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TECHNICAL REPORT

Optical fibres – Reliability – Power law theory

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FOREWORD

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IEC 62048, which is a technical report, has been prepared by subcommittee 86A: Fibres and cables, of IEC technical committee 86: Fibre optics.

This second edition cancels and replaces the first edition published in 2002, and constitutes a technical revision. The main changes with respect to the previous edition are listed below:

– correction to the FIT equation in addition to all call-outs and derivations;
– insertion of a new section explaining how to numerically calculate bends and tension;
– editorial corrections of inconsistencies.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
86A/1357/DTR	86A/1375/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
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OPTICAL FIBRES –

Reliability – Power law theory

1 Scope

This technical report provides guidelines and formulae to estimate the reliability of fibre under a constant service stress. It is based on a power law for crack growth which is derived empirically, but there are other laws which have a more physical basis (for example, the exponential law). All these laws generally fit short-term experimental data well but lead to different long-term predictions. The power law has been selected as the most reasonable representation of fatigue behaviour by the experts of several standard-formulating bodies.

Reliability is expressed as an expected lifetime or as an expected failure rate. The results cannot be used for specifications or for the comparison of the quality of different fibres. This document develops the theory behind the experimental principles used in measuring the fibre parameters needed in the reliability formulae. Much of the theory is taken from the referenced literature and is presented here in a unified manner. The primary results are formulae for lifetime or for failure rate, given in terms of the measurable parameters. Conversely, an allowed maximum service stress or extreme value of another parameter may be calculated for an acceptable lifetime or failure rate.

For readers interested only in the final results of this technical report – a summary of the formulae used and numerical examples in the calculation of fibre reliability – Clauses 5 and 6 are sufficient and self-contained. Readers wanting a detailed background with algebraic derivations will find this in Clauses 7 to 12. An attempt is made to unify the approach and the notation to make it easier for the reader to follow the theory. Also, it should ensure that the notation is consistent in all test procedures. Clause 13 has a limited set of mostly theoretical references, but it is not necessary to read them to follow the analytical development in this technical report.

NOTE Clauses 7 to 11 reference the B-value, and this is done for theoretical completeness only. There are as yet no agreed methods for measuring B, so Clause 12 gives only a brief analytical outline of some proposed methods and furthermore develops theoretical results for the special case in which β can be neglected.

2 Symbols

Table 1 provides a list of symbols found in this document. Each symbol is first defined in the subclause or paragraph indicated in the final column of the table.

Table 1 – Symbols

Symbol	Unit	Name	Subclause or paragraph
a		Crack size (11.1)	7.1
A	µm	Flaw depth	7.1
a_f	µm	Radius of glass fibre	10.3
b	dimensionless	Bend designation	12.1.2
B	GPa^2 x s	Crack strength preservation parameter or B-value	7.1
B_0	GPa^2 x s	Transitional B-value at the slow-unloading/fast-unloading boundary	9.4
c	dimensionless	Non-linearity term for stress versus strain	7.4
C	dimensionless	Additive dimensionless prooftest term or C-value	10.6

Symbol	Unit	Name	Subclause or paragraph
C_a	dimensionless	Average additive dimensionless prooftest term or C-value	12.1.1
C_0	dimensionless	Transitional value of C at the slow-unloading/fast-unloading boundary	10.6
D	Mm	Fibre-axe separation in two-point bending	10.3.3
E	GPa	Young's modulus	7.4
E_0	GPa	Zero-stress Young's modulus	7.4
$f(h)$	dimensionless	Cumulative number of failures as a function of the number of hours h	11.4
F	dimensionless	Fibre failure probability	11.1
F_p	dimensionless	Fibre failure probability during prooftesting	12.1.2
h	dimensionless	proportionality constant	10.1.1
i	dimensionless	Rank order, sorted by increasing failure stress	5.3.2
I		Strength integral over the sample surface (assuming interior flaws are negligible)	10.2.1
$K_I(t)$	GPa x µm$^{1/2}$	Stress intensity factor	7.1
$K_{Ic}(t)$	GPa x µm$^{1/2}$	Critical stress intensity factor	7.1
L	km	Fibre effective length under uniform stress, or equivalent tensile length	10.2.1
L_b	km	Fibre length in uniform bend	10.3.2
L_p	km	Mean survival length, or survival length, during prooftesting	10.6
L_0	km	Gauge length, reference length	10.2.1
m	dimensionless	"Inert" Weibull parameter or m-value	10.2.1
m_d	dimensionless	m-value under dynamic fatigue	10.5
m_s	dimensionless	m-value under static fatigue	10.4
n	dimensionless	Stress corrosion susceptibility parameter or n-value	10.2
N	dimensionless	Total number of specimens tested	5.3.2
N_p	km^{-1}	Mean breakrate per unit length during prooftesting	10.6
$N(S)$	km^{-1}	Flaws per unit length not exceeding inert strength S	10.2.1
P	dimensionless	Fibre survival probability	10.2.1
P_i	dimensionless	Fibre survival probability of each strip	6.2.4
P_p	dimensionless	Fibre survival probability after prooftesting	10.6
R	m	Fibre bend radius	10.3.2
S	GPa	Strength	10.1.1
S_{min}	GPa	Minimum initial strength	10.5
$S(t)$	GPa	"Inert" strength of a crack	7.1
S_p	GPa	Strength after prooftesting	9.3
S_{pmin}	GPa	Minimum strength after prooftesting	9.4
S_u	GPa	Strength after unloading	9.2
S_{umin}	GPa	Minimum strength after unloading	9.3
S_0	GPa	Weibull gauge strength	10.2.1
t	s	Variable of time	7.1
$\hat{t}$	s	Critical survival time	9.3.2
t_d	s	Time to failure under dynamic fatigue, or prooftesting dwelltime	8.2.1, 9.2

Symbol	Unit	Name	Subclause or paragraph
t_f	s	Lifetime (time to failure) under constant stress or static fatigue testing	7.2, 8.1
t_{fp}	s	Lifetime after prooftesting	10.8
$t_{fp\text{min}}$	s	Minimum lifetime for certain survival after prooftesting	10.8
$t_f(1)$	dimensionless	Intercept on a static fatigue plot	8.1
t_l	ms	Prooftest loadtime	9.2
t_p	ms	Effective prooftime	9.3
t_u	ms	Prooftest unloadtime	9.2
t_y	years	Service time in years	6.1
$t0$	dimensionless	Static Weibull time-scaling parameter	10.4
V	µm/s	Crack growth velocity	7.1
V_C	µm/s	Critical crack growth velocity	7.1
w_i	dimensionless	Weibull cumulative probability ordinate scale	5.3.2
$wout_i$	dimensionless	Median Weibull cumulative probability ordinate scale	5.3.2
x	dimensionless	Factor relating bend length to equivalent tensile length	10.3.2
Y	dimensionless	Crack geometry shape parameter	7.1
z	dimensionless	Length factor	11.1
α	dimensionless	Ratio of prooftest unload parameters to crack parameters	9.4
β	$\text{GPa}^n\text{-s-km}^{(n-2)/m}$	Weibull β-value	10.4, 10.5
β_i	$\text{GPa}^n\text{-s-km}^{(n-2)/m}$	Weibull β-value for each failure stress	5.3.2
ε	dimensionless	Strain corresponding to a particular stress	7.4
λi	$\text{km}^{-1}\text{-yr.}^{-1}$	Breaks/length-time (instantaneous failure rate)	11.1
λa	$\text{km}^{-1}\text{-yr.}^{-1}$	Averaged failure rate	11.2
$\sigma(t)$	GPa	Stress applied to a crack	7.1
σ_a	GPa	Applied stress under static fatigue testing and lifetime	8.1, 11.2
$\dot{\sigma}_a$	GPa/s	Applied stress rate under dynamic fatigue testing	8.2.1
σ_b	GPa	Maximum bend stress	6.3.4
σf	GPa	Failure stress under dynamic fatigue testing, without prooftesting	8.2.1
σfp	GPa	Failure stress after prooftesting	10.8
σfp_{min}	GPa	Minimum failure stress after prooftesting	10.8
$\sigma f(1)$	dimensionless	Intercept on a dynamic fatigue plot	8.2.1
σp	GPa	Prooftest stress	9.2
σ_{max}	GPa	(Non-failing) maximum stress	5.3.2
σ_u	GPa	Applied stress during unloading	9.2
$\dot{\sigma}_u$	GPa/s	Positive unloading stress rate	9.2
$\sigma 0$	GPa	Dynamic Weibull stress-scaling parameter	10.5

3 General approach

First, the equivalence of the growth of an individual crack and its associated weakening is shown. This is related to applied stress or strain as an arbitrary function of time. Applied stress can be taken to fracture, from which the lifetime of the crack is calculated. Next, the destructive tests of static and dynamic fatigue are reviewed, along with their relationship to each other. These tests measure parameters useful in the theory. This also shows the difference between "inert" strength and "dynamic" strength.

The above single-crack theory is then extended to a statistical distribution of many cracks. This is done in terms of a survival (or failure) Weibull probability distribution in strength. It can allow for several deployment geometries in testing and service. The inert distribution and the distributions obtained by static or dynamic fatigue testing are derived for before and after prooftesting. The latter is sometimes done with approximations that may not require knowing the B-value explicitly. Finally, the various parameters measured by the above testing are related to formulae for fibre reliability, that is, lifetime and failure rate.

Some of the main assumptions in the development are as indicated below.

– The relationship between the stress intensity factor, applied stress and flaw size is given by Equation (29); while at fracture, the relationship between the critical stress intensity factor, strength, and flaw depth is given by Equation (30).
– The crack growth velocity is related to the stress intensity factor by Equation (32).
– The Weibull distribution of stress (before any prooftesting) is unimodal according to Equations (85) and (86), or bimodal according to Equation (91). The (m, S_0) pair appropriate to the desired survival probability level and length must be used. Deployment lengths will differ upon the application such as fibre on reels, in cable, splice trays, or within a connector or other component. Because of the low failure probabilities desired, however, the low-strength extrinsic mode must usually be used.
– The values of the fatigue parameters, both static and dynamic, depend upon the fibre environment, fibre ageing, and fibre preconditioning prior to testing. In theory, they are taken to be independent of time, so that some engineering judgement is needed to decide the practical values to be used in the calculations. This also implies that the corresponding static and dynamic parameters equal each other (for the same environment and time duration).
– Zero-stress ageing is not accounted for. Since the above parameters are independent of time, the strength decreases due only to stress fatigue following the power law according to 7.1.

4 Formula types

The formulae utilise parameters obtained from fatigue testing-to-failure, and from prooftesting with potential random failures. In the service condition of interest, a fibre of effective length L (dependent upon deployment geometry) is subjected to a constant applied service stress that does not change with time. (This stress is tensile, including bending stress. Torsional or compressive stresses are not covered.) The lifetime as a function of failure probability or failure rate as a function of time are given.

The formulae assume a Weibull distribution with parameters that vary among fibre types and perhaps among fibres of the same type. Moreover, they change with environment and applied stress levels. The Weibull distribution may have several nominally linear terms depending upon several levels of flaw strength. It is important that the Weibull parameters for the term of interest be used in the formulae. These are obtained from fatigue measurements. Generally, the low-strength region near the prooftest stress and below is of interest, and measurements must be on long fibre gauge lengths and with many samples, so that the total fibre length tested is large. Parameters measured for a small number of short samples, characterizing the high-strength region, will differ from the preceding ones. They must not be used in the formulae to extrapolate to lower-strength lower-probability regions.

Within the above power-law assumptions, the equations of Clauses 7 to 11 are algebraically "exact". However, in some applications, certain terms may be negligible, and more approximate and simpler algebraic equations are given in Clause 12. This has the advantage that the B-value, for which there is yet no standard test method and which has been reported to span several orders of magnitude, is not required.

Even with these formulae, there is no assured way of accurately predicting fibre reliability. Some fibres may break before the most conservative of predictions, while others may last longer than the most pessimistic of predictions. After fibre manufacture, fatigue or damage may occur due to cabling, installation, or operation; this usually cannot be accounted for in the theory. A start on estimating these effects could be made by measuring the parameters of fibres after each of these stages, but this is not commonly done.

For convenience in assisting the reader to find the derivations of equations, if desired, the formulae summarized in Clauses 5 and 6 include the indication in brackets of the equations listed in Clauses 7 to 12. However, it is not necessary to refer to the derivations to be able to follow Clauses 5 and 6.

5 Measuring parameters for fibre reliability

5.1 General

This clause outlines how the parameters in the reliability (lifetime and failure rate) equations are obtained in the approximation of the small B-value. Prooftest parameters are obtained from testing the full length of fibre to be deployed. By contrast, both static and dynamic fatigue procedures use many short-length test samples. These are used to obtain "linear" Weibull plots of the cumulative failure probability F scaled as $\ln\text{-}\ln\frac{1}{P}$ (where $P = 1 - F$ is the survival probability) versus the ln of a suitable variable (failure time or failure stress). For situations in which the plot may be fitted to two or more straight line parts, that part closest to the anticipated service stress should be used in obtaining the needed parameters.

5.2 Length and equivalent length

The testing and service geometries may differ from each other. The symbol L_0 is the gauge length in static or dynamic fatigue testing, whereas L is the in-service length subjected to constant applied service stress. The gauge length equals the actual length only for the case of longitudinal tension. Other geometries require equivalent lengths.

For uniform bending (for example, mandrel wrap), the in-service bend length L_b is replaced by an approximate equivalent in-service tensile length L given by Equation (97).

$$L \approx 0{,}4\frac{L_b}{\sqrt{x}} \quad (1)$$

The same relationship holds between the gauge bend length L_{b0} and the equivalent gauge length L_0. In this equation there is the factor Equation (98).

$$x = \frac{mn}{n-2} = m_s n = \frac{m_d n}{n+1} \quad (2)$$

using inert, static fatigue, and dynamic fatigue parameters, respectively, as obtained below.

For two-point bending, the equivalent length depends upon the applied stress in a complex way. Computation of the equivalent in-service length for an arbitrary applied service stress is difficult. The equivalent gauge length is approximately 10 μm to 30 μm, depending upon the failure stress.

5.3 Reliability parameters

5.3.1 General

This subclause outlines methods that are commonly used to derive reliability parameters.

5.3.2 Prooftesting

- Obtain the composite prooftest parameter $\sigma_p^n t_p$, where σ_p is the actual prooftest stress during dwell, and n is the stress-corrosion susceptibility parameter (or n-value). The effective prooftime is given by Equation (64).

$$t_p = t_d + \frac{t_l + t_u}{n+1} \qquad (3)$$

  obtained from the loadtime t_l, the dwelltime t_d, and the unloadtime t_u.

- (Optional) If from prooftesting the mean number of breaks N_p per length or the mean survival length L_p during prooftesting is known, calculate Equations (172) and (173).

$$\beta = \frac{\sigma_p^n t_p}{N_p^{\frac{n-2}{m}}} = \sigma_p^n t_p L_p^{\frac{n-2}{m}} \qquad (4)$$

where

$$\frac{m}{n-2} = m_s = \frac{m_d}{n+1} \qquad (5)$$

If this is not possible, obtain β as a fit parameter in 5.2.2, 5.2.3, or 5.3.

5.3.3 Static fatigue

- Obtain the static Weibull plot of scaled probability versus the natural log of failure times t_f for any particular constant applied stress σ_a (Equation (174)).

$$\ln \frac{1}{P_p(t_f)} = \left[\left(t_f \sigma_a^n + t_p \sigma_p^n \right)^{m_s} - \left(t_p \sigma_p^n \right)^{m_s} \right] \frac{L}{\beta^{m_s}} \qquad (6)$$

  Determine parameters m_s and β from the characteristics of the plot.

- Obtain the best-fit straight line to the log of failure times versus the log of applied stresses (Equation (48)).

$$\log t_f(\sigma_a) \approx \log t_f(1) - n \ \log \sigma_a \qquad (7)$$

Measure the static stress-corrosion susceptibility parameter as the negative slope $-n$ of this line. The term $t_f(1)$ is the "intercept" of this line on the ordinate axis, that is, the value of failure time where the applied stress is unity. (This value will depend on the units used, and may require a straight-line extrapolation beyond the data points. It does not have the dimension of time.)

5.3.4 Dynamic fatigue

IEC 60793-1-31[1] describes how to measure both short-length and long-length strength distributions of optical fibres.

- Obtain the dynamic Weibull plot of scaled probability versus natural log of failure stresses σ_f for any particular constant applied stress rate $\dot{\sigma}_a$ (Equation (175)).

$$\ln \frac{1}{P_p(\sigma_f)} = \left\{ \left[\frac{\sigma_f^{n+1}}{(n+1)\dot{\sigma}_a} + \sigma_p^n \, t_p \right]^{\frac{m_d}{n+1}} - \left(\sigma_p^n \, t_p \right)^{\frac{m_d}{n+1}} \right\} \frac{L}{\beta^{\frac{m_d}{n+1}}} \tag{8}$$

  Determine parameters m_d and β from the characteristics of the plot.

- Obtain the best-fit straight line to the log of failure stresses versus the log of applied stress rates (Equation (53)).

$$\log \sigma_f(\dot{\sigma}_a) \approx \log \sigma_f(1) + \frac{\log \dot{\sigma}_a}{n+1} \tag{9}$$

Measure the dynamic stress-corrosion susceptibility parameter from the slope $\frac{1}{n+1}$ of this line.

The term $\sigma_f(1)$ is the "intercept" of this line on the ordinate axis, that is, the value of failure stress where the applied stress rate is unity. (This value will depend on the units used, and may require a straight-line extrapolation beyond the data points. It does not have the dimension of stress.)

5.4 Parameters for the low-strength region

5.4.1 General

This subclause describes the way to measure the strength distribution at sufficiently low probability to represent the distribution of failure strengths near the prooftest stress level for the second mode of the Weibull distribution (shown as the extrinsic region in Figure 14). Normally, the fibre population has been prooftested once according to Clause 9.

NOTE These implementations are used only for characterization and not for specification.

5.4.2 Variable prooftest stress

This method (briefly mentioned in 9.5) subjects a full length of fibre to a certain prooftest stress, another length to a higher prooftest stress, and so on for several increasing levels of prooftest stress. The mean survival length L_p (or number of breaks N_p per unit length) is counted for each length and stress level. This resembles a static fatigue test in which the failure stress (the prooftest stress σ_p) varies. However, the failure time does not exceed the fixed prooftime t_p. The n-values are obtained by the fatigue measurements of 5.3.

First, consider the case in which there is no initial prooftest at manufacture. From Equations (171) and (173) one has

[1] IEC 60793-1-31:2001, *Optical fibres – Part 1-31: Measurement methods and test procedures – Tensile strength.*

$$\ln L_p + m_s(n \ln \sigma_p + \ln t_p - \ln \beta) = 0 \quad (10)$$

so a logarithmic plot of mean survival length versus prooftest stress should be close to a straight line. The slope is $-nm_s$, while the stress and length "intercepts" are $\frac{1}{n}(\ln \beta - \ln t_p)$ and $m_s(\ln t_p - \ln \beta)$, respectively.

In Reference [11][2], fibres with a 400 µm jacket and initial lengths of 10 km to 15 km were used, with five proofstrains of 0,8 % to 3,5 %. There was no other initial prooftest. Equation (10) is equivalent to Equations (18) to (20) of Reference [11] with $C = \beta^{-m_s}$. With a duration time t_d of 1 s, it was found that nm_s = 2,07, so that with n = 20, one has m_s = 0,1035. Also, $m_s \ln t_p + \ln C$ = –2,09, so that $\beta^{m_s} = 8{,}085$ GPa^{nm_s} x s^{m_s} x km.

More common is the case in which there is an initial prooftest at manufacture. If the second prooftest stress is significantly above the first, then Equation (10) can still be used.

In Reference [12], the prooftest stress level at manufacture was not stated. A minimum sample length of 10 km or 20 km was used, and each sample was subjected to a different one of five prooftest stress levels between 1 GPa and 4 GPa. The prooftest speed was reduced to minimize breakage during the start-up acceleration period, so the duration time t_d was normalized to 1 s using n = 23. The failure probabilities F per meter were calculated for each stress and plotted to fit the straight line of the form

$$\ln\left(\frac{1}{1-F}\right) = M \ln \sigma_p + \ln K \quad (11)$$

With another "ln" on the left (apparently missing), this is equivalent to Equation (101) for static fatigue (ignoring the initial prooftesting) if

$$M \equiv nm_s \text{ and } K \equiv \left(\frac{t_d}{\beta}\right)^{m_s} \quad (12)$$

From this it was determined that M = 1,69, so we find m_s = 0,0735, and K = 0,000418, so that $\beta^{m_s} = 2{,}392$ GPa^{nm_s} x s^{m_s} x km.

5.4.3 Dynamic fatigue

This is a form of dynamic testing with censoring, as mentioned in 8.2.2, and with more details on the apparatus given in Reference [5].

A specimen is a single gauge length L_0 of fibre. (A recommended gauge length L_0 is longer than 1 m; for example, 10 m to 20 m.) A sample is a group of specimens from a given population of fibres.

Each specimen is loaded to a failure stress σ_f, or, with censoring, to a (non-failing) maximum stress σ_{max} (for example, 2,4 GPa, about 3,2 % strain from Equation (44)). The recommended strain rate $\dot{\sigma}_a$ is fast (for example, greater than 200 %min, about 2,6 GPa/s, from Equation (43)). The sample size should be large enough to provide an adequate representation of the second Weibull mode (for example, so that 1 km of the total specimens fail).

2 Numbers in square brackets refer to the Bibliography.

The following data are recorded

– the total number of specimens tested: N, whether or not failure occurred;
– the failure stress values of those specimens that failed: σ_{fi} in GPa. Here i is the rank order, sorted by increasing failure stress.
– the stress rate (converted from strain rate): $\dot{\sigma}_a$ in GPa/s.
– the gauge length of the specimens: L_0 in km.

A Weibull plot in the form of Figure 1 may also be presented (without the curve fits). The points are measurements from about 0,8 GPa to 2,4 GPa, for an acrylate-coated fused silica fibre with a cladding diameter of 125 µm.

Calculation of Weibull parameters

Here the data of the measurement is analysed. According to Equation (86), the Weibull cumulative probability ordinate scale is of the form

$$\ln\left[\ln\left(\frac{1}{1-F}\right)\right],$$

where F = 1 – P is the cumulative failure probability.

Hence compute

$$w_i = \ln\left[-\ln\left(1-\frac{i}{N+1}\right)\right] \tag{13}$$

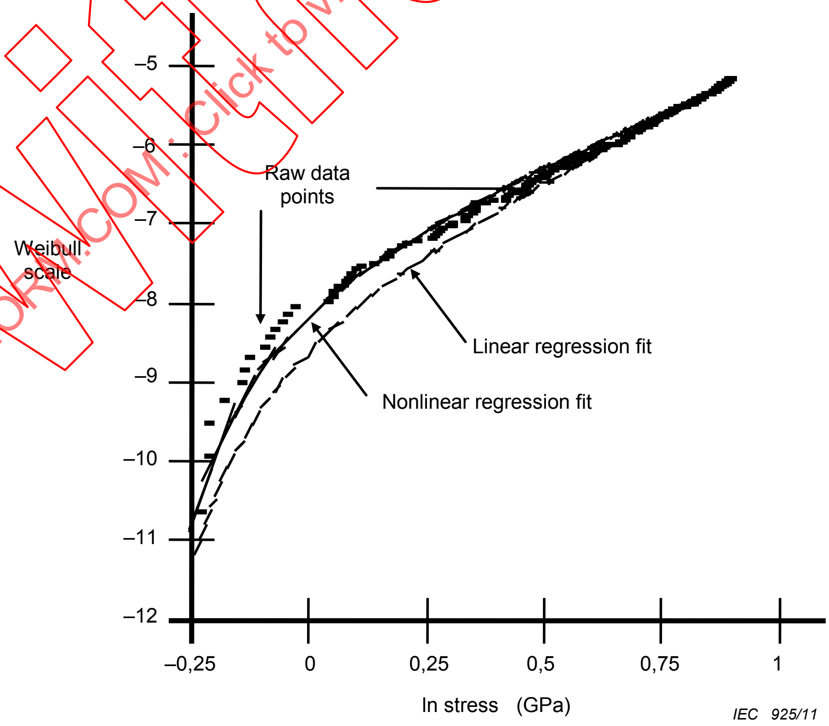

Figure 1 – Weibull dynamic fatigue plot near the prooftest stress level

Exponentiate this and use a rearranged form of Equation (138).

$$\exp w_i = \frac{\sigma_{fi}^{m_d} L_0}{\left[\dot{\sigma}_a (n+1)\beta\right]^{\frac{m_d}{n+1}}} \times \left\{ \left[1 + \frac{\dot{\sigma}_a (n+1)\sigma_p^n t_p}{\sigma_{fi}^{n+1}}\right]^{\frac{m_d}{n+1}} - \left[\frac{\dot{\sigma}_a (n+1)\sigma_p^n t_p}{\sigma_{fi}^{n+1}}\right]^{\frac{m_d}{n+1}} \right\} \tag{14}$$

Here the length L becomes the testing gauge length L_0.

Two fits are used on this equation.

a) Linear regression fit

For large failure stresses, the term in Equation (14) that is enclosed in curled brackets { } approaches one. The equation approaches the "linear" form of the usual Equations (108) and (110) without prooftesting.

$$w_i = A + m_d \ln \sigma_{fi} \tag{15}$$

where

$$\ln \beta = \frac{n+1}{m_d}(\ln L_0 - A) - \ln\left[(n+1)\dot{\sigma}_a\right] \tag{16}$$

Hence for failure stress greater than some selected value, find m_d and A such that the least squares error of w_i is minimized.

This procedure produces the linear regression fit in Figure 1. Clearly, it would be better to have a closer fit at the left side of this data set, where the values are closer to the applications of interest.

b) Non-linear regression fit

This uses lower-stress data, where the curvature is apparent. Given a value of m_d, a value of β is computed for each failure stress, and a value from the middle of the data of interest produces a curve that goes through that area. The result produces a sum of squared errors for the entire curve, and m_d is varied to minimize the sum. The steps are indicated below.

- Define a range of data in which the fit is to be forced. Here it is constructed using ln σ_{fi} values between 0 and 0,5.
- Select a value of m_d.
- Compute ln β_i for each failure stress in the defined region using Equation (14).
- Set ln β equal to the median of the computed ln β_i values.
- Compute $wout_i$ using Equation (14) and the above value of β.
- Compute the squared errors $(w_i - wout_i)^2$, and then compute the sum of squared errors.
- Vary m_d as in the second step, and repeat the remaining steps to minimize the sum.

The procedure produces the non-linear regression fit in Figure 1.

5.5 Measured numerical values

In this subclause, experimental values resulting from the measurements of 5.4.3 are obtained. They will be used in the calculations of Clause 6.

In 5.3.2, the composite prooftest parameter is $\sigma_p^n t_p = 8{,}8849 \times 10^{-5}$ GPa^n x s, with a nominal proof stress of σ_p = 0,69 GPa (ln σ_p = –0,37 in Figure 1). From 5.2.3 n = 20 is obtained, and in 5.3.3 the stress rate is $\dot{\sigma}_a$ = 4,59477 GPa/s for a gauge length L_0 = 20 m. The non-linear fit gives the values m_d = 2,359 and ln β = 25,499. Note that β has the unit $\text{GPa}^n \times \text{s} \times \text{km}^{\frac{n+1}{m_d}}$.

The static value will also be needed in the following subclause (Equation (114))

$$m_s = \frac{m_d}{n+1} \tag{17}$$

which according to the above subclause equals 0,11233. Note too that β now has the unit GPa^n x s x $\text{km}^{\frac{1}{m_s}}$ and that β^{m_s} = 17,538 GPa^{nm_s} x s^{m_s} x km.

The two values are in reasonable agreement with those of 5.4.2.

6 Examples of numerical calculations

6.1 General

The numerical values experimentally obtained in 5.5 are used in the calculations below. The results will be conservative since the explicit B-value is neglected; lower failure rates and longer lifetimes would be obtained if this value were included. (The degree of improvement increases as B increases.) The results of the calculations can be quite sensitive to the choice of parameter values. These values are related to each other, and a change in one parameter will affect the values of other parameters. Ideally, the parameter values should be obtained on the basis of experimentation.

If the failure probability $F = 1 - P$ is 10^{-3} or less (generally the region of practical interest), the term $\ln\frac{1}{P}$ in the formulae below may be replaced by F to an accuracy of 0,5 % or better, but this has not been done in the numerical results given here. Set Equation (160)

$$t_f(\text{sec}) = 31\,577\,600\ t_y(\text{yr}) \tag{18}$$

Finally, the notation "5,45 10-10" means "5,45 $\times 10^{-10}$".

6.2 Failure rate calculations

6.2.1 General

Compute the failure rates λ that would occur at various static stress levels σ_a as a function of service time t_y in years.

6.2.2 FIT rate formulae

The instantaneous failure rate is [Equation (178)]

$$\lambda_i\left(t_f\right) = 3{,}6 \times 10^{12}\, m_s \left(\frac{\sigma_a}{\beta}\right)^{m_s} \left[t_f + t_p\left(\frac{\sigma_p}{\sigma_a}\right)^n\right]^{m_s - 1} \tag{19}$$

The averaged failure rate is [Equation (178)]

$$\lambda_a(t_f) = \frac{3{,}6 \times 10^{12}}{t_f} \left\langle 1 - \exp\left\{ \left[\left(t_p \sigma_p^n\right)^{m_s} - \left(t_f \sigma_a^n + t_p \sigma_p^n\right)^{m_s} \right] \frac{L}{\beta^{m_s}} \right\} \right\rangle \quad (20)$$

6.2.3 Long lengths in tension

Use several applied stresses, as a fraction of the nominal prooftest stress, in the above equations for a fibre length L = 1 km. As a function of time out to 50 years, these give the instantaneous and averaged failure rate plots of Figures 2 and 3, respectively. The corresponding values are given in Table 2. (Zero time is avoided because of the singularity shown there of the averaged failure rate.)

Two points are worth noting. First, all the failure rates are almost constant over time, especially at the lower stresses. This indicates an almost linear increase in total cumulative failures with time. Secondly, and because of this, there is little practical difference in the values between the instantaneous and averaged failure rates.

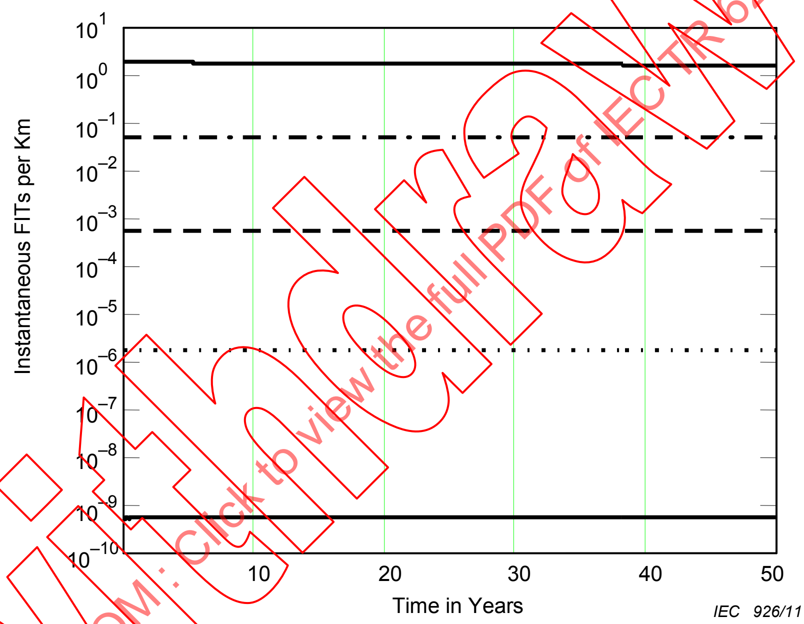

Figure 2 – Instantaneous FIT rates per fibre km versus time for applied stress/prooftest stress percentages (bottom to top): 10 %, 15 %, 20 %, 25 %, 30 %

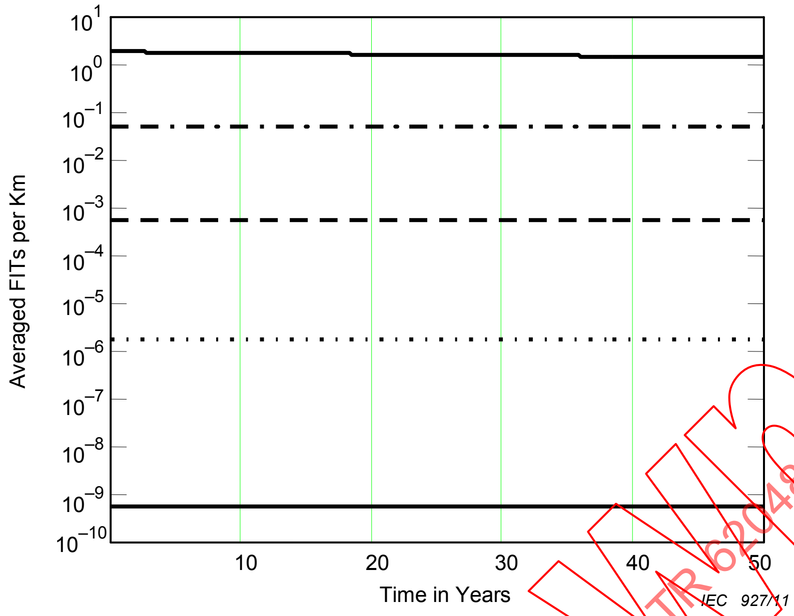

Figure 3 – Averaged FIT rates per fibre km versus time for applied stress/prooftest stress percentages (bottom to top): 10 %, 15 %, 20 %, 25 %, 30 %

Table 2 – FIT rates of Figures 2 and 3 at various times

Applied stress as a % of prooftest stress	Instantaneous FIT/km						Averaged FIT/km					
	1 year	10 years	20 years	30 years	40 years	50 years	1 year	10 years	20 years	30 years	40 years	50 years
10	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}	5,45 10^{-10}
15	1,81 10^{-10}	1,81 10^{-10}	1,81 10^{-10}	1,81 10^{-10}	1,81 10^{-6}	1,81 10^{-6}	1,81 10^{-6}	1,81 10^{-6}	1,81 10^{-6}	1,81 10^{-6}	1,81 10^{-6}	1,81 10^{-6}
20	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}	5,71 10^{-4}
25	4,95 10^{-2}	4,95 10^{-2}	4,95 10^{-2}	4,93 10^{-2}	4,92 10^{-2}	4,91 10^{-2}	4,95 10^{-2}	4,95 10^{-2}	4,94 10^{-2}	4,94 10^{-2}	4,94 10^{-2}	4,93 10^{-2}
30	1,89	1,78	1,68	1,59	1,51	1,44	1,89	1,84	1,78	1,73	1,69	1,64

6.2.4 Short lengths in uniform bending

From Equations (97) and (98), reduce the actual bend length L_B by the bend-length factor

$$\frac{L}{L_B} = \frac{0{,}4}{\sqrt{nm_s}} \qquad (21)$$

which equals 0,266 86 here. From the non-linear Equation (94), the maximum applied stress at the outside fibre surface under bend is

$$\sigma_a(D) = \frac{70{,}33}{8D}\left(1 + \frac{9}{32D}\right) \qquad (22)$$

where D is the bend diameter in millimetres. Use several bend diameters in the equations of 6.2.2 for a bent fibre length of 1 m. As a function of time out to 50 years, these give the instantaneous and averaged failure rate plots of Figures 4 and 5, respectively. The corresponding values are given in Table 3.

The "% of prooftest stress" number is the percentage that the maximum stress (at the outside of the bend) is of the 0,69 GPa prooftest stress. Compared with the tensile case of the previous subclause, the constancy of the failure rate with time no longer holds at these higher stresses, especially at shorter times. Also, more significant differences between the instantaneous and averaged values appear, especially at shorter times and tighter bends.

Finally, Table 4 gives results when the non-linear term is not included in Equation (22). As expected, the failure rates are smaller, since the applied stress is slightly underestimated. Moreover, the percentage deviation error from the more correct values of Table 3 increases as the bend diameter decreases and stress increases.

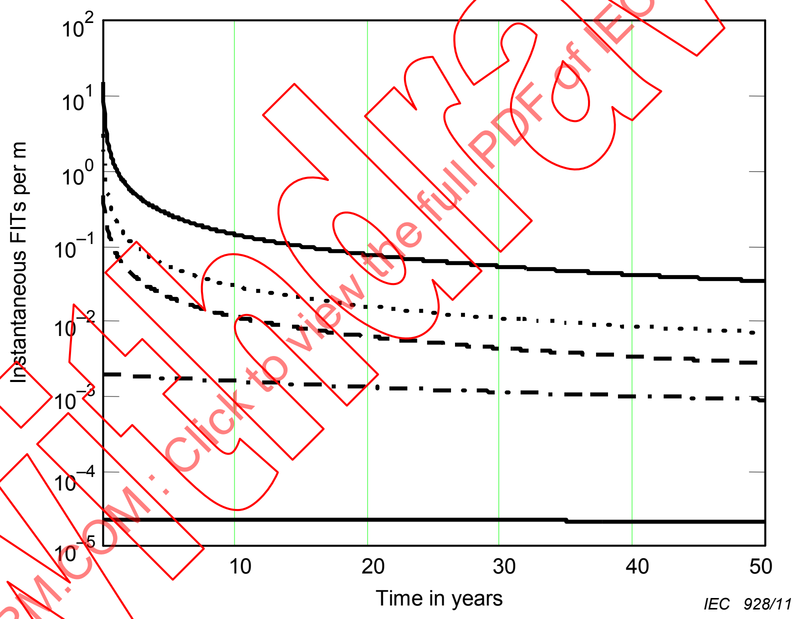

Figure 4 – Instantaneous FIT rates per bent fibre metre versus time (top to bottom): 10 mm, 20 mm, 30 mm, 40 mm, 50 mm

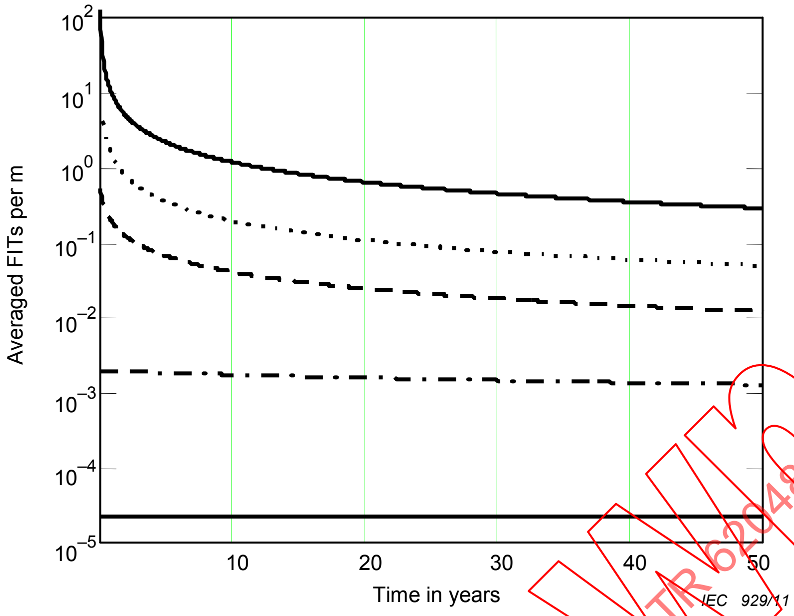

Figure 5 – Averaged FIT rates per bent fibre metre versus time for bend diameters (top to bottom): 10 mm, 20 mm, 30 mm, 40 mm, 50 mm

Table 3 – FIT rates of Figures 4 and 5 at various times

Bend diameter (mm), % of prooftest stress	Instantaneous FIT/m in bend						Averaged FIT/m in bend					
	1 year	10 years	20 years	30 years	40 years	50 years	1 year	10 years	20 years	30 years	40 years	50 years
10, 131,0	1,08	1,40 10^{-1}	7,57 10^{-2}	5,28 10^{-2}	4,09 10^{-2}	3,35 10^{-2}	9,01	1,19 10^{-1}	6,43 10^{-1}	4,50 10^{-1}	3,49 10^{-1}	2,86 10^{-1}
20, 64,6	2,21 10^{-1}	2,86 10^{-2}	1,55 10^{-2}	1,08 10^{-2}	8,35 10^{-3}	6,85 10^{-3}	1,36	1,94 10^{-1}	1,07 10^{-1}	7,57 10^{-2}	5,92 10^{-2}	4,88 10^{-2}
30, 42,9	8,03 10^{-2}	1,13 10^{-2}	6,12 10^{-3}	4,28 10^{-3}	3,33 10^{-3}	2,72 10^{-3}	1,83 10^{-1}	4,06 10^{-2}	2,44 10^{-2}	1,79 10^{-2}	1,44 10^{-2}	1,21 10^{-2}
40, 32,1	1,88 10^{-3}	1,55 10^{-3}	1,30 10^{-3}	1,12 10^{-3}	9,87 10^{-4}	8,84 10^{-4}	1,91 10^{-3}	1,73 10^{-3}	1,57 10^{-3}	1,45 10^{-3}	1,35 10^{-3}	1,27 10^{-3}
50, 25,6	2,17 10^{-5}	2,16 10^{-5}	2,15 10^{-5}	2,15 10^{-5}	2,14 10^{-5}	2,14 10^{-5}	2,17 10^{-5}	2,16 10^{-5}	2,16 10^{-5}	2,16 10^{-5}	2,15 10^{-5}	2,15 10^{-5}

Table 4 – FIT rates of Table 3 neglecting stress versus strain non-linearity

Bend diameter (mm), % of prooftest stress	Instantaneous FIT/m in bend						Averaged FIT/m in bend					
	1 year	10 years	20 years	30 years	40 years	50 years	1 year	10 years	20 years	30 years	40 years	50 years
10, 127,4	1,02	1,32 10^{-1}	7,11 10^{-2}	4,96 10^{-2}	3,84 10^{-2}	3,15 10^{-2}	8,43	1,11 10^{-1}	6,02 10^{-1}	4,21 10^{-1}	3,27 10^{-1}	2,68 10^{-1}
20, 63,7	2,14 10^{-1}	2,77 10^{-2}	1,50 10^{-2}	1,03 10^{-2}	8,10 10^{-3}	6,64 10^{-3}	1,30	1,86 10^{-1}	1,03 10^{-1}	7,28 10^{-2}	5,69 10^{-2}	4,70 10^{-2}
30, 42,5	7,73 10^{-2}	1,10 10^{-2}	5,99 10^{-3}	4,19 10^{-3}	3,25 10^{-3}	2,66 10^{-3}	1,68 10^{-1}	3,85 10^{-2}	2,32 10^{-2}	1,71 10^{-2}	1,38 10^{-2}	1,16 10^{-2}
40, 31,9	1,64 10^{-3}	1,38 10^{-3}	1,18 10^{-3}	1,03 10^{-3}	9,15 10^{-4}	8,25 10^{-4}	1,66 10^{-3}	1,52 10^{-3}	1,40 10^{-3}	1,30 10^{-3}	1,22 10^{-3}	1,15 10^{-3}
50, 25,5	1,94 10^{-5}	1,93 10^{-5}	1,93 10^{-5}	1,92 10^{-5}	1,92 10^{-5}	1,91 10^{-5}	1,94 10^{-5}	1,93 10^{-5}	1,93 10^{-5}	1,93 10^{-5}	1,93 10^{-5}	1,92 10^{-5}

6.3 Lifetime calculations

6.3.1 General

Compute the lifetimes that would occur at various static stress levels σ_a as a function of failure probability F.

6.3.2 Lifetime formulae

The lifetime is Equation (177):

$$t_f = \left\{ \left[\frac{\beta^{m_s}}{L} \ln \frac{1}{P} + \left(\sigma_p^n t_p \right)^{m_s} \right]^{\frac{1}{m_s}} - \sigma_p^n t_p \right\} \sigma_a^{-n} \qquad (23)$$

6.3.3 Long lengths in tension

Use several applied stresses, as a fraction of the nominal prooftest stress σ_p = 0,69 GPa, in the above equation for a fibre length L = 1 km. As a function of failure probability, this gives the lifetime plots of Figure 6. The corresponding values are given in Table 5.

For this example, it appears that an applied stress that is 30 % of the prooftest stress is unacceptable at any failure probability level, whereas the 10 % and 15 % values are always acceptable.

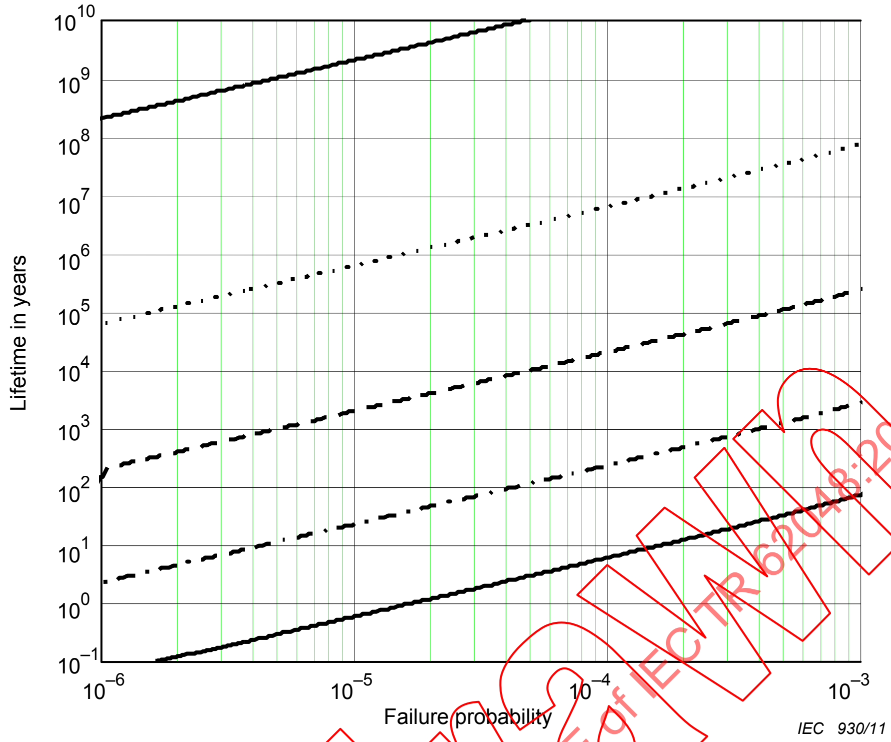

Figure 6 – 1-km lifetime versus failure probability for applied stress/prooftest stress percentages (top to bottom): 10 %, 15 %, 20 %, 25 %, 30 %

6.3.4 Short lengths in uniform bending

Calculating failure probability for bent fibre can be accomplished using the length under bend. Use Equation (21) for the bend-length factor and Equation (22) for the maximum applied stress at the outside fibre surface under bend described in this section or using the numerical approach described in the bend with tension described in the next section. Both methods produce similar results for large diameter bends (greater than 15 mm). For the length under bend estimation use several bend diameters in Equation (177) for a bent fibre length of 1 m. As a function of failure probability, this gives the lifetime plots of Figure 7. The corresponding values are given in Table 6.

Compared with the long length in tension of the previous part, at these higher stresses, there is a greater and non-linear variation in the lifetime with failure probability. It appears that the 30 mm, 40 mm, and 50 mm bends are acceptable at almost all the failure probabilities.

Finally, Table 7 gives results when the non-linear term is not included in Equation (22). As expected, the lifetimes are larger, since the applied stress is slightly underestimated. Moreover, the percentage deviation error from the more correct values of Table 6 increases as the bend diameter decreases and stress increases.

Table 5 – One kilometer lifetimes of Figure 6 for various failure probabilities

Applied stress as a % of prooftest stress	Failure probability			
	10^{+3}	10^{+4}	10^{+5}	10^{+6}
10	2,56 10^{11}	2,14 10^{10}	2,10 10^{+9}	2,10 10^{+8}
15	7,70 10^{+7}	6,43 10^{+6}	6,31 10^{+5}	6,30 10^{+4}
20	2,44 10^{+5}	2,04 10^{+4}	2,00 10^{+3}	2,00 10^{+2}
25	2,82 10^{+3}	2,35 10^{+2}	2,31 10	2,30
30	7,35 10	6,13	6,02 10^{-1}	6,01 10^{-2}

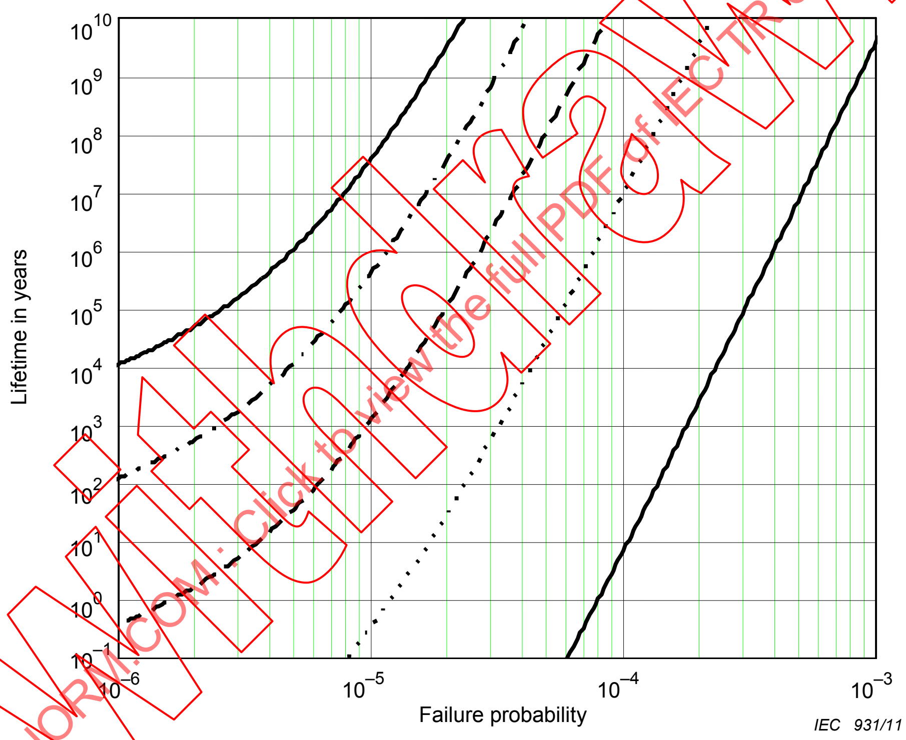

Figure 7 – Lifetimes per bent fibre metre versus failure probability for bend diameters (bottom-right to top-left): 10 mm, 20 mm, 30 mm, 40 mm, 50 mm

Table 6 – One-meter lifetimes of Figure 7 for various failure probabilities

Bend diameter mm	% of prooftest stress	Failure probability			
		10^{-3}	10^{-4}	10^{-5}	10^{-6}
10	131,0	3,83 10^{+9}	7,23	2,57 10^{-7}	7,69 10^{-11}
20	64,6	5,28 10^{15}	9,99 10^{+6}	3,54 10^{-1}	1,06 10^{-4}
30	42,9	1,93 10^{19}	3,64 10^{10}	1,29 10^{+3}	3,87 10^{-1}
40	32,1	6,37 10^{+2}1	1,20 10^{3}	4,27 10^{+5}	1,28 10^{+2}
50	25,6	5,68 10^{+23}	1,07 10^{15}	3,81 10^{+7}	1,14 10^{+4}

Table 7 – Lifetimes of Table 6 neglecting stress versus strain non-linearity

Bend diameter mm	% of prooftest stress	Failure probability			
		10^{-3}	10^{-4}	10^{-5}	10^{-6}
10	127,4	6,66 10^{+9}	1,26 10	4,47 10^{-7}	1,34 10^{-1}0
20	63,7	6,99 10^{15}	1,32 10^{+7}	4,69 10^{-1}	1,40 10^{-4}
30	42,5	2,32 10^{19}	4,39 10^{10}	1,56 10^{+3}	4,67 10^{-1}
40	31,9	7,33 10^{+22}	1,38 10^{13}	4,91 10^{+5}	1,47 10^{+2}
50	25,5	6,35 10^{+23}	1,20 10^{15}	4,26 10^{+7}	1,28 10^{+4}

6.3.5 Short lengths with uniform bending and tension

Optical cables are traditionally designed to separate bending forces from axial tensions. This assumption is not valid for drop cables used in building applications as described in Reference [22]. These small diameter low fibre count cables may be routed through existing construction with practices similar to copper and may be subject to bends and tension simultaneously. Under these demanding conditions, the strain from all sources should be taken into account to accurately predict mechanical lifetime at the bend.

The resulting failure probability when bends and tension are present can be calculated using the strip calculation. The difference in the calculation is that instead of using Equation 21 for effective length, the calculation is done for a number of surface strips, each of which has a different static stress. The probability of surviving the load period is calculated for each strip. The sum of these probabilities is the probability that all strips will survive. One minus probability of overall survival yields probability of failure and thus average failure rate.

Equation 21 is derived from assumptions that include a uniform Weibull slope and which do not include the proof test. Doing the strip calculation allows taking the actual distribution into account. It also allows the introduction of tensile load to bending.

The strip calculation

Calculate the maximum bend stress, σ_b, as a function of bend diameter, D, as

$$\sigma_b(D) = \frac{70,33}{8D}\left(1+\frac{9}{32D}\right) \qquad (24)$$

Define the number of strips as n_s and designate the strips with index, i, ranging from 0 to n_{s1}. It has been found that i should be at least 20 to get reasonable results. Define $\theta = i2\pi/n_s$. A tensile load stress, σ_t, in addition to bend stress is also allowed. Calculate the stress for each strip, σ_i, as:

$$\sigma_i = \max\left[\sigma_b \cos(\theta_i) + \sigma_t, 0\right] \qquad (25)$$

Calculate the log of the probability of survival for each strip, $\ln(P_i)$, as:

$$\ln(P_i) = \left[\left(t_p \sigma_p^n\right)^{m_s} - \left(t_f \sigma_i^n + t_p \sigma_p^n\right)^{m_s}\right]\frac{L/n_s}{\beta^{m_s}} \qquad (26)$$

where L (km) is the length of fibre under bend and tension. Calculate the probability of failure, F, as:

$$F = 1 - \exp\left[\sum_i \ln(P_i)\right] \qquad (27)$$

Calculate average FIT/L as:

$$FIT/L = \frac{3,6\times 10^{12}}{t_f}F \qquad (28)$$

This calculation can be used with no tension and will provide reasonable agreement with the failure rates described in the previous section. The tensile load option allows one to calculate the failure rate for the combination of bend and tension. Table 8 shows the result when 30 % proof test is added to each radii for a 30 year case.

Table 8 – Bend plus 30 % of proof test tension for 30 years

Bend diameter	Failure prob/m	Average FIT/m	Average FIT/turn
10	1,87 10^{-4}	7,10 10^{-1}	2,23 10^{-2}
15	9,00 10^{-5}	3,42 10^{-1}	1,61 10^{-2}
20	5,53 10^{-5}	2,10 10^{-1}	1,32 10^{-2}
30	2,90 10^{-5}	1,10 10^{-1}	1,04 10^{-2}
40	1,88 10^{-5}	7,15 10^{-2}	8,99 10^{-2}

Except for the purpose of comparison, it would probably be better to calculate failure probability in a configuration similar to deployment such as one ¼ or ½ turn. For a 10mm diameter full turn, this works out to around 6 ppm.

7 Fibre weakening and failure

7.1 General

Individual cracks in the glass are considered first. Their statistical nature is treated in Clause 10.

7.2 Crack growth and weakening

The theory of fibre strength, as shown in Reference [1], follows the theory of brittle materials. It assumes that very small imperfections or cracks are distributed along the length of glass. For a silica-based fibre, the critical cracks are mostly at the surface where they are vulnerable to attack and weakening by moisture, dust, chemicals, etc. For silica-based optical fibres, polymer coatings around the glass or a hermetic film (for example, amorphous carbon film, plus coating) on the glass are intended to slow these effects.

The stress intensity factor at a crack tip is defined as a function of time t to be

$$K_I(t) = Y\sigma(t)a^{\frac{1}{2}}(t) \tag{29}$$

assumed to hold in any environment of interest.

Here

Y is a dimensionless crack geometry shape parameter (assumed to be constant),

σ is the positive applied stress, and

a is the flaw "depth", that is, the flaw size normal to the direction of applied stress. The distribution of crack sizes is statistical, as will be discussed in Clause 10.

In the ideal inert environmental condition of low temperature (for example, liquid N_2), or zero humidity, or high vacuum, cracks will grow only at the critical velocity and will then fracture. For a crack of constant size, the applied-stress intensity factor in Equation (29) varies proportionally to the time-varying stress applied to the crack. The factor has a maximum value K_{Ic}, called the "critical stress intensity factor or fracture toughness". This occurs when the applied stress increases to its allowable maximum given by the inert strength S of the crack (typically above 15 GPa for a short length of pristine fibre), so at this instant the time-varying Equation (29) becomes

$$K_{Ic} = YSa^{\frac{1}{2}} \tag{30}$$

At this point, catastrophic failure occurs. Because of the required environment, inert strength is difficult to measure, and this has led to non-unique values obtained experimentally. We typically will use the term "strength" rather than "inert strength" to mean the limiting value of failure stress that would be measured for "instant fracture" under static or dynamic fatigue. (This is described mathematically in 8.4.3.) The value may have a dependence on the environment in which this measurement would occur.

For the fibre in a non-inert or active (ambient or hostile) environment, such as at higher temperatures and with humidity, water, or chemical species, any applied stress will cause crack growth to occur. This is called stress corrosion, since hydrolysis of silica bonds occurs. The values of the shape parameter Y and the fracture toughness K_{Ic} are uncertain to at least 10 %, but are assumed constant for that type of crack over a wide range of environments.

Then Equation (30) establishes a one-to-one relationship between the increasing crack size in the active environment with the decreasing strength that could be approximately measured in an inert environment, or for instant fracture. Numerically, with assumed values of $Y \approx 1{,}24$ for an elliptical crack under tension and $K_{Ic} \approx 0{,}8$ MPa·m$^{1/2}$, Equation (30) is

$$a(\mu\text{m}) = 0{,}42[S(\text{GPa})]^{-2} \tag{31}$$

A strength of 0,7 GPa in the proof-stress region corresponds to a crack size of about 0,88 µm. In the high-strength region, 7 GPa corresponds to a crack size of only 8,8 nm. The latter is not much larger than the tetrahedral structural units of the glass, so the concept of a "crack" may be more useful as a model rather than being physically significant in this region.

In a non-inert environment, the rate of crack growth or crack growth velocity V is assumed to be related to the stress intensity factor by the empirical equation

$$\frac{da}{dt} = V(t) = AK_I^n(t) = V_c\left[\frac{K_I(t)}{K_{Ic}}\right]^n \tag{32}$$

This contains the critical crack growth velocity at the instant of failure

$$V_c = AK_{Ic}^n \tag{33}$$

Here, A is a material scaling parameter that depends on the environment through the critical crack growth velocity. For example, V_c is expected to increase as the partial pressure of water increases. The dimensionless exponent n is the crack's stress corrosion susceptibility parameter, or n-value for short. The power-law relationship of Equation (32) holds in the linear region I of the $\log a$ versus $\log K_I$ curve, where n is the slope of the line. This may have little physical basis, but is justified by the results it produces in approximating experimental results. Both n and A (which includes a dependence upon n) depend upon the particular environment.

In the treatment below, we will generally speak of strength rather than crack size, although they are interchangeable via Equations (30) and (31). To calculate how a crack weakens (loses strength) as stress is applied, one substitutes the stress intensity factor of Equation (33) into Equation (32) for crack growth and eliminates crack size via Equation (30). This gives the strength variation with applied stress

$$\frac{dS^{n-2}(t)}{dt} = -\frac{\sigma^n(t)}{B} \tag{34}$$

Here one defines the crack's strength preservation parameter or B-value (for short)

$$B^{-1} = \left(\frac{n}{2}-1\right)AY^2K_{Ic}^{n-2} = \left(\frac{n}{2}-1\right)V_c\left(\frac{Y}{K_{Ic}}\right)^2 \tag{35}$$

The parameter depends upon the environment through the earlier-defined parameters in this equation. It will not be necessary later in this document to individually know them all to be able to calculate B. However, to measure it is difficult (see 12.4), and quoted values may range from 10^{-8} to 1 $GPa^2 \cdot s$.

Equation (34) can be integrated so that if the fibre is subjected to a stress history $\sigma(t)$, a particular crack at an initial time 0 and of an initial strength $S(0)$ will weaken to a strength $S(t)$ given in

$$S^{n-2}(t) = S^{n-2}(0) - \frac{1}{B}\int_0^t \sigma^n(\bar{t})d\bar{t} \quad \text{with } \sigma(t) < S(t) \tag{36}$$

or

$$\frac{S(t)}{S(0)} = \left\{1 - \frac{S^2(0)}{B}\int_0^t \left[\frac{\sigma(\bar{t})}{S(0)}\right]^n d\bar{t}\right\}^{\frac{1}{n-2}} \tag{37}$$

The flaw does not break so long as its remaining strength exceeds the applied stress. The last term on the right side accounts for the dynamic characteristics of the applied stress and the fibre's response to it via the environmentally dependent parameters n and B. Note that in Equations (36) and (37) crack weakening (and growth) occurs so long as n exceeds 2, though common values may range from 15 to 30 for typical fibres to over a hundred for hermetic fibres. Moreover, a particular applied stress history will have a smaller weakening effect on a fibre having a higher value of B (in keeping with its name).

7.3 Crack fracture

In the previous subclause, the crack grew and weakened but did not break. With fracture, the critical failure condition of Equation (30) occurs just when the final strength in Equations (36) and (37) equals the applied stress at the instant of failure t_f, that is

$$t = t_f \text{ and } S(t_f) = \sigma(t_f) \tag{38}$$

(In another interpretation the crack size has grown to the critical value given by Equations (30) and (31).) Using this condition and the strength degradation Equations (36) and (37), one obtains the general lifetime equation

$$\sigma^{n-2}(t_f) = S^{n-2}(0) - \frac{1}{B}\int_0^{t_f} \sigma^n(t)dt \tag{39}$$

or

$$\frac{S(0)}{\sigma(t_f)} = \left\{1 + \frac{\sigma^2(t_f)}{B}\int_0^{t_f} \left[\frac{\sigma(t)}{\sigma(t_f)}\right]^n dt\right\}^{\frac{1}{n-2}} \tag{40}$$

The ratio under the integral sign becomes unity at fracture. Either form of this equation implicitly gives the lifetime t_f for any stress history $\sigma(t)$ in a given fixed environment.

Further simplification is useful for those cases in which the left side in Equation (39) is negligible, that is the fracture stress is somewhat less than the initial strength. For example, that fracture stress term is negligible (less than ~1 % of the other two terms) when

$$\sigma(t_f) \le 0{,}01^{\frac{1}{n-2}} S(0) \tag{41}$$

For the examples n = 15 and 30, this holds if the applied fracture stress is less than 70 % and 85 % of the initial strength, respectively. Hence if sufficient crack weakening (crack growth) has occurred, the general lifetime equation of Equation (39) is simply

$$\int_0^{t_f} \sigma^n(t)dt = BS^{n-2}(0) \tag{42}$$

An advantage of this form is that now B and $Sn^{-2}(0)$ do not need to be known separately but only in a product. (Also, it results in the linearized plots below of Equation (46) for static fatigue and Equation (47) for dynamic fatigue, so this approximation is often used in the literature.)

7.4 Features of the general results

- Crack weakening in Equations (36) and (37) and crack failure in Equations (39) and (40) depend on the magnitude of the applied stress and upon the duration of the stress application, as well as the crack parameters n, B.
- These general results can be applied to many different practical problems concerning fibre weakening or failure due to applied stress. Some of these problems will be treated below; they include prooftesting, which is intended to be non-destructive, the destructive tests of static fatigue and dynamic fatigue, and predicting service lifetime or failure rate.
- Equations (40) and (41) for crack weakening may be applied repeatedly, for example, to the prooftesting during manufacture, to destructive fatigue testing, to a dormant shipping period of almost zero stress, to the cabling process, and to field deployment. The strength decrease during each period (from beginning to end) may thus be calculated.
- All results assume that the strength decreases with time as a function of the applied stress alone. For a fixed environment, the fatigue parameters are assumed to remain unchanged with time. This is a limitation because they have been observed to change with temperature and humidity, and no mapping functions of these parameters versus environment have been agreed upon.
- Strength can change in harsh environments, even when no stress is applied; this is called zero-stress ageing. In the intrinsic region, the small-flaw strength often decreases; whereas in the prooftest region, the strength of the larger flaws sometimes increases. Experiments may sometimes mix the two phenomena, and, if the effects are not separated, the analysis of results can produce incorrect estimates of the fatigue parameters.
- For reliability estimates, careful engineering judgement is required concerning the load-time history and the environmental conditions the fibre experienced before installation, the strength distribution, the fatigue parameters, the ageing characteristics, and the stresses and environments expected in the field application.

7.5 Stress and strain

Instead of using stress, one may speak of the associated fractional increase in length, the strain ε. The two are related in References [2] and [3] by the quadratic relationship

$$\sigma(\varepsilon) = E_0(1+c\varepsilon)\varepsilon \tag{43}$$

Here E_0 is the zero-strain Young's modulus which has values of 70,3 GPa to 73,8 GPa. The non-linearity term c has ranged from 3 to 3,3 for axial strain; we will use the former here. Due to asymmetric stress distributions, the value changes in other geometries are discussed in Reference [4] and briefly in 10.3. In this technical report, stress will be used rather than strain. Inverting Equation (43) gives strain in terms of stress

$$\varepsilon(\sigma) = \frac{\left(E_0^2 + 4E_0 c\sigma\right)^{\frac{1}{2}} - E_0}{2E_0 c} \qquad (44)$$

A stress of 0,7 GPa, typically used as a prooftest stress, corresponds to a strain of 0,92 % to 0,97 %.

We assume a perfectly circular glass fibre with a diameter of 125 µm and ignore the coating contribution to load. Then 1 GPa stress is equivalent to a force of 12,272 N or a load of 1 251,4 gf.

8 Fatigue testing

8.1 General

We now examine the means by which the crack parameters mentioned in Clause 7 may be measured. One method is by fatigue testing applied destructively to numerous fibre specimens of a fixed gauge length. This allows for the measurement of n and the product BS^{n-2}. There are two types of fatigue testing, and the subscripts s and d refer to parameters measured under static and dynamic conditions, respectively.

8.2 Static fatigue

In static fatigue, a fibre specimen is subjected to a constant applied stress σ_a until the weakest crack of initial strength S breaks at an observed failure time t_f. This stress history is shown schematically in Figure 8. From Equations (39) and (40) the failure lifetime is

$$t_f(\sigma_a) = B_s \sigma_a^{-n_s}\left(S^{n_s-2} - \sigma_a^{n_s-2}\right) \qquad (45)$$

If the stress-to-strength ratio is sufficiently small, such as in Equation (41), or if the approximate lifetime in Equation (42) is used, then this has the reduced form

$$t_f(\sigma_a) \approx B_s S^{n_s-2} \sigma_a^{-n_s} = t_f(1)\sigma_a^{-n_s} \qquad (46)$$

This has the unit-stress "intercept" value

$$t_f(1) = B_s S^{n_s-2} \qquad (47)$$

which depends upon the units used. (Note that this intercept does not have the dimension of time.)

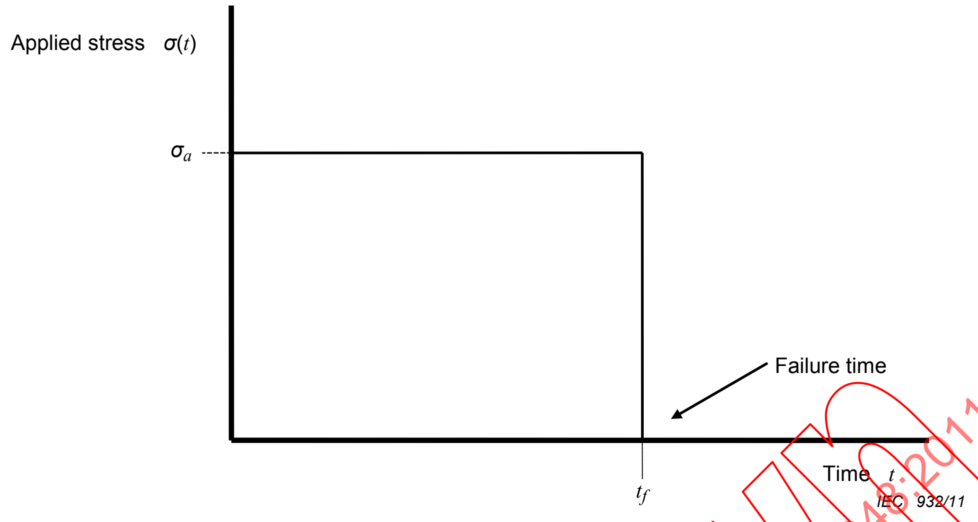

Figure 8 – Static fatigue: applied stress versus time for a particular applied stress

In static fatigue testing, different stresses are applied to different sets of specimens. From Equation (46), a ln-ln plot of failure times versus applied stresses gives

$$\log t_f(\sigma_a) \approx \log t_f(1) - n_s \log \sigma_a \tag{48}$$

The data should lead to a best-fit straight line with a negative slope of $-n_s$ and a vertical intercept of $\log t_f(1)$. (This value will depend on the units used, and may require a straight-line extrapolation beyond the data points.) This data is shown schematically in Figure 9. According to Equation (45), non-linearities are expected for large applied stress or weak flaws. As discussed in 8.4.4, this can lead to errors in measuring n_s.

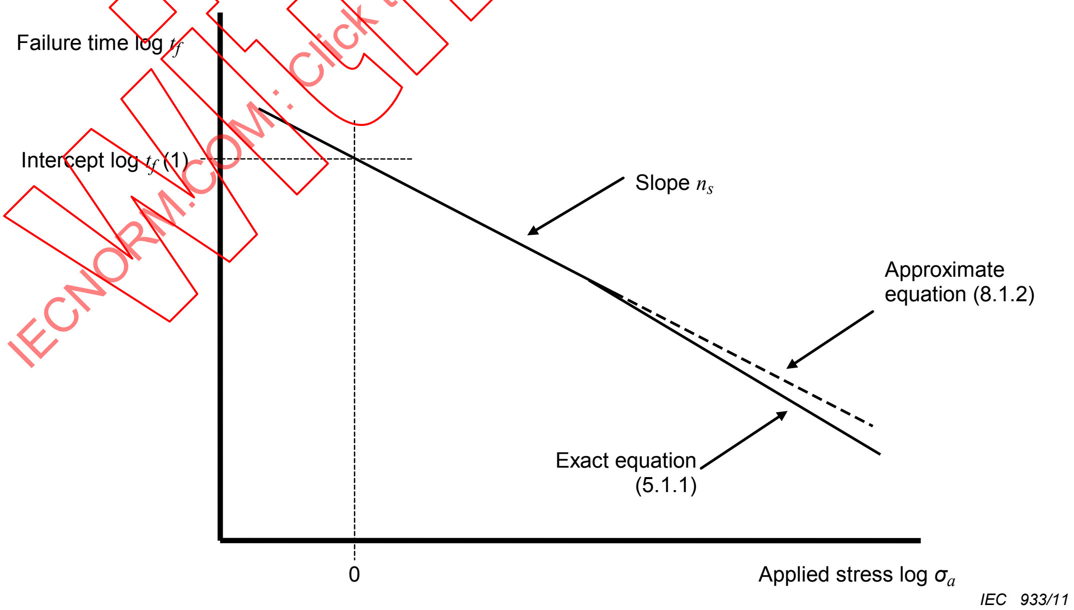

Figure 9 – Static fatigue: schematic data of failure time versus applied stress

8.3 Dynamic fatigue

8.3.1 General

This test is performed in either of two ways. In one, the stress is applied until the fibre breaks. In the other, the stress is applied to a maximum value or until the fibre breaks, whichever occurs first.

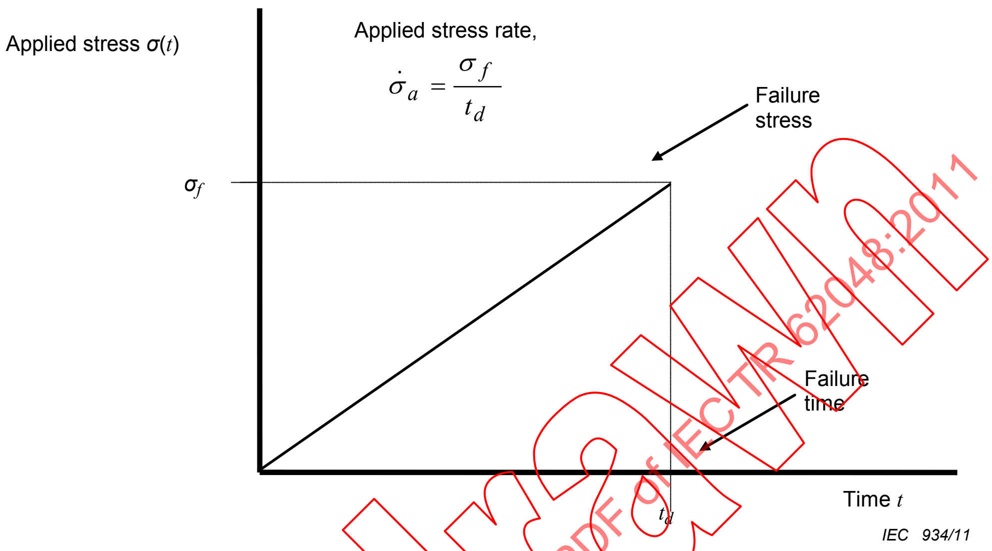

Figure 10 – Dynamic fatigue: applied stress versus time for a particular applied stress rate

8.3.2 Fatigue to breakage

In the more common dynamic fatigue a fibre specimen is subjected to a constant applied stress rate $\dot{\sigma}_a$ until the weakest crack of initial strength S breaks at an observed failure time t_d, or dynamic failure stress

$$\sigma_f = \dot{\sigma}_a t_d \tag{49}$$

This stress history is shown schematically in Figure 10. (Do not confuse t_d with prooftesting dwelltime defined in 9.2.) From Equations (39) and (40), the failure stress is

$$\sigma_f(\dot{\sigma}_a) = \left[(n_d + 1)B_d \dot{\sigma}_a \left(S^{n_d - 2} - \sigma_f^{n_d - 2}\right)\right]^{\frac{1}{n_d + 1}} \tag{50}$$

If the stress-to-strength ratio is sufficiently small according to Equation (41), or if the approximate lifetime of Equation (42) is used, then this has the reduced form

$$\sigma_f(\dot{\sigma}_a) \approx \left[(n_d + 1)B_d \dot{\sigma}_a S^{n_d - 2}\right]^{\frac{1}{n_d + 1}} = \sigma_f(1)\dot{\sigma}_a^{\frac{1}{n_d + 1}} \tag{51}$$

This has the unit-stress-rate "intercept" value

$$\sigma_f(1) = \left[(n_d + 1)B_d S^{n_d - 2}\right]^{\frac{1}{n_d + 1}} \tag{52}$$

which depends upon the units used. (Note that this intercept does not have the dimension of stress.) (One can also work with t_d versus σ_f or $\dot{\sigma}_a$, but these formats are less common.)

In dynamic fatigue testing, different stress rates are applied to different sets of specimens. From Equation (51), a ln-ln plot of failure stresses versus stress rates gives

$$\log \sigma_f(\dot{\sigma}_a) \approx \log \sigma_f(1) + \frac{\log \dot{\sigma}_a}{n_d + 1} \tag{53}$$

The data should lead to a best-fit straight line with a slope of $(n_d + 1)^{-1}$ and a vertical intercept of $\log\sigma_f(1)$. (This value will depend on the units used and may require a straight-line extrapolation beyond the data points.) This data is shown schematically in Figure 11. According to Equation (50), non-linearities are expected for large failure stresses or weak flaws. As discussed in 8.4.4, this can lead to errors in measuring n_d.

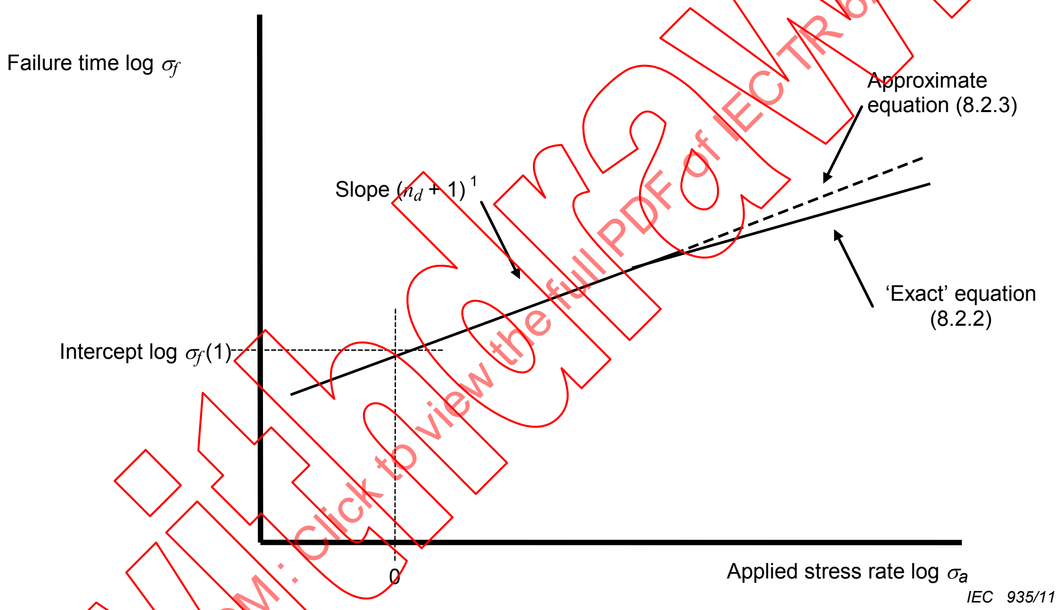

Figure 11 – Dynamic fatigue: schematic data of failure time versus applied stress rate

8.3.3 Fatigue to a maximum stress

For long-term reliability prediction, it is useful to obtain data in the low-strength low-fracture-probability portion of the Weibull distribution discussed in 10.2.2, 10.5, and 10.8. Except for fibre deployments that involve very high stresses, very tight bends, or rather high allowable failure probabilities, it is these cracks that are more important. One method of characterizing these cracks is dynamic fatigue testing with censoring.

According to Reference [5], the method differs in several ways from the usual dynamic fatigue testing. The gauge length is longer, and more samples are used. Both these factors mean that a very long length of fibre is tested. Only one high strain rate is applied, and the resulting applied stress is limited to a maximum value σ_{max} within the lower mode of the Weibull strength distribution. The stress history is similar to that of Figure 10 with a constant stress rate, but with the highest stress being either σ_f with breakage or σ_{max} without breakage. The latter is more common, and this means that a smaller fraction of fibre from a longer sampled length is broken.

More details of the measurements and calculations are given in 8.4.2 and 8.4.3.

8.4 Comparisons of static and dynamic fatigue

8.4.1 Intercepts and parameters obtained

The parameters n, B obtained statically or dynamically should, in principle, be equal for the same environment. With a few exceptions for the n-value, we will drop the subscripts s and d from this point forward. Then Equations (47) and (52) relate the intercepts as

$$BS^{n-2} = t_f(1) = \frac{\sigma_f^{n+1}(1)}{n+1} \tag{54}$$

8.4.2 Time duration

Static fatigue experiments are usually carried out over longer periods of time (days to months) and may be conducted in a hostile environment (elevated temperature, humidity, and possible chemical species). Dynamic fatigue is usually carried out over shorter periods of time (seconds to hours), often (but not always) in an inert or ambient environment. For many applications, it is necessary to assess the strength distribution in the neighbourhood of the prooftest stress, usually by dynamic fatigue testing.

Solving Equation (46) for $t_f(1)$ and Equation (51) for $\sigma_f(1)$, and using Equation (54) gives

$$\sigma_a^n t_f = \frac{\sigma_f^{n+1}}{(n+1)\dot{\sigma}_a} \tag{55}$$

This suggests that if the static applied stress is taken equivalent to the dynamic failure stress, then as in Reference [6]

$$t_f = \frac{\sigma_f}{(n+1)\dot{\sigma}_a} = \frac{t_d}{n+1} \tag{56}$$

where Equation (49) has been used. This means that an "effective" static failure time results from dividing the dynamic failure time by $n + 1$.

In an extended approach, two flaws of different initial inert strengths S_1, S_2 prior to static and dynamic fatigue are equivalent, if from the "exact" static lifetime Equation (45)

$$\left(\frac{S_1}{\sigma_a}\right)^{n-2} = 1 + \frac{\sigma_a^2 t_f}{B} \tag{57}$$

and from the "exact" dynamic failure stress Equation (50)

$$\left(\frac{S_2}{\sigma_f}\right)^{n-2} = 1 + \frac{\sigma_f^3}{(n+1)B\dot{\sigma}_a} \tag{58}$$

Equating the left sides leaves

$$t_f = \frac{\sigma_f^3}{\sigma_a^2 (n+1)\dot{\sigma}_a} \tag{59}$$

Equation (55) was applied to dynamic and fatigue results taken at several laboratories on the same fibre measured in various deployment geometries in Reference [6]. When the non-linear corrections of Equation (43) and the area corrections of 10.2 were incorporated, the static and dynamic n-values in a static fatigue type plot fell on the same line. However the slope of the line varied to give n-values ranging from 17 at high stress/short time to 40 at low stress/long time.

8.4.3 Dynamic and inert strengths

From Equations (49) and (50), the ratio of inert strength to failure stress (or "dynamic strength") is

$$\frac{S}{\sigma_f} = \left[1 + \frac{\sigma_f^3}{(n+1)B\sigma_a}\right]^{\frac{1}{n-2}} = \left[1 + \frac{\sigma_f^2 t_d}{(n+1)B}\right]^{\frac{1}{n-2}} \tag{60}$$

which approaches unity as the stress rate (or B-value) increases. This explains why high stress rates are sometimes used to experimentally estimate inert strength.

8.4.4 Plot non-linearities

In static fatigue, the failure time decreases as the constant applied stress increases, according to Equation (45). The applied stress approaches the "inert" strength as the failure time vanishes. For the plot of ln t_f versus ln σ_a according to Equation (48), the more exact Equation (45) leads to a

$$\text{static fatigue logarithmic slope} = -\frac{n_s - 2\left(\frac{\sigma_a}{S}\right)^{n_s-2}}{1-\left(\frac{\sigma_a}{S}\right)^{n_s-2}} \tag{61}$$

As the applied stress increases, the plot of Figure 9 will curve downwards such that the absolute value of the slope increases from n_s to higher values, so that the apparent n_s is larger.

In dynamic fatigue, the failure stress or dynamic strength increases with stress rate, according to Equation (50). For example, if n_d = 15 and 30, increasing the stress rate by a factor of one thousand (which is often done in dynamic fatigue testing) increases the dynamic strength by 54 % and 25 %, respectively. Moreover, the proportionality factors relating the two strengths depend upon the environment (through B and n). These factors impact the manner in which vendors or users can specify "strength".

For the plot of ln σ_f versus ln $\dot{\sigma}_a$ according to Equation (53), the more exact Equation (50) leads to a

$$\text{dynamic fatigue logarithmic slope} = \frac{1}{(n_d + 1)\left[1 + \dfrac{(n_d - 2)B_d \dot{\sigma}_a}{\sigma_f^3}\right]} \qquad (62)$$

As the applied stress rate increases, the plot of Figure 11 will flatten somewhat such that the value of the slope decreases from $(n_d + 1)^{-1}$ to smaller values, so that the apparent n_d is larger.

Because n increases with longer duration tests, as discussed in the last paragraph of 8.3.2, increased steepness of the static fatigue curve at very low stresses and decreased steepness of the dynamic fatigue curve at very low stress rates are observed. In these cases, the static and dynamic n-values converge.

8.4.5 Environments

The dependence of crack parameters on environment is under continuing investigation. An understanding of the effects of temperature, humidity, and other environments on crack growth would permit the "mapping" from one environment to another. It would also permit the use of shorter-term "accelerated ageing" in harsh environments to predict reliability for longer-term service in more benign environments.

In a truly inert environment, the failure stress is independent of the stress rate. This is equivalent to putting either n or B equal to infinity in Equations (51) through (53).

9 Prooftesting

9.1 General

This clause shows how prooftesting reduces "infant mortalities" and determines important reliability features.

NOTE This clause references the B-value, and this is done for theoretical completeness only. There are as yet no agreed methods for measuring B, and Clause 9 develops theoretical results for the special case in which B can be neglected.

9.2 The prooftest cycle

Prooftesting requires that a nominally constant prooftest stress σ_p be applied sequentially along the full length of the fibre. Unlike fatigue testing, it is not performed necessarily to failure, although, as discussed in 10.6, a breakrate (failures per unit length) N_p is statistically expected. This is done during fibre manufacturing, on-line as part of the fibre drawing and coating process, or off-line as part of the testing process.

The stress history of prooftest stressing, shown schematically in Figure 12, is

– stress loading from near-zero to the prooftest stress, during a loadtime t_l,
– constant prooftest stress σ_p during a dwelltime t_d (symbol not to be confused with the dynamic fatigue failure time defined in 10.3.),
– stress unloading from the prooftest stress back down to near-zero, during an unloadtime t_u.

Consider a fibre prooftested at a fibre speed s. The loading/unloading time across a quarter turn of a wheel of diameter D is then $t_{l,u} = \frac{\pi D}{4s}$. The fibre dwell-length is st_d.

As shown in Figure 12 and References [7] to [10], it is assumed that the load and unload processes are essentially linear. The load and unload portions are similar to dynamic fatigue, and the dwell is somewhat similar to static fatigue. The differences are that the maximum applied stress (the prooftest stress) is limited and that breakage does not necessarily occur. Since this topic is particularly complex, equivalence to equations in the references (often with varying approaches and notations) are given in the development below.

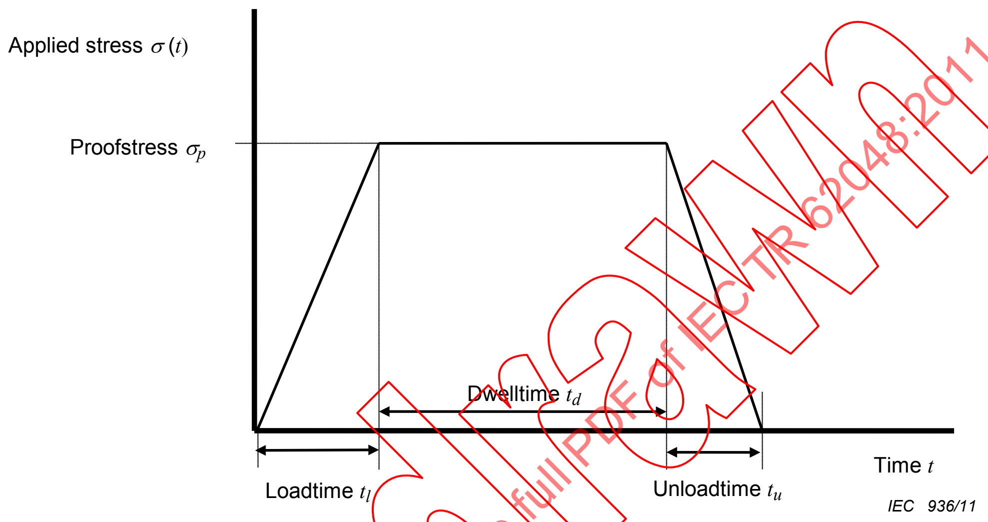

Figure 12 – Prooftesting: applied stress versus time

9.3 Crack weakening during prooftesting

Equations (36) and (37) for weakening show that for a crack that does not break, its initial "inert" strength S before prooftesting has reduced to an "inert" strength S_p after prooftesting, where

$$S_p^{n-2} = S^{n-2} - \frac{\sigma_p^n t_p}{B} \qquad (63)$$

Here the effective prooftime is given by

$$t_p = t_d + \frac{t_l + t_u}{n+1} \qquad (64)$$

(This is equivalent to Equation (7) of Reference [7].) In this equation, the loadtime t_l and unloadtime t_u in the fraction contribute little to the effective prooftime. As an example, if $n \geq 20$, and neither the loadtime nor unloadtime exceeds 10 % of the dwelltime, the fraction in Equation (64) constitutes less than 1 % of the effective prooftime. This shows that the dwelltime t_d should be kept small so as to minimize the fatigue of the surviving cracks.

A crack that has an initial strength (before prooftesting) not exceeding the prooftest stress will weaken and break during loading. A stronger crack will weaken and will break during the dwell if its strength degrades to the prooftest stress. If it survives the dwell, it has a strength $S_u(0)$ just before unloading at least equal to the prooftest stress; however, further weakening occurs during unloading. During a time t into the unloading, the applied stress is

$$\sigma_u(t) = \sigma_p\left(1 - \frac{t}{t_u}\right) = \sigma_p - \dot{\sigma}_u t \tag{65}$$

where the (positive) unloading rate is

$$\dot{\sigma}_u = \frac{\sigma_p}{t_u} \tag{66}$$

Using Equations (65) and (66) in Equations (36) and (37), the monotonically decreasing crack strength during unloading is obtained as

$$S_u^{n-2}(t) = S_u^{n-2}(0) - \frac{\sigma_p^n t_u}{(n+1)B}\left[1 - \left(1 - \frac{t}{t_u}\right)^{n+1}\right] \tag{67}$$

(This is equivalent to Equation (7) of Reference [8].)

9.4 Minimum strength after prooftesting

9.4.1 General

For the weakest crack just surviving the prooftest, the minimum final strength from Equation (63) equals the minimum strength after unloading from Equation (67), so that

$$S_{p\min} = S_{u\min}(t_u) \tag{68}$$

There are two situations by which this strength is determined, depending upon whether the unloading process is "fast" or "slow." This relates to the dimensionless quantity

$$\alpha = \frac{\sigma_p^2 t_u}{(n-2)B} = \frac{\sigma_p^3}{(n-2)B\dot{\sigma}_u} \tag{69}$$

explicitly containing fibre and prooftest unloading parameters. As shown below, it is important whether this quantity is larger or smaller than unity.

9.4.2 Fast unloading

Here the weakest crack just before unloading has its strength equal to the prooftest stress, that is,

$$S_{u\min}(0) = \sigma_\text{p} \tag{70}$$

During unloading, the decreasing strength from Equation (67) is larger than the rapidly decreasing unload stress of Equation (65), so that fracture does not occur. The rates of decrease for both strength and applied stress just before unloading must satisfy

$$\left|\frac{dS_u(t)}{dt}\right|_{t=0} \leq \dot{\sigma}_u \tag{71}$$

Applying Equation (66) to Equation (67) and using Equation (69) implies that $\alpha \le 1$. Then Equations (67) and (70) give the minimum crack strength after prooftesting

$$S_{p\min} = \sigma_p \left[1 - \frac{\sigma_p^2 t_u}{(n+1)B} \right]^{\frac{1}{n-2}} = \sigma_p \left(1 - \frac{n-2}{n+1} \alpha \right)^{\frac{1}{n-2}} \quad (72)$$

for $\alpha \le 1$ or $t_u \le (n-2)\dfrac{B}{\sigma_p^2}$

(This is equivalent to Equation (9) of Reference [8].) For fast unloading, $B \ge B_0$.

9.4.3 Slow unloading

Here the weakest crack just before unloading has its strength exceeding the prooftest stress, that is,

$$S_{u\min}(0) \ge \sigma_p \quad (73)$$

For failure at some time $\hat{t}$ during unloading, the decreasing strength of Equation (67) equals the decreasing unloading strength of Equation (65), that is,

$$S_{u\min}(\hat{t}) = \sigma_u(\hat{t}) = \sigma_{\min} \le \sigma_p \quad (74)$$

where σ_{min} is a minimum value of the failure stress. Then putting Equations (68) and (74) into Equation (67) gives the minimum crack strength after prooftesting

$$S_{p\min} = \sigma_{\min} \left[1 - \frac{\sigma_{\min}^2 t_u}{(n+1)B} \right]^{\frac{1}{n-2}} \quad (75)$$

(This is equivalent to Equation (9) of Reference [7].) The unknown minimum failure stress $\sigma_{\min}$ can be determined by noting that at the critical survival time $\hat{t}$, the strength and stress curves are equal but do not quite intersect. They are tangential, so that at that point

$$\left| \frac{dS_u(t)}{dt} \right|_{t=\hat{t}} = \dot{\sigma}_u \quad (76)$$

Equations (76), (67) and (69) leave

$$\sigma_{\min}^3 = (n-2)B\dot{\sigma}_u = \frac{\sigma_p^3}{\alpha} \quad (77)$$

This and Equation (74) imply that $\alpha \geq 1$, and Equation (75) gives

$$S_{p\min} = \left(\frac{3}{n+1}\right)^{\frac{1}{n-2}} \left[(n-2)B\dot{\sigma}_u\right]^{\frac{1}{3}} = \sigma_p \left(\frac{3}{n+1}\right)^{\frac{1}{n-2}} \alpha^{-\frac{1}{3}} \tag{78}$$

$$\text{for } \alpha \geq 1 \text{ or } t_u \geq (n-2)\frac{B}{\sigma_p^2}$$

(These are equivalent to Equations (10) and (11) of Reference [7], and to Equation (10) of Reference [8].) For slow unloading, $B \leq B_0$.

Note that here the unloading rate is the only prooftest (non-crack) parameter determining the minimum surviving strength (which is now independent of the prooftest stress itself). Equations (72) and (78) both show the importance of minimizing the unloadtime t_u as stated in IEC 60793-1-3.

9.4.4 Boundary condition

At the boundary between the fast-unload and slow-unload conditions, α =1. Then Equation (69) gives the transitional B-value.

$$B_0 = \frac{\sigma_p^2 t_u}{n-2} \tag{79}$$

which can also be solved for the unloadtime. From Equations (72) and (78), the minimum surviving strength at the boundary is

$$S_{p\min} = \sigma_p \left(\frac{3}{n+1}\right)^{\frac{1}{n-2}} \tag{80}$$

9.5 Varying the prooftest stress

One effect of increasing (similar considerations apply for decreasing) the prooftest stress is to increase the minimum surviving strength according to Equations (72) and (78). Another is to increase the fibre breakrate (or decrease the survival length), as will be shown in Clause 10. This has been used in References [11] and [12] to probe the low-strength distribution over long fibre lengths, and is detailed in 5.4.1.

10 Weibull probability

10.1 General

So far, single cracks with deterministic values of particular parameters have been considered. Now it is assumed that for a particular type of crack most parameters are constant but that strengths are statistically distributed in value.

10.2 Strength statistics in uniform tension

10.2.1 Unimodal probability distribution

We assume a fibre of circular geometry that is uniform along its length. When simple longitudinal tension is applied to it, there are two degrees of stress uniformity, both along the fibre length and in the fibre cross-part. (Fibre volume or surface area may instead be used as the prime dimension.) Call $N(S)$ the cumulative number of flaws per unit length having an "inert" strength equal to or less than S. If a fibre length L is put under stress, call $P(S,L)$ the cumulative survival probability up to the strength S. (A parallel derivation can be made in terms of the flaw size a.) In this "weakest link" model, the incremental probability that the fibre fails in the strength interval S to $S + dS$ is

$$[1 - P(S + dS,L)] - [1 - P(S,L)] \qquad (81)$$

This equals the probability of survival to the failure strength times a quantity proportional to the number of flaws in that strength interval

$$P(S,L)hL[N(S + dS) - N(S)] \qquad (82)$$

where h is a proportionality constant. Equating Equations (81) and (82), integrating and using the boundary condition of no flaws of zero strength, leads to the Weibull survival distribution

$$P(S,L) = e^{-hLN(S)} \qquad (83)$$

(Usually, the literature speaks in terms of the cumulative failure probability $F = 1 - P$. We use P to simplify the notation.) Often it is found (or assumed) that

$$N(S) \propto S^m \qquad (84)$$

where m is the Weibull parameter for the inert environment.

Using Equations (83) and (84) leaves the cumulative survival probability, as in Reference [13]

$$P(S,L) = \exp\left[-\left(\frac{S}{S_0}\right)^m \frac{L}{L_0}\right] \qquad (85)$$

The boundary conditions are such that survival is certain at zero strength or length, and failure is certain at infinite strength or length. Here S_0 is the Weibull strength parameter measured at a "gauge" length L_0, corresponding to a survival probability of e^{-1} or 36,8 %. The "inert" m-value is related to the variance or 'width' of the distribution with respect to strength and determines the variation of strength with fibre length. Furthermore,

$$\ln\ln\frac{1}{P(S,L)} = m\ln\frac{S}{S_0} + \ln\frac{L}{L_0} \qquad (86)$$

so that for a fixed length, a scaled cumulative failure probability plot versus inert strength leaves a straight line of slope m. Increasing the length will increase the probability intercept. (Prooftesting will distort the distribution, as discussed in 10.6.)

The probability distribution function, a continuous "histogram" of the survival population, is

$$p(S,L) = -\frac{\partial P}{\partial S} = \frac{m}{S_0}\frac{L}{L_0}\left(\frac{S}{S_0}\right)^{m-1} P(S,L) \qquad (87)$$

This is related to the number of flaws per unit length having an "inert" strength S given by

$$\frac{\partial N}{\partial S} = \frac{p(S,L)}{LP(S,L)} = \frac{m}{S_0 L_0}\left(\frac{S}{S_0}\right)^{m-1} \qquad (88)$$

Here Equations (83) through (85) and Equation (87) have been used.

For a given survival probability, the strength can be predicted for another length in terms of the gauge values

$$S(l,P) = S_0\left(L_0, e^{-1}\right)\left(\frac{L_0}{L}\ln\frac{1}{P}\right)^{\frac{1}{m}} \qquad (89)$$

Similarly, for a given strength, the survival probabilities of both lengths are related by

$$L_0 \ln P\,(S, L) = L \ln P\,(S, L_0) \qquad (90)$$

Extrapolation to longer fibre lengths in the above equations is uncertain, since weaker flaws of a "different type" may appear. They will have a different set of values for the Weibull parameters as per the bimodal distribution below.

10.2.2 Bimodal probability distribution

If there are two flaw types, there are two main ways of statistically describing them according to Reference [14]. With concurrent flaw distributions, both flaw types are present in all specimens. This is the case with specimens from one manufacturer. Here, the survival probability distribution is the product of the individual survival probability distributions for each flaw type. With exclusive flaw distributions, a given specimen may contain only one flaw type. An example would be specimens from two manufacturers. In this case, the survival probability distribution is the sum of the individual survival probability distributions for each flaw type. Partially concurrent distributions can also be modelled.

Fibre flaws are likely of the concurrent type so that the Weibull assumption of Equation (85) becomes

$$P(S,L) = \exp\left[-\left(\frac{S}{S_{01}}\right)^{m_1}\frac{L}{L_{01}} - \left(\frac{S}{S_{02}}\right)^{m_2}\frac{L}{L_{02}}\right] \qquad (91)$$

Here (m_1, S_{01}) and (m_2, S_{02}) characterize each flaw, while gauge lengths L_{01}, L_{02} provide a weighting factor for the two flaw types. This type of distribution can describe the "knee" found in plotting measured data. The steeper distribution of high-strength breaks is attributable to silica bonding, and this is the region of "intrinsic" flaws. The lower-slope distribution of low-strength "extrinsic" flaws is introduced during the manufacturing process. Prooftesting will generally affect only the latter segment of the population distribution.

Plots with negative curvature can be fit to exclusive or partially concurrent distributions.

10.3 Strength statistics in other geometries

10.3.1 Stress non-uniformity

It is usually assumed that the glass fibre is perfectly circular, with a radius a_f constant with length L (although non-uniformities can be corrected for in principle). The effect of the coating on stress calculation is often neglected or only approximately accounted for. The treatment in the text until now has been for longitudinal tension in which the stress is uniformly applied along the fibre length. Stress then equals the fraction [tension/(πa_f^2)]. With non-uniform stress, the exponent in the survival probability in Equation (85) for unimodal distributions has $S^m L$ replaced by the integral over the sample surface (assuming interior flaws are negligible)

$$I = \int_A S^m(A)dA \tag{92}$$

Two configurations have commonly been treated mathematically, as in References [15] and [16].

10.3.2 Uniform bending

Here a bend of radius R (with respect to the fibre axis) is applied uniformly along a fibre length L_b, as around a mandrel. The resulting applied stress is uniform along the bent fibre length, but it varies on the fibre surface as

$$\sigma_a(\theta) = \sigma_{max}\sin\theta \tag{93}$$

In the cross-part of the fibre, the angle θ is with the axis perpendicular to the bend plane and passing through the fibre axis. The maximum stress occurs along a line on the outside of the fibre bend and in the plane of the bend. It has the value

$$\sigma_{max}(R) = E_0\left(1+\frac{9}{4}\frac{a_f}{R}\right)\frac{a_f}{R} \tag{94}$$

using the non-linear effect of Equation (43) with a geometrical correction from Reference [4]. For a fibre with a diameter $2a_f$ = 125 µm, the error in calculating stress while neglecting the non-linear term exceeds 1 % when the bend diameter $2R$ is less than about 28 mm. Also, a maximum stress of 0,7 GPa corresponds to a bend diameter of about 13 mm (depending upon the assumed value of Young's modulus).

The incremental area at the above stress is

$$dA = L_b a_f d\theta \tag{95}$$

Integration according to Equation (92) leaves the Weibull forms similar to Equation (85) and to the later Equations (100), and (107), but with the bend length replaced by the equivalent tensile length

$$L = \frac{L_b}{\pi}\int_0^{\frac{\pi}{2}} \sin^x\theta\, d\,\theta \tag{96}$$

This defines the length in uniform straight tension at the maximum stress σ_{max}, giving the same probability distribution as for the real length under uniform bending. This result applies to the approximate Weibull static distribution of Equation (101) and to the approximate Weibull dynamic distribution of Equation (108). It does not apply to those distributions after prooftesting in Equations (122), (132), and (138).

A better fit for Equation (96) is

$$L \approx 0{,}4 \frac{L_b}{\sqrt{x}} \tag{97}$$

a result independent of the bend radius. In this equation there is the factor

$$x = \frac{mn}{n-2} = m_s n = \frac{m_d n}{n+1} \tag{98}$$

using inert, static fatigue, and dynamic fatigue parameters, respectively. The equivalent length is some fraction of the real bend length.

10.3.3 Two-point bending

Here the fibre is bent between parallel flat plates in a "U" fashion. The tension is non-uniform, both in the fibre cross-part and length, and reaches a maximum value at the mid-point of the outside of the bend. Similar to Equation (94) and as in Reference [4], that value is about

$$\sigma_{max}(D) = 2{,}4 E_0 \left(1 + 0{,}51 \frac{a_f}{D}\right) \frac{a_f}{D} \tag{99}$$

where D is the separation of the fibre axes in the straight portions of the "U". Moreover, the equivalent tensile length decreases as that stress increases, and the analysis becomes very complicated, as in Reference [15]. One calculates a length range of only 10 µm to 30 µm at typical failure strains, but the precise equivalent length is difficult to determine.

Now consider how the fatigue testing of Clause 8, carried out in non-inert active (ambient or hostile) environments, affects the inert Weibull probability distribution introduced in 10.2. Then add the prooftesting of Clause 9, which affects mainly the low-strength extrinsic region of the bimodal distribution. We show how both static and dynamic fatigue testing preserve the linear nature of the distribution, but prooftesting distorts and truncates it.

However, in fatigue testing, the applied stress and the failure stress are usually larger than the prooftest stress. This is not so for most practical longer-term service conditions. The Weibull parameters obtained this way apply only to the high-strength segment of a bimodal distribution and cannot be extrapolated to lower failure probability values. To obtain values that apply to the lower-strength segment including the prooftest stress region, longer fibre lengths must be tested. An example is given in 8.3.3.

10.4 Weibull static fatigue before prooftesting

For a particular value of stress σ_a applied to a fibre length L, there is some statistical variation in the measured failure lifetimes t_f. The initial strength prior to static fatigue is given by Equation (57), which substituted into the Weibull probability Equation (85) leaves the static survival probability

$$P(t_f) = \exp\left[-\left(\frac{t_f + \frac{B}{\sigma_a^2}}{t_0}\right)^{m_s}\right] \quad (100)$$

$$\approx \exp\left[-\left(\frac{t_f}{t_0}\right)^{m_s}\right] \text{ when } t_f >> \frac{B}{\sigma_a^2} \quad (101)$$

(The usual approximate form in Equation (101) is equivalent to Equation (5) of Reference [17].) Here the static Weibull parameter is related to the "inert" m-value (not usually known) via

$$m_s = \frac{m}{n-2} \quad (102)$$

The Weibull time-scaling parameter is

$$t_0 = \frac{\beta}{L^{\frac{1}{m_s}} \sigma_a^n} \quad (103)$$

where

$$\beta = BS_0^{n-2} L_0^{\frac{1}{m_s}} \quad (104)$$

These depend upon several crack and measurement parameters and can be calculated from static fatigue testing on gauge fibre lengths L_0 at several applied stresses according to 8.2.

From any probability value, one can calculate

$$t_0 = \left(t_f + \frac{B}{\sigma_a^2}\right)\left[\ln \frac{1}{P(t_f)}\right]^{\frac{1}{m_s}} \quad (105)$$

A convenient point is the "intercept" for unity failure time; convenient probability values are e^{-1} (36,8 %) or ½ (corresponding to the median lifetime). Furthermore, from Equation (100) one obtains

$$\ln\text{-}\ln \frac{1}{P(t_f)} = m_s \ln\left[\frac{t_f + \frac{B}{\sigma_a^2}}{t_0}\right] \quad (106)$$

so that a scaled cumulative failure probability plot versus failure time leaves a straight line of slope m_s. This preserves the form of the inert Equation (85) and is shown schematically in Figure 13 for a bimodal distribution.

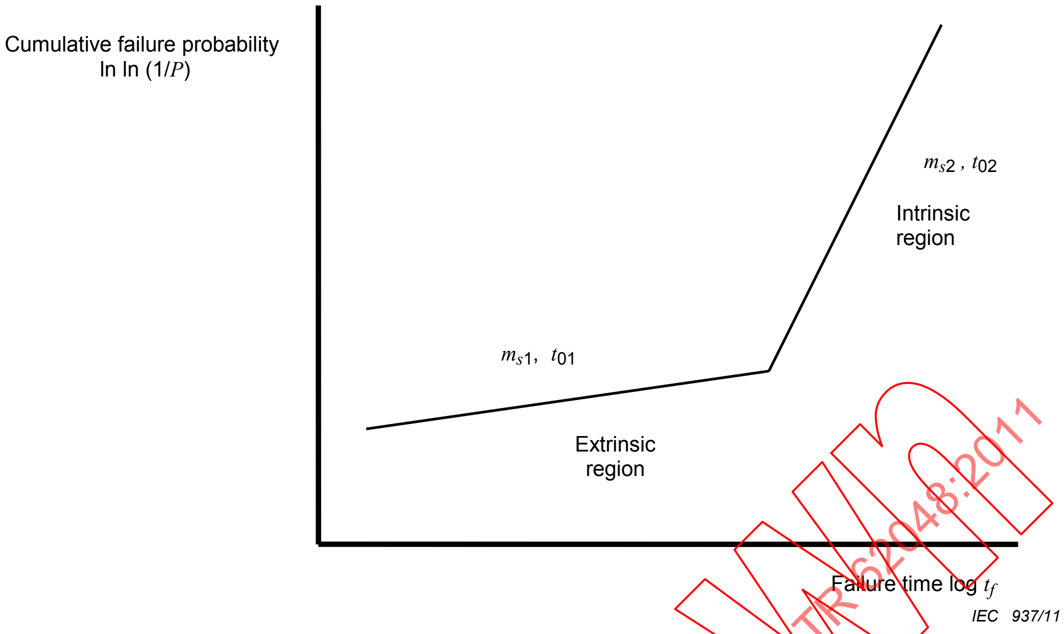

Figure 13 – Static fatigue schematic Weibull plot

10.5 Weibull dynamic fatigue before prooftesting

For a particular value of stress rate $\dot{\sigma}_a$ applied to a fibre length L, there is some statistical variation in the measured failure stresses σ_f. The initial strength prior to dynamic fatigue is given by Equation (58) which, substituted into the Weibull probability Equation (85), leaves the dynamic survival probability

$$P(\sigma_f) = \exp\left\langle -\left\{ \frac{\sigma_f}{\sigma_0}\left[1 + \frac{(n+1)B\sigma_a}{\sigma_f^3}\right]^{\frac{1}{n+1}} \right\}^{m_d} \right\rangle \tag{107}$$

$$\approx \exp\left[-\left(\frac{\sigma_f}{\sigma_0}\right)^{m_d}\right] \text{ when } \sigma_f^3 >> (n+1)B\sigma_a \tag{108}$$

(The usual approximate form in Equation (108) is equivalent to Equation (6) of Reference [17].) Here the dynamic Weibull parameter is related to the "inert" m-value (not usually known) via

$$m_d = \left(\frac{n+1}{n-2}\right)m \tag{109}$$

The Weibull stress-scaling parameter is

$$\sigma_0 \frac{[(n+1)\beta\dot{\sigma}_a]^{\frac{1}{n+1}}}{L^{\frac{1}{m_d}}} \tag{110}$$

where

$$\beta = BS_0^{n-2} L_0^{\frac{n+1}{m_d}} \tag{111}$$

These depend upon several crack and measurement parameters and can be calculated from dynamic fatigue testing on a fibre length L_0 at several applied stress rates, according to 8.3.

From any probability value, one can calculate

$$\sigma_0 = \sigma_f \left[\ln \frac{1}{P(\sigma_f)} \right]^{-\frac{1}{m_d}} \left[1 + \frac{(n+1)B\sigma_a}{\sigma_f^3} \right] \tag{112}$$

A convenient point is the "intercept" for unity failure stress; convenient probability values are e^{-1} (36,8 %) or ½ (corresponding to the median failure stress). Furthermore, from Equation (107), one obtains

$$\text{ln-ln} \frac{1}{P(\sigma_f)} = m_d \ln \frac{\sigma_f}{\sigma_0} + \frac{m_d}{n+1} \left[1 + \frac{(n+1)B\sigma_a}{\sigma_f^3} \right] \tag{113}$$

so that a scaled cumulative failure probability plot versus failure stress leaves a straight line of slope m_d. This preserves the form of the inert Equation (85) and is shown schematically in Figure 14 for a bimodal distribution.

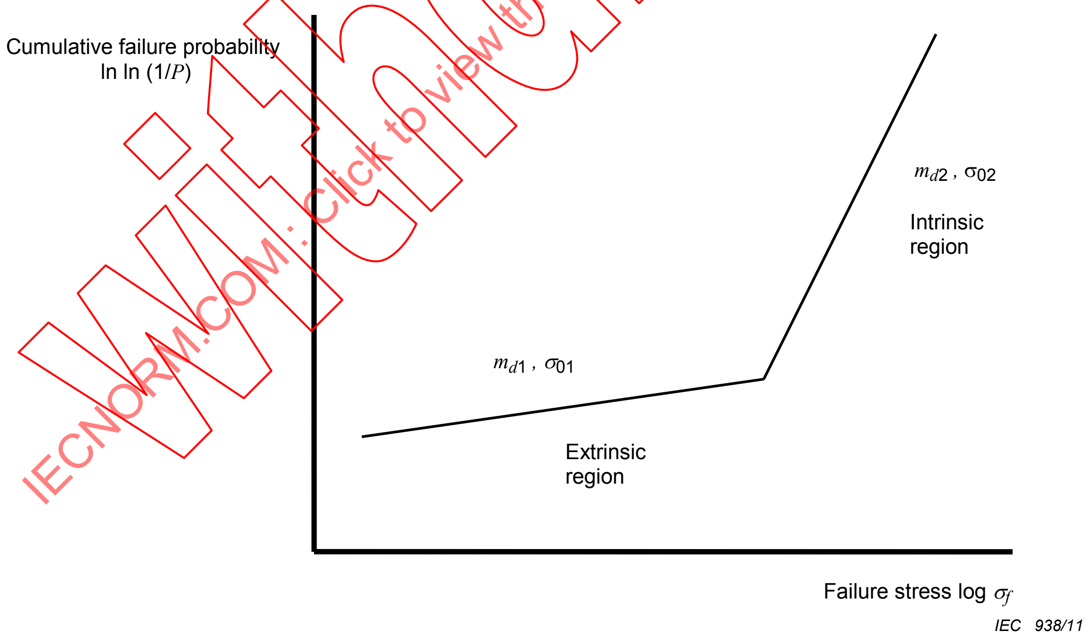

Figure 14 – Dynamic fatigue schematic Weibull plot

From Equations (102) and (109), the static and dynamic Weibull parameters are related by

$$m_d = (n+1)m_s \tag{114}$$

Then comparing Equations (103) and (110), the static and dynamic Weibull parameters are related by

$$\sigma_0^{n+1} = (n+1)\dot{\sigma}_a \sigma_a^n \; t_0 \tag{115}$$

This is analogous to the relation of static and dynamic plot intercepts given by Equation (54).

10.6 Weibull after prooftesting

Here it is shown how the "inert" strength distributions of 10.2 are modified by the prooftest procedure. Fatigue testing probes fibre survival probability by breaking many samples of limited gauge length. On the other hand, prooftesting is applied to the whole fibre length with the intent of breaking only those cracks with strengths below some specified minimum value. As discussed below, this enhances the survival probability of the remaining usable fibre.

As a function of final strength S_p after prooftesting, the enhanced survival probability P_p equals the survival probability as a function of initial strength S before prooftesting, divided by the survival probability as a function of the minimum initial strength $S_{\min}$ at the prooftest stress

$$P_p(S_p) = \frac{P(S)}{P(S_{\min})} \tag{116}$$

Now we use the inert Weibull Equation (85), though the bimodal Equation (91) usually applies. However, in the comparatively weak "extrinsic" flaw region around the prooftest level, the first unimodal term there will predominate, and one has

$$\ln\frac{1}{P_p(S_p)} = \left(\frac{S^m - S_{\min}^m}{S_0^m}\right)\frac{L}{L_0} \tag{117}$$

For simplification, we will suppress the several variables implicit in P_p, and will use $\ln\frac{1}{P_p}$ to eliminate the exponential functions and reduce the number of brackets.

The right side of such an equation is always non-negative, and zero for certain survival P_p = 1. If the failure probability $F_p = 1 - P_p < 10^{-3}$, which is generally the region of practical interest, the term $\ln\frac{1}{P_p}$ may be replaced by F_p to within an accuracy of 0,5 % or better.

The minimum initial strength at the prooftest stress level is related to the number of failures per unit length (breakrate) during prooftesting, obtained by Equations (84) and (85)

$$N_p = \left(\frac{S_{\min}}{S_0}\right)^m \frac{1}{L_0} \tag{118}$$

Then Equation (117) becomes

$$\ln\frac{1}{P_p(S_p)} = N_p L\left[\left(\frac{S}{S_{\min}}\right)^m L_0\right] \tag{119}$$

(This form is equivalent to Equation (19) of Reference [8].) Taking the mean with respect to length shows that N_p is the mean breakrate, ideally a small number. It is convenient to define the mean survival length after prooftesting

$$L_p = \frac{1}{N_p} = \left(\frac{S_0}{S_{\min}}\right)^m L_0 \tag{120}$$

using Equation (118), ideally a large number.

For the above equations, Equation (63) relates strength before prooftesting to strength after prooftesting; it also relates their minimum values as

$$S_{p\min}^{n-2} = S_{\min}^{n-2} - \frac{\sigma_p^n t_p}{B} \tag{121}$$

where the minimum strength for cracks just surviving the prooftest stress is given by Equations (72) or (77), depending upon the value of α. By use of Equations (63) and (121), Equations (117) and (119) become the survival probability distribution after prooftesting

$$\ln\frac{1}{P_p(S_p)} = \left\{\left(BS_p^{n-2} + \sigma_p^n t_p\right)^{\frac{m}{n-2}} - \left[\sigma_p^n t_p(1+C)\right]^{\frac{m}{n-2}}\right\}\frac{L}{\beta^{\frac{m}{n-2}}} \tag{122}$$

(Equation (122) is equivalent to Equation (19) of Reference [8].) The additive dimensionless positive new term is defined as

$$C = \frac{BS_{p\min}^{n-2}}{\sigma_p^n t_p} \tag{123}$$

(This term is equivalent to Equation (20) of Reference [8].) For fast unloading this is

$$C = \frac{\dfrac{B}{\sigma_p^2} - \dfrac{t_u}{n+1}}{t_p} \quad \text{when } t_u \le (n-2)\frac{B}{\sigma_p^2} \tag{124}$$

and for slow unloading this is

$$C = \frac{3}{(n+1)t_p}\left[\left(\frac{n-2}{t_u}\right)^{n-2}\left(\frac{B}{\sigma_p^2}\right)^{n+1}\right]^{\frac{1}{3}} \quad \text{when } t_u \ge (n-2)\frac{B}{\sigma_p^2} \tag{125}$$

At the fast/slow unloading transition point, one has the transitional value

$$C_0 = \frac{3}{n+1}\frac{B_0}{\sigma_p^2 t_p} = \frac{3}{(n+1)(n=2)}\frac{t_u}{t_p} \quad \text{when } t_u = (n-2)\frac{B_0}{\sigma_p^2} \tag{126}$$

B_0 is the transitional B-value of Equation (79). To obtain the above, Equation (69) for α and Equation (64) for t_p have been used.

By Equation (123), prooftesting ensures an inert strength exceeding

$$S_p \geq S_{p\min} = \left(\frac{\sigma_p^n t_p}{B} C\right)^{\frac{1}{n-2}} \tag{127}$$

(This agrees with Equations (72) and (78) for minimum surviving strength.) This, along with Equations (121), (118), and (120), gives the prooftesting mean breakrate and reciprocal mean survival length as

$$N_p = \left[\frac{\sigma_p^n t_p}{\beta}(1+C)\right]^{\frac{m}{n-2}} = \frac{1}{L_p} \tag{128}$$

The β-value can be obtained from prooftest parameters as

$$\beta = \frac{\sigma_p^n t_p (1+C)}{N_p^{\frac{n-2}{m}}} = \sigma_p^n t_p (1+C) L_p^{\frac{n-2}{m}} \tag{129}$$

Alternatively, the β-value can be obtained from fatigue testing, as in 10.7 and 10.8.

Unlike the situation of Equation (90) for inert strength without prooftesting, a post-prooftest Weibull plot is not linear. From Equation (122) such a plot has the slope

$$\text{slope}\,(S_p) = \frac{mCS_p^{n-2}\left(CS_p^{n-2} + S_{p\min}^{n-2}\right)^{\frac{m}{n-2}-1}}{\left(CS_p^{n-2} + S_{p\min}^{n-2}\right)^{\frac{m}{n-2}} - S_{p\min}^m (1+C)^{\frac{m}{n-2}}} \tag{130}$$

The effect of prooftesting is such that as the inert strength increases from the truncated value of Equation (127), the slope rapidly decreases from infinity to $n - 2$ and then towards m, the slope without prooftesting. For bimodal distributions at lower strengths, usually $m < n - 2$; at higher strengths, usually $m > n - 2$, according to Reference [9].

Note that as the prooftest stress and prooftime go to zero, so does the minimum strength. The probability distribution of Equation (122) reduces to the linear form of Equation (86) before prooftesting.

10.7 Weibull static fatigue after prooftesting

The post-prooftest stress strength similar to Equation (57) is given by

$$S_p^{n-2} = \sigma_a^{n-2}\left(1+\frac{\sigma_a^2 t_{fp}}{B}\right) \tag{131}$$

Substituting this into the survival probability of Equation (122) gives the static Weibull distribution after prooftesting

$$\ln\frac{1}{P_p(t_{fp})} = \left\{\left[\left(t_{fp}+\frac{B}{\sigma_a^2}\right)\sigma_a^n + t_p\sigma_p^n\right]^{m_s} - \left[t_p\sigma_p^n(1+C)\right]^{m_s}\right\}\frac{L}{\beta^{m_s}} \tag{132}$$

From Equations (104) and (129),

$$\beta^{m_s} = B^{m_s}S_0^m L = \frac{\left[\sigma_p^n t_p(1+C)\right]}{N_p} = \left[\sigma_p^n t_p(1+C)\right]^{m_s} L_p \tag{133}$$

In Equation (132) the numerator is zero for certain survival P_p = 1, so that prooftesting ensures a static failure time

$$t_{fp} \geq t_{fp\min} = t_p\left(\frac{\sigma_p}{\sigma_a}\right)C - \frac{B}{\sigma_a^2} \tag{134}$$

(This result can be obtained also directly from Equations (127) and (131).) With Equation (124) for fast unloading this becomes

$$t_{fp\min} = \left(\frac{\sigma_p}{\sigma_a}\right)^n\left(\frac{B}{\sigma_p^2} - \frac{t_u}{n+1}\right) - \frac{B}{\sigma_a^2} \quad \text{where } t_u \leq (n-2)\frac{B}{\sigma_p^2} \tag{135}$$

and with Equation (125) for slow unloading this becomes

$$t_{fp\min} = \frac{3}{n+1}\left(\frac{\sigma_p}{\sigma_a}\right)^n\left[\left(\frac{n-2}{t_u}\right)^{n-2}\left(\frac{B}{\sigma_p^2}\right)^{n+1}\right]^{\frac{1}{3}} - \frac{B}{\sigma_a^2} \quad \text{when } t_u \geq (n-2)\frac{B}{\sigma_a^2} \tag{136}$$

Unlike the situation of Equation (106) for static fatigue without prooftesting, a post-prooftest Weibull plot is not linear. The effect of prooftesting is such that, as the static failure time increases from the truncated value of Equation (134), the slope rapidly decreases from infinity to unity and then towards m_s (the slope without prooftesting). For bimodal distributions at lower failure times, usually $m_s < 1$; at higher failure times, usually $m_s > 1$. As the prooftest stress and prooftimes go to zero, so does the minimum failure time. The probability distribution of Equation (132) reduces to the linear form of Equation (106) before prooftesting.

With C << 1, the result of Equation (132) and a less exact Equation (131) would be approximately equivalent to a result [B] given in Reference [19].

10.8 Weibull dynamic fatigue after prooftesting

The post-prooftest stress strength similar to Equation (58) is given by

$$S_p^{n-2} = \sigma_{fp}^{n-2}\left[1+\frac{\sigma_{fp}^{3}}{(n+1)B\sigma_a}\right] \tag{137}$$

Substituting this into the survival probability of Equation (122) gives the dynamic Weibull distribution after prooftesting

$$\ln\frac{1}{P_p(\sigma_{fp})} = \left\{\left[\frac{\sigma_{fp}^{n+1}}{(n+1)\sigma_a} + B\sigma_{fp}^{n-2} + \sigma_p^n t_p\right]^{\frac{m_d}{n+1}} - \left[\sigma_p^n t_p(1+C)\right]^{\frac{m_d}{n+1}}\right\}\frac{L}{\frac{m_d}{\beta^{n+1}}} \tag{138}$$

From Equations (111) and (129),

$$\frac{m_d}{\beta^{n+1}} = \frac{\left[\sigma_p^n t_p(1+C)\right]^{\frac{m_d}{n+1}}}{N_p} = \left[\sigma_p^n t_p(1+C)\right]^{\frac{m_d}{n+1}} L_p \tag{139}$$

In Equation (138) the numerator is zero for certain survival P_p = 1, so that prooftesting ensures a dynamic failure stress contained in

$$\frac{\sigma_{fp\min}^{n+1}}{(n+1)\sigma_a} + B\sigma_{fp\min}^{n-2} = \sigma_p^n t_p C \tag{140}$$

(This result can be obtained also directly from Equations (127) and (137).)

If the minimum failure stress satisfies the inequality in Equation (108), then the approximate solution is

$$\sigma_{fp\min} \approx \left[(n+1)\dot{\sigma}_a \sigma_p^n t_p C\right]^{\frac{1}{n+1}} \tag{141}$$

With Equation (140) and Equation (124) for fast unloading this becomes

$$\sigma_{fp\min} \approx \left\{\sigma_a\sigma_p^n\left[(n+1)\frac{B}{\sigma_p^2} - t_u\right]\right\}^{\frac{1}{n+1}} \quad \text{when } t_u \le (n-2)\frac{B}{\sigma_p^2} \tag{142}$$

and with Equation (125) for slow unloading this becomes

$$\sigma_{fp\min} \approx \left\langle 3\sigma_a \left\{ \left[\frac{(n-2)\sigma_p}{t_u} \right]^{n-2} B^{n+1} \right\}^{\frac{1}{3}} \right\rangle^{\frac{1}{n+1}} \quad \text{when } t_u \geq (n-2)\frac{B}{\sigma_p^2} \tag{143}$$

Unlike the situation of Equation (113) for dynamic fatigue without prooftesting, a post-prooftest Weibull plot is not linear. The effect of prooftesting is such that as the failure stress increases from the truncated value of Equation (141), the slope rapidly decreases from infinity to $n + 1$ and then towards m_d (the slope without prooftesting). For bimodal distributions at lower failure stresses, usually $m_d < n + 1$; at higher failure stresses, usually $m_d > n + 1$. Note that as the prooftest stress and prooftimes go to zero, so does the minimum failure stress. The probability distribution of Equation (138) reduces to the linear form Equation (113) before prooftesting.

With C << 1, the result of Equation (138) and a less exact Equation (137) would be approximately equivalent to a result [A] given in Reference [19].

11 Reliability prediction

11.1 Reliability under general stress and constant stress

The theory given in the previous subclauses assumes that fibre crack parameters n and B do not change with time, although crack size a and strength S do change. Estimates of these parameters are obtained from the fatigue testing described in Clause 10. In addition to the items in 7.4, these estimates depend upon experimental conditions such as test duration, stress level, flaw initial strength, and environmental conditions. Careful and informed engineering judgement is required for reliability design.

In-service lifetime and in-service failure rate can be calculated for various fibre stress histories as follows. For a crack subjected to an arbitrary stress history to failure, the failure lifetime is implicitly contained in the general lifetime Equations (39) and (40) or (42). In principle, Equation (45) or (46) could be used for lifetime prediction, since the intercept is directly obtained by static fatigue and Equation (47), or is indirectly obtained by dynamic fatigue and Equation (54). However, crack strengths are statistically distributed along the fibre length according to Equation (85). This means that the failure times or the equivalent failure rates must also be statistically distributed.

Examples of stress histories and geometries are constant tension, as in a buried cable or in a bend within a splice housing, or variable tension, as due to temperature cycles, wind, or fibre payout from a bobbin or reel. However, a fibre that is subjected to a time-invariant constant applied service stress is the commonest situation for which reliability calculations are made.

The preceding static fatigue probabilities explicitly contain failure time, so that lifetime may be extracted. As an alternative to lifetime, one may calculate the failure rate per unit length and per unit time. The instantaneous value, derived from Equation (8) of Reference [20] and Equation (20) of Reference [21], is given by

$$\lambda_i(t_f) = \frac{\partial^2 F}{P \partial t_f} = \frac{-\partial^2 P}{P \partial t_f} \tag{144}$$