TECHNICAL REPORT

IEC TR 61282-3

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Fibre optic communication system design guides –

Part 3: Calculation of link polarization mode dispersion

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FIBRE OPTIC COMMUNICATION SYSTEM DESIGN GUIDES –

Part 3: Calculation of link polarization mode dispersion

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IEC 61282-3, which is a technical report, has been prepared by subcommittee 86C: Fibre optic systems and active devices, of IEC technical committee 86: Fibre optics.

This second edition cancels and replaces the first edition published in 2002. It is a technical revision that includes the following significant changes:

a) the title has been changed to better reflect its applicability to links;
b) Equations (1) and (2) were simplified in order to align with agreements in the ITU-T.

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The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
86C/701/DTR	86C/720/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts of the IEC 61282 series, published under the general title *Fibre optic communication system design guides,* can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
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INTRODUCTION

Polarization mode dispersion (*PMD*) is usually described in terms of a differential group delay (*DGD*), which is the time difference between the principal states of polarization of an optical signal at a particular wavelength and time. *PMD* in cabled fibres and optical components causes an optical pulse to spread in the time domain, which may impair the performance of a fibre optic telecommunication system, as defined in IEC 61281-1.

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FIBRE OPTIC COMMUNICATION SYSTEM DESIGN GUIDES –

Part 3: Calculation of link polarization mode dispersion

1 Scope

This part of IEC 61282 provides guidelines for the calculation of polarization mode dispersion (*PMD*) in fibre optic systems to accommodate the statistical variation of *PMD* and differential group delay (*DGD*) in optical fibre cables and components.

This technical report describes methods for calculating *PMD* due to optical fibre cables and optical components in an optical link. The calculations are compatible with those documented in the outdoor optical fibre cable specification IEC 60794-3. Example calculations are given to illustrate the methods for calculating total optical link *PMD* from typical cable and optical component data. The calculations include the statistics of concatenating individual optical fibre cables drawn from a specified distribution. The calculations assume that all components have *PMD* equal to the maximum specified value.

The calculations described cover first order *PMD* only. The following subject areas are currently beyond the scope of this technical report, but remain under study:

– calculation of second and higher order *PMD*;
– accommodation of components with polarization dependent loss (PDL) – if it is assumed that PDL is negligible in optical fibre cables;
– system impairments (power penalty) due to *PMD*;
– interaction with chromatic dispersion and other nonlinear effects.

Measurement of *PMD* is beyond the scope of this technical report. Methods of measurement of *PMD* of optical fibre and cable are given in IEC 60793-1-48. The measurement of optical amplifier *PMD* is in IEC 61290-11-1. The measurement of component *PMD* is in IEC 61300-3-32. Measurement of link *PMD* is given in 61280-4-4. A general theory and guidance on measurements is given in 61282-9.

2 Basic design models for total system *PMD* performance

2.1 Notation

For cabled fibre and components with randomly varying *DGD*, the *PMD* frequency domain measurement is based on averaging the individual *DGD* values for a range of wavelengths. The probability density function of *DGD* values is known to be Maxwell for fibre, and is assumed to be Maxwell, in effect, for components. The single parameter for the Maxwell distribution scales with the *PMD* value.

For long fibre and cable (typically longer than 500 m to 1 000 m), the *PMD* value is divided by the square root of the length to obtain the *PMD* coefficient. For components, the *PMD* value is reported without normalization. The following terms and meanings will be used to distinguish the various expressions:

– *DGD* value — The differential group delay at a time and wavelength (ps)
– *PMD* value — The wavelength average of *DGD* values (ps)
– *PMD* coefficient — The length normalised *PMD* (ps/sqrt(km))
– *DGD* coefficient — The length normalised *DGD* (ps/sqrt(km))

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NOTE The term "*DGD* coefficient" is used only in some of the calculations. The physical square root length dependence of the *PMD* value does not apply to *DGD*.

Deterministic components are those for which the *DGD* may vary with wavelength, but not appreciably with time. The variation in wavelength may be complex, depending on the number and characteristics of the sub-components within. For these types of components, either the maximum *DGD* is reported or the wavelength average is reported as the *PMD* value. For components with multiple paths, such as an optical demultiplexer, the maximum *DGD* of the different paths should be reported as the *PMD* value.

2.2 Calculation of system *PMD*

PMD values of randomly varying elements can be added in quadrature. Annex A shows the basis of this, as well as one basis for concluding that the Maxwell distribution is appropriate to describe the distribution of *DGD* values. Annex A describes the concatenation in terms of the addition of rotated polarization dispersion vectors (pdv) which are, for randomly varying components, assumed to be random in magnitude and direction over both time and wavelength.

For deterministic components, the evolution of the pdv with wavelength may be quite complex, but for each wavelength, there is a value that does not vary appreciably with time. Analysis of the relationships in Annex A shows that deterministic components that are randomly aligned in combination with random elements behave like random components.

For randomly varying components such as fibre, the statistics of *DGD* variation imply that there is little wavelength dependence of the *PMD* value. This leads to an equivalence between *PMD* measurement methods such as Jones Matrix Eigenanalysis (JME) and interferometric methods (IT) where the wavelength ranges of the two are different. For deterministic elements, there can be distinct dependence of both the *DGD* and *PMD* on the wavelength range. Therefore for these elements, when doing calculations which combine both randomly varying and deterministic elements, the combined values are only representative of the wavelength overlap.

The relationships of Annex A also show an analysis for an assumption that the deterministic components are randomly aligned. For this assumption, the *DGD* values are time randomised across the wavelengths by the fibre. The random alignment of these components with respect to the other elements leads to the following conclusions for deterministic components.

– The quadrature addition of *PMD* values can be used to calculate the contribution to system *PMD*.
– The Maxwell distribution can conservatively be used to describe the variation in *DGD* across time and wavelength.

The following two subclauses provide equations to calculate: a) the maximum *PMD* value for the link, b) the maximum *DGD* value for the link. In both cases, the maximum is defined in terms of a probability level that takes into account the statistics of the concatenation of individual cables drawn from a specified distribution of optical fibre cable. For maximum *DGD*, these statistics are combined with the Maxwell statistics of *DGD* variation. Clause 3 provides methods of calculating the relevant statistics for the contribution of optical fibre cable, which are used in combination with the component values below.

2.2.1 Link maximum *PMD*

The total maximum *PMD* value of a fibre optic system including optical fibre cables and other optical components is given by the following:

$$PMD_{\text{tot}} = \left[L_{\text{link}} PMD_Q^2 + \sum_i PMD_{Ci}^2 \right]^{1/2} \quad (1)$$

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where

PMD_{tot} is the total link PMD value (ps);

PMD_Q is the link design value of the concatenated optical fibre cable (ps/√km);

L_{link} is the link length (km);

PMD_{Ci} is the PMD value of the i^{th} optical component (ps);

The link design value, PMD_Q, (see 3.1) defines a maximum in terms of the probability, Q, for links with at least M individual cable sections.

NOTE The PMD_Q parameter is not related to the Q factor used in bit error ratio calculations.

For a link for which the individual cabling sections have been measured, the term $L_{link} PMD_Q^2$ can be replaced by $\sum_i PMD_i^2$, where PMD_i is the PMD value (ps) of the i^{th} section.

The validity of these equations has been demonstrated empirically for systems composed of concatenated optical fibre cables [2].

2.2.2 Calculation of system maximum *DGD*

The total maximum DGD value of a fibre optic system including optical fibre cable and other optical components is given by one of the following:

$$DGD_{\text{max tot}} = \left[DGD_{\text{max}F}^2 + S^2 \sum_i PMD_{Ci}^2 \right]^{1/2} \tag{2}$$

where

$DGD_{\text{max tot}}$ is the maximum link DGD (ps);

$DGD_{\text{max}F}$ is the maximum concatenated optical fibre cable DGD (ps) (see below);

S is the Maxwell adjustment factor (see below);

PMD_{Ci} is the PMD value of the i^{th} component (ps);

For a statistical specification of optical fibre cable, the maximum DGD is defined by a probability, P_F, and reference length (see 3.2). It is computed from the convolution of the distribution of the concatenated link PMD distribution and the Maxwell distribution of DGD values. For a link that has been measured, such as described below Equation (1), the term $DGD_{\text{max}F}$ is replaced by $\left[S^2 \sum_i PMD_i^2 \right]^{1/2}$.

The S parameter relates to the probability, P_C, that a random component DGD value exceeds $S \cdot PMD_C$, assuming the Maxwell distribution. Table 1 shows the relationship of S to probability when the PMD value is defined as the wavelength average.

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Table 1 – Probability based on wavelength average

S	**Probability**
3,0	4,2E-05
3,1	2,0E-05
3,2	9,2E-06
3,3	4,1E-06
3,4	1,8E-06
3,5	7,7E-07
3,6	3,2E-07
3,7	1,3E-07
3,775	6,5E-08
3,8	5,1E-08
3,9	2,0E-08
4,0	7,4E-09
4,1	2,7E-09
4,2	9,6E-10
4,3	3,3E-10
4,4	1,1E-10
4,5	3,7E-11

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Annex B shows that the probability that a system DGD value, DGD_{tot}, exceeds DGD_{maxtot} is bounded by the sum of the two probabilities as:

$$P\left(DGD_{\text{tot}} > DGD_{\text{maxtot}}\right) \le P_F + P_C \qquad (3)$$

NOTE The notation $P(\,)$ indicates a probability statement relative to the inequality within the parenthesis.

The above equations are applicable to all links with length less than the reference length. An adjustment for longer lengths is included in 3.2. Equation (2) is relevant for the assumption that deterministic components are randomly aligned. The multiplication of the deterministic PMD values with the S parameter treats these elements as though their DGD values are distributed as Maxwell – a conservative assumption that allows the quadrature addition.

Equation (3) illustrates that the total probability of exceeding some overall maximum can be bounded by an addition that does not depend on the relative magnitude of $DGD\text{max}_F$ and $S{\cdot}PMD_C$. Given an overall probability target, one approach is to allocate half the overall allowed probability to fibre and half to components. Annex C provides a worked example.

3 Calculation of cabled fibre *PMD*

3.1 General

PMD is a stochastic attribute that varies in magnitude randomly over time and wavelength. The variation in the DGD value is described by a Maxwell probability density function that can be characterised by a single parameter, the PMD value (see Equation (16) in 3.3.1). This parameter may be the average of the DGD values measured across a wavelength band, or it may be the r.m.s. value of these DGD values, depending on the definition chosen. For mode coupled fibre, the PMD coefficient is the PMD value divided by the square root of length.

In accordance with the outdoor Sectional Specification for outdoor optical fibre cable, IEC 60794-3, the PMD of cabled fibre should be specified/characterised on a statistical basis, not on an individual fibre basis. Two methods for this specification are proposed: Method 1 can be used to obtain PMD_Q, used in 2.2.1, and Method 2 can be used to obtain $DGD_{\text{max}F}$ and P_F, used in 2.2.2.

In the ITU Recommendation G.652 (and others), Method 1 forms a normative requirement and Method 2 is used to determine functionality for system performance, which is specified in terms of DGD_{maxtot} in Recommendations such as G.691 or G.959.1.

Subclause 3.4 shows how specification values for each method can be selected so the two methods are nearly equivalent.

Method 1 relies on the fact that the mean PMD coefficient of an optical link is the root mean square (quadrature average) of the mean PMD coefficients of the cabled fibres comprising the link. Method 2 assumes the same relationship.

Let x_i and L_i be the PMD coefficient (ps/√km) and length, respectively, of a fibre in the i^{th} cable in a concatenated link of N cables. The PMD coefficient, x_N (ps/√km), of this link is:

$$x_N = \left[\frac{\sum_{i=1}^{N} L_i x_i^2}{\sum_{i=1}^{N} L_i} \right]^{1/2} = \left[\frac{1}{L_{\text{Link}}} \sum_{i=1}^{N} L_i x_i^2 \right]^{1/2} \tag{4}$$

If it is assumed that all cable section lengths are less than some common value, L_{Cab}, and simultaneously reducing the number of assumed cable sections to $M = L_{\text{Link}}/L_{\text{Cab}}$, then, for a link comprised of equal-length cables, $L_i = L_{\text{cable}}$, Equation (4) becomes

$$x_N \leq x_M = \left[\frac{L_{\text{Cab}}}{L_{\text{Link}}} \sum_{i=1}^{M} x_i^2 \right]^{1/2} = \left[\frac{1}{M} \sum_{i=1}^{M} x_i^2 \right]^{1/2} \tag{5}$$

The variation in the concatenated link PMD coefficient, x_M, will be less than the variation in the individual cable sections, x_i, because of the averaging of the concatenated fibres.

Method 1 should be used with Equation (1) of 2.2.1. In Method 1, the manufacturer supplies a maximum PMD link design value, PMD_Q, that serves as a statistical upper bound for the PMD coefficient of the concatenated fibres comprising an optical cable link. For this case, the upper bound for the PMD value of the concatenation of optical fibre cables, $PMD_{F\text{Tot}}$, in Equation (1) becomes:

$$PMD_{F\text{Tot}} = PMD_Q \sqrt{L_{\text{Link}}} \tag{6}$$

Unless otherwise specified in the cabled fibre detail specification, the PMD link design value shall be less than 0,5 ps/√km, and the probability that a PMD coefficient of a link comprised of at least 20 cables will exceed the link design value shall be less than 10^{-4}. The link design value shall be computed using a method agreed upon between the buyer and cable manufacturer (see 3.2 for examples).

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Because Method 1 provides a statistical upper bound on the *PMD* of concatenated links, approved *PMD* measurement methods can be used on installed cable links to determine whether their *PMD* complies with the statistical upper bound stated by the manufacturer. Furthermore, the upper bound can be used to compute the effect of the link *PMD* on the performance of any type of transmission system and is a more realistic indication of the maximum *PMD* likely to be encountered in a concatenated link than the value that would be obtained using a worst-case *PMD* value.

Method 2 should be used with Equations (2) and (3) of 2.2.2. Method 2 combines the *PMD* density function of the concatenated links with the Maxwell probability density function of *DGD* values to compute an estimate of the probability that the *DGD* of a concatenated link at a given wavelength exceeds a specified value for a defined reference link.

The specification is that the probability that the *DGD* over the link exceeds a given value, $DGD\text{max}_F$, shall be less than some maximum, P_F. One useful reference system consists of a concatenated link of 400 km comprised of forty 10 km cable sections. For such a reference link, a value such as $DGD_{\text{max}F}$ = 25 ps for $P_F \leq 6{,}5\times10^{-8}$ has been used to justify the maximum value on PMD_Q of 0,5 ps/km$^{1/2}$. For other lengths and bit rates, different values have been selected in the ITU Recommendations and IEC Specifications.

NOTE Subclause 3.4 shows conditions under which the specifications of the two methods are nearly equivalent.

3.2 Method 1: Calculating PMD_Q, the *PMD* link design value

3.2.1 Determining the probability distribution of the link *PMD* coefficients

Equation (5) shows that the *PMD* coefficient, x_M, of a particular concatenated link can be derived from the *PMD* coefficients of the individual cable sections, x_i, comprising that link. The probability distribution of the link *PMD* coefficients depends on the distribution of the cable *PMD* coefficients and the number of cable sections comprising the link.

The following clauses describe three methods that can be used to estimate the distribution of the link *PMD* coefficients. One method is numerical [1][1], and two are analytic [4]. Of the two analytic methods, the first assumes a specific analytic function for the distribution of the cable *PMD* coefficients, while the second method makes no such assumptions but invokes an extension of the central limit theorem.

3.2.1.1 Monte Carlo numeric method [1]

The Monte Carlo method can be used to determine the probability density, f_{link}, of the concatenated link *PMD* coefficients without making any assumption about its form. This method simulates the process of building links by sampling the measured cable population repeatedly. *PMD* coefficients are measured on a sufficiently large number of cabled fibres so as to characterise the underlying distribution. This data is then used to compute the *PMD* coefficient for a single fibre-path in a concatenated link.

Computation of the link *PMD* coefficient is made by randomly selecting *M* values from the measured cabled *PMD* coefficients, and adding them on an r.m.s. basis (in quadrature) according to Equation (5). The computed link *PMD* coefficient is placed in a table or a histogram of values derived from other random samplings. The process is repeated until a sufficient number of link *PMD* values has been computed to produce a high density (0,001 ps/√km) histogram of the concatenated link *PMD* coefficient distribution. If used directly, without any additional characterization, the number of resamples should be at least 100 000.

[1] Figures in square brackets refer to the Bibliography.

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Because of the central limit theorem, the histogram of link *PMD* coefficients will tend to converge to distributions that can be described with a minimum of two parameters. Hence, the histogram can be fit to a parametric distribution that enables extrapolation to probability levels that are smaller than what would be implied by the sample size. The two parameters will invariably represent two aspects of the distributions: the central value and the variability about the central value. A choice of probability distributions can be made on the basis of the shape of the histogram. Typical distributions could include lognormal (the log of the link *PMD* coefficients is Gaussian) or one that is derived from the Gamma distribution.

3.2.1.2 Gamma distribution analytic method [4]

The Gamma family of distributions can often be used to represent the distributions of both the measured cable *PMD* coefficients and the link *PMD* coefficients. If one assumes that the square of the measured cable *PMD* coefficients, x_i, is distributed as a Gamma random variable, the probability density of the cabled *PMD* coefficients is given by

$$f_{\text{cable}}(x;\alpha,\beta) = \frac{2\beta^{\alpha} x^{2\alpha-1}}{\Gamma(\alpha)} \exp\left(-\beta x^2\right) \tag{7}$$

where x is a possible value of the cable *PMD* coefficient, $\Gamma(\ )$ is the Gamma function, and the two parameters α and β control the shape of the density. Standard fitting techniques, such as the method of maximum likelihood, can be used to fit Equation (7) to measured cable *PMD* data to find values for α and β.

The probability density of the link *PMD* coefficients, x_M, of M concatenated equal cable lengths has the same form as Equation (7), but with α and β replaced by $M\alpha$ and $M\beta$:

$$f_{\text{Link}}(x;\alpha,\beta,M) = \frac{2(M\beta)^{M\alpha} x^{2M\alpha-1}}{\Gamma(M\alpha)} \exp\left(-M\beta x^2\right) \tag{8}$$

Consequently, the α and β parameters found by fitting Equation (7) to the measured cable *PMD* coefficients can be used in Equation (8) to describe the probability density of the link *PMD* coefficients.

3.2.1.3 Model-independent analytic method [4]

A more general alternative to the one described in 3.2.1.2 can be used that does not make any assumptions regarding the form of the density function that describes the measured *PMD* coefficients of the cabled fibre.

After measuring the *PMD* coefficients, x_i, on N cabled fibres, compute the mean, variance and third moment of their squares

$$\mu_1 = \frac{1}{N}\sum_{i=1}^{N} x_i^2 \qquad \mu_2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i^2-\mu_1\right)^2 \qquad \mu_3 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i^2-\mu_1\right)^3 \tag{9}$$

Let x_M be a random variable representing the link *PMD* coefficient of a fibre-path formed from the concatenation of M equal-length cables, and let u be a possible value of x_M. Invoking the extended central limit theorem [5], it can be shown that the distribution of the link *PMD* coefficients is approximated by:

$$f_{\text{Link}}(u;M) = \Phi[z(u)] + \frac{t}{M^{1/2}}\phi[z(u)]\left[1-z(u)^2\right] \tag{10}$$

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where

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \qquad \Phi(z) = \int_{-\infty}^{z} \phi(y)dy$$

and

$$z(u) = \left(\frac{M}{\mu_2}\right)^{1/2}\left(u^2 - \mu_1\right), \quad t = \frac{\mu_3}{6\mu_2^{3/2}}$$

Differentiating Equation (10) with respect to u provides an approximation to the link PMD coefficient probability density function.

3.2.2 Determining the value of PMD_Q

The density functions found for the link PMD coefficients using one of the three methods described in 3.2.1 can now be used to compute the PMD link design value. For a concatenated link comprised of M cables, the PMD link design value, PMD_Q, is defined as the value that the link PMD coefficient, x_M, exceeds with probability Q:

$$P\left(x_M > PMD_Q\right) = Q \tag{11}$$

It follows, that for $N > M$, the probability that x_N exceeds PMD_Q is less than Q:

$$P\left(x_N > PMD_Q\right) < Q \tag{12}$$

For discussion purposes, an assumption is made that $N \geq 20$ (the link contains at least 20 cables) and that $Q = 10^{-4}$ (the probability that the link PMD exceeds the PMD design value is less than 0,0001). The following subclauses discuss how PMD_Q can be found using the cable PMD density functions obtained in 3.2.1.

3.2.2.1 Determining the *PMD* link design value from the Monte Carlo density of 3.2.1.1

To obtain probability levels of $Q = 10^{-4}$ using a pure numeric approach requires Monte Carlo simulations of at least 10^5 samples. Once this is complete, PMD_Q can be interpolated from the associated cumulative probability density function.

Alternatively, the histogram of the link PMD coefficients can be fit with a parametric distribution to enable extrapolation to lower probability levels than the measurement resampling would otherwise allow. A choice of probability distributions can be made on the basis of the shape of the histogram. Typical distributions could include lognormal (the log of the link PMD coefficients is Gaussian) or one that is derived from the Gamma distribution. After the function is fit, the value for PMD_Q at the Q^{th} quantile can be computed.

3.2.2.2 Determining the *PMD* link design value from the Gamma density of 3.2.1.2

A good approximation for the link PMD coefficient, x_Q, for M cables at the 10^{-4} quantile is given by:

$$PMD_Q = \frac{2{,}004 + 0{,}975\sqrt{M\alpha}}{\sqrt{M\beta}} \tag{13}$$

where the α and β parameters were those found in 3.2.1.2.

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For the 288 randomly selected scaled cabled fibres reported in [6], α = 0,979 and β = 48,6, and PMD_Q = 0,20 ps/√km.

3.2.2.3 Determining the *PMD* link design value using the model-independent method of 3.2.1.3

The moments computed in 3.2.1.3 can be used to compute the link *PMD* coefficient, x_Q. For a link comprised of *M* cables, x_Q at the Q^{th} quantile can be approximated by [5]:

$$PMD_Q = \left[\mu_1 + z_Q \left(\frac{\mu_2}{M} \right)^{1/2} + \frac{\mu_3}{6M\mu_2} \left(z_Q^2 - 1 \right) \right]^{1/2} \tag{14}$$

where z_Q is the Q^{th} quantile of the standard normal distribution. For $N > M = 20$ cables and $Q=10^{-4}$, z_Q = 3,72, the *PMD* design value becomes:

$$PMD_Q = \left[\mu_1 + 0{,}832\sqrt{\mu_2} + 0{,}107 \frac{\mu_3}{\mu_2} \right]^{1/2} \tag{15}$$

For the 288 randomly selected scaled cabled fibres reported in [6],

$$\mu_1 = 2{,}2 \times 10^{-2} \qquad \mu_2 = 7{,}43 \times 10^{-4} \qquad \mu_3 = 8{,}26 \times 10^{-5}$$

and Equation (15) produces PMD_Q = 0,23 ps/√km.

3.3 Method 2: Calculating the probability of exceeding DGD_{max}

PMD induced impairment of an optical signal occurs when the *DGD* at the signal's wavelength is too high. Since *DGD* varies randomly with time and wavelength, some means of imposing an upper limit, defined in terms of a low probability value, is necessary for system design. This upper limit is usually associated with a receiver sensitivity penalty. The probability can be associated with a potential *PMD*-induced impairment time (min/year/circuit). (See Annex D.)

One means of calculating an upper limit on *DGD* is to multiply the upper limit on the *PMD* value by a Maxwell adjustment factor, i.e. 3 (see Table 1 of 2.2.2). This could also be done with the upper limit represented by PMD_Q. When this is done, one is in effect assuming that the bulk of the distribution is very close to the upper limit. In reality, the bulk of the distribution is usually well away from the upper limit. Method 2 is intended to provide metrics and methods to take this into account. Because Method 2 takes into account the statistics of the individual optical fibre cables and their concatenation, as well as the combined statistics of *DGD* variation, the values calculated for system design, Equation (2), are substantially reduced from the "worst-case" values – both in the value of maximum *DGD* and the probability of exceeding it.

The following figure shows several distributions of concatenated link *PMD* coefficient. Each distribution just passes the default criteria of M = 20, Q = 10^{-4}, and $PMD_Q \leq$ 0,5 ps/√km.

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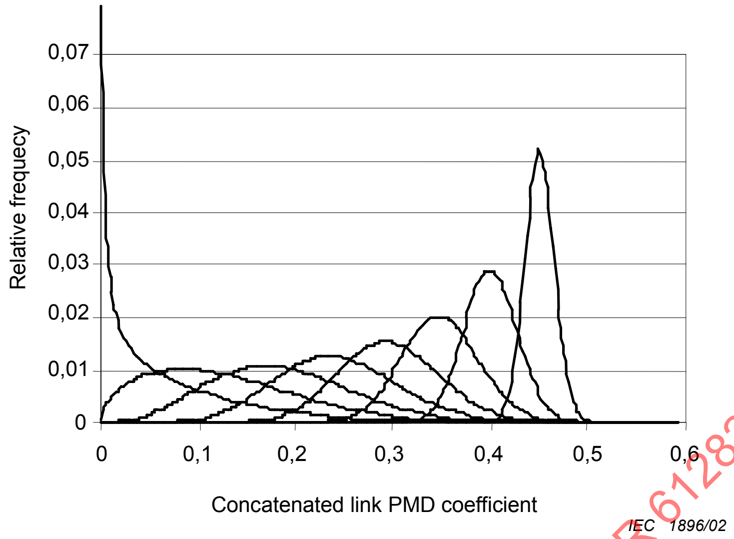

NOTE The above distributions are representative of the Gamma type distribution defined in 3.2.1.2.

Figure 1 – Various passing distributions

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The leftmost distributions should provide better *DGD* performance than the rightmost distributions. Method 2 assigns value to producing a distribution that is more to the left. 3.3.1 provides a means to link Method 1 and Method 2 so that, for most practical situations, passing the default Method 1 criterion will imply passing a default Method 2 criterion.

3.3.1 Combining link and Maxwell variations

DGD coefficient (ps/√km) values, X_M, vary randomly with time and wavelength according to the Maxwell probability density function:

$$f_{\mathrm{Max}}(X_M; x_M) = 2\left(\frac{4}{\pi x_M^2}\right)^{3/2} \frac{X_M^2}{\Gamma(3/2)} \exp\left[-\frac{4}{\pi}\left(\frac{X_M}{x_M}\right)^2\right] \quad (16)$$

where x_M is the *PMD* coefficient of a concatenated link comprised of M cables as given by Equations (4) or (5). The distribution of *DGD* values (over the length) is obtained by multiplying the *DGD* coefficient values with the square root of the link length.

To combine the variations in the concatenated link *PMD* coefficient with the Maxwell variation into a value to be used in a system design, a reference link is defined. Performance on the reference link can then be generalised to other links. The reference link is defined with two parameters, the overall link length, L_{REF}, and the cable section length, L_{Cab}, which is assumed to be constant for all cable sections.

Let $f_{\mathrm{Link}}(xi)$ be the discretised probability density function (histogram) of the values of the concatenated link *PMD* coefficient values defined by the analysis of the distribution of the measured *PMD* coefficient values and Equation (4). Any of the methods of 3.2.1 for determining the probability density function of the concatenated link can be used.

Let X_{max} be some DGD coefficient value (ps/√km) that is to be used in system design. The probability, P_F, of exceeding X_{max} is:

$$P_F = \sum_i f_{\text{Link}}(x_i)\left[1 - \int_0^{X_{max}} f_{\text{Max}}(Y;x_i)dY\right] = \sum_i f_{\text{Link}}(x_i)\left[1 - \int_0^{\frac{4}{\pi}\left(\frac{X_M}{x_M}\right)^2} \frac{y^{3/2-1}}{\Gamma(3/2)}\exp(-y)dy\right] \quad (17)$$

NOTE The rightmost integral is just the standard gamma function that can be calculated within many spreadsheets.

The maximum DGD over the link derived from optical fibre cable, $DGD\text{max}_F$, is the product of the square root of the reference link length and X_{Max}:

$$DGD_{\text{max}\,F} = X_{\text{Max}}\sqrt{L_{\text{REF}}} \quad (18)$$

For Method 1, the probability is pre-set and the associated PMD_Q value is calculated and required to be less than a specified value. For Method 2, $DGD_{\text{max}F}$ is pre-set and the probability value, P_F is calculated and required to be less than a specified value.

For link lengths less than the reference length, the DGD and probability relationship will be conservative as long as either the installed lengths are less than L_{Cab} or the cable lengths measured to obtain the distribution are less than L_{Cab}. The reduction in averaging because of the reduced number of cable lengths is offset by the decrease in overall length. For link lengths greater than the reference length, the maximum DGD to optical fibre cable should be adjusted as:

$$DGD_{\text{adj}\,F} = DGD_{\text{max}\,F}\sqrt{\frac{L_{\text{Link}}}{L_{\text{REF}}}} \quad (19)$$

3.3.2 Convolution: Theory of Method 2

The calculation principle is derived from extending the worst case approach. With this approach, the link PMD distribution is assumed to be a Dirac function and the DGD distribution is represented as a Maxwell distribution. The probability that the Maxwell distribution exceeds $DGD_{\text{max}F}$ yields P_F. These distributions are represented in Figure 2.

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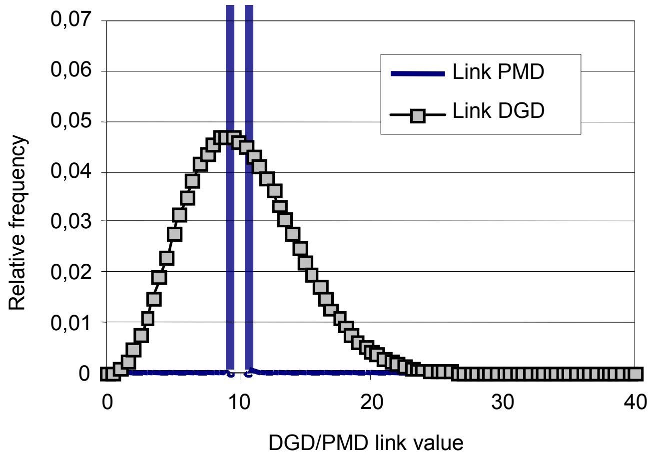

IEC 1897/02

Figure 2 – Worst case approach assumption

NOTE Though not shown, the Dirac function illustrated in Figure 2 extends to a relative frequency value of 1,0.

Suppose the link *PMD* distribution could be represented by two Dirac functions, each with a magnitude of 0,5. This would represent a situation where half the links were at one value and the other half at another value. The *DGD* probability density function of the combined distribution would be the weighted total of the two individual Maxwell distributions. Figure 3 illustrates this case.

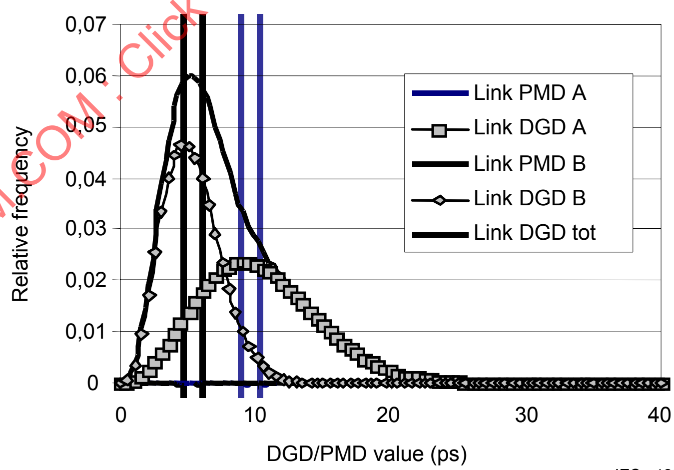

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Figure 3 – Convolution of two Diracs

NOTE Though not shown, the two Dirac functions representing *PMD* value distributions in Figure 3 extend to relative frequency values of 0,5.

In this example, the probability of *DGD* exceeding 30 ps is reduced by just a little less than a factor of two, compared to the result associated with Figure 2.

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Full convolution extends this notion to a complete distribution of link *PMD* coefficients. For the Monte Carlo technique, the histogram of link *PMD* coefficients may be considered as a collection of Dirac functions. For the continuous models, the probability density function is reduced to a histogram by integrating the curve over the region that is represented as a single histogram bin. The probability that $DGD_{\text{max}F}$ is exceeded is calculated for each of the histogram bins (using the bin maximum). The weighted total yields P_F.

3.4 Equivalence of methods

Method 1 might be considered most practical for commercial specification because it can be interpreted in terms of the defined measurements. Method 2 provides the most direct information on the possible signal impairments. This subclause shows how the two methods can be compared and establishes near equivalence of the default specifications.

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The method for determining equivalence of statistical criteria relies on a parametric model and is based on the following.

– A process can be characterised by parameters relevant to an assumed parametric distribution type.

– Given these parameters, any statistical criterion can be evaluated to determine whether the process distribution is conforming or not.

– For each criterion, the mathematical space of all possible parameters can be segmented into two regions: conforming and not.

– The boundary between the two regions will form a curve, or envelope, in at least two dimensions. Parameter values falling on one side of the envelope are conforming. Those on the other side are not.

– Processes that are on the conforming side of the envelopes of two criteria pass both criteria. Processes that are on the non-conforming side of the envelopes of two criteria fail both criteria.

– Criteria that pass and/or fail the same process distributions are considered equivalent. In this case, the two envelopes will overlay one another.

3.4.1 Equivalence of the default specifications

Figure 4 shows the envelopes for the default specifications for the two methods, based on the Gamma distribution as defined in 3.2.1.2 and illustrated in Figure 1. The x-axis represents the quadrature average of the process ($=\sqrt{\alpha/\beta}$). The y-axis represents a sort of standard deviation metric for the Gamma distribution.

The default criteria are:

Method 1		**Method 2**	
M	20	L_{REF}	400 km
Q	10^{-4}	L_{Cab}	10 km
PMD_Q	≤ 0,5 ps/√km	$DGD\text{max}_F$	25 ps
P_F	≤ 6,5 × 10^{-8}		

Each envelope of Figure 4 is built by looping through the possible values of the overall process quadrature average. For each possible value, the β parameter is varied to find the value that just passes the relevant specification. A plot of the relationship of the overall process quadrature average versus $1/2\sqrt{\beta}$ yields the envelope. Processes for which the parameters fall below the envelope pass the specification. Processes for which the parameters fall above the envelope fail.

For the region of most practical interest – where the overall process quadrature average is less then 0,2 ps/√km – if the process passes the default method 1 specification, it is passing the default method 2 criterion. Hence, the method 2 parameters can safely be used in the formulas of 2.2.1 and 2.2.2

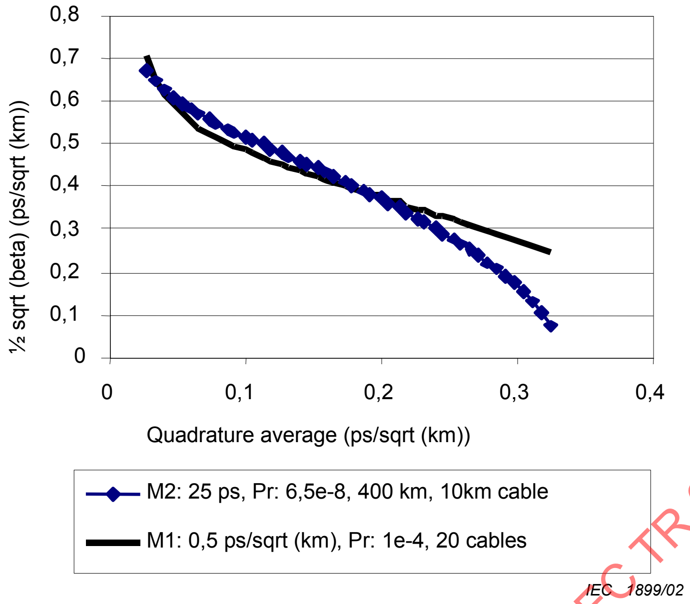

Figure 4 – Equivalence envelopes for method 1/2 defaults

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3.4.2 Discussion regarding the basis of the default specifications for Method 2

Equation (3) of 2.2.2 shows that the overall probability that the combined *DGD* of optical fibre cable and components exceeds DGD_{maxTot} is contained by the sum of the probabilities: P_F and P_C. If these probabilities are both set to 6,5 × 10^{-8}, their sum is 1,3 × 10^{-7}, a value that should provide an appropriately low potential *PMD* induced impairment time (see Annex D).

The default specification for Method 1 was agreed on the basis of a combination of factors including an analysis of the ITU-T system Recommendations in conjunction with the maximum *DGD* that is implied..

The default specification for Method 2 was derived so that:

- the above probability objective was met;
- near equivalence with the default Method 1 specification was achieved;
- $DGD\mathrm{max}_F$ is low enough to allow practical system designs. (See Annex C for a worked example.)

3.4.3 Calculation of the parameters of Figure 4

This calculation for the parameters of the Gamma-type distribution defined in 3.2.1.2 is called the method of moments. Because it is based on the assumption that $M\alpha > 5$, it is best to use it in conjunction with the Monte Carlo method defined in 3.2.1.1.

Define x_I to be one of N computed concatenated link *PMD* coefficient values based on M cables per link. The quadrature average, v, is calculated as:

$$v = \left(\frac{\alpha}{\beta}\right)^{1/2} = \left(\frac{1}{N}\sum_{i=1}^{N} x_i^2\right)^{1/2} \qquad (20)$$

The parameter relating to the standard deviation is calculated as:

$$\frac{1}{2\sqrt{M\beta}} = \left[\frac{1}{N-1} \sum_{i=1}^{N} (x_i - v)^2 \right]^{1/2} \tag{21}$$

Multiplying the result of Equation (21) by $\sqrt{M}$ will yield the value to be plotted on the y-axis of Figure 4. One can also easily compute the individual values of α and β using the values of Equations (20) and (21). Since the result is based on the Monte Carlo, the assumption $M\alpha > 5$ can always be met by increasing M.

4 Calculation of optical component *PMD*

Optical components such as dispersion accommodation devices or optical fibre amplifiers also have small *PMD*. These devices are characterised as random or deterministic. The distinction is primarily based on the behaviour of the curve of *DGD* versus wavelength and how this curve varies with time.

For random components, small changes in temperature will produce random variations in the *DGD* curve versus time. When the collection of *DGD* values are plotted with a histogram, the histogram will follow the Maxwell distribution. Some dispersion accommodation devices have been characterised as random [7].

For deterministic components, small changes in temperature will not produce random variations in the relationship of *DGD* versus wavelength. Some optical fibre amplifier devices have been characterised as deterministic [8].

Some deterministic components are simple in that they typically include a few individual optical elements and the *DGD* versus wavelength curve is either flat or has a simple sinusoidal shape. Some deterministic components are complex in that they might be comprised of several individual optical elements and the *DGD* curve versus wavelength is irregular. For these components, a histogram of the *DGD* values will begin to approach a Maxwell distribution.

For both types of component, there are at least two metrics of particular interest:

– *PMD* value: the average of *DGD* values across a specified wavelength range;

– DGD_{max}: the maximum *DGD* across a specified wavelength range.

The usual form of specification is just a maximum value, although statistical treatments, such as those defined for optical fibre cable, could be defined for component *PMD*. The number and types of components would make standardization difficult.

The concatenation formulas are given in 2.2.1, Equation (1).

5 Summary of acronyms and symbols

Table 2 contains a list of the acronyms and definitions used in the body of this technical report. Table 3 contains a list of symbols and the clause in which they are defined.

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Table 2 – Acronyms and definitions

Acronym	Definition
PMD	Polarization mode dispersion
DGD	Differential group delay
Pdv	Polarization dispersion vector
PMD_Q	Link design value
$DGD_{\mathrm{max}F}$	Maximum *DGD* induced by optical fibre cable
DGD_{maxTot}	Maximum *DGD* of the link

Table 3 – Symbols and clause of definition

Symbol	Defining subclause
PMD_{Tot}	2.2.1
PMD_Q	2.2.1
Q	2.2.1
L_{Link}	2.2.1
PMD_{Ci}	2.2.1
PMD_{Dj}	2.2.1
$PMD_{D\mathrm{last}}$	2.2.1
M	2.2.1
DGD_{maxtot}	2.2.2
$DGD_{\mathrm{max}F}$	2.2.2
$DGD_{\mathrm{max}Dj}$	2.2.2
$DGD_{\mathrm{max}D\mathrm{last}}$	2.2.2
P_F	2.2.2
P_C	2.2.2
S	2.2.2
X_N	3.1
X_M	3.1
L_i	3.1
L_{Cab}	3.1
$PMD_{F\mathrm{Tot}}$	3.1
f_{cable}	3.2.1.2
f_{Link}	3.2.1.2
α, β, Γ	3.2.1.2
μ_1, μ_2, μ_3, Φ, φ, z	3.2.1.3
X_{Max}	3.3.1
$DGD_{\mathrm{adj}\ F}$	3.3.1
L_{REF}	3.3.1
v	3.4.3

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Annex A
(informative)

PMD concatenation fundamentals

This annex describes the mathematics for concatenating fibre optic elements (fibre or components) in terms of the polarization dispersion vector, as described by Foschini and Poole [9]. The fundamental relationships are defined, followed by developments related to concatenation of first random then deterministic elements.

A.1 Definitions

The polarization dispersion vector, Ω, and birefringence vector, W, are defined by the evolution of the output stokes vector, s, as a function of position within the element, z, and optical angular frequency, ω. For any position and frequency, there is a rotation matrix, R, also a function of position and frequency, that maps an input stokes vector, s_0, to the output as:

$$s = Rs_0 \tag{A.1}$$

Using the notation, R^T, for the transpose (and inverse) of R, the derivatives of s with respect to position and frequency, assuming that s_0 does not depend on frequency, are given as:

$$\frac{ds}{dz} = \frac{dR}{dz} R^T s = W \times s \tag{A.2}$$

$$\frac{ds}{d\omega} = \frac{dR}{d\omega} R^T s = \Omega \times s \tag{A.3}$$

These equations define W and Ω. The following additional relationships are useful:

$$\frac{d\Omega}{dz} = \frac{dW}{d\omega} + W \times \Omega \tag{A.4}$$

$$\Delta\tau = \|\Omega\| \tag{A.5}$$

Equation (A.4) can be derived from Equations (A.2) and (A.3). Equation (A.5) gives the *DGD* as the length of the polarization dispersion vector.

For W fixed over position, the rotation matrix is defined in terms of a unit rotation vector, y, and angular displacement, γ, as:

$$y = \frac{W}{\|W\|} \qquad \gamma = z\|W\| \tag{A.6}$$

$$R = yy^T(1 - \cos(\gamma)) + I\cos(\gamma) + [y\times]\sin(\gamma) \tag{A.7}$$

where I is the identity matrix and $[y\times]$ is the matrix that completes the cross product operation.

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Furthermore, for this case, the pdv can be written in terms of W and various cross products.

$$f^2 = W \cdot W \qquad P = W \cdot \frac{dW}{d\omega} \tag{A.8}$$

$$\Omega = \frac{1}{f^2}\left[zPW + (1-\cos(fz))W \times \frac{dW}{d\omega} + \frac{\sin(fz)}{f}\left(f^2 \frac{dW}{d\omega} - PW\right)\right] \tag{A.9}$$

A.2 Concatenation – General

The concatenation of two elements, A through B, can be given in terms of the initial and final stokes vectors as:

$$s_{AB} = R_B R_A s_0 \tag{A.10}$$

The derivative of Equation (A.10) is

$$\frac{ds_{AB}}{d\omega} = \frac{dR_B}{d\omega} R_A s_0 + R_B \frac{dR_A}{d\omega} s_0 \tag{A.11}$$

$$= \frac{dR_B}{d\omega} R_B^T R_B R_A s_0 + R_B \frac{dR_A}{d\omega} R_A^T R_B^T R_B R_A s_0 \tag{A.12}$$

$$= \Omega_B \times s_{AB} + R_B\left(\Omega_A \times R_B^T\right) s_{AB} \tag{A.13}$$

$$= \left(\Omega_B + R_B \Omega_A\right) \times s_{AB} \tag{A.14}$$

Since (A.14) is in the form of the definition of the pdv, the pdv for AB, Ω_{AB}, is:

$$\Omega_{AB} = \Omega_B + R_B \Omega_A \tag{A.15}$$

Equation (A.15) can be extended through an arbitrary number of optical elements by recursive application of rotation on the last output and addition.

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A.3 Application to random elements

Foschini and Poole [9] concluded that for long enough lengths of optical fibre over which mode coupling is occurring, the contents of the polarization dispersion vector can be described as Gaussian identically distributed independent random variables. Extension of Equation (A.15) leads to the conclusion that the pdv of the concatenation will also have the same distribution, but with increased variance due to the addition of several random vectors. (The random rotation of a Gaussian random vector is a Gaussian random vector with the same variance.) In particular, define the variance of the vector element values of the ith pdv as $\sigma_i^2/3$. The variance of the vector element values of the concatenated pdv are given as:

$$\sigma_{\text{tot}}^2 = \sum_i \sigma_i^2 / 3 \tag{A.16}$$

This is related to the *PMD* (r.m.s.) value as:

$$PMD_{\text{tot}} = \left\langle \Delta\tau^2 \right\rangle^{1/2} = \langle \Omega \bullet \Omega \rangle^{1/2} = \left(3\sigma_{\text{tot}}^2\right)^{1/2} = \left(\sum_i \sigma_i^2\right)^{1/2} \qquad \text{(A.17)}$$

The same relationship occurs on the individual sections with $PMD_i = \sigma_i$ so the quadrature addition of the *PMD* values of individual random elements is justified for the concatenated *PMD* value. If one assumes that the *PMD* of the several elements are equal and that their lengths are also equal, Equation (A17) yields the square root dependence of *PMD* on overall length – hence the square root length normalization used for the *PMD* coefficient.

The *DGD* values are the length of the pdv. If the vector element values are Gaussian independent identically distributed random values, the *DGD* is the square root of the sum of squares of three Gaussians. This is the definition of the Maxwell distribution, which is also the square root of a chi-square random variable with three degrees of freedom.

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A.4 Application to deterministic elements

If the values of the birefringence vector, *W*, of several deterministic components are aligned just right, the sum of the pdv of the concatenation will just be the linear sum of the individual pdvs. In this case, the components are said to be aligned.

Analysis of Equation (9) for the case of these components following a series of random elements leads to adding the *PMD* of the random elements to the linear addition of the deterministic components. Similarly, the worst case random *DGD* is added to the sum of worst case deterministic *DGD* values.

When the pdvs of the deterministic elements are randomly aligned, the rotations will become random and the length of the concatenated pdv will form a distribution. This distribution will be representative of the *DGD* values across wavelengths and concatenations, but for a given concatenation of deterministic elements, the particular values are not expected to change with time. To evaluate this distribution, a series of Monte Carlos were completed. The different Monte Carlos each represent the concatenation of a different number of randomly oriented vectors of a common length. The vector length, L_v, is given in terms of the number of vectors concatenated, n_v, as:

$$L_v = \frac{1}{\sqrt{n_v}} \qquad \text{(A.18)}$$

The quadrature sum of the individual pdv values is therefore one and the worst case orientation value is equal to $\sqrt{n_v}$. This allows overlaying the distributions formed by adding different numbers of components and a comparison with a Maxwell distribution from a random component with a *PMD* value of one. Figure A.1 shows the result laid out as a series of histograms. Figure A.2 shows the cumulative probability in the tail of the distributions.

For all but the Maxwell distribution, the cumulative probabilities are bounded by $\sqrt{n_v}$, as expected. In addition, for all cases, the Maxwell distribution extends to larger values than the rotated vector result. This is the sense in which the Maxwell distribution is conservative with regard to estimating the distribution of randomly oriented but fixed length vectors.

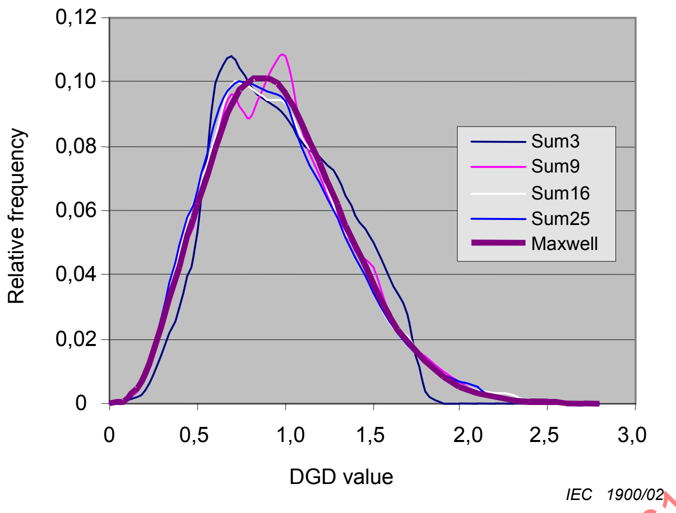

Figure A.1 – Sum of randomly rotated elements

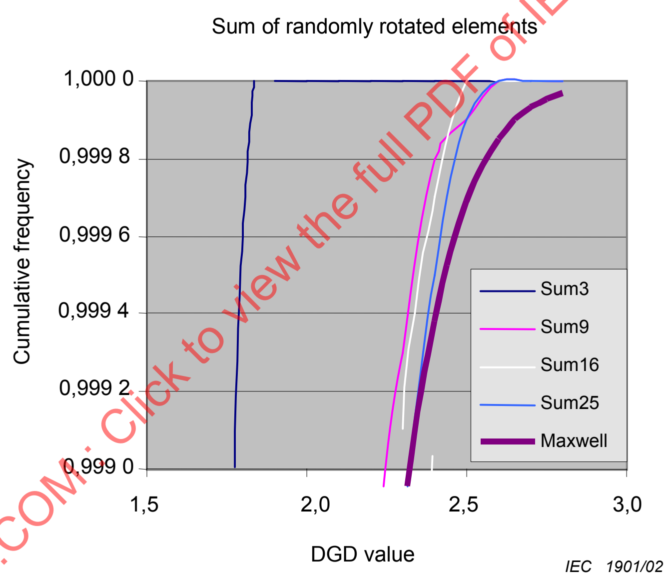

Figure A.2 – Sum of randomly rotated elements

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So the application of the Maxwell distribution in connection with the *PMD* value will yield a maximum estimated *DGD* that is larger than actual for a concatenation of randomly oriented deterministic elements of equal *PMD*. Since, for most link designs, the value of the deterministic components is assumed to be the maximum specified value, this equal size assumption is also conservative.

For a particular orientation on a particular link, the concatenation of deterministic components will be the same over time and this might be a concern. Examination of Equation (9) yields a surprising result in this regard. If a deterministic element is combined with a fibre or other random component, the rotation (R_B) of that element is applied to the pdv of the deterministic component. Since this rotation is random, the effect is to randomise the effect of the deterministic element over both time and wavelength.

The overall contribution is then not fixed and it can be considered as just another random optical element – with the exception that the probability estimates will be higher than actual. The probability treatment that is applicable for random components is therefore appropriate for deterministic components.

Annex B
(informative)

Combining Maxwell extrema from two populations

B.1 General

The purpose of this annex is to demonstrate the validity of Equations (2) and (3) of Clause 2. Equation (2) combines the *DGD* extrema of two populations, optical fibre cable and random components, to obtain a combined extreme value. The two populations are each characterised by probability levels associated with the separate extrema. Inequality 3 asserts that the probability that a *DGD* value of the concatenation of the two populations exceeds the combined extreme value is bounded by the sum of the two separate probabilities.

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The analysis shows that only two assumptions are needed:

- The distribution of *DGD* is Maxwell-like in that only one parameter describes it.
- The parameter of the concatenation is given as the quadrature total of the fibre and component parameters.

This demonstration is done in four steps:

- define the Maxwell distribution;
- define a convolution;
- define a double convolution;
- valuate the characteristics of a double convolution.

B.2 Maxwell distribution definitions

The probability that a random *DGD* value, $\Delta\tau$, exceeds a given maximum *DGD*, D, depends on the *PMD* value, d, according to:

$$P(\Delta\tau > D; d) = 1 - \int_0^D 2\left(\frac{4}{\pi d^2}\right)^{3/2} \frac{x^2}{\Gamma(3/2)} \exp\left[-\frac{4}{\pi}\left(\frac{x}{d}\right)^2\right] dx \qquad \text{(B.1)}$$

To simplify notation, a function, $P_{\mathrm{Max}}(S)$, is defined as:

$$P_{\mathrm{Max}}(S) = 1 - \int_0^S 2\left(\frac{4}{\pi}\right)^{3/2} \frac{x^2}{\Gamma(3/2)} \exp\left[-\frac{4}{\pi}x^2\right] dx \qquad \text{(B.2)}$$

Then

$$P(\Delta\tau > D; d) = P_{\mathrm{Max}}\left(\frac{D}{d}\right) \qquad \text{(B.3)}$$

NOTE $P_{\mathrm{Max}}(S)$ is monotonically decreasing in S.

B.3 Convolution definition

This describes the convolution of the Maxwell distribution with a distribution of *PMD* values. The resultant distribution is not Maxwell, but does have the characteristic of describing the distribution of *DGD* values across all possible *PMD* values. The definition is done in terms of a discretised distribution, or histogram, of *PMD* values for simplicity. Since the granularity of the histogram is arbitrary and in practice, defined by the measurement resolution, the simplification, relative to using integral expressions, is warranted.

The discretised distribution of *PMD* values is described in terms of a histogram with the "bin" *PMD* values given as d_i and relative frequency given as p_i for the i^{th} bin. The combined probability that a given maximum *DGD* value, D, is exceeded is:

$$P(\Delta\tau > D) = \sum_i p_i P_{\max}\left(\frac{D}{d_i}\right) \tag{B.4}$$

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B.4 Convolution of optical fibre cable and random components

This clause outlines the notation for the double convolution of cables and random components. For optical fibre cable, the distribution of *PMD* values is that of the concatenated link, as derived from Clause 3. For random components, a distributional notation will be maintained for generality, but in practice, the specification values of all components will be combined in quadrature to form one value. The component distribution is then a histogram with only one non-zero bin.

Let f_i and p_{fi} represent the histogram of optical fibre cable *PMD* values. Let c_j and p_{cj} represent the histogram of component *PMD* values. The two distributions are assumed to be independent. Given a fibre *PMD* value, f, and a component *PMD* value, c, the *PMD* value of the concatenation, d, is assumed to follow:

$$d = \left(f^2 + c^2\right)^{1/2} \tag{B.5}$$

Further assume that the extreme values and probability limits are provided separately for optical fibre cable and components as:

$$P(\Delta\tau > F) = \sum_i p_{fi} P_{\max}\left(\frac{F}{f_i}\right) < P_F \text{ for optical fibre cable} \tag{B.6}$$

$$P(\Delta\tau > C) = \sum_j p_{cj} P_{\max}\left(\frac{C}{c_j}\right) < P_C \text{ for components} \tag{B.7}$$

The problem is to determine the probability limit, P_D, for the concatenation of cable and components relative to a maximum, D, given as:

$$D = \left(F^2 + C^2\right)^{1/2} \tag{B.8}$$

If there were access to the full distributions of both cable and components, this probability could be calculated as:

$$P_D = P(\Delta\tau > D) = \sum_i \sum_j p_{fi} p_{cj} P_{\max}\left[\frac{\left(F^2 + C^2\right)^{1/2}}{\left(f_i^2 + c_j^2\right)^{1/2}}\right] \quad \text{(B.9)}$$

Equation (B.9) is the double convolution of cable and components.

B.5 Evaluation of the double convolution

This clause will demonstrate the following relationship for the problem defined in Clause B.4.

$$P_D < P_F + P_D \quad \text{(B.10)}$$

The formulas and values for cables and components are given separately from Equations (B.6) and (B.7). These are combined and rewritten as:

$$P_F + P_C \geq \sum_i p_{fi} P_{\max}\left(\frac{F}{f_i}\right) + \sum_j p_{cj} P_{\max}\left(\frac{C}{c_j}\right) = \sum_i \sum_j p_{fi} p_{cj}\left[P_{\max}\left(\frac{F}{f_i}\right) + P_{\max}\left(\frac{C}{c_j}\right)\right] \quad \text{(B.11)}$$

The last expression in (B.11) is due to the definition of the histogram probabilities: $1 = \sum_i p_{fi} = \sum_j p_{cj}$.

The asserted inequality, (B.10), can now be written as:

$$P_D = \sum_i \sum_j p_{fi} p_{cj} P_{\max}\left[\frac{\left(F^2 + C^2\right)^{1/2}}{\left(f_i^2 + c_j^2\right)^{1/2}}\right] \leq \sum_i \sum_j p_{fi} p_{cj}\left[P_{\max}\left(\frac{F}{f_i}\right) + P_{\max}\left(\frac{C}{c_j}\right)\right] \leq P_F + P_C \quad \text{(B.12)}$$

Inequality (B.12) will be true if, for each i, j pair, the following is true:

$$P_{\max}\left[\frac{\left(F^2 + C^2\right)^{1/2}}{\left(f_i^2 + c_j^2\right)^{1/2}}\right] \leq P_{\max}\left(\frac{F}{f_i}\right) + P_{\max}\left(\frac{C}{c_j}\right) \quad \text{(B.13)}$$

Inequality (B.13) will be true if the left side is smaller than either of the terms on the right side. Since $P_{\max}(S)$ is decreasing in S, this will be true if either of the following two inequalities is true:

$$\frac{F^2 + C^2}{f_i^2 + c_j^2} \geq \frac{F^2}{f_i^2} \quad \text{(B.14)}$$

or

$$\frac{F^2 + C^2}{f_i^2 + c_j^2} \geq \frac{C^2}{c_j^2} \quad \text{(B.15)}$$

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