INTERNATIONAL STANDARD

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Third edition
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Calculation of load capacity of bevel gears —

Part 3: Calculation of tooth root strength

Calcul de la capacité de charge des engrenages coniques —

Partie 3: Calcul de la résistance du pied de dent

Reference number
ISO 10300-3:2023(E)

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

This third edition cancels and replaces the second edition (ISO 10300-3:2014), which has been technically revised.

The main changes are as follows:

— Table 1 has been inserted;

— Table 2 has been inserted;

— Figure 4 — surface condition factor, $Y_{R,relT}$, for permissible stress number relative to standard test gear dimensions has been removed;

— Figure 5 — relative notch sensitivity factor with respect to standard test gear dimensions has been removed;

— new Figure 5 — life factor, Y_{NT} (standard reference test gears) has been added;

— Figure 7 — size factor, Y_X, for permissible tooth root stress has been removed.

A list of all parts in the ISO 10300 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user’s national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

When ISO 10300:2001 (all parts) became due for its first revision, the opportunity was taken to include hypoid gears, since previously the series only allowed for calculating the load capacity of bevel gears without offset axes. The former structure is retained, i.e. three parts of the ISO 10300 series, together with ISO 6336-5, and it is intended to establish general principles and procedures for rating of bevel gears. Moreover, ISO 10300 (all parts) is designed to facilitate the application of future knowledge and developments, as well as the exchange of information gained from experience.

In view of the decision for ISO 10300 (all parts) to cover hypoid gears also, it was agreed to include a separate clause: “Gear tooth rating formulae — Method B2” in this document, while the former methods B and B1 were combined into one method, i.e. method B1. So, it became necessary to present a new, clearer structure of the three parts, which is illustrated in ISO 10300-1:2023, Figure 1.

NOTE ISO 10300 (all parts) gives no preferences in terms of when to use method B1 and when to use method B2.

Failure of gear teeth by tooth root breakage can be brought about in many ways; severe instantaneous overloads, excessive macropitting, case crushing and bending fatigue are a few. The strength ratings determined by the formulae in this document are based on cantilever projection theory modified to consider the following:

— compressive stress at the tooth roots caused by the radial component of the tooth load;

— non-uniform moment distribution of the load, resulting from the inclined contact lines on the teeth of spiral bevel gears;

— stress concentration at the tooth root fillet;

— load sharing between adjacent contacting teeth;

— lack of smoothness due to a low contact ratio.

The formulae are used to determine a load rating, which prevents tooth root fracture during the design life of the bevel gear. Nevertheless, if there is insufficient material under the teeth (in the rim), a fracture can occur from the root through the rim of the gear blank or to the bore (a type of failure not covered by this document). Moreover, it is possible that special applications require additional blank material to support the load.

Surface distress (pitting or wear) can limit the strength rating, either due to stress concentration around large sharp cornered pits, or due to wear steps on the tooth surface. Neither of these effects is considered in this document.

In most cases, the maximum tensile stress at the tooth root (arising from bending at the root when the load is applied to the tooth flank) can be used as a determinant criterion for the assessment of the tooth root strength. If the permissible stress number is exceeded, the teeth can break.

When calculating the tooth root stresses of straight bevel gears, this document starts from the assumption that the load is applied at the tooth tip of the virtual cylindrical gear. The load is subsequently converted to the outer point of single tooth contact. The procedure thus corresponds to method C for the tooth root stress of cylindrical gears (see ISO 6336-3[1]).

For spiral bevel and hypoid gears with a high face contact ratio of $\varepsilon_{v\beta} > 1$ (method B1) or with a modified contact ratio of $\varepsilon_{v\gamma} > 2$ (method B2), the midpoint in the zone of action is regarded as the critical point of load application.

The breakage of a tooth generally means the end of a gear's life. It is often the case that all gear teeth are destroyed as a consequence of the breakage of a single tooth. A safety factor, S_F, against tooth root breakage higher than the safety factor against damage due to macropitting is, therefore, generally to be preferred (see ISO 10300-1).

Calculation of load capacity of bevel gears —

Part 3:
Calculation of tooth root strength

1 Scope

This document specifies the fundamental formulae for use in the tooth root stress calculation of straight and helical (skew), Zerol and spiral bevel gears including hypoid gears, with a minimum rim thickness under the root of 3,5 m_{mn}. All load influences on tooth root stress are included, insofar as they are the result of load transmitted by the gearing and able to be evaluated quantitatively. Stresses, such as those caused by the shrink fitting of gear rims, which are superposed on stresses due to tooth loading, are intended to be considered in the calculation of the tooth root stress, σ_F, or the permissible tooth root stress σ_{FP}. This document is not applicable in the assessment of tooth flank fracture.

The formulae in this document are based on virtual cylindrical gears and restricted to bevel gears whose virtual cylindrical gears have transverse contact ratios of $\varepsilon_{v\alpha} < 2$. The results are valid within the range of the applied factors as specified in ISO 10300-1. The bending strength formulae are applicable to fractures at the tooth fillet, but not to those on the active flank surfaces, to failures of the gear rim or of the gear blank through the web and hub.

This document does not apply to stress levels above those permitted for 10^3 cycles, as stresses in that range can exceed the elastic limit of the gear tooth.

NOTE This document is not applicable to bevel gears which have an inadequate contact pattern under load.

The user is cautioned that when the formulae are used for large average mean spiral angles $(\beta_{m1} + \beta_{m2})/2 > 45°$, for effective pressure angles $\alpha_e > 30°$ and/or for large facewidths $b > 13\ m_{mn}$, the calculated results of this document should be confirmed by experience.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 701, *International gear notation — Symbols for geometrical data*

ISO 1122-1, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

ISO 6336-5, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials*

ISO 10300-1:2023, *Calculation of load capacity of bevel gears — Part 1: Introduction and general influence factors*

ISO 10300-2:2023, *Calculation of load capacity of bevel gears — Part 2: Calculation of surface durability (macropitting)*

ISO 17485, *Bevel gears — ISO system of accuracy*

ISO 23509:2016, *Bevel and hypoid gear geometry*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 1122-1, ISO 23509 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
tooth root breakage
failure of gear teeth at the tooth root by static or dynamic overload

3.2
nominal tooth root stress
σ_{F0}
bending stress in the critical section of the tooth root calculated for the critical point of load application for error-free gears loaded by a constant nominal torque

3.3
tooth root stress
σ_F
determinant bending stress in the critical section of the tooth root calculated for the critical point of load application including the load factors which consider static and dynamic loads and load distribution

3.4
nominal stress number
$\boldsymbol{\sigma}_{\mathbf{F,lim}}$
maximum tooth root stress of standardized test gears and determined at standardized operating conditions as specified in ISO 6336-5

3.5
allowable stress number (bending)
σ_{FE}
maximum bending stress of the un-notched test piece under the assumption that the material is fully elastic

3.6
permissible tooth root stress
σ_{FP}
maximum tooth root stress of the evaluated gear set including all influence factors

4 Symbols, general subscripts and abbreviated terms

For the purposes of this document, the symbols given in ISO 701, ISO 17485, ISO 23509, and the following shall apply.

Table 1 — Symbols

Symbol	Description or term	Unit
a_{BS}	Auxiliary value	—
b	Facewidth	mm
b_a	Developed length of one tooth as facewidth of the calculation model	mm
b_{BS}	Auxiliary value	—
b_{ce}	Calculated effective facewidth	mm
b_k	Mean facewidth	mm

Table 1 *(continued)*

Symbol	Description or term	Unit
b_v	Facewidth of virtual cylindrical gears	mm
$b_{v,rel}$	Relative facewidth of virtual cylindrical gear pair	—
C_{mm}	Conversion factor used in Formula (202), C_{mm} = 25,4 mm	mm
c_{BS}	Auxiliary value	—
d_{van}	Tip diameter of virtual cylindrical gear in normal section	mm
d_{vbn}	Base diameter of virtual cylindrical gear in normal section	mm
E, G, H	Auxiliary quantities for tooth form factor (method B1)	—
F_{mt}	Nominal tangential force at mid-facewidth of the reference cone	N
F_{vmt}	Nominal tangential force of virtual cylindrical gears	N
g_J	Relative length of action to point of load application (method B2)	—
g_{f0}	Auxiliary term	mm
g_{rb}	Relative distance from blade edge to centre line	—
$g_{v\alpha n}$	Relative length of action in normal section	—
$g_{v\alpha n,hyp}$	Relative length of action in normal section for hypoid gears	—
g_{xb}	Auxiliary term	—
g_{yb}	Auxiliary term	—
g_{za}	Auxiliary term	—
g_{zb}	Auxiliary term	—
g_η	Relative length of action within the contact ellipse	—
g_0	Auxiliary term	—
g_1	Intermediate value	mm
h_{a0}	Tool addendum	mm
h_{Fa}	Bending moment arm for tooth root stress (load application at tooth tip)	mm
h_{fm}	Mean dedendum	mm
h_m	Mean whole depth used for bevel spiral angle factor	mm
h_N	Relative load height from critical section (method B2)	—
h_{vfm}	Relative mean virtual dedendum	—
$h_{1,2,3,4}$	Auxiliary values	—
K_A	Application factor	—
$K_{F\alpha}$	Transverse load factor for bending stress	—
$K_{F\beta}$	Face load factor for bending stress	—
K_v	Dynamic factor	—
k'	Contact shift factor	—
L	Factor to calculate the stress correction factor according to Dolan and Broghamer	—
$L_{a,D,C}$	Auxiliary value	—
l_{bb}	Part of the model's facewidth covered by the contact line	mm
l_{bm}	Theoretical length of middle contact line	mm
M	Factor to calculate the stress correction factor according to Dolan and Broghamer	—
m_{et}	Outer transverse module	mm
m_{mn}	Mean normal module	mm
m_{mt}	Mean transverse module	mm
N_L	Number of load cycles	—
O	Factor to calculate the stress correction factor according to Dolan and Broghamer	—
q_s	Notch parameter	—

Table 1 *(continued)*

Symbol	Description or term	Unit
R_{CL2}	Relative radius from tool centre to critical pinion coast side fillet point	—
R_{DL2}	Relative radius from tool centre to critical pinion drive side fillet point	—
R_m	Mean cone distance	mm
Rz	Mean roughness	µm
r_{L1o}	Relative pinion radius to fillet point	—
r_{L2o}	Relative wheel radius to pinion fillet point	—
r_{mf}	Relative tooth fillet radius at the root in mean section	—
r_{mpt}	Mean pitch radius	mm
r_{my0}	Mean transverse radius to point of load application (method B2)	mm
r_{va}	Relative mean virtual tip radius	—
r_{vn}	Relative mean virtual pitch radius	—
Δr_{y0}	Relative distance from pitch circle to the pinion point of load application and the wheel tooth centre line	—
S_F	Safety factor for bending stress (against tooth root breakage)	—
$S_{F,min}$	Minimum safety factor for bending stress	—
s_{Fn}	Tooth root chord in calculation section	mm
s_N	Relative horizontal distance from centreline to critical fillet point (method B2)	—
s_{mn}	Mean normal circular thickness	mm
s_{pr}	Amount of protuberance	mm
s_{vmn}	Relative virtual tooth thickness	—
W_{m2}	Wheel mean slot width	mm
x_{hm}	Profile shift coefficient	—
x_{sm}	Thickness modification coefficient	—
x_N	Tooth strength factor (method B2)	—
x_{oo}	Distance from mean section to point of load application	mm
x_1	Relative horizontal distance from pitch circle to fillet point	—
Y_A	Root stress adjustment factor (method B2)	—
Y_{BS}	Bevel spiral angle factor	—
Y_{Fa}	Tooth form factor for load application at the tooth tip (method B1)	—
Y_{FS}	Combined tooth form factor for generated gears	—
Y_f	Stress concentration and stress correction factor (method B2)	—
Y_J	Bending strength geometry factor (method B2)	—
Y_{LS}	Load sharing factor (bending)	—
Y_{NT}	Life factor (bending)	—
Y_P	Combined geometry factor (method B2)	—
Y_R	Surface condition at the root fillet	—
Y_{RT}	Surface condition at the test gear	—
$Y_{R,relT}$	Relative surface condition factor	—
Y_{Sa}	Stress correction factor for load application at the tooth tip	—
Y_{ST}	Stress correction factor for dimensions of the standard test gear	—
Y_X	Size factor for tooth root stress	—
$Y_{1,2}$	Tooth form factor of pinion and wheel (method B2)	—
$Y_{\delta,relT}$	Relative notch sensitivity factor	—
Y_ε	Contact ratio factor for bending (method B1)	—

Table 1 *(continued)*

Symbol	Description or term	Unit
y_J	Relative location of point of load application for maximum bending stress on path of action (method B2)	—
y_1	Relative vertical distance from pitch circle to fillet point	—
y_3	Relative distance from the beginning of the path of action to the point of load application on path of action for maximum root stress	—
Z_{LS}	Load sharing factor (method B1)	—
z_{vn}	Number of teeth of virtual cylindrical gear in normal section	—
α_{Cnf}	Coast flank pressure angle in wheel root coordinates	°
α_{Dnf}	Drive flank pressure angle in wheel root coordinates	°
α_{Fan}	Load application angle at tooth tip of virtual cylindrical gear (method B1)	°
α_L	Normal pressure angle at point of load application (method B2)	°
α_{LN}	Generated pressure angle at fillet point	°
α_a	Adjusted pressure angle (method B2)	°
α_{an}	Normal pressure angle at tooth tip	°
$\alpha_{eD,C}$	Effective pressure angle for drive side/coast side	°
α_f	Limit pressure angle in wheel root coordinates (method B2)	°
α_h	Auxiliary term	°
α_{lim}	Limit pressure angle	°
$\alpha_{nD,C}$	Generated pressure angle for drive side/coast side	°
β_a	Intermediate angle	°
β_v	Helix angle of virtual gear (method B1), virtual spiral angle (method B2)	°
β_{vb}	Helix angle at base circle of virtual cylindrical gear	°
γ_a	Auxiliary angle for tooth form and tooth correction factor	°
ε_N	Load sharing ratio for bending (method B2)	—
ε_b	Lengthwise load sharing factor	—
ε_f	Profile load sharing factor	—
$\varepsilon_{v\alpha}$	Transverse contact ratio of virtual cylindrical gears	—
$\varepsilon_{v\alpha n}$	Transverse contact ratio of virtual cylindrical gears in normal section	—
$\varepsilon_{v\alpha n,hyp}$	Transverse contact ratio of virtual cylindrical gears in normal section for hypoid gears	—
$\varepsilon_{v\beta}$	Face contact ratio of virtual cylindrical gears	—
$\varepsilon_{v\gamma}$	Virtual contact ratio (method B1), modified contact ratio (method B2)	—
θ	Auxiliary angle for tooth form and tooth correction factors	rad
θ_{D1}	Wheel angle from centreline to pinion tip on drive side	rad
$\Delta\theta$	Wheel angle between fillet points	°
ϑ	Auxiliary value	—
μ	Relative distance from centreline to tool critical fillet point	—
ξ	Assumed angle in locating weakest section	rad
ξ_h	One half of angle subtended by normal circular tooth thickness at point of load application	rad
ρ_{a0}	Cutter edge radius	mm
ρ_F	Fillet radius at point of contact of 30° tangent	mm
σ_B	Tensile strength (corresponds to R_m of ISO 6892-1)	N/mm^2
σ_{FE}	Allowable stress number (bending)	N/mm^2
σ_{FP}	Permissible tooth root stress	N/mm^2

Table 1 *(continued)*

Symbol	Description or term	Unit
$\sigma_{F,lim}$	Nominal stress number (bending)	N/mm^2
ρ_{Fn}	Fillet radius at point of contact of 30° tangent in normal section	mm
ρ_{fP}	Root fillet radius of basic rack for cylindrical gears	mm
σ_S	Yield stress (corresponds to R_p of ISO 6892-1)	N/mm^2
$\sigma_{0,2}$	Proof stress (0,2 % permanent set) (corresponds to $R_{p0,2}$ of ISO 6892-1)	N/mm^2
$\rho_{\Delta red}$	Radius of curvature change	—
χ^X	Relative stress drop in notch root	mm^{-1}
χ_T^X	Relative stress drop in notch root of standardized test gear	mm^{-1}

Table 2 — General subscripts

Subscripts	Description
0	Tool
1	Pinion
2	Wheel
A, B, B1, B2, C	Value according to method A, B, B1, B2 or C
D	Drive flank
C	Coast flank
T	Relative to standardized test gear dimensions
(1), (2)	Trials of interpolation

Table 3 — Abbreviated terms in accordance with ISO 6336-5

Abbreviated term	Material	Type
St	Normalized low carbon steels/cast steels	Wrought normalized low carbon steels
St (cast.)		Cast steels
GTS (perl.)	Cast iron materials	Black malleable cast iron (perlitic structure)
GGG (perl., bai., ferr.)		Nodular cast iron (perlitic, bainitic, ferritic structure)
GG		Grey cast iron
V	Through hardened wrought steels	Carbon steels, alloy steels
V (cast)	Through hardened cast steels	Carbon steels, alloy steels
Eh	Case-hardened wrought steels	
IF	Flame or induction hardened wrought or cast steels	
NT (nitr.)	Nitrided wrought steels/nitriding steels/ through hardening steels, nitrided	Nitriding steels
NV (nitr.)		Through hardening steels
NV (nitrocar.)	Wrought steels, nitrocarburized	Through hardening steels

5 General rating procedure

There are two main methods for determining tooth bending strength of bevel and hypoid gears: method B1 and method B2. They are provided in Clauses 6 and 7, while Clause 8 contains those influence factors which are equal for both methods. Method B1 contains one set of formulae for both, bevel and hypoid gears. Method B2 has different sets of formulae for bevel gears and for hypoid gears (see 7.4.3 for general aspects).

With both methods, the capability of a gear tooth to resist tooth root stresses shall be determined by the comparison of the following stress values:

— tooth root stress σ_F, based on the geometry of the tooth, the accuracy of its manufacture, the rigidity of the gear blanks, bearings and housing, and the operating torque, expressed by the tooth root stress formula (see 6.1 and 7.1);

— permissible tooth root stress σ_{FP}, based on the bending stress number, $\sigma_{F,lim}$, of a standard test gear and the effect of the operating conditions under which the gears operate, expressed by the permissible tooth root stress formula (see 6.2 and 7.2).

NOTE In respect of the permissible tooth root stress, reference is made to a stress "number", a designation adopted because pure stress, as determined by laboratory testing, is not calculated by the formulae in this document. Instead, an arbitrary value is calculated and used in this document, with accompanying changes to the allowable stress number in order to maintain consistency for design comparison.

The ratio of the permissible root stress and the calculated root stress is the safety factor S_F. The value of the minimum safety factor for tooth root stress, $S_{F,min}$, should be ≥ 1,3 for spiral bevel gears. For straight bevel gears, or where $\beta_m \leq 5°$, $S_{F,min}$ should be ≥ 1,5.

The gear designer and customer should agree on the value of the minimum safety factor.

Tooth root breakage usually ends transmission service life. The destruction of all gears in a transmission can be a consequence of the breakage of one tooth, then, the drive train between input and output shafts is interrupted. Therefore, the chosen value of the safety factor, S_F, against tooth root breakage should be carefully chosen to fulfil the application requirements (see ISO 10300-1 for general comments on the choice of safety factor).

6 Gear tooth rating formulae — Method B1

6.1 Tooth root stress formula

The calculation of the tooth root stress is based on the maximum bending stress at the tooth root. It is determined separately for pinion (suffix 1) and wheel (suffix 2) in accordance with Formula (1); in the case of hypoid gears, additionally for drive flank (suffix D) and coast flank (suffix C):

$$\sigma_{F\text{-}B1} = \sigma_{F0\text{-}B1} \cdot K_A \cdot K_v \cdot K_{F\beta} \cdot K_{F\alpha} < \sigma_{FP\text{-}B1} \tag{1}$$

with the load factors K_A, K_v, $K_{F\beta}$, $K_{F\alpha}$, which shall be as specified in ISO 10300-1.

The nominal tooth root stress is defined as the maximum bending stress at the tooth root (30° tangent to the root fillet):

$$\sigma_{F0\text{-}B1} = \frac{F_{vmt}}{b_v \cdot m_{mn}} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\varepsilon \cdot Y_{BS} \cdot Y_{LS} \tag{2}$$

where

F_{vmt} is the nominal tangential force of the virtual cylindrical gear in accordance with ISO 10300-1:2023, Formula (2);

b_v is the facewidth of the virtual cylindrical gear calculated for the active flank, drive or coast side, as specified in ISO 10300-1:2023, Annex A;

Y_{Fa} is the tooth form factor (see 6.4.1) which accounts for the influence of the tooth form on the nominal bending stress at the tooth root for load application at the tooth tip;

Y_{Sa} is the stress correction factor (see 6.4.2) which accounts for the stress increasing notch effect in the root fillet, as well as for the radial component of the tooth load and the fact that the stress condition in the critical root section is complex, but not for the influence of the bending moment arm;

Y_{ε} is the contact ratio factor (see 6.4.3) which accounts for the conversion of the root stress determined for the load application at the tooth tip to the determinant position;

Y_{BS} is the bevel spiral angle factor, which accounts for smaller values for l_{bm} compared to the total facewidth, b_v, and the inclined lines of contact (see 6.4.4);

Y_{LS} is the load sharing factor, which accounts for load distribution between two or more pairs of teeth (see 6.4.5).

The determinant position of load application is:

a) the outer point of single tooth contact, if $\varepsilon_{v\beta} = 0$;

b) the midpoint of the zone of action, if $\varepsilon_{v\beta} \geq 1$;

c) interpolation between a) and b), if $0 < \varepsilon_{v\beta} < 1$.

6.2 Permissible tooth root stress

The permissible tooth root stress, σ_{FP}, shall be calculated separately for pinion and wheel. The values should preferably be evaluated on the basis of the strength of a standard test gear instead of a prismatic specimen, which deviates too much with respect to similarity in geometry, course of movement and manufacture.

$$\sigma_{FP\text{-}B1} = \sigma_{FE} \cdot Y_{NT} \cdot Y_{\delta,relT\text{-}B1} \cdot Y_{R,relT\text{-}B1} \cdot Y_X \tag{3}$$

$$\sigma_{FP\text{-}B1} = \sigma_{F,lim} \cdot Y_{ST} \cdot Y_{NT} \cdot Y_{\delta,relT\text{-}B1} \cdot Y_{R,relT\text{-}B1} \cdot Y_X \tag{4}$$

where

σ_{FE} is the allowable stress number (tooth root);

σ_{FE} $= \sigma_{F,lim1,2} \cdot Y_{ST}$, is the basic bending strength of the un-notched specimen under the assumption that the material (including heat treatment) is fully elastic;

$\sigma_{F,lim}$ is the nominal stress number (bending) of the test gear, which accounts for material, heat treatment and surface influence at test gear dimensions as specified in ISO 6336-5;

Y_{ST} is the stress correction factor for the dimensions of the standard test gear, $Y_{ST} = 2{,}0$;

$Y_{\delta,relT\text{-}B1}$ is the relative notch sensitivity factor for the permissible stress number, related to the conditions at the standard test gear (see 6.5.2), $Y_{\delta,relT} = Y_{\delta}/Y_{\delta T}$ accounts for the notch sensitivity of the material;

$Y_{R,relT\text{-}B1}$ is the relative surface condition factor (see 6.5.1), $Y_{R,relT} = Y_R/Y_{RT}$ accounts for the surface condition at the root fillet, related to the conditions at the test gear;

Y_X is the size factor for tooth root strength, which accounts for the influence of the module on the tooth root strength (see 8.1);

Y_{NT} is the life factor, which accounts for the influence of required numbers of cycles of operation (see 8.2).

6.3 Calculated safety factor

The evaluated tooth root stress, σ_F, shall be ≤ σ_{FP}, which is the permissible tooth root stress. The calculated safety factor against tooth root breakage shall be determined separately for pinion and wheel:

$$S_{F\text{-}B1} = \frac{\sigma_{FP\text{-}B1}}{\sigma_{F\text{-}B1}} > S_{F,min} \tag{5}$$

NOTE This is the calculated safety factor with respect to the transmitted torque.

Considerations in reference to the safety factors and the risk (probability) of failure are given in ISO 10300-1:2023, 5.2.

6.4 Tooth root stress factors

6.4.1 Tooth form factor, Y_{Fa}

6.4.1.1 General

The tooth form factor, Y_{Fa}, accounts for the influence of the tooth form on the nominal tooth root stress in the case of load application at the tooth tip. It is determined separately for pinion and wheel. In doing so, the possibility to manufacture bevel and hypoid gears with different pressure angles at drive and coast side shall be considered (see Figure 1).

In the case of gears with tip and root relief, the actual bending moment arm is slightly smaller, but this should be neglected, and the calculation is on the safe side.

Bevel gears without offset generally have octoid teeth and tip and root relief. However, deviations from an involute profile are small, especially in view of the tooth root chord and bending moment arm, and thus both, tip and root relief, are neglected when calculating the tooth form factor.

The distance between the contact points of the 30° tangents at the root fillets of the tooth profile of the virtual cylindrical gear is taken as the critical section for calculation (see Figure 1).

By method B1 of ISO 10300, the tooth form factor, Y_{Fa}, and stress correction factor, Y_{Sa}, are determined for the nominal gear without deviations.

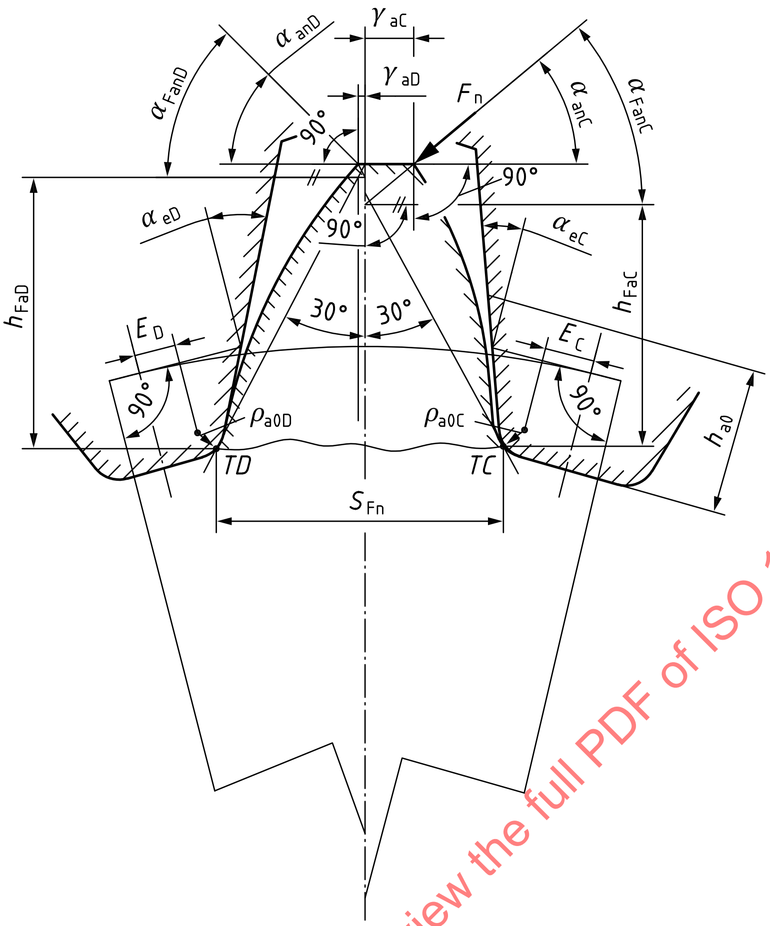


Figure 1 — Tooth root chordal thickness, s_{Fn}, and bending moment arm, h_{Fa}, in normal section for load application, F_n, at the tooth tip of the virtual cylindrical gear

6.4.1.2 Tooth form factor for generated gears

6.4.1.2.1 General

The tooth form factor, Y_{Fa}, of generated bevel gears is calculated with parameters of the active flank of the virtual cylindrical gear in normal section which includes the corresponding effective pressure angle α_{eD} or α_{eC} (see ISO 10300-1:2023, Annex A). However, the direction of the normal force, F_n, in relation to the tangential force, F_{vmt}, is given by the generated pressure angle α_{nD} or α_{nC}.

The tooth form factor, Y_{Fa}, and its parameters shall be determined for the pinion (suffix 1) and the wheel (suffix 2) separately:

$$Y_{FaD,C} = \frac{6 \cdot \dfrac{h_{FaD,C}}{m_{mn}} \cdot \cos\alpha_{FanD,C}}{\left(\dfrac{s_{Fn}}{m_{mn}}\right)^2 \cdot \cos\alpha_{nD,C}} \tag{6}$$

where

$h_{FaD,C}$ and $h_{FanD,C}$ see 6.4.1.2.5;

$\alpha_n = \alpha_{nD}$ = generated pressure angle for drive side (specified in ISO 23509);

$\alpha_n = \alpha_{nC}$ = generated pressure angle for coast side (specified in ISO 23509).

See Figure 1 for the explanation of parameters. See ISO 6336-3[1] for more information about the tooth form factor.

6.4.1.2.2 Auxiliary quantities

For the calculation of the tooth root chord, s_{Fn}, and the bending moment arm, h_{Fa}, firstly, the auxiliary quantities *E, G, H* and ϑ shall be determined.

The parameter, *E*, is calculated for the magnitudes of the active flank. For generated gears, the effective pressure angle $\alpha_e = \alpha_{eD}$ for the drive side and $\alpha_e = \alpha_{eC}$ (see ISO 23509) for the coast side, respectively, are used in Formula (7). The cutter edge radii ρ_{a0D} and ρ_{a0C} as well as the protuberance, $s_{prD,C}$, can be different, but not h_{a0}, which is the tool addendum:

NOTE Formulae (7) to (22) are only valid if there is a real mean normal top land (positive value) of the virtual cylindrical gear.

$$E_{D,C} = \left(\frac{\pi}{4} - x_{sm} \right) \cdot m_{mn} - h_{a0} \cdot \tan \alpha_{eD,C} - \frac{\rho_{a0D,C} \cdot \left(1 - \sin \alpha_{eD,C} \right) - s_{prD,C}}{\cos \alpha_{eD,C}} \quad (7)$$

$$G_{D,C} = \frac{\rho_{a0D,C}}{m_{mn}} - \frac{h_{a0}}{m_{mn}} + x_{hm} \quad (8)$$

$$H_{D,C} = \frac{2}{z_{vnD,C}} \cdot \left(\frac{\pi}{2} - \frac{E_{D,C}}{m_{mn}} \right) - \frac{\pi}{3} \quad (9)$$

$$\vartheta_{D,C} = \frac{2 \cdot G_{D,C}}{z_{vnD,C}} \cdot \tan \vartheta_{D,C} - H_{D,C} \quad (10)$$

For the solution of the transcendent Formula (10), $\vartheta = \pi/6$ should be inserted as the initial value. A suggested value for the difference ($\vartheta_{new} - \vartheta$) is 0,000 001. In most cases, the calculation already converges after a few iterations.

6.4.1.2.3 Tooth root chordal thickness, s_{Fn}

The tooth root chords s_{FnD} and s_{FnC} are calculated for pinion and wheel, each with the corresponding geometry data for the drive flank and the coast flank:

$$s_{FnD,C} = m_{mn} \cdot z_{vnD,C} \cdot \sin \left(\frac{\pi}{3} - \vartheta_{D,C} \right) + m_{mn} \cdot \sqrt{3} \cdot \left(\frac{G_{D,C}}{\cos \vartheta_{D,C}} - \frac{\rho_{a0D,C}}{m_{mn}} \right) \quad (11)$$

Then, the respective tooth root chord s_{Fn} for pinion or wheel results in:

$$s_{Fn} = 0{,}5 \cdot s_{FnD} + 0{,}5 \cdot s_{FnC} \quad (12)$$

6.4.1.2.4 Fillet radius, ρ_F, at contact point of 30° tangent

The fillet radius, ρ_F, is calculated with the corresponding geometry data for the drive flank and the coast flank:

$$\rho_{FD,C} = \rho_{a0D,C} + \frac{2 \cdot G_{D,C}^2 \cdot m_{mn}}{\cos \vartheta_{D,C} \cdot \left(z_{vnD,C} \cdot \left(\cos \vartheta_{D,C} \right)^2 - 2 \cdot G_{D,C} \right)} \quad (13)$$

6.4.1.2.5 Bending moment arm, h_{Fa}

The bending moment arm, h_{Fa}, is calculated with geometry data referring to the drive flank and to the coast flank:

$$h_{FaD,C} = \frac{m_{mn}}{2} \cdot \left[\left(\cos\gamma_{aD,C} - \sin\gamma_{aD,C} \cdot \tan\alpha_{FanD,C} \right) \cdot \frac{d_{vanD,C}}{m_{mn}} - z_{vnD,C} \cdot \cos\left(\frac{\pi}{3} - \vartheta_{D,C} \right) - \frac{G_{D,C}}{\cos\vartheta_{D,C}} + \frac{\rho_{a0D,C}}{m_{mn}} \right] \quad (14)$$

where

$$\alpha_{FanD,C} = \alpha_{anD,C} - \gamma_{aD,C} \quad (15)$$

$$\alpha_{anD,C} = \arccos\left(\frac{d_{vbnD,C}}{d_{vanD,C}} \right) \quad (16)$$

$$\gamma_{aD,C} = \frac{1}{z_{vnD,C}} \cdot \left[\frac{\pi}{2} + 2\left(x_{hm} \cdot \tan\alpha_{eD,C} + x_{sm} \right) \right] + \mathrm{inv}\alpha_{eD,C} - \mathrm{inv}\alpha_{anD,C} \quad (17)$$

Data of the virtual cylindrical gears (pinion and wheel) in normal section, d_{van}, d_{vbn} and z_{vn}, are specified in ISO 10300-1:2023, A.3. Dimensions at the basic rack profile of the tooth are shown in Figure 2.

At the design stage, the tooth form factor Y_{Fa} for bevel gears without offset may be calculated for a basic rack profile of the tool with the following data α_n = 20°, h_{a0}/m_{mn} = 1,25, and ρ_{a0}/m_{mn} = 0,25.

6.4.1.3 Tooth form factor for non-generated gears

The tooth form factor, Y_{Fa}, for non-generated gears should be considered separately. In this case of form cutting, the slot profile of the wheel is identical to the tool profile and so the tooth form factor can directly be determined (see Figure 2).

The tooth form factor of the mating pinion, which is manufactured by an adapted generating process, is approximated by the formulae according to 6.4.1.2.

Tooth root thickness of the wheel (suffix 2):

$$s_{\text{FnD,C}} = \pi \cdot m_{\text{mn}} - 2 \cdot E_{\text{D,C}} - 2 \cdot \rho_{\text{a0D,C}} \cdot \cos 30° \quad (18)$$

where

$$E_{\text{D,C}} = \left(\frac{\pi}{4} - x_{\text{sm}} \right) \cdot m_{\text{mn}} - h_{\text{a0}} \cdot \tan \alpha_{\text{nD,C}} - \frac{\rho_{\text{a0D,C}} \cdot \left(1 - \sin \alpha_{\text{nD,C}}\right) - s_{\text{prD,C}}}{\cos \alpha_{\text{nD,C}}} \quad (19)$$

The tooth root chord s_{Fn} is then calculated by:

$$s_{\text{Fn}} = 0{,}5 \cdot s_{\text{FnD}} + 0{,}5 \cdot s_{\text{FnC}} \quad (20)$$

Fillet radius at contact point of 30° tangent:

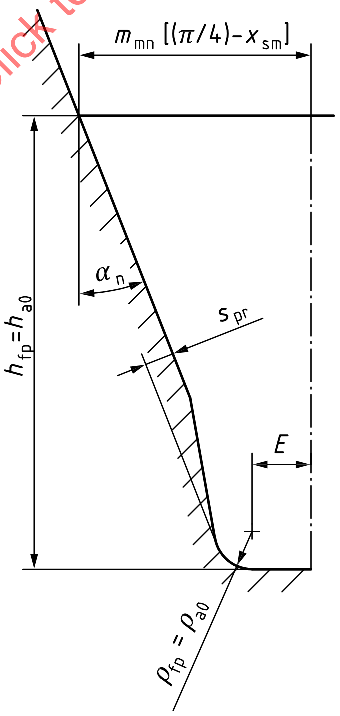
$$\rho_{\text{FD,C}} = \rho_{\text{a0D,C}} \quad (21)$$

Bending moment arm:

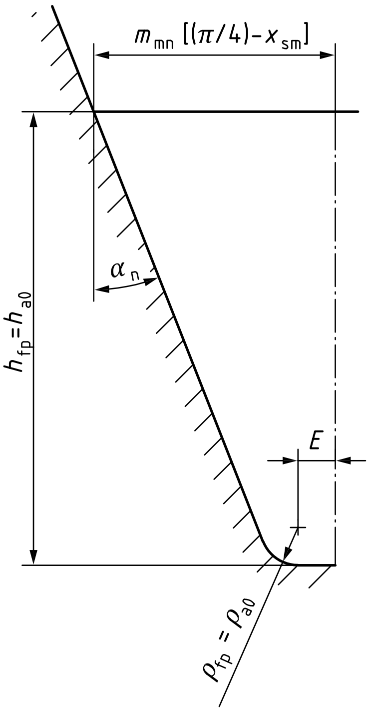
$$h_{\text{FaD,C}} = h_{\text{a0}} - \frac{\rho_{\text{a0D,C}}}{2} + m_{\text{mn}} - \left(\frac{\pi}{4} + x_{\text{sm}} - \tan \alpha_{\text{nD,C}} \right) \cdot m_{\text{mn}} \cdot \tan \alpha_{\text{nD,C}} \quad (22)$$

Tooth form factor of the wheel according to Formula (6) with $\alpha_{\text{FanD,C}} = \alpha_{\text{nD,C}}$:

$$Y_{\text{FaD,C}} = \frac{6 \cdot \dfrac{h_{\text{FaD,C}}}{m_{\text{mn}}}}{\left(\dfrac{s_{\text{Fn}}}{m_{\text{mn}}} \right)^2} \quad (23)$$



a) With protuberance



b) Without protuberance

Figure 2 — Dimensions at the basic rack profile of the tooth

6.4.2 Stress correction factor for load at tooth tip, Y_{Sa}

The stress correction factor for load at tooth tip, Y_{Sa}, accounts for the stress increasing notch effect in the root fillet as well as for other stress components which arise beside the tooth root stress (see ISO 6336-3[1] for additional information).

$$Y_{SaD,C} = \left(1{,}2 + 0{,}13 \cdot L_{aD,C}\right) \cdot q_{sD,C}^{\left(\frac{1}{1{,}21 + 2{,}3/L_{aD,C}}\right)} \tag{24}$$

$$L_{aD,C} = \frac{s_{Fn}}{h_{FaD,C}} \tag{25}$$

$$q_{sD,C} = \frac{s_{Fn}}{2 \cdot \rho_{FD,C}} \tag{26}$$

where

s_{Fn} is calculated for generated or non-generated gears according to Formula (12) or Formula (20);

h_{Fa} is calculated for generated or non-generated gears according to Formula (14) or Formula (22);

ρ_F is calculated for generated or non-generated gears according to Formula (13) or Formula (21).

The range of validity of Formula (26) is $1 \le q_s < 8$ (see ISO 6336-3[1] for the influence of grinding notches).

6.4.3 Contact ratio factor, Y_ε

The contact ratio factor, Y_ε, converts the load application at the tooth tip, where the tooth form factor, Y_{Fa}, and stress correction factor, Y_{Sa}, apply, to the determinant point of load application.

There are three ranges for $\varepsilon_{v\beta}$ to calculate Y_ε:

a) for $\varepsilon_{v\beta} = 0$:

$$Y_\varepsilon = 0{,}25 + \frac{0{,}75}{\varepsilon_{v\alpha}} \geq 0{,}625 \quad (27)$$

b) for $0 < \varepsilon_{v\beta} < 1$:

$$Y_\varepsilon = 0{,}25 + \frac{0{,}75}{\varepsilon_{v\alpha}} - \varepsilon_{v\beta} \cdot \left(\frac{0{,}75}{\varepsilon_{v\alpha}} - 0{,}375 \right) \geq 0{,}625 \quad (28)$$

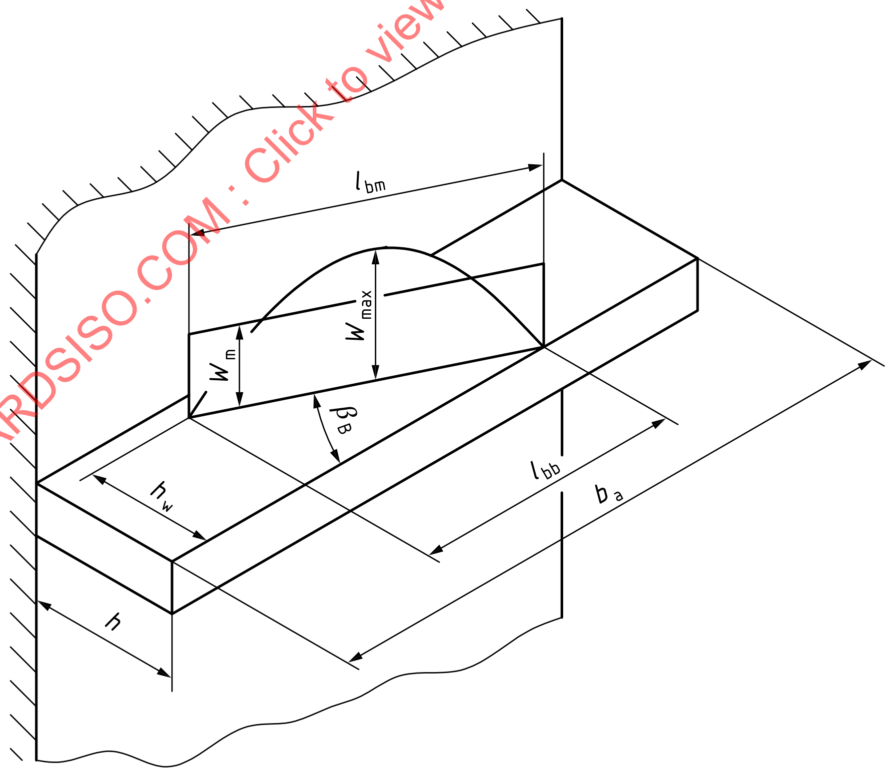
c) for $\varepsilon_{v\beta} \geq 1$:

$$Y_\varepsilon = 0{,}625 \quad (29)$$

6.4.4 Bevel spiral angle factor, Y_{BS}

The bevel spiral angle factor, Y_{BS}, accounts for the non-uniform distribution of the tooth root stress along the facewidth. The stress distribution depends on the inclination of the contact lines due to the spiral angle. With an increasing spiral angle, the inclination angle also increases till the contact lines are limited by tip and root of the teeth. Thus, the facewidth is not completely used to carry the load. This leads to a higher stress maximum in the tooth root in the middle of the facewidth (see Figure 3), where a tooth developed into a plane is replaced by a cantilever beam.

The bevel spiral angle factor Y_{BS} is not validated for bevel gears with $\beta_m = 0°$, for example straight bevel gears. In this case the assumption $Y_{BS} = 1{,}0$ may be considered in the calculation.



NOTE All dimensions in this figure are projected in a plane.

Figure 3 — Definition of geometric parameters of tooth model

Y_{BS} is given by the following empirical formulae [i.e. Formulae (30) to (33)]:

$$Y_{BS} = \frac{a_{BS}}{c_{BS}} \cdot \left(\frac{l_{bb}}{b_a} - 1{,}05 \cdot b_{BS} \right)^2 + 1 \tag{30}$$

$$a_{BS} = -0{,}0182 \cdot \left(\frac{2 \cdot b_a}{h_{m1} + h_{m2}} \right)^2 + 0{,}4736 \cdot \left(\frac{2 \cdot b_a}{h_{m1} + h_{m2}} \right) - 0{,}32 \tag{31}$$

$$b_{BS} = -0{,}0032 \cdot \left(\frac{2 \cdot b_a}{h_{m1} + h_{m2}} \right)^2 + 0{,}0526 \cdot \left(\frac{2 \cdot b_a}{h_{m1} + h_{m2}} \right) + 0{,}712 \tag{32}$$

$$c_{BS} = -0{,}0050 \cdot \left(\frac{2 \cdot b_a}{h_{m1} + h_{m2}} \right)^2 + 0{,}0850 \cdot \left(\frac{2 \cdot b_a}{h_{m1} + h_{m2}} \right) + 0{,}54 \tag{33}$$

with auxiliary values a_{BS}, b_{BS}, c_{BS} and mean whole tooth depth, h_m, as specified in ISO 23509.

The developed length of one tooth as facewidth of the calculation model:

$$b_a = b_v / \cos \beta_v \tag{34}$$

Part of the model's facewidth covered by the contact line:

$$l_{bb} = l_{bm} \cdot \frac{\cos \beta_{vb}}{\cos \beta_v} \tag{35}$$

6.4.5 Load sharing factor, Y_{LS}

The load sharing factor, Y_{LS}, for bending accounts for load sharing between two or more pairs of teeth:

$$Y_{LS} = Z_{LS}^2 \tag{36}$$

with load sharing factor, Z_{LS}, in accordance with ISO 10300-2:2023, 6.4.2 shall apply.

6.5 Permissible tooth root stress factors

6.5.1 Relative surface condition factor, $Y_{R,relT\text{-}B1}$

The tooth root strength depends on the surface condition at the root predominantly on the roughness in the root fillet. The surface condition factor, $Y_{R,relT}$, accounts for this dependence related to standard test gear conditions with Rz = 10 µm (see ISO 6336-3[1] for general remarks) and is determined separately for pinion (suffix 1) and wheel (suffix 2). If no surface condition factors determined according to method A are available, method B described in Clause 6 shall be used.

This method is only valid if there are no scratches or similar defects deeper than 2 Rz. The relative surface condition factor, $Y_{R,relT}$, determined by tests with test specimens, is calculated as a function of roughness Rz and material.

Formulae (37) to (42) shall be used depending on two ranges of roughness.

Range Rz < 1 µm:

a) For through hardened and case-hardened steels:

$$Y_{R,relT} = 1{,}12 \tag{37}$$

b) For non-hardened steels:

$$Y_{R,relT} = 1{,}07 \quad (38)$$

c) For grey cast iron, nitrided and nitro carburized steels:

$$Y_{R,relT} = 1{,}025 \quad (39)$$

Range 1 µm ≤ Rz ≤ 40 µm:

a) For through hardened and case-hardened steels:

$$Y_{R,relT} = \frac{Y_R}{Y_{RT}} = 1{,}674 - 0{,}529 \cdot (Rz+1)^{1/10} \quad (40)$$

b) For non-hardened steels:

$$Y_{R,relT} = \frac{Y_R}{Y_{RT}} = 5{,}306 - 4{,}203 \cdot (Rz+1)^{1/100} \quad (41)$$

c) For grey cast iron, nitrided and nitro carburized:

$$Y_{R,relT} = \frac{Y_R}{Y_{RT}} = 4{,}299 - 3{,}259 \cdot (Rz+1)^{1/200} \quad (42)$$

6.5.2 Relative notch sensitivity factor, $Y_{\delta,relT\text{-}B1}$

6.5.2.1 General

The dynamic notch sensitivity factor, Y_δ, indicates the amount by which the theoretical stress peak exceeds the permissible stress number in the case of fatigue breakage. It is a function of the material and relative stress drop. It is possible to calculate the notch sensitivity factor on the basis of strength values determined at un-notched or notched specimens, or at test gears. If more exact test results (method A) are not available, method B described in Clause 6 shall be used.

The calculation of permissible tooth root stresses of bevel gears is based on bending strength values determined for both, bevel and cylindrical test gears. Therefore, the relative notch sensitivity factor, $Y_{\delta,relT}$, is the ratio between the sensitivity factor of the gear to be calculated and the sensitivity factor of the standard test gear.

6.5.2.2 $Y_{\delta,relT\text{-}B1}$ for reference stress

In order to calculate the relative notch sensitivity factor, $Y_{\delta,relT}$ according to method B1, Formulae (43) and (44) shall be used:

$$Y_{\delta,relT1,2} = \frac{1+\sqrt{\rho' \cdot \chi_{1,2}^{X}}}{1+\sqrt{\rho' \cdot \chi_{T}^{X}}} \quad (43)$$

where ρ' shall be taken from Table 4 as a function of the material;

$$\chi_{1,2}^{X} = \frac{1}{5} \cdot (1 + 2 \cdot q_{s1,2}) \quad (44)$$

χ_T^X is derived by using q_{sT} = 2,5 from test gear.

$$\chi_T^X = 1{,}2 \quad (45)$$

Table 4 — Slip layer thickness ρ′

No.	Material		Slip layer thickness ρ' [mm]
1	GG	σ_B = 150 N/mm²	0,312 4
2	GG, GGG (ferr.)	σ_B = 300 N/mm²	0,309 5
3	NT (nitr.), NV (nitr.), NV (nitrocar.)	for all hardnesses	0,100 5
4	St	σ_S = 300 N/mm²	0,083 3
5	St	σ_S = 400 N/mm²	0,044 5
6	V, GTS, GGG (perl., bai.)	$\sigma_{0,2}$ = 500 N/mm²	0,028 1
7	V, GTS, GGG (perl., bai.)	$\sigma_{0,2}$ = 600 N/mm²	0,019 4
8	V, GTS, GGG (perl., bai.)	$\sigma_{0,2}$ = 800 N/mm²	0,006 4
9	V, GTS, GGG (perl., bai.)	$\sigma_{0,2}$ = 1 000 N/mm²	0,001 4
10	Eh, IF (root)	for all hardnesses	0,003 0

6.5.2.3 $Y_{\delta,\text{relT-B1}}$ for static stress

$Y_{\delta,\text{relT}}$ for static stress can be calculated using Formulae (46) to (51).

a) For St with well-defined yield point:

$$Y_{\delta,\text{relT-B1}} = \frac{1+0{,}93\cdot(Y_{Sa}-1)\cdot\sqrt[4]{\frac{200}{\sigma_S}}}{1+0{,}93\cdot\sqrt[4]{\frac{200}{\sigma_S}}} \quad (46)$$

b) For St with steadily increasing elongation curve and 0,2 % proof stress, V and GGG (perl., bai.):

$$Y_{\delta,\text{relT-B1}} = \frac{1+0{,}82\cdot(Y_{Sa}-1)\cdot\sqrt[4]{\frac{300}{\sigma_{0,2}}}}{1+0{,}82\cdot\sqrt[4]{\frac{200}{\sigma_{0,2}}}} \quad (47)$$

These values are only valid if the local stresses do not reach the yield point.

c) For Eh and IF (root) with stress up to crack initiation:

$$Y_{\delta,\text{relT-B1}} = 0{,}44\cdot Y_{Sa} + 0{,}12 \quad (48)$$

d) For NT and NV with stress up to crack initiation:

$$Y_{\delta,\text{relT-B1}} = 0{,}20\cdot Y_{Sa} + 0{,}60 \quad (49)$$

e) For GTS with stress up to crack initiation:

$$Y_{\delta,\text{relT-B1}} = 0{,}075\cdot Y_{Sa} + 0{,}85 \quad (50)$$

f) For GG and GGG (ferr.) with stress up to fracture limit:

$$Y_{\delta,\text{relT-B1}} = 1{,}0 \quad (51)$$

7 Gear tooth rating formulae — Method B2

7.1 Tooth root stress formula

The tooth root stress is determined separately for pinion (suffix 1) and wheel (suffix 2):

$$\sigma_{\text{F-B2}} = \sigma_{\text{F0-B2}} \cdot K_{\text{A}} \cdot K_{\text{v}} \cdot K_{\text{F}\beta} \cdot K_{\text{F}\alpha} < \sigma_{\text{FP-B2}} \tag{52}$$

with load factors K_A, K_v, $K_{F\beta}$ and $K_{F\alpha}$, which shall be as specified in ISO 10300-1.

The tooth root stress $\sigma_{F0\text{-}B2}$ is defined as the maximum tensile stress arising at the tooth root due to the nominal torque when an error-free gear is loaded.

When applying method B2, the combined geometry factor Y_P replaces the factors Y_{Fa}, Y_{Sa}, Y_ε, Y_{BS} and Y_{LS} of method B1 in the tooth root stress formula:

$$\sigma_{\text{F0-B2}} = \frac{F_{\text{mt1,2}}}{b_{1,2} \cdot m_{\text{mn}}} \cdot Y_{\text{P1,2}} \tag{53}$$

The value of Y_P is determined by Formula (54):

$$Y_{\text{P1,2}} = \frac{Y_{\text{A1,2}}}{Y_{\text{J1,2}}} \cdot \frac{m_{\text{mt1,2}} \cdot m_{\text{mn}}}{m_{\text{et2}}^2} \tag{54}$$

Substitution in Formula (53):

$$\sigma_{\text{F0-B2}} = \frac{F_{\text{mt1,2}}}{b_{1,2}} \cdot \frac{m_{\text{mt1,2}}}{m_{\text{et2}}^2} \cdot \frac{Y_{\text{A1,2}}}{Y_{\text{J1,2}}} \tag{55}$$

where

F_{mt} is the nominal tangential force of bevel gears in accordance with ISO 10300-1:2023, 6.1;

Y_A is the root stress adjustment factor for method B2 (see 7.4.7);

Y_J is the bending strength geometry factor for method B2 (see 7.4.3).

The bending strength geometry factor, Y_J, evaluates the shape of the tooth, the position at which the most damaging load is applied, the stress concentration due to the geometric shape of the root fillet, the sharing of load between adjacent pairs of teeth, the tooth thickness balance between the wheel and mating pinion, the effective facewidth due to lengthwise crowning of the teeth, and the buttressing effect of an extended facewidth on one member of the pair. Both the tangential (bending) and radial (compressive) components of the tooth load are included.

7.2 Permissible tooth root stress

The permissible tooth root stress, σ_{FP}, is determined separately for pinion and wheel. It should be calculated on the basis of the strength determined at an actual gear. In this way, the reference value for geometrical similarity, course of movement and manufacture lies within the field of application:

$$\sigma_{\text{FP-B2}} = \sigma_{\text{FE}} \cdot Y_{\text{NT}} \cdot Y_{\delta,\text{relT-B2}} \cdot Y_{\text{R,relT-B2}} \cdot Y_{\text{X}} \tag{56}$$

$$\sigma_{\text{FP-B2}} = \sigma_{\text{F,lim}} \cdot Y_{\text{ST}} \cdot Y_{\text{NT}} \cdot Y_{\delta,\text{relT-B2}} \cdot Y_{\text{R,relT-B2}} \cdot Y_{\text{X}} \tag{57}$$

where

σ_{FE} is the allowable stress number (bending);

σ_{FE} $= \sigma_{F,lim1,2} \cdot Y_{ST}$, is the basic bending strength of the un-notched specimen under the assumption that the material (including heat treatment) is fully elastic;

$\sigma_{F,lim}$ is the nominal stress number (bending) of the standard test gear, which accounts for material, heat treatment, and surface influence at test gear dimensions as specified in ISO 6336-5;

Y_{ST} is the stress correction factor for the dimensions of the standard test gear, $Y_{ST} = 2,0$;

$Y_{\delta,relT\text{-}B2}$ is the relative notch sensitivity factor (see 7.5.2) for the bending stress number related to the conditions at the standard test gear ($Y_{\delta,relT} = Y_\delta / Y_{\delta T}$ accounts for the notch sensitivity of the material);

$Y_{R,relT\text{-}B2}$ is the relative surface condition factor (see 7.5.1) ($Y_{R,relT} = Y_R / Y_{RT}$ accounts for the surface condition at the root fillet, related to the conditions at the test gear);

Y_X is the size factor for tooth root strength (see 8.1), which accounts for the influence of the module on the tooth root strength;

Y_{NT} is the life factor, which accounts for the influence of required numbers of cycles of operation (see 8.2).

7.3 Calculated safety factor

The determined tooth root stress, σ_F, shall be ≤ σ_{FP}, which is the permissible tooth root stress. The calculated safety factor against tooth root breakage shall be determined separately for pinion and wheel, on the basis of the bending stress number determined for the standard test gear:

$$S_{F\text{-}B2} = \frac{\sigma_{FP\text{-}B2}}{\sigma_{F\text{-}B2}} > S_{F\,min} \tag{58}$$

NOTE This is the calculated safety factor with respect to the transmitted torque.

Considerations in reference to the safety factors and the risk (probability) of failure are given in ISO 10300-1:2023, 5.2.

7.4 Tooth root stress factors

7.4.1 General

To calculate the bending strength geometry factor, Y_J, Formula (59) in 7.4.3 should be used. Because of the complexity of the calculation, computerization is recommended.

ANSI/AGMA 2003[4] contains graphs for the bevel geometry factor, Y_J, for straight, Zerol and spiral bevel gears for a series of gear designs, based on the smaller of the facewidth to be chosen $b = 0{,}3 \cdot Re$ or $b = 10 \cdot m_{et}$. Corresponding graphs for hypoid gears can be found in AGMA 932.[5] These may be used whenever the tooth proportions and thickness, facewidths, tool edge radii, pressure and spiral angles of the design, and driving with the concave side, correspond to those in the graphs.

7.4.2 Stress parabola according to Lewis

The basis for method B2 is the Lewis formula applied to a virtual cylindrical gear, which has been defined in transverse section as specified in ISO 10300-1:2023, Annex B, with the following additions and modifications:

— the tooth strength is considered in the normal section rather than in the transverse section;

— the position of the point of load application is determined by taking into account theoretical lines of contact, tooth bearing modifications and experimental evidence;

— the amount of load carried by one tooth is estimated based on tooth bearing modification and contact ratio;

— the radial component of the normal load is considered;

— a stress concentration factor based on experimental data are applied;

— the concept of effective facewidth is used.

Bending stress shall be calculated assuming the tooth shaped beam is simulated by a parabola tangent to the tooth profile at the most highly stressed section. Figure 4 shows a layout for the cases of: a) no load sharing and b) load sharing.

7.4.3 Basic formula of geometry factor, Y_J

The parameters for calculating the geometry factor, Y_J are the same for bevel and hypoid gears. However, the calculation procedures are different. See 7.4.4 for bevel gears without hypoid offset or 7.4.5 for hypoid gears.

The bevel geometry factor, Y_J, is calculated using Formula (59):

$$Y_{J1,2} = \frac{Y_{1,2}}{Y_{f1,2} \cdot \varepsilon_N \cdot Y_i} \cdot \frac{r_{my01,2}}{r_{mpt1,2}} \cdot \frac{b_{ce1,2}}{b_{1,2}} \cdot \frac{m_{mt1,2}}{m_{et2}} \tag{59}$$

where

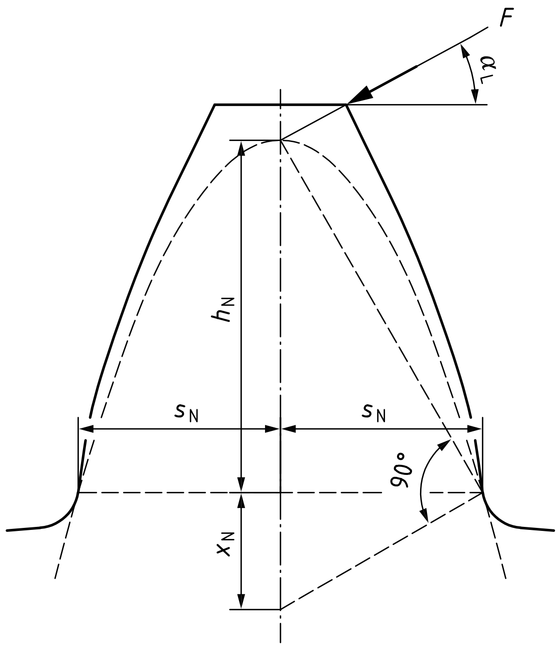
$Y_{1,2}$ is the tooth form factor of pinion and wheel (see 7.4.4.4 for bevel gears and 7.4.5.4 for hypoid gears);

ε_N is the load sharing ratio (see 7.4.4.3 and 7.4.5.2, respectively);

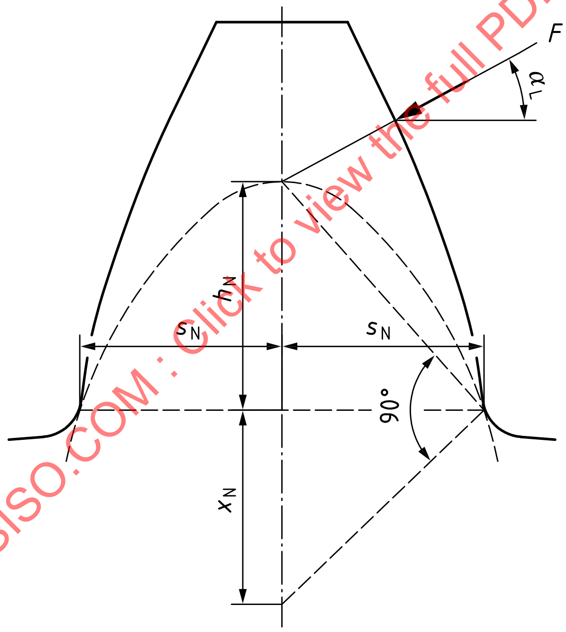
$r_{my01,2}$ is the mean transverse radius to point of load application for pinion or wheel, in millimetres (see 7.4.4.2 and 7.4.5.5);

$r_{mpt1,2}$ is the mean transverse pitch radius, in millimetres (see ISO 23509);

$Y_{f1,2}$ is the stress concentration and correction factor (see 7.6.2);



a) No load sharing



b) Load sharing

Figure 4 — Stress parabola according to Lewis

7.4.4 Geometry factor, Y_J, for bevel gears (for hypoid gears, see 7.4.5)

7.4.4.1 Point of load application for maximum tooth root stress, y_3

For most straight, Zerol and spiral bevel gears, the maximum tooth root stress occurs at the equivalent of the highest point of single tooth contact when the modified contact ratio is ≤ 2. When the modified contact ratio is > 2, it is assumed that the contact line passes through the centre of the path of action. For statically loaded straight bevel and Zerol bevel gears, such as those used in automotive differentials, the load is applied at the tip of the tooth. In any case, the relative position is measured along the path of action from its centre, and is designated by y_J. Its relative distance from the beginning of the path of action is designated by y_3.

So, there are three cases for the determination of y_J:

when $\varepsilon_{v\gamma} \le 2{,}0$:

$$y_J = \frac{\pi \cdot m_{mn} \cdot \cos\alpha_a}{m_{et2}} - \frac{g_\eta}{2} \tag{60}$$

for α_a ISO 10300-1:2023, Formula (B.16), and for g_η ISO 10300-2:2023, Formula (34) apply

when $\varepsilon_{v\gamma} > 2{,}0$:

$$y_J = 0 \tag{61}$$

for statically loaded straight bevel and Zerol bevel gears (tip loading):

$$y_J = \frac{g_\eta}{2} \tag{62}$$

where

$$g_\eta^2 = g_{v\alpha n}^2 \cdot \cos^4\beta_{vb} + b_{v,rel}^2 \cdot (\sin\beta_{vb})^2 \tag{63}$$

The determination of the distance, y_3, depends on the type of bevel gears:

for straight bevel and Zerol bevel gears:

$$y_3 = \frac{g_{v\alpha n}}{2} + \frac{g_{v\alpha n}^2 \cdot y_J}{g_\eta^2} \tag{64}$$

for spiral bevel pinions:

$$y_{31} = \frac{g_{v\alpha n}}{2} + \frac{g_{v\alpha n}^2 \cdot y_J \cdot \cos^2\beta_{vb} + b_{v,rel} \cdot g_{v\alpha n} \cdot g_J \cdot k' \cdot \sin\beta_{vb}}{g_\eta^2} \tag{65}$$

for spiral bevel wheels:

$$y_{32} = \frac{g_{v\alpha n}}{2} + \frac{g_{v\alpha n}^2 \cdot y_J \cdot (\cos\beta_{vb})^2 - b_{v,rel} \cdot g_{v\alpha n} \cdot g_J \cdot k' \cdot \sin\beta_{vb}}{g_\eta^2} \tag{66}$$

where

$$g_J = \sqrt{g_\eta^2 - 4 \cdot y_J^2} \tag{67}$$

k' is the contact shift factor [see ISO 10300-1:2023, Formula (B.26)].

7.4.4.2 Transverse radius to point of load application, $r_{my0\,1,2}$

Since the point of load application does not usually lie in the mean section of the tooth, the actual radius is determined using Formulae (68) to (75). The distance from mean section to point of load application, $x_{oo1,2}$, measured in the lengthwise direction along the tooth, is calculated depending on the type of gear:

a) for straight and Zerol bevel gears:

$$x_{oo1,2} = \frac{b_{v,rel} \cdot g_{v\alpha n} \cdot g_J \cdot k' \cdot m_{et2}}{g_\eta^2} \quad (68)$$

b) for spiral bevel pinion:

$$x_{oo1} = \frac{b_{v,rel} \cdot g_{v\alpha n} \cdot g_J \cdot k' \cdot (\cos\beta_{vb})^2 \cdot m_{et2} - b_{v,rel}^2 \cdot y_J \cdot \sin\beta_{vb} \cdot m_{et2}}{g_\eta^2} \quad (69)$$

c) for spiral bevel wheel:

$$x_{oo2} = \frac{b_{v,rel} \cdot g_{v\alpha n} \cdot g_J \cdot k' \cdot (\cos\beta_{vb})^2 \cdot m_{et2} + b_{v,rel}^2 \cdot y_J \cdot \sin\beta_{vb} \cdot m_{et2}}{g_\eta^2} \quad (70)$$

The normal pressure angle at point of load application, $\alpha_{L1,2}$, for pinion and wheel is derived from:

$$\tan\alpha_{L1,2} = \frac{y_{31,2} + a_{vn} \cdot \sin\alpha_n - \sqrt{r_{va2,1}^2 - r_{vbn2,1}^2}}{r_{vbn1,2}} \quad (71)$$

The rotation angle, $\xi_{h1,2}$, used in bending strength calculations for pinion and wheel, $\xi_{h1,2}$, is:

$$\xi_{h1,2} = \frac{s_{vmn1,2}}{2 \cdot r_{vn1,2}} - \mathrm{inv}\alpha_{L1,2} + \sin\alpha_n \quad (72)$$

Relative distance from pitch circle to the pinion point of load application and the wheel tooth centreline is:

$$\Delta r_{y01,2} = \frac{r_{vbn1,2}}{\cos\alpha_{h1,2}} - r_{vn1,2} \quad (73)$$

where

$$\alpha_{h1,2} = \alpha_{L1,2} - \xi_{h1,2} \quad (74)$$

Mean transverse radius to point of load application, in millimetres:

$$r_{my01,2} = r_{mpt1,2} \cdot \left(\frac{R_m + x_{oo1,2}}{R_m} \right) + \Delta r_{y01,2} \cdot m_{et2} \quad (75)$$

7.4.4.3 Load sharing ratio, ε_N

The load sharing ratio, ε_N, is used to calculate the proportion of the total load carried on the tooth being analysed. It is given by Formulae (76), (77) and (78):

$$g'^3_J = g_J^3 + \sum_{k=1}^{k=x} \sqrt{\left[g_J^2 - 4 \cdot k \cdot \frac{\pi \cdot m_{mn} \cdot \cos\alpha_a}{m_{et2}} \cdot \left(k \cdot \frac{\pi \cdot m_{mn} \cdot \cos\alpha_a}{m_{et2}} + 2 \cdot y_J \right) \right]^3} + \sum_{k=1}^{k=y} \sqrt{\left[g_J^2 - 4 \cdot k \cdot \frac{\pi \cdot m_{mn} \cdot \cos\alpha_a}{m_{et2}} \cdot \left(k \cdot \frac{\pi \cdot m_{mn} \cdot \cos\alpha_a}{m_{et2}} - 2 \cdot y_J \right) \right]^3} \quad (76)$$

In Formula (76), k is a positive integer, which has successive values from 1 to x or y, generating all real terms (positive values under the radical) in each series. Imaginary terms (negative values under the radical) shall be ignored. For most designs, x and y are not greater than 2.

The load sharing ratio is:

$$\varepsilon_{N} = \frac{g_{J}^{3}}{g_{J}^{\prime 3}} \tag{77}$$

For statically loaded straight and Zerol bevel gears:

$$\varepsilon_{N} = 1{,}0 \tag{78}$$

7.4.4.4 Tooth form factor, $Y_{1,2}$

The tooth form factor incorporates both the radial and tangential components of the normal load. Since this factor defines the weakest section, its value shall be determined by iteration for pinion and wheel separately.

$$g_{01,2} = 0{,}5 \cdot s_{vmn1,2} + h_{vfm1,2} \cdot \tan\alpha_{n} + \rho_{va01,2} \cdot \left(\frac{1 - \sin\alpha_{n}}{\cos\alpha_{n}}\right) \tag{79}$$

$$g_{yb1,2} = h_{vfm1,2} - \rho_{va01,2} \tag{80}$$

$$g_{f01,2(1)} = g_{01,2} + g_{yb1,2} \tag{81}$$

Start of iteration with $g_{f01,2(1)}$ as initial value:

$$\xi_{1,2} = \frac{g_{f01,2}}{r_{vn1,2}} \tag{82}$$

$$g_{xb1,2} = g_{f01,2} - g_{01,2} \tag{83}$$

$$g_{za1,2} = g_{yb1,2} \cdot \cos\xi_{1,2} - g_{xb1,2} \cdot \sin\xi_{1,2} \tag{84}$$

$$g_{zb1,2} = g_{yb1,2} \cdot \sin\xi_{1,2} + g_{xb1,2} \cdot \cos\xi_{1,2} \tag{85}$$

$$\tan\tau_{1,2} = \frac{g_{za1,2}}{g_{zb1,2}} \tag{86}$$

$$s_{N1,2} = r_{vn1,2} \cdot \sin\xi_{1,2} - \rho_{va01,2} \cdot \cos\tau_{1,2} - g_{zb1,2} \tag{87}$$

$$h_{N1,2} = \Delta r_{y01,2} + r_{vn1,2} \cdot \left(1 - \cos\xi_{1,2}\right) + \rho_{va01,2} \cdot \sin\tau_{1,2} + g_{za1,2} \tag{88}$$

Change $g_{f01,2}$ until

$$\frac{s_{N1,2} \cdot \cot\tau_{1,2}}{h_{N1,2}} = 2{,}0 \pm 0{,}001 \tag{89}$$

For the second trial, make $g_{f01,2(2)} = g_{f01,2(1)} + 0{,}005 \cdot m_{et2}$. For the third and subsequent trials, interpolate. For this calculation, m_{et2} shall be used without unit.

End of iteration.

Tooth strength factor:

$$x_{N1,2} = \frac{s_{N1,2}^2}{h_{N1,2}} \tag{90}$$

Tooth form factor for bevel gears without hypoid offset:

$$Y_{1,2} = \frac{2}{3} \cdot \left[\frac{1}{\left(\frac{1}{x_{N1,2}} - \frac{\tan \alpha_{h1,2}}{3 \cdot s_{N1,2}} \right)} \right] \tag{91}$$

7.4.5 Geometry factor, Y_J, for hypoid gears

7.4.5.1 Initial formulae

Tooth surface points are calculated using the function $e^{\theta \cdot \tan \alpha_f}$ to approximate the tooth surfaces.

Drive flank pressure angle in wheel root coordinates:

$$\alpha_{Dnf} = \alpha_{nD} - \theta_{f2} \cdot \sin \beta_{m2} \tag{92}$$

where θ_{f2} is the dedendum angle of the wheel (given in ISO 23509:2016, Table C.5).

Coast flank pressure angle in wheel root coordinates:

$$\alpha_{Cnf} = \alpha_{nC} + \theta_{f2} \cdot \sin \beta_{m2} \tag{93}$$

Average pressure angle unbalance:

$$\Delta\alpha_1 = \frac{(\alpha_{Dnf} - \alpha_{Cnf})}{2,0} \tag{94}$$

Limit pressure angle in wheel root coordinates:

$$\alpha_f = \alpha_{lim} - \theta_{f2} \cdot \sin \beta_{m2} \tag{95}$$

Relative distance from blade edge to centreline:

$$g_{rb} = \frac{\left[h_{fm2} \cdot \tan\left(\frac{\alpha_{nD} + \alpha_{nC}}{2,0} \right) + \frac{W_{m2}}{2,0} \right] \cdot \cos\left(\frac{\alpha_{nD} + \alpha_{nC}}{2,0} \right)}{m_{et2}} \tag{96}$$

where W_{m2} is wheel mean slot width (see ISO 23509:2016, Figure 16).

Intermediate value:

$$\eta_D = \tan \alpha_{Dnf} \cdot \left(\frac{g_{rb}}{\sin \alpha_{Dnf}} - h_{vfm2} \right) \tag{97}$$

Intermediate value:

$$\eta_C = -\tan \alpha_{Cnf} \cdot \left(\frac{g_{rb}}{\sin \alpha_{Cnf}} - h_{vfm2} \right) \tag{98}$$

Intermediate angle:

$$\tan \beta_a = \frac{\dfrac{W_{m2}}{2{,}0 \cdot m_{et2}} - \rho_{va02} \cdot \left[\sec\left(\dfrac{\alpha_{nD} + \alpha_{nC}}{2{,}0} \right) - \tan\left(\dfrac{\alpha_{nD} + \alpha_{nC}}{2{,}0} \right) \right]}{h_{vfm2} - \rho_{va02}} \tag{99}$$

Intermediate angle:

$$(\beta_D - \Delta\alpha) = \beta_a - \Delta\alpha_1 \tag{100}$$

Intermediate angle:

$$(\beta_C - \Delta\alpha) = -\beta_a - \Delta\alpha_1 \tag{101}$$

Intermediate value:

$$g_1 = \frac{h_{vfm2} - \rho_{va02}}{\cos \beta_a} \tag{102}$$

Wheel angle between centreline and fillet point on drive side:

$$\tan \Delta\theta_D = \frac{g_1 \cdot \sin(\beta_D - \Delta\alpha)}{r_{vn2} - g_1 \cdot \cos(\beta_D - \Delta\alpha)} \tag{103}$$

Wheel angle between centreline and fillet point on coast side:

$$\tan \Delta\theta_C = \frac{g_1 \cdot \sin(\beta_C - \Delta\alpha)}{r_{vn2} - g_1 \cdot \cos(\beta_C - \Delta\alpha)} \tag{104}$$

Wheel angle between fillet points:

$$\Delta\theta_2 = \frac{\theta_{v2} + \Delta\theta_D + \Delta\theta_C}{2{,}0} \tag{105}$$

where θ_{v2} is angular pitch of virtual cylindrical wheel [see ISO 10300-1:2023, Formula (B.11)].

Relative vertical distance from pitch circle to fillet point:

$$y_1 = r_{vn2} - \frac{[r_{vn2} - g_1 \cdot \cos(\beta_D - \Delta\alpha)] \cdot \cos(\Delta\theta_2 - \Delta\theta_D)}{\cos\Delta\theta_D} \quad (106)$$

Relative horizontal distance from centreline to fillet point:

$$x_1 = \frac{[r_{vn2} - g_1 \cdot \cos(\beta_D - \Delta\alpha)] \cdot \sin(\Delta\theta_2 - \Delta\theta_D)}{\cos\Delta\theta_D} \quad (107)$$

Generated pressure angle of wheel at fillet point [required for Formula (155)]:

$$\alpha_{LN2} = \alpha_{Dnf} - \Delta\theta_2 \quad (108)$$

Relative distance from centreline to tool critical drive side fillet point:

$$\mu_{D1} = \eta_D + \tan\alpha_{Dnf} \cdot (h_{vfm1} + h_{vfm2}) + \rho_{va01} \cdot (\sec\alpha_{Dnf} - \tan\alpha_{Dnf}) \quad (109)$$

Relative distance from centreline to tool critical coast side fillet point:

$$\mu_{C1} = \eta_C + \tan\alpha_{Cnf} \cdot (h_{vfm1} + h_{vfm2}) + \rho_{va01} \cdot (\sec\alpha_{Cnf} - \tan\alpha_{Cnf}) + \frac{j_{en}}{m_{et2}} \quad (110)$$

Wheel angle between centreline and critical pinion drive side fillet point:

$$\tan\theta_{DLS} = \frac{\mu_{D1}}{r_{vn2} + h_{vfm1}} \quad (111)$$

Wheel angle between centreline and critical pinion coast side fillet point:

$$\tan\theta_{CLS} = \frac{\mu_{C1}}{r_{vn2} + h_{vfm1}} \quad (112)$$

Relative radius from tool centre to critical pinion drive side fillet point:

$$R_{DL2} = \frac{r_{vn2} + h_{vfm1}}{\cos\theta_{DLS}} \quad (113)$$

Relative radius from tool centre to critical pinion coast side fillet point:

$$R_{CL2} = \frac{r_{vn2} + h_{vfm1}}{\cos\theta_{CLS}} \quad (114)$$

Wheel angle from centreline to pinion tip on drive side, θ_{D1}:

For start of iteration, assume $\theta_{D1} = \theta_{v2}$.

$$\Delta r_1 = r_{vn2} \cdot \left(e^{\theta_{D1} \cdot \tan\alpha_f} - 1{,}0\right) \quad (115)$$

$$h_1 = (r_{vn2} + \Delta r_1) \cdot \sin(\alpha_{Dnf} + \theta_{D1}) - (r_{vn2} \cdot \sin\alpha_{Dnf} - g_{rb}) \quad (116)$$

$$h_{1o} = \sqrt{r_{va1}^2 - (r_{vn1} - \Delta r_1)^2 \cdot \cos^2(\alpha_{Dnf} + \theta_{D1})} - (r_{vn1} - \Delta r_1) \cdot \sin(\alpha_{Dnf} + \theta_{D1}) \quad (117)$$

Change θ_{D1} until $h_{1o} = h_1$ which is the end of iteration.

Wheel angle from centreline to tooth surface at critical fillet point on drive side, θ_{D2o}:

For the start of iteration, assume $\theta_{D2o} = 1/2 \cdot \theta_{v2}$.

$$\mu_{D1o} = r_{vn2} \cdot e^{\theta_{D2o} \cdot \tan\alpha_f} \cdot \sin\theta_{D2o} \tag{118}$$

Change θ_{D2o} until $u_{D1o} = u_{D1}$ which is the end of iteration.

Wheel angle from centreline to tooth surface at critical fillet point on coast side, θ_{C2o}:

For start of iteration, assume $\theta_{C2o} = -1/2 \cdot \theta_{v2}$.

$$\mu_{C1o} = r_{vn2} \cdot e^{\theta_{C2o} \cdot \tan\alpha_f} \cdot \sin\theta_{C2o} \tag{119}$$

Change θ_{C2o} until $\mu_{C1o} = \mu_{C1}$ which is the end of iteration.

Pinion angle from centreline to tooth surface at critical drive side fillet point, θ_{D1o}:

Formula (120) shall be solved for θ_{D1o}.

$$r_{vn2} \cdot \left(e^{\theta_{D2o} \cdot \tan\alpha_f} - 1{,}0\right) = r_{vn1} \cdot \left(1{,}0 - e^{\theta_{D1o} \cdot \tan\alpha_f}\right) \tag{120}$$

Pinion angle from centreline to tooth surface at critical coast side fillet point, θ_{C1o}:

Formula (121) shall be solved for θ_{C1o}.

$$r_{vn2} \cdot \left(e^{\theta_{C2o} \cdot \tan\alpha_f} - 1{,}0\right) = r_{vn1} \cdot \left(1{,}0 - e^{\theta_{C1o} \cdot \tan\alpha_f}\right) \tag{121}$$

Wheel difference angle between tool and surface at drive side fillet point:

$$\Delta\theta_{D2o} = \theta_{DLS} - \theta_{D2o} \tag{122}$$

Wheel difference angle between tool and surface at coast side fillet point:

$$\Delta\theta_{C2o} = \theta_{CLS} - \theta_{C2o} \tag{123}$$

Pinion difference angle between tool and surface at drive side fillet point:

$$\tan\Delta\theta_{D1o} = -\frac{R_{DL2} \cdot \sin\Delta\theta_{D2o}}{r_{vn2} + r_{vn1} - R_{DL2} \cdot \cos\Delta\theta_{D2o}} \tag{124}$$

Pinion difference angle between tool and surface at coast side fillet point:

$$\tan\Delta\theta_{C1o} = -\frac{R_{CL2} \cdot \sin\Delta\theta_{C2o}}{r_{vn2} + r_{vn1} - R_{CL2} \cdot \cos\Delta\theta_{C2o}} \tag{125}$$

Pinion angle unbalance between fillet points:

$$\Delta\theta_1 = \frac{\theta_{D1o} + \theta_{C1o} + \Delta\theta_{D1o} + \Delta\theta_{C1o}}{2} \tag{126}$$

Pinion angle from centreline to pinion tip, θ_{Do}:

Formula (127) shall be solved for θ_{Do}.

$$\Delta r_1 = r_{vn1} \cdot \left(1{,}0 - e^{\theta_{Do} \cdot \tan\alpha_f}\right) \tag{127}$$

Wheel angle from centreline to tooth surface at pitch point on drive side, θ_D:

For start of iteration, assume $\theta_D = -1/3 \cdot \theta_{v2}$.

$$\Delta r = r_{vn2} \cdot \left(e^{\theta_D \cdot \tan \alpha_f} - 1,0 \right) \tag{128}$$

$$h = (r_{vn2} + \Delta r) \cdot \sin(\alpha_{Dnf} + \theta_D) - (r_{vn2} \cdot \sin \alpha_{Dnf} - g_{rb}) \tag{129}$$

Change θ_D until $h = 0,0$ which is the end of iteration

The wheel angle from centreline to fillet point on drive flank, θ_{D2}, is evaluated by iteration. The initial value, θ_{D2}, should be determined depending on the amount of $(r_{vn2} + \Delta r)$:

a) $(r_{vn2} + \Delta r) > r_{va2}$

$$\theta_{D2} = 0,8 \cdot \theta_D \tag{130}$$

b) $(r_{vn2} + \Delta r) = r_{va2}$

$$\theta_{D2} = 1,0 \cdot \theta_D \tag{131}$$

c) $(r_{vn2} + \Delta r) < r_{va2}$

$$\theta_{D2} = 1,2 \cdot \theta_D \tag{132}$$

Start of iteration:

$$\Delta r_2 = r_{vn2} \cdot \left(e^{\theta_{D2} \cdot \tan \alpha_f} - 1,0 \right) \tag{133}$$

$$h_2 = (r_{vn2} + \Delta r_2) \cdot \sin(\alpha_{Dnf} + \theta_{D2}) - (r_{vn2} \cdot \sin \alpha_{Dnf} - g_{rb}) \tag{134}$$

$$h_{2o} = \pm \sqrt{r_{va1}^2 - (r_{vn1} + \Delta r_2)^2 \cdot \cos^2(\alpha_{Dnf} + \theta_{D2})} + (r_{vn1} + \Delta r_2) \cdot \sin(\alpha_{Dnf} + \theta_{D2}) \tag{135}$$

with +sign, if $(r_{vn2} \cdot \sin \alpha_{Dnf} - g_{rb}) \leq 0,0$ or −sign, if $(r_{vn2} \cdot \sin \alpha_{Dnf} - g_{rb}) \geq 0,0$

Change θ_{D2} until $h_2 = h_{2o}$ which is the end of iteration.

7.4.5.2 Load sharing ratio, ε_N, for hypoid gears

Load sharing ratio for hypoid gears:

$$\varepsilon_N = 1,0 \tag{136}$$

7.4.5.3 Tooth strength factor, $x_{N1,2}$, for hypoid gears

Relative length of action from pinion tip to pitch circle in normal section:

$$g_{v\alpha n1} = \sqrt{h_1^2 + (\Delta r_1 - \Delta r)^2 - 2,0 \cdot h_1 \cdot (\Delta r_1 - \Delta r) \cdot \sin(\alpha_{Dnf} + \theta_{D1})} \quad (137)$$

Relative length of action from wheel tip to pitch circle in normal section:

$$g_{v\alpha n2} = \sqrt{h_2^2 + (\Delta r_2 - \Delta r)^2 - 2,0 \cdot h_2 \cdot (\Delta r_2 - \Delta r) \cdot \sin(\alpha_{Dnf} + \theta_{D2})} \quad (138)$$

Relative length of action in normal section for hypoid gears:

$$g_{v\alpha n,hyp} = g_{v\alpha n1} + g_{v\alpha n2} \quad (139)$$

Profile contact ratio in mean normal section for hypoid gears:

$$\varepsilon_{v\alpha n,hyp} = g_{v\alpha n,hyp} / p_{mn} \quad (140)$$

Modified contact ratio for hypoid gears:

$$\varepsilon_{v\gamma} = \sqrt{\varepsilon_{v\alpha n,hyp}^2 + \varepsilon_{v\beta}^2} \quad (141)$$

The profile load sharing factor is

a) when $\varepsilon_{v\gamma} < 2,0$:

$$\varepsilon_f = 1,0 - 0,5 \cdot \varepsilon_{v\gamma} \quad (142)$$

b) when $\varepsilon_{v\gamma} \geq 2,0$:

$$\varepsilon_f = 0,0 \quad (143)$$

The lengthwise load sharing factor is

a) when $\varepsilon_{v\gamma} < 2,0$:

$$\varepsilon_b = 2,0 \cdot \sqrt{\varepsilon_{v\gamma} - 1,0} \quad (144)$$

b) when $\varepsilon_{v\gamma} \geq 2,0$:

$$\varepsilon_b = \varepsilon_{v\gamma} \quad (145)$$

Relative length of action from pinion tip to point of load application:

$$g_{\mathrm{v}\alpha\mathrm{n}3} = \left| \frac{p_{\mathrm{mn}} \cdot \varepsilon_{\mathrm{v}\alpha\mathrm{n,hyp}}^2}{\varepsilon_{\mathrm{v}\gamma}^2} \cdot \left(\frac{0{,}5 \cdot \varepsilon_{\mathrm{v}\gamma}^2}{\varepsilon_{\mathrm{v}\alpha\mathrm{n,hyp}}} - \frac{\varepsilon_{\mathrm{v}\beta} \cdot \varepsilon_{\mathrm{b}} \cdot k'}{\varepsilon_{\mathrm{v}\alpha}} + \varepsilon_{\mathrm{f}} \right) - g_{\mathrm{v}\alpha\mathrm{n}1} \right| \tag{146}$$

Relative length of action from wheel tip to point of load application:

$$g_{\mathrm{v}\alpha\mathrm{n}4} = \left| \frac{p_{\mathrm{mn}} \cdot \varepsilon_{\mathrm{v}\alpha\mathrm{n,hyp}}^2}{\varepsilon_{\mathrm{v}\gamma}^2} \cdot \left(\frac{0{,}5 \cdot \varepsilon_{\mathrm{v}\gamma}^2}{\varepsilon_{\mathrm{v}\alpha\mathrm{n,hyp}}} - \frac{\varepsilon_{\mathrm{v}\beta} \cdot \varepsilon_{\mathrm{b}} \cdot k'}{\varepsilon_{\mathrm{v}\alpha}} + \varepsilon_{\mathrm{f}} \right) - g_{\mathrm{v}\alpha\mathrm{n}2} \right| \tag{147}$$

Relative length of action to point of load application:

$$g_{\mathrm{J}1,2} = g_{\mathrm{v}\alpha\mathrm{n,hyp}} - g_{\mathrm{v}\alpha\mathrm{n}3,4} \tag{148}$$

Wheel angle from pinion tip to point of load application, θ_{D3}:

For start of iteration, assume $\theta_{\mathrm{D3}} = -1/2 \cdot \theta_{\mathrm{v2}}$.

$$\Delta r_3 = r_{\mathrm{vn}2} \cdot \left(e^{\theta_{\mathrm{D3}} \cdot \tan\alpha_{\mathrm{f}}} - 1{,}0 \right) \tag{149}$$

$$h_3 = (r_{\mathrm{vn}2} + \Delta r_3) \cdot \sin(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D3}}) - (r_{\mathrm{vn}2} \cdot \sin\alpha_{\mathrm{Dnf}} - g_{\mathrm{rb}}) \tag{150}$$

$$h_{3\mathrm{o}} = \sqrt{g_{\mathrm{v}\alpha\mathrm{n}3}^2 - (\Delta r_3 - \Delta r)^2 \cdot \cos^2(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D3}})} - |\Delta r_3 - \Delta r| \cdot \sin(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D3}}) \tag{151}$$

Change θ_{D3} until $h_3 = h_{3\mathrm{o}}$ which is the end of iteration.

Pinion angle from wheel tip to point of load application, θ_{D4}:

For start of iteration, assume $\theta_{\mathrm{D4}} = 1/3 \cdot \theta_{\mathrm{v2}}$.

$$\Delta r_4 = r_{\mathrm{vn}2} \cdot \left(e^{\theta_{\mathrm{D4}} \cdot \tan\alpha_{\mathrm{f}}} - 1{,}0 \right) \tag{152}$$

$$h_4 = (r_{\mathrm{vn}2} + \Delta r_4) \cdot \sin(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D4}}) - (r_{\mathrm{vn}2} \cdot \sin\alpha_{\mathrm{Dnf}} - g_{\mathrm{rb}}) \tag{153}$$

$$h_{4\mathrm{o}} = \sqrt{g_{\mathrm{v}\alpha\mathrm{n}4}^2 - (\Delta r_4 - \Delta r)^2 \cdot \cos^2(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D4}})} - |\Delta r_4 - \Delta r| \cdot \sin(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D4}}) \tag{154}$$

Change θ_{D4} until $h_4 = h_{4\mathrm{o}}$ which is the end of iteration.

Relative distance from pitch circle to point of load application, Δr_{LN2}:

$$\Delta r_{\mathrm{LN2}} = \frac{(r_{\mathrm{vn}2} + \Delta r_3) \cdot \cos(\alpha_{\mathrm{Dnf}} + \theta_{\mathrm{D3}})}{\cos\alpha_{\mathrm{LN2}}} - r_{\mathrm{vn}2} \tag{155}$$

Angle between centreline and line from point of load application and fillet point on wheel, $\alpha_{2\mathrm{oo}}$:

For start of iteration, assume $\alpha_{2\mathrm{oo}} = 2{,}0 \cdot \alpha_{\mathrm{Dnf}}$.

Relative horizontal distance from centreline to critical fillet point:

$$s_{N2} = x_1 - \rho_{va02} \cdot \cos\alpha_{2oo} \tag{156}$$

Relative vertical distance from pitch circle to critical fillet point:

$$y_2 = y_1 + \rho_{va02} \cdot \sin\alpha_{2oo} \tag{157}$$

Relative wheel load height at weakest section:

$$h_{N2} = y_2 + \Delta r_{LN2} \tag{158}$$

Auxiliary value:

$$h_{N2o} = \frac{s_{N2}}{2{,}0 \cdot \tan\alpha_{2oo}} \tag{159}$$

Change α_{2oo} until $h_{N2} = h_{N2o}$ which is the end of iteration.

At this stage, the wheel tooth strength factor is calculated by:

$$x_{N2} = \frac{s_{N2}^2}{h_{N2}} \tag{160}$$

For the pinion angle from pitch point to point of load application, θ_{D5}:

Formula (161) shall be solved for θ_{D5}.

$$\Delta r_4 = r_{vn1} \cdot \left(1{,}0 - e^{\theta_{D5} \cdot \tan\alpha_f}\right) \tag{161}$$

Pinion pressure angle at point of load application:

$$\alpha_{LN1} = \alpha_{vnf} + \theta_{D4} - \theta_{D5} + \Delta\theta_1 \tag{162}$$

Relative pinion radial distance to point of load application:

$$r_{41o} = \frac{(r_{vn1} - \Delta r_4) \cdot \cos(\alpha_{Dnf} + \theta_{D4})}{\cos\alpha_{LN1}} \tag{163}$$

Start of iteration comprising Formulae (164) to (177):

$\alpha_{Do} = \alpha_{nD}$ should be used as initial value.

Wheel angle between centreline and pinion fillet, θ_{D2oo}:

For an enclosed iteration, assume $\theta_{D2oo} = 1/2 \cdot \theta_{v2}$.

$$\Delta r_5 = r_{vn2} \cdot e^{\theta_{D2oo} \cdot \tan\alpha_f} \tag{164}$$

$$\mu_{D2} = \frac{r_{vn2} + h_{vfm1} - \rho_{va01} - \Delta r_5 \cdot \cos\theta_{D2oo}}{\tan\alpha_{Do}} \tag{165}$$

$$\mu_D = \Delta r_5 \cdot \sin\theta_{D2oo} \tag{166}$$

Change θ_{D2oo} until $\mu_D = \mu_{D1} + \mu_{D2}$ which is the end of the enclosed iteration.

For the pinion angle between centreline and pinion fillet solve Formula (167) for θ_{D1oo}:

$$r_{\mathrm{vn2}} \cdot \left(e^{\theta_{\mathrm{D2oo}} \cdot \tan\alpha_{\mathrm{f}}} - 1{,}0\right) = r_{\mathrm{vn1}} \cdot \left(1{,}0 - e^{\theta_{\mathrm{D1oo}} \cdot \tan\alpha_{\mathrm{f}}}\right) \tag{167}$$

Wheel rotation through path of action:

$$\tan\theta_{\mathrm{L2o}} = \frac{\mu_{\mathrm{D1}} - \rho_{\mathrm{va01}} \cdot \cos\alpha_{\mathrm{Do}}}{r_{\mathrm{vn2}} + h_{\mathrm{vfm1}} - \rho_{\mathrm{va01}} + \rho_{\mathrm{va01}} \cdot \sin\alpha_{\mathrm{Do}}} \tag{168}$$

Wheel angle difference between path of action and tooth surface at pinion fillet:

$$\Delta\theta_{\mathrm{D2oo}} = \theta_{\mathrm{L2o}} - \theta_{\mathrm{D2oo}} \tag{169}$$

Relative wheel radius to pinion fillet point:

$$r_{\mathrm{L2o}} = \frac{r_{\mathrm{vn2}} + h_{\mathrm{vfm1}} - \rho_{\mathrm{va01}} + \rho_{\mathrm{va01}} \cdot \sin\alpha_{\mathrm{Do}}}{\cos\theta_{\mathrm{L2o}}} \tag{170}$$

Pinion angle to fillet point:

$$\tan\Delta\theta_{\mathrm{D1oo}} = -\frac{r_{\mathrm{L2o}} \cdot \sin\Delta\theta_{\mathrm{D2oo}}}{r_{\mathrm{vn2}} + r_{\mathrm{vn1}} - r_{\mathrm{L2o}} \cdot \cos\Delta\theta_{\mathrm{D2oo}}} \tag{171}$$

Relative pinion radius to fillet point:

$$r_{\mathrm{L1o}} = \frac{r_{\mathrm{vn2}} + r_{\mathrm{vn1}} - r_{\mathrm{L2o}} \cdot \cos\Delta\theta_{\mathrm{D2oo}}}{\cos\Delta\theta_{\mathrm{D1oo}}} \tag{172}$$

Pinion angle from centreline to fillet point:

$$(\Delta\theta_1 - \theta_{\mathrm{L1o}}) = \Delta\theta_1 - \theta_{\mathrm{D1oo}} - \Delta\theta_{\mathrm{D1oo}} \tag{173}$$

Angle between centreline and line from point of load application and fillet point on pinion, α_1

$$\alpha_1 = \alpha_{\mathrm{Do}} - \theta_{\mathrm{D2oo}} + \theta_{\mathrm{D1oo}} \tag{174}$$

Relative horizontal distance from centreline to critical fillet point:

$$s_{\mathrm{N1}} = r_{\mathrm{L1o}} \cdot \sin(\Delta\theta_1 - \theta_{\mathrm{L1o}}) \tag{175}$$

Relative pinion load height at weakest section:

$$h_{\mathrm{N1}} = r_{\mathrm{41o}} - r_{\mathrm{L1o}} \cdot \cos(\Delta\theta_1 - \theta_{\mathrm{L1o}}) \tag{176}$$

Auxiliary value:

$$h_{\mathrm{N1o}} = \frac{s_{\mathrm{N1}}}{2{,}0 \cdot \tan\alpha_1} \tag{177}$$

Change α_{Do} until $h_{\mathrm{N1}} = h_{\mathrm{N1o}}$

End of iteration.

At this stage, the pinion tooth strength factor is calculated by:

$$x_{\mathrm{N1}} = \frac{s_{\mathrm{N1}}^2}{h_{\mathrm{N1}}} \tag{178}$$