equations (C.9), (C.10), (C.11) and (C.12):

$$\hat{A} = \bar{y} - \hat{B}\bar{x} \tag{C.9}$$

$$\hat{B} = \sum(x_i - \bar{x})(y_i - \bar{y}) / \sum(x_i - \bar{x})^2 \tag{C.10}$$

$$\tilde{\sigma}_1^2 = \sum(y_i - \hat{y}_i)^2 / (n-2) = \text{RSS}/(n-2) \tag{C.11}$$

where

$$\text{RSS} = \sum y_i^2 - \hat{A} \times \sum y_i - \hat{B} \times \sum x_i \times y_i \tag{C.12}$$

NOTE RSS is the residual sum of squares.

C.2.1.4 Determination of estimates for parameters c and d

Calculate the following using Equations (C.13) and (C.14):

$$\hat{c} = \left(\hat{A}\hat{B}^{-1} + \lg_{10} 60\right) \tag{C.13}$$

$$\hat{d} = -\hat{B}^{-1} \tag{C.14}$$

C.2.2 Procedure for Line 2

C.2.2.1 Determination of X_i, Y_i, $\bar{X}$ and $\bar{Y}$

Calculate X_i using Equation (C.15):

$$X_i = 1/\left\{1 + \exp\left[-\lg_{10}(\text{Time}) - \hat{c}\right]/\hat{d}\right\} \tag{C.15}$$

NOTE Values for c and d are given as Equations (C.13) and (C.14).

$$Y_i = \lg_{10}(\text{stiffness}) \tag{C.16}$$

$$\bar{X} = \left(\sum X_i\right)/n \tag{C.17}$$

$$\bar{Y} = \left(\sum Y_i\right)/n \tag{C.18}$$

C.2.2.2 Determination of the least squares estimates for $\hat{a}$ and $\hat{b}$ and unbiased estimate for $\tilde{\sigma}_1^2$

The estimates are derived using Equations (C.19), (C.20), (C.21) and (C.22):

$$\hat{b} = \sum\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right) / \sum\left(X_i - \bar{X}\right)^2 = \left(n \times \sum XY - \sum X \times \sum Y\right) / \left(n \times \sum X^2 - \sum X \times \sum X\right) \quad \text{(C.19)}$$

$$\hat{a} = \bar{Y} - \hat{b} \times \bar{X} \quad \text{(C.20)}$$

$$\tilde{\sigma}_1^2 = \sum\left(Y_i - \hat{Y}_i\right)^2 / (n-2) = \text{RSS} / (n-2) \quad \text{(C.21)}$$

where

$$\text{RSS} = \sum Y_i^2 - \hat{a} \times \sum Y_i - \hat{b} \times \sum X_i \times Y_i \quad \text{(C.22)}$$

Check that the following constraints are met: $\hat{a} + \hat{b} > Y_i > \hat{a}$

C.2.2.3 Determination of confidence and prediction intervals

Calculate the variances of $\hat{a}$ and $\hat{b}$ using Equations (C.23) and (C.24):

$$\text{var}\left(\hat{a}\right) = \left(\tilde{\sigma}_2^2 \sum X_i^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(\tilde{\sigma}_2{}^2 \sum X_i^2\right) \Big/ \left[n \sum\left(X_i - \bar{X}\right)^2\right] \quad \text{(C.23)}$$

$$\text{var}\left(\hat{b}\right) = \left(n\tilde{\sigma}_2^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(n\tilde{\sigma}_2{}^2\right) \Big/ \left[n \sum\left(X_i - \bar{X}\right)^2\right] \quad \text{(C.24)}$$

Calculate the estimated standard error, ε, of $\hat{a}$ and $\hat{b}$ given as Equations (C.25) and (C.26):

$$\varepsilon\left(\hat{a}\right) = \sqrt{\text{var}\left(\hat{a}\right)} \quad \text{(C.25)}$$

$$\varepsilon\left(\hat{b}\right) = \sqrt{\text{var}\left(\hat{b}\right)} \quad \text{(C.26)}$$

Formulae for 100 % confidence and prediction intervals about the fitted Line 2 as functions of X are given as Equations (C.27) and (C.28) respectively:

$$\text{Confidence interval } \mu_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[\frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum\left(X_i - \bar{X}\right)^2}\right]} \quad \text{(C.27)}$$

$$\text{Prediction interval } Y_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[1 + \frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum\left(X_i - \bar{X}\right)^2}\right]} \quad \text{(C.28)}$$

where

$$\hat{Y}_X = \hat{\mu}_X = \hat{a} + \hat{b}X \quad \text{(C.29)}$$

C.2.2.4 Student's t test for a and b

To test whether $\hat{a}$ or $\hat{b}$ is equal to 0, perform the following calculations:

$$\Pr\left(|t| < t_P\right) = P;$$

t has Student's t-distribution on $(n - 2)$ degrees of freedom.

From Statistical tables t boundary for $P = 90\ \% = 1{,}771$.

From Statistical tables t boundary for $P = 95\ \% = 2{,}160$.

If Equations (C.30) and (C.31) give t values greater than the applicable one of the above bounds, it is certain that $\hat{a}$ and $\hat{b}$ are not equal to 0.

$$t \text{ for } \hat{a} = \hat{a}/\varepsilon(\hat{a}) \qquad \text{(C.30)}$$

$$t \text{ for } \hat{b} = \hat{b}/\varepsilon(\hat{b}) \qquad \text{(C.31)}$$

C.2.2.5 Calculation of long-term (50-year) stiffness

All the formulae in C.2.1 and C.2.2 are standard formulae for straight-line regressions. From the values given in Tables C.4 and C.5, the estimated long-term stiffness and its confidence and prediction limits can be determined using Equations (C.32), (C.33), (C.34), (C.35) and (C.36):

Using Equation (C.29), the extrapolated long-term stiffness is given as Equation (C.32):

$$\hat{Y}_X = \hat{\mu}_X = \hat{a} + \hat{b}X \qquad \text{(C.32)}$$

Using Equation (C.27), the confidence limits for $\mu_{50\ \text{years}}$ are given as Equation (C.33):

$$\mu_{50\ \text{years}} = \hat{\mu}_X \pm \mu_{\text{bounds}} \qquad \text{(C.33)}$$

Using Equation (C.28), the prediction limits are given as Equation (C.34):

$$\hat{Y}_{50\ \text{years}} = Y_X \pm Y_{Y\ \text{bounds}} \qquad \text{(C.34)}$$

Transforming these values back to stiffness gives:

Extrapolated long-term stiffness $= 10^{\hat{Y}_{50\ \text{years}}}$

90 % confidence limits for extrapolated long-term stiffness are given as Equation (C.35):

$$\mu(S)_{50\ \text{years}} = 10^{\hat{\mu}_X \pm \mu_{\text{bounds}}} \qquad \text{(C.35)}$$

90 % prediction limits for extrapolated long-term stiffness are given as Equation (C.36):

$$S_{50\ \text{years}} = 10^{Y_X \pm Y_{Y\ \text{bounds}}} \qquad \text{(C.36)}$$

C.3 Validation of statistical procedures by an example calculation

Use the data given in Table C.1 for the calculation procedures described in C.2.1 to C.2.2.5 to ensure that the statistical procedures to be used in conjunction with this method give results to within ± 0,1 % of the values given in this example (n = 15).

C.3.1 Procedure for Line 1

C.3.1.1 Determination of derived values for Y_i, x_i and y_i

$$Y_i = \lg_{10}\left(S_i\right) \quad \text{(C.37)}$$

NOTE 1 See derived values in Table C.1.

$$x_i = \lg_{10}(\text{minutes} + 1) \quad \text{(C.38)}$$

NOTE 2 See derived values in Table C.1.

$$y_i = \ln\left[\left(a + b - Y_i\right) / \left(Y_i - a\right)\right] \quad \text{(C.39)}$$

NOTE 3 See derived values in Table C.1.

$$\bar{x} = \left(\sum x_i\right) / n = 48{,}465\ 540/15 = 3{,}231\ 036 \quad \text{(C.40)}$$

$$\bar{y} = \left(\sum y_i\right) / n = -2{,}512\ 828/15 = -0{,}167\ 522 \quad \text{(C.41)}$$

C.3.1.2 Determination of parameters a and b

For values of Y_i, see Table C.1.

$$a_0 = 0{,}995 \text{ min.}\left(Y_i\right) = 0{,}995 \times 3{,}696\ 793 = 3{,}678\ 309 \quad \text{(C.42)}$$

$$a_0 + b_0 = 1{,}005 \text{ max.}\left(Y_i\right) = 1{,}005 \times 3{,}852\ 114 = 3{,}871\ 375 \quad \text{(C.43)}$$

$$b_0 = \left(a_0 + b_0\right) - a_0 = 3{,}871\ 375 - 3{,}678\ 309 = 0{,}193\ 066 \quad \text{(C.44)}$$

C.3.1.3 Determination of the least squares estimates for A and B and unbiased estimate for $\tilde{\sigma}_1^2$

Calculate the following:

$$\hat{A} = \bar{y} - \hat{B}\bar{x} = -0{,}167\ 5 - 0{,}831\ 9 \times 3{,}231\ 036 = -2{,}855\ 5 \quad \text{(C.45)}$$

$$\hat{B} = \sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) / \sum\left(x_i - \bar{x}\right)^2 = \left(n \times \sum xy - \sum x \times \sum y\right) \div \left[n \times \sum x^2 - \left(\sum x\right)^2\right] \quad \text{(C.46)}$$

Using values from Table C.2,

$$\hat{B} = \left(15 \times 17{,}8 - 48{,}47 \times 2{,}51\right) / \left[15 \times 187{,}75 - \left(2\ 349{,}34\right)\right] = 0{,}831\ 9$$

Using values for A and B in this subclause and from Table C.2,

$$\tilde{\sigma}_1^2 = \sum\left(y_i - \hat{y}_i\right)^2 / \left(n - 2\right) = \text{RSS} / \left(n - 2\right) = 0{,}665\ 3 / 13 = 0{,}051\ 2 \quad \text{(C.47)}$$

where

$$\text{RSS} = \sum y_i^2 - \hat{A} \times \sum y_i - \hat{B} \times \sum x_i \times y_i \quad \text{(C.48)}$$

C.3.1.4 Determination of estimates for parameters c and d

Using the values for $\hat{A}$ and $\hat{B}$ calculated above, the estimated values for $\hat{c}$ and $\hat{d}$ are:

$$\hat{c} = \left(\hat{A}\hat{B}^{-1} + \lg_{10} 60\right) = 1{,}653\ 53 \quad \text{(C.49)}$$

$$\hat{d} = -\hat{B}^{-1} = -1{,}202 \quad \text{(C.50)}$$

C.3.2 Procedure for Line 2

C.3.2.1 Determination of X_i, Y_i, $\bar{X}$ and $\bar{Y}$

Calculate the following:

$$X_i = 1/\left\{1 + \exp\left[-\lg_{10}(\text{Time}) - \hat{c}\right]/\hat{d}\right\} \quad \text{(C.51)}$$

NOTE 1 See derived values in Table C.3.

NOTE 2 Values for c and d are given in Equations (C.49) and (C.50).

$$Y_i = \lg_{10}(\text{stiffness}) \quad \text{(C.52)}$$

NOTE 3 See derived values in Table C.3.

$$\bar{X} = \left(\sum X_i\right)/n = 7{,}966\ 259/15 = 0{,}531\ 084 \quad \text{(C.53)}$$

$$\bar{Y} = \left(\sum Y_i\right)/n = 56{,}728\ 211/15 = 3{,}781\ 881 \quad \text{(C.54)}$$

C.3.2.2 Determination of the least squares estimates for $\hat{a}$ and $\hat{b}$ and unbiased estimate for $\tilde{\sigma}_1^2$

The estimates are derived using Equations (C.55), (C.56), (C.57) and (C.58):

$$\hat{b} = \sum\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)/\sum\left(X_i - \bar{X}\right)^2 = \left(n \times \sum XY - \sum X \times \sum Y\right)/\left(n \times \sum X^2 - \sum X \times \sum X\right) \quad \text{(C.55)}$$

$$\hat{b} = (15 \times 30{,}302\ 557 - 7{,}966\ 259 \times 56{,}728\ 211)/(15 \times 5{,}146\ 065 - 7{,}966\ 259 \times 7{,}966\ 259)$$
$$= 2{,}626\ 734/13{,}729\ 693 = 0{,}191\ 318$$

$$\hat{a} = \bar{Y} - \hat{b} \times \bar{X} \quad \text{(C.56)}$$

$$\hat{a} = 3{,}781\ 881 - 0{,}191\ 318 \times 0{,}531\ 083\ 4 = 3{,}680\ 275$$

$$\tilde{\sigma}_1^2 = \sum\left(Y_i - \hat{Y}_i\right)^2/(n-2) = \text{RSS}/(n-2) \quad \text{(C.57)}$$

$$\text{RSS} = \sum Y_i^2 - \hat{a} \times \sum Y_i - \hat{b} \times \sum X_i \times Y_i =$$

$$\sum Y^2 - \hat{a} \times \sum Y - \hat{b} \times \sum XY = 214{,}573\,977 - 3{,}680\,275 \times 56{,}728\,211 - 0{,}191\,318 \times 30{,}302\,557 =$$
$$= 0{,}001\,136$$

$$\tilde{\sigma}_2^2 = 0{,}001\,136/13 = 0{,}000\,087 \qquad \text{(C.58)}$$

Check that the following constraints are met: $\hat{a} + \hat{b} > Y_i > \hat{a}$

max. $Y_i = 3{,}852\,114$, min. $Y_i = 3{,}696\,793$, $\hat{a} + \hat{b} = 3{,}680\,275 + 0{,}191\,318 = 3{,}871\,593$ and $\hat{a} = 3{,}680\,275$

From inspection of these values, the constraints are satisfied.

C.3.2.3 Determination of confidence and prediction intervals

Using Equations (C.59) and (C.60), determine the variances of $\hat{a}$ and $\hat{b}$:

$$\text{var}(\hat{a}) = \left(\tilde{\sigma}_2^2 \sum X_i^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(\tilde{\sigma}_2{}^2 \sum X_i{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2\right] \qquad \text{(C.59)}$$

$$\text{var}(\hat{a}) = 0{,}000\,033$$

$$\text{var}(\hat{b}) = \left(n\tilde{\sigma}_2^2\right) \Big/ \left[n \sum X_i^2 - \left(\sum X_i\right)^2\right] = \left(n\tilde{\sigma}_2{}^2\right) \Big/ \left[n \sum \left(X_i - \bar{X}\right)^2\right] \qquad \text{(C.60)}$$

$$\text{var}(\hat{b}) = 0{,}000\,097$$

Using Equations (C.61) and (C.62), compute the estimated standard errors, ε, of $\hat{a}$ and $\hat{b}$:

$$\varepsilon(\hat{a}) = \sqrt{\text{var}(\hat{a})} \qquad \text{(C.61)}$$

$$\varepsilon(\hat{a}) = \sqrt{0{,}000\,033} = 0{,}005\,756$$

$$\varepsilon(\hat{b}) = \sqrt{\text{var}(\hat{b})} \qquad \text{(C.62)}$$

$$\varepsilon(\hat{b}) = \sqrt{0{,}000\,097} = 0{,}009\,828$$

Formulae for 100 % confidence and prediction intervals about the fitted Line 2 as functions of X are given as Equations (C.63) and (C.64) respectively:

$$\text{Confidence interval } \mu_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[\frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum \left(X_i - \bar{X}\right)^2}\right]} \qquad \text{(C.63)}$$

$$\text{Prediction interval } Y_X = \hat{\mu}_X \pm t_P \tilde{\sigma}_2 \sqrt{\left[1 + \frac{1}{n} + \frac{\left(X - \bar{X}\right)^2}{\sum \left(X_i - \bar{X}\right)^2}\right]} \qquad \text{(C.64)}$$

where

extrapolated long-term stiffness is $\hat{\mu}_X = \hat{a} + \hat{b}X$ (C.65)

$t_P = 1{,}771$ for $P = 90$ %

In Table C.4, the confidence intervals are shown as μ_L and μ_U and the prediction intervals as Y_L and Y_U.

In Table C.5, the confidence intervals are shown as $S\mu_L$ and $S\mu_U$ and the prediction intervals as S_L and S_U.

C.3.2.4 Student's t test for a and b

To test whether $\hat{a}$ or $\hat{b}$ is equal to 0, the following calculations are performed:

$$\Pr\left(|t| < t_P\right) = P;$$

t has Student's t-distribution on $(n - 2)$ degrees of freedom.

From Statistical tables t boundary for $P = 90$ % = 1,771

From Statistical tables t boundary for $P = 95$ % = 2,160

If Equations (C.66) and (C.67) give t values greater than the applicable one of the above bounds, it is certain that $\hat{a}$ and $\hat{b}$ are not equal to 0.

t for $\hat{a} = \hat{a}/\varepsilon(\hat{a})$ (C.66)

t for $\hat{a} = 3{,}680\,3/0{,}005\,756 = 639{,}337\,53$

t for $\hat{b} = \hat{b}/\varepsilon(\hat{b})$ (C.67)

t for $\hat{b} = 0{,}191\,3/0{,}009\,828 = 19{,}466\,632\,3$

By inspection, $\hat{a}$ and $\hat{b}$ are not equal to 0.

C.3.2.5 Calculation of long-term (50-year) stiffness

All of the formulae in C.3 are standard formulae for straight-line regressions. From the values given in Tables C.4 and C.5, the estimated long-term stiffness and its confidence and prediction limits are:

Using Equation (C.65), the extrapolated long-term stiffness is given as Equation (C.68):

$$\hat{Y}_X = \hat{\mu}_X = \hat{a} + \hat{b}X \quad \text{(C.68)}$$

$$\hat{Y}_X = \hat{\mu}_X = 3{,}680\,3 + 0{,}191\,3 \times 0{,}35 = 3{,}686\,968$$

Using Equation (C.63), the confidence limits for $\mu_{50\text{ years}}$ are given as Equation (C.69):

$$\mu_{50\text{years}} = \hat{\mu}_X \pm \mu_{\text{bounds}} \quad \text{(C.69)}$$

$$\mu_{50\text{years}} = 3{,}686\,968 \pm 0{,}009\,6 = \left(3\,677\,322 \quad 3\,696\,614\right)$$

Using Equation (C.64), the prediction limits for $\hat{Y}_{50\text{years}}$ are given as Equation (C.70):

$$\hat{Y}_{50\text{years}} = Y_X \pm Y_{Y\ \text{bounds}} \tag{C.70}$$

$$\hat{Y}_{50\text{years}} = 3{,}686\ 968 \pm 0{,}019\ 2 = (3\ 667\ 725 \quad 3\ 706\ 21)$$

Transforming these values back to stiffness gives:

Extrapolated long-term stiffness is equal to:

$$S_{50\ \text{years}} = 10^{\hat{Y}_{50\ \text{years}}} \tag{C.71}$$

$$S_{50\ \text{years}} = 10^{3{,}686\ 968} = 486\ 4\ \text{N/m}^2$$

90 % confidence limits for extrapolated long-term stiffness, $\mu(S)_{50\ \text{years}}$, is given as Equation (C.72):

$$\mu(S)_{50\ \text{years}} = 10^{\hat{\mu}_X \pm \mu_{\text{bounds}}} \tag{C.72}$$

90 % confidence limits for extrapolated long-term stiffness, $\mu(S)_{50\ \text{years}}$, is given as Equation (C.73):

$$\mu(S)_{50\ \text{years}} = 10^{3{,}686\ 968 \pm 0{,}009\ 6} = (4\ 757{,}497\ 3) \tag{C.73}$$

90 % prediction limits for extrapolated long-term stiffness, $Y(S)_{50\ \text{years}}$, is given as Equation (C.74):

$$Y(S)_{50\ \text{years}} = 10^{Y_X \pm Y_{Y\ \text{bounds}}} \tag{C.74}$$

90 % prediction limits for extrapolated long-term stiffness, $Y(S)_{50\ \text{years}}$, is given as Equation (C.75):

$$Y(S)_{50\ \text{years}} = 10^{3{,}686\ 968 \pm 0{,}019\ 2} = (4\ 653 \quad 508\ 4) \tag{C.75}$$

C.4 Description and comments on data and model

A sequential linearized procedure is utilized in this annex so as to meet the requirement for easily accessible explicit formulae. This procedure is almost certainly only marginally sub-optimal for the particular purposes of this International Standard. In particular, it can be observed that for predicting the value of S at $S_{50\ \text{years}}$, which is the extrapolated long-term stiffness normally at 50 years, only the parameter a plus the associated estimates of measurement error for S and a are important. The four-parameter model for this procedure is shown as Equation (C.76):

$$Y_i = \lg_{10}(S_i) = a + b \Big/ \Big\langle 1 + \exp\big\{-\big[\lg_{10}(T_i) - c\big]\big/ d\big\}\Big\rangle \quad i = 1, \ldots, n \tag{C.76}$$

where

S is the stiffness, expressed in newtons per square metre (N/m^2);

T is time, expressed in hours (h);

i is the index for observations.

This model is linear in the parameters a and b but non-linear in the parameters c and d. This means that a fully developed statistical analysis designed to produce all the required estimates and intervals would require much algebraic development coupled with a ‘black-box’ use of professional statistical packages. However, an alternative sequential linearized procedure is utilized in this annex to meet the requirement for easily accessible explicit formulae.

C.4.1 Line 1

Line 1 is a rewrite of the model given as Equation (C.76) in order to display time as a function of stiffness, followed by the addition of a simple random error term so as to complete the full specification for a standard straight-line regression model. It is obtained by transforming the Y-axis using preliminary estimates for a and b [see Equation (C.77)].

$$\text{Line 1}\ y_i = \lg_e\left[\left(a+b-Y_i\right)/\left(Y_i-a\right)\right] = A + Bx_i + e_{1,\,i} \quad i = 1, n \tag{C.77}$$

where

$e_{1,i} \sim N\left(0, \sigma_1{}^2\right)$ is the added random error term.

NOTE 1 This represents normally distributed measurement and sample piece variability under nominally constant experimental conditions.

$$x_i = \lg_{10}\left(60T_i + 1\right) \approx \lg_{10} 60 + \lg_{10} T$$

NOTE 2 The additional '1 min' is a variant on the standard adaptation to ensure that zeroes on the Time and lg (Time) axes roughly correspond.

So that

$$A = \left(c + \lg_{10} 60\right)/d;$$

$$B = -1/d.$$

and hence

$$c = -\left(AB^{-1} + \lg_{10} 60\right);$$

$$d = -B^{-1}.$$

Given initial estimates for a and b, which are easily obtained via the observed maximum and minimum values for stiffness, this straight-line regression model can then be used to estimate c and d.

However, as it stands, this model for Line 1 requires $a + b > Y_i > a$ and this might not hold for the initial estimates. But assuming the model fits the data well, in which case measurement errors are small, it seems reasonable to replace y_i with Equation (C.78):

$$y_i = \lg_e\left\{\text{abs}\left[\left(a+b-Y_i\right)/\left(Y_i-a\right)\right]\right\} \tag{C.78}$$

Suitable initial estimates for a and b can be obtained from the data by setting:

$$a_0 + b_0 = 1{,}005\ \text{max.}\left(Y_i\right);$$

$$a_0 = 0{,}995\ \text{min.}\left(Y_i\right);$$

$$\text{So}\ b_0 = \left(a_0 + b_0\right) - a_0.$$

Alternatively, values for a, b, c and d can be obtained using suitable statistical software which can compute a set of best-fit values for these variables from the data by iteration on an appropriately specified least-squares criterion. Such software may not, however, supply standard errors or confidence intervals for these best-fit values.

C.4.2 Line 2

Line 2 is obtained by rewriting the basic model [Equation (C.76)] as a simple straight-line dependence of stiffness on a transformed Time axis using the Line 1 estimates for c and d [see Equation (C.79)]:

$$Y_i = a + bX_i + e_{2i} \tag{C.79}$$

where

$X_i = \left\langle 1 + \exp\left\{-\left[\lg_{10}\left(T_i\right) - c\right]/d\right\}\right\rangle^{-1}$;

$e_{2,i} \sim N\left(0, \sigma_2^2\right)$ represents random measurement and sample piece variability error.

NOTE This represents normally distributed measurement and sample piece variability under nominally constant experimental conditions.

Given the estimates for c and d obtained using either the Line 1 procedure described above or a suitable statistics package, this straight-line regression model can then be used to re-estimate a and b. Confidence and prediction intervals about this line can then be constructed using standard statistical techniques for linear models and hence, by back transformation, about the curve for S as a function of X or Time.

C.4.3 Analysis and formulae

Using the data set given in Table C.1, the sequential procedure is demonstrated and fully described in this subclause.

WARNING — The calculations described in this annex should only be performed using suitable statistical software packages or appropriate spreadsheet software. This is to avoid errors which are known to occur when attempts are made to perform them using pocket calculators.

C.4.3.1 Line 1

The least-squares estimates for A and B and the unbiased estimate for $\tilde{\sigma}_1^2$ are given as Equations (C.80) and (C.81).

C.4.3.1.1 Computation of $\hat{B}$

$\hat{B}$ is computed using Equation (C.80):

$$\hat{B} = \sum(x_i - \bar{x})(y_i - \bar{y}) \Big/ \sum(x_i - \bar{x})^2 = \left(n \times \sum xy - \sum x \times \sum y\right) \Big/ \left[n \times \sum x^2 - \left(\sum x\right)^2\right]$$
$$= (15 \times 17{,}80 - 48{,}47 \times 2{,}51) \Big/ \left[15 \times 187{,}75 - (2\,349{,}34)\right] = 0{,}831\,9 \tag{C.80}$$

C.4.3.1.2 Computation of $\hat{A}$

$\hat{A}$ is computed using Equation (C.81):

$$\hat{A} = \bar{y} - \hat{B}\bar{x} \tag{C.81}$$

where

$\bar{y} = \sum y/n = -2{,}51/15 = -0{,}167\,5$;

$\bar{x} = \sum x/n = 48{,}47/15 = 3{,}231\,036$;

$\hat{B} = 0{,}831\,9$.

NOTE See Equation (C.80).

$$\hat{A} = 0{,}167\,5 - 0{,}831\,9 \times 3{,}231\,036 = -2{,}855\,5 \tag{C.82}$$

C.4.3.1.3 Computation of $\tilde{\sigma}_1{}^2$

$\tilde{\sigma}_1{}^2$ is given as Equation (C.83):

$$\tilde{\sigma}_1^2 = \sum (y_i - \hat{y}_i)^2 \big/ (n-2) \quad \text{(C.83)}$$

where

$$\sum (y_i - \hat{y}_i)^2 = \sum y^2 - A \times \sum y - B \times \sum xy \big/ n - 2 = 22{,}65 - -2{,}855\,5 \times -2{,}51 - 0{,}831\,9 \times 17{,}8 = 0{,}665\,3$$

$$\tilde{\sigma}_1^2 = 0{,}665\,3/13 = 0{,}051\,2 \quad \text{(C.84)}$$

Table C.1 — Raw data and derived values

count i	Raw data		Derived values				
	Time, hours	Stiffness S	$\lg_{10}(S)$ Y_i	$\lg_{10}$(mins+1) x_i	Linearized Y y_i	Linearized time X_i	$\lg_{10}$(Time) T_i
0	0	*	*	0,000 000	*	*	*
1	0,10	7 114	3,852 114	0,845 098	−2,199 873	0,900 981 089	−1,000 000
2	0,27	6 935	3,841 046	1,235 528	−1,680 067	0,864 045 469	−0,568 636
3	0,50	6 824	3,834 039	1,491 362	−1,428 181	0,835 713 323	−0,301 030
4	1,00	6 698	3,825 945	1,785 330	−1,178 593	0,798 385 481	0,000 000
5	3,28	6 533	3,815 113	2,296 226	−0,888 531	0,720 523 173	0,515 874
6	7,28	6 453	3,809 762	2,641 276	−0,757 777	0,659 034 121	0,862 131
7	20,0	6 307	3,799 823	3,079 543	−0,529 608	0,572 939 842	1,301 030
8	45,9	6 199	3,792 322	3,440 122	−0,366 192	0,498 426 628	1,661 813
9	72,0	6 133	3,787 673	3,635 584	−0,267 424	0,457 861 654	1,857 332
10	166	5 692	3,755 265	3,998 303	0,411 303	0,384 436 030	2,220 108
11	219	5 508	3,740 994	4,118 628	0,732 338	0,361 035 235	2,340 444
12	384	5 393	3,731 830	4,362 501	0,958 300	0,315 663 484	2,584 331
13	504	5 364	3,729 489	4,480 596	1,019 680	0,294 833 214	2,702 431
14	3 000	5 200	3,716 003	5,255 275	1,416 309	0,179 974 497	3,477 121
15	10 520	4 975	3,696 793	5,800 168	2,245 487	0,122 406 128	4,022 016
16	438 300	*	*	7,419 923	*	0,034 979 805	5,641 771

NOTE 1 Linearized $Y = y_i = \ln\left\langle ABS\left\{\left[a_0 + b_0 - \lg_{10}(S)\right] / \left[\lg_{10}(S) - a_0\right]\right\}\right\rangle$.

NOTE 2 Linearized time $X_i = 1/\left\langle 1 + \exp\left\{-\left[\lg_{10}(\text{Time})\right] - c/d\right\}\right\rangle$.

NOTE 3 The raw data in this table are the same as those in Table 5.

NOTE 4 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values.

C.4.3.1.4 Computation of a, b, c and d

The values for a, b, c and d can be found by either using the procedure described in C.4 or by the use of suitable statistical software.

C.4.3.1.4.1 Using estimation procedure

Assuming measurements of stiffness for up to 10 000 h are available, suitable initial estimates for a and b are obtained from the data by using Equations (C.85), (C.86), and (C.87) (see also C.2.1):

$$a + b = 1{,}005 \times \text{max.}\left(Y_i\right) = 1{,}005 \times 3{,}852\,114 = 3{,}871\,375 \quad \text{(C.85)}$$

$$a = 0{,}995\ \text{min.}\left(Y_i\right) = 0{,}995 \times 3{,}696\,793 = 3{,}678\,309 \quad \text{(C.86)}$$

$$b = 3{,}871\,375 - 3{,}678\,309 = 0{,}193\,066 \quad \text{(C.87)}$$

For these data:

a is $\lg_{10}$ [stiffness at 50 years (S_{50})], therefore S_{50} = 10^3,678 309 = 4 768 N/m²;

b is change in $\lg_{10}$(stiffness) between initial and 50 years so, b = $\lg_{10}$[of the ratio (S_0/S_{50})];

but 10^b = 10^0,193 066 = 1,559 79

therefore, initial stiffness, S_0 = 1,559 79 × S_{50} = 1,559 79 × 4 768 = 7 437 N/m².

Alternatively, $a + b$ is $\lg_{10}(S_0)$ so S_0 = 10^3,871 375 = S_0 = 7 437 N/m².

Hence, the implied estimates for c and d are:

$$\hat{c} = -\left[A/B - \lg_{10}\left(60\right)\right] = 3{,}432\,504 - 1{,}778\,151\,3 = 1{,}653\,53 \quad \text{(C.88)}$$

$$\hat{d} = -B^{-1} = -0{,}831\,9^{-1} = -1{,}202\,011 \quad \text{(C.89)}$$

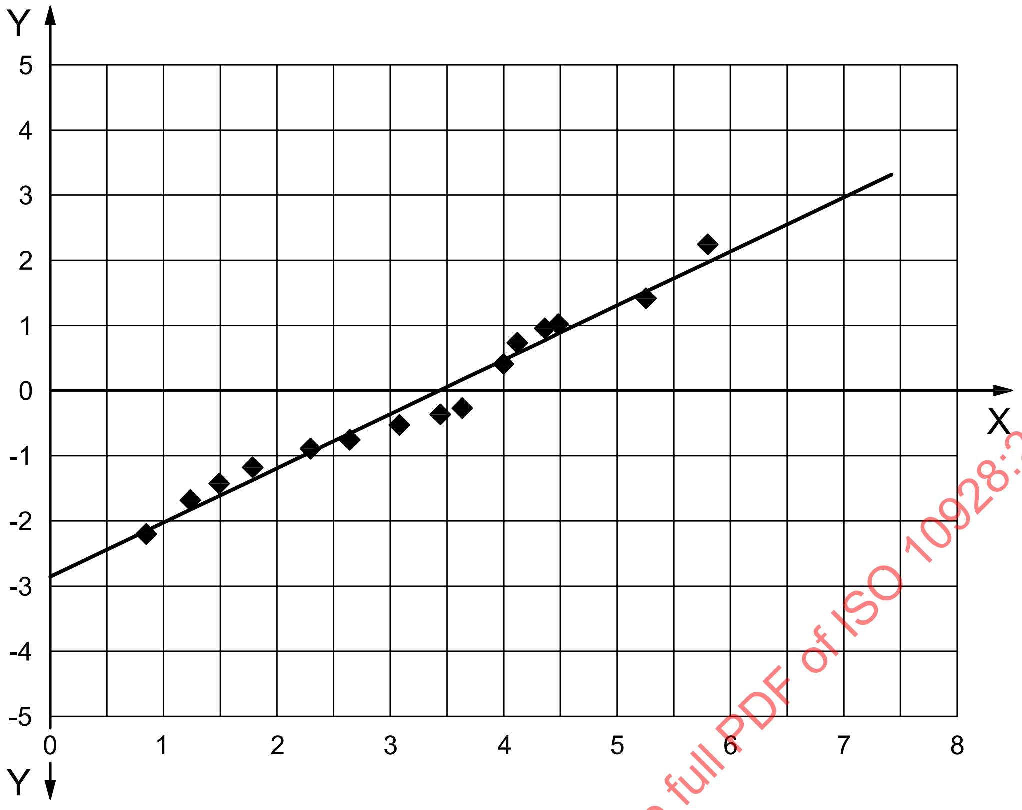
where

$\hat{c} = \lg_{10}\left(\text{half-period}\right)$;

NOTE The half-period is the time at which $\lg_{10}$ (stiffness) achieves 50 % of its overall loss.

$\hat{d}$ is the negative inverse of the slope of the line in Figure C.1.

The fitted parameters are statistically significant and the fitted line is shown in Figure C.1.



X-axis $\lg_{10}$(minutes + 1)
Y-axis transformed stiffness
trend-line $(\hat{y} = A + Bx)$
■ data points

NOTE See Table C.2 for details of the data used to produce this graph.

Figure C.1 — Line 1 – Straight-line fit

C.4.3.1.4.2 Using statistical software

By using initial estimates for a, b and c of 1 and –1 for d and the data in Table C.1, the software was run for up to 1 000 iterations and the following values were produced:

$a = 3{,}675$

$b = 0{,}193$

$a + b = 3{,}871$

$c = 1{,}654$

$d = -1{,}202$

These values can be seen to be very similar to the estimated values obtained in C.3.1.4 and C.3.1.3.

Table C.2 — Line 1 data results for Figure C.1

Count i	x_i	y_i	x^2	y^2	xy		y_{hat} $A + Bx$	Residuals $y - y_{hat}$
0	0,000 000	−2,855 548	0,000 000	8,154 155	0,000 000		−2,855 548	0,000 000
1	0,845 098	−2,199 873	0,714 191	4,839 439	−1,859 108		−2,152 478	−0,047 395
2	1,235 528	−1,680 067	1,526 531	2,822 626	−2,075 771		−1,827 663	0,147 596
3	1,491 362	−1,428 181	2,224 160	2,039 700	−2,129 934		−1,614 826	0,186 645
4	1,785 330	−1,178 593	3,187 403	1,389 081	−2,104 177		−1,370 262	0,191 669
5	2,296 226	−0,888 531	5,272 655	0,789 487	−2,040 267		−0,945 227	0,056 696
6	2,641 276	−0,757 777	6,976 338	0,574 226	−2,001 498		−0,658 167	−0,099 610
7	3,079 543	−0,529 608	9,483 585	0,280 485	−1,630 951		−0,293 555	−0,236 053
8	3,440 122	−0,366 192	11,834 437	0,134 097	−1,259 746		0,006 425	−0,372 617
9	3,635 584	−0,267 424	13,217 473	0,071 516	−0,972 242		0,169 038	−0,436 462
10	3,998 303	0,411 303	15,986 426	0,169 170	1,644 515		0,470 798	−0,059 495
11	4,118 628	0,732 338	16,963 100	0,536 319	3,016 228		0,570 901	0,161 437
12	4,362 501	0,958 300	19,031 418	0,918 339	4,180 586		0,773 789	0,184 512
13	4,480 596	1,019 680	20,075 742	1,039 746	4,568 772		0,872 036	0,147 643
14	5,255 275	1,416 309	27,617 914	2,005 933	7,443 096		1,516 522	−0,100 213
15	5,800 168	2,245 487	33,641 945	5,042 213	13,024 202		1,969 840	0,275 647
16	7,419 923	3,317 378	55,055 253	11,004 998	24,614 690		3,317 378	0,000 000
Totals	48,465 540	-2,512 828	187,753 317	22,652 377	17,803 704		37,464 846	0,000 000

NOTE 1 $x_i = \lg_{10}(\text{minutes} + 1)$.

NOTE 2 y_i = linearized Y.

NOTE 3 The values given in rows 0 and 16 are calculated values, while the values in rows 1 to 15 inclusive are measured values or derived from measured values. The values in rows 0 and 16 are not included in the totals below row 16.

C.4.3.1.5 Other derived variables associated with Line 1

$$S_x = n \times \sum x_i^2 - \left(\sum x_i\right)^2 = 467{,}39 \quad \text{(C.90)}$$

$$S_y = n \times \sum y_i^2 - \left(\sum y_i\right)^2 = 333{,}47 \quad \text{(C.91)}$$

$$S_{xy} = n \times \sum x_i y_i - \left(\sum x_i \times \sum y_i\right) = 388{,}84 \quad \text{(C.92)}$$

$$\bar{x} = \sum x_i / n = 3{,}231\,036 \quad \text{(C.93)}$$

$$\bar{y} = \sum y_i / n = -0{,}167\,522 \quad \text{(C.94)}$$

C.4.3.2 Line 2

C.4.3.2.1 Computation of a, b, and $\hat{\sigma}_2{}^2$

NOTE 1 The data used in these calculations are shown in Table C.3.

The least-squares estimates for a and b and the unbiased estimate for $\tilde{\sigma}_2^2$ are given as Equation (C.95):

$$\begin{aligned}\hat{b} &= \sum(X_i - \bar{X})(Y_i - \bar{Y}) \Big/ \sum(X_i - \bar{X})^2 = \left(n \times \sum XY - \sum X \times \sum Y\right) \Big/ \left(n \times \sum X^2 - \sum X \times \sum X\right); \\ \hat{b} &= (15 \times 30{,}302\ 557 - 7{,}966\ 259 \times 56{,}728\ 211) / (15 \times 5{,}146\ 065 - 7{,}966\ 259 \times 7{,}966\ 259) = \\ &= 2{,}626\ 734 / 13{,}729\ 693 = 0{,}191\ 318\end{aligned} \tag{C.95}$$

Using the estimates for c and d, Equations (C.96), (C.97), (C.98), (C.99), (C.100) and (C.101) are derived using Line 1:

$$\bar{Y} = \sum Y / n = 56{,}728\ 211/15 = 3{,}781\ 881 \tag{C.96}$$

$$\bar{X} = \sum X / n = 7{,}966\ 259/15 = 0{,}531\ 084 \tag{C.97}$$

$$\hat{a} = \bar{Y} - \hat{b} \times \bar{X} = 3{,}781\ 881 - 0{,}191\ 318 \times 0{,}531\ 083\ 4 = 3{,}680\ 275 \tag{C.98}$$

$$\tilde{\sigma}_2^2 = \sum(Y_i - \hat{Y}_i)^2 \big/ (n-2) = \text{RSS}/n - 2 \tag{C.99}$$

$$\begin{aligned}\text{RSS} &= \sum Y^2 - \hat{a} \times \sum Y - \hat{b} \times \sum XY = \\ &= 214{,}573\ 977 - 3{,}680\ 275 \times 56{,}728\ 211 - 0{,}191\ 318 \times 30{,}302\ 557 = 0{,}001\ 136\end{aligned} \tag{C.100}$$

$$\hat{\sigma}_2{}^2 = 0{,}001\ 136/13 = 0{,}000\ 087 \tag{C.101}$$

NOTE 2 For the data observed, these estimates satisfy the model constraint $a + b > Y_i > a$.