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TECHNICAL REPORT

Multicore and symmetrical pair/quad cables for digital communications – Part 1-2: Electrical transmission characteristics and test methods of symmetrical pair/quad cables

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MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES FOR DIGITAL COMMUNICATIONS – PART 1-2: ELECTRICAL TRANSMISSION CHARACTERISTICS AND TEST METHODS OF SYMMETRICAL PAIR/QUAD CABLES

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IEC 61156-1-2, which is a technical report, has been prepared by subcommittee 46C: Wires and symmetric cables, of IEC technical committee 46: Cables, wires, waveguides, R.F. connectors, R.F. and microwave passive components and accessories.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
46C/853/DTR	46C/889/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts of the IEC 61156 series, under the general title: *Multicore and symmetrical pair/quad cables for digital communications*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES FOR DIGITAL COMMUNICATIONS – PART 1-2: ELECTRICAL TRANSMISSION CHARACTERISTICS AND TEST METHODS OF SYMMETRICAL PAIR/QUAD CABLES

1 Scope

This technical report is a revision of the symmetrical pair/quad electrical transmission characteristics present in IEC 61156-1:2002 (Edition 2) and not carried into IEC 61156-1:2007 (Edition 3).

This technical report includes the following topics from IEC 61156-1:2002:

– the characteristic impedance test methods and function fitting procedures of 3.3.6;
– Annex A covering basic transmission line equations and test methods;
– Annex B covering the open/short-circuit method;
– Annex C covering unbalance attenuation.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-726, *International Electrotechnical Vocabulary – Part 726: Transmission lines and waveguides*

IEC 61156-1:2007, *Multicore and symmetrical pair/quad cables for digital communications – Part 1: Generic specification*

IEC/TR 62152, *Background of terms and definitions of cascaded two-ports*

3 Terms, definitions, symbols, units and abbreviated terms

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-726 and IEC/TR 62152 apply.

3.2 Symbols, units and abbreviated terms

For the purposes of this document, the following symbols, units and abbreviated terms apply.

Transmission line equation electrical symbols and related terms and symbols:

R	pair resistance (Ω/m)
L	pair inductance (H/m)
G	pair conductance (S/m)
C	pair capacitance (F/m)
α	attenuation coefficient (Np/m)
β	phase coefficient (rad/m)

γ	propagation coefficient (Np/m, rad/m)
v_P	phase velocity of cable (m/s)
v_G	group velocity of cable (m/s)
τ_P	phase delay time (s/m)
τ_G	group delay time (s/m)
Z_C	complex characteristic impedance, or mean characteristic impedance if the pair is homogeneous or free of structure (also used to represent a function fitted result) (Ω)
$\angle Z_C$	angle of the characteristic impedance in radians
Z_∞	high frequency asymptotic value of the characteristic impedance (Ω)
l	length (m)
j	imaginary denominator
Re	real part operator for a complex variable
Im	imaginary part operator for a complex variable
ω	radian frequency (rad/s)
f	frequency (Hz)
R'	first derivative of R with respect to ω
C'	first derivative of C with respect to ω
L'	first derivative of L with respect to ω
R_0	d.c. resistance of a round solid wire with radius r (Ω/m)
R_C	constant with frequency component of resistance which is about 1/4 of the d.c. resistance (Ω/m)
R_S	square-root of frequency component of resistance (Ω/m)
L_E	external (free space) inductance (H/m)
L_I	internal inductance whose reactance equals the surface resistance at high frequencies (H/m)
σ	specific conductivity of the wire material (S/m)
ρ	resistivity of the wire material (Ω/m^2)
μ	permeability of the wire material (H/m)
r	radius of the wire (m)
δ	skin depth (not to be confused with the dissipation factor tan δ) (m) $\delta = \frac{1}{\sqrt{\pi f \mu\sigma}}$
tan δ	dissipation factor tan $\delta = G/(\omega C)$
q	forward echo coefficient at the far end of the cable at a resonant frequency
p	reflection coefficient measured from the near end of the cable at a resonant frequency, $p = 10^{-PSRL/20} = \left\|\frac{Z_{CM} - Z_C}{Z_{CM} + Z_C}\right\|$

A_Q forward echo attenuation at a resonant frequency (dB)

$A_Q = -20 \log |q|$

$PSRL$ structural return loss at a resonant frequency (dB)

$PSRL = -20 \log |p|$

K $= 2\alpha l - 1$ when $2\alpha l >> 1$ (Np)

A_Q $= 2 \times PSRL - 20 \log(2\alpha l - 1)$ (dB) where $2\alpha l$ is in Np

Z_{OC} complex measured open circuit impedance (Ω)

Z_{SC} complex measured short circuit impedance (Ω)

Z_{CM} characteristic impedance as measured (with structure) (Ω)

$$Z_{CM} = \sqrt{Z_{SC} Z_{OC}}$$

Z_{MEAS} complex measured impedance (open or short) (Ω)

Z_{IN} input impedance of the cable when it is terminated by Z_L (Ω)

Z_{OUT} output impedance of the cable when the input of the cable is terminated by Z_G (Ω)

Z_{CN} nominal characteristic impedance of a cable and is the specified Z_C value at a given frequency with tolerance and the structural return loss SRL limits in dB in a frequency range (Ω)

Z_N nominal (reference) impedance of the link and/or terminals (the system) between which the cable is operating (Ω)

Z_R (nominal) reference impedance that is used in measurement. Normally (for actual return loss results), $Z_R = Z_N$. When using a return loss measurement to approximate SRL, it is practical to choose Z_R to give the best balance in the given frequency range (Ω)

Z_T terminated impedance measurement made with the opposite end of the cable pair terminated in the reference impedance Z_R (Ω)

ς reflection coefficient measured in the terminated measurement method

$$\varsigma = \frac{Z_R - Z_C}{Z_R + Z_C}$$

Z_G termination at the cable input when defining the output impedance of the cable Z_{OUT} (Ω)

Z_L termination at the cable output when defining the input impedance of the cable Z_{IN} (Ω)

L_0, L_1, L_2, L_3 least squares fit coefficients for angle of the characteristic impedance

K_0, K_1, K_2, K_3 least squares fit coefficients of the characteristic impedance

$|Z_C|$ fitted magnitude of the characteristic impedance (Ω)

$|Z_{CM}|$ measured magnitude of the characteristic impedance (Ω)

$\angle(V_{1N})$ input angle relative to a reference angle in radians

$\angle(V_{1F})$ output angle relative to the same reference angle in radians

k multiple of 2π radians

S_{11} reflection coefficient measured with an S parameter test set

RL	return loss (dB)
SRL	structural return loss (dB)

Attenuation unbalance electrical symbols:

TA	transverse asymmetry
LA	longitudinal asymmetry
R_1, R_2	resistance of one conductor per unit length (Ω)
L_1, L_2	inductance of one conductor per unit length (H)
C_1, C_2	capacitance of one conductor to earth (F)
G_1, G_2	conductance of one conductor to earth (S)
α_u	unbalance attenuation (dB)
T_u	unbalance coupling transfer function
Z_{com}	characteristic impedance of the common-mode circuit (Ω)
Z_{diff}	characteristic impedance of the differential-mode circuit (Ω)
Z_{unbal}	unbalance impedance (Ω)
ℓ	length of transmission line (m)
x	length coordinate (m)
γ_{com}	propagation factor of the common-mode circuit (Np/m, rad/m)
γ_{diff}	propagation factor of the differential-mode circuit (Np/m, rad/m)
α_{diff}	operational differential-mode attenuation of the cable (dB)
α_{com}	operational common-mode attenuation of the cable (dB)
ΔR	resistance unbalance of the sample length (Ω)
ΔL	inductance unbalance of the sample length (H)
ΔC	capacitance unbalance to earth (F)
ΔG	conductance unbalance to earth (S)
S	summing function
U_{diff}	voltage in the differential-mode circuit (V)
U_{com}	voltage in the common-mode circuit (V)
n, f	index to designate the near end and far end, respectively

4 Basic transmission line equations

4.1 Introduction

A review of the relationships between the propagation coefficient and characteristic impedance and the primary parameters *R, L, G* and *C* is useful here. Characteristic impedance is commonly thought of as being a magnitude quantity. While this concept may suffice for high frequency applications, this quantity is actually a complex one consisting of real and imaginary components or magnitude and angle. The associated propagation coefficient is readily viewed as being complex, consisting of the real attenuation and imaginary phase coefficient components. The four secondary components are readily related to the primary components. Frequency dependence of these parameters is also developed.

The cable pair parameters are represented as frequency domain dependent quantities. The measurement methods are based on frequency domain techniques. Measurement methods based on time domain techniques and combinations of time and frequency while useful in many cases are not covered here. The present-day availability of excellent frequency domain equipment such as the network analysers and impedance meters supports the frequency domain approach.

4.2 Characteristic impedance and propagation coefficient equations

4.2.1 General

The frequency domain of the complex characteristic impedance Z_C relates to the primary parameters as:

$$Z_C = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (1)$$

The propagation coefficient, γ, relates to the primary parameters as:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (2)$$

4.2.2 Propagation coefficient

4.2.2.1 Attenuation and phase coefficients

Equation (2) is separated into its real and imaginary parts, the attenuation coefficient α and the phase coefficient β:

$$\alpha = \sqrt{-\frac{1}{2}(\omega^2 LC - RG) + \frac{1}{2}\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}} \qquad (3)$$

$$\beta = \sqrt{\frac{1}{2}(\omega^2 LC - RG) + \frac{1}{2}\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}} \qquad (4)$$

Further, by factoring out $\omega\sqrt{LC}$ we obtain:

$$\beta = \omega\sqrt{LC}\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \qquad (5)$$

It can be shown that:

$$\alpha\beta = \omega\sqrt{LC}\left(\frac{R}{2}\sqrt{\frac{C}{L}}\right) \qquad (6)$$

4.2.2.2 Equations useful at high frequencies

From Equations (5) and (6) we can solve for α and thus obtain for α and β the following expressions, valid within the entire frequency range:

$$\alpha = \frac{\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}}{\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}}} \quad (7)$$

$$\beta = \omega\sqrt{LC}\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \quad (8)$$

Equations (7) and (8) are well suited for evaluation of high frequencies.

4.2.2.3 Equations useful at low frequencies

For low frequency evaluations, the expressions given by Equations (9) and (10) are suitable.

$$\alpha = \sqrt{\frac{\omega RC}{2}}\sqrt{\left(\frac{G}{\omega C} - \frac{\omega L}{R}\right) + \sqrt{\left(1 + \frac{\omega^2 L^2}{R^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \quad (9)$$

$$\beta = \sqrt{\frac{\omega RC}{2}}\sqrt{\left(\frac{\omega L}{R} - \frac{G}{\omega C}\right) + \sqrt{\left(1 + \frac{\omega^2 L^2}{R^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}} \quad (10)$$

4.2.3 Characteristic impedance

4.2.3.1 Real and imaginary parts

The characteristic impedance Z_C can also be separated into its real and imaginary parts as developed in Equations (11) and (12).

$$Z_C = Re\ Z_C + j\ Im\ Z_C = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \frac{\alpha + j\beta}{G + j\omega C} \quad (11)$$

$$Z_C = \frac{\frac{1}{\omega C}\left[\left(\beta + \frac{G}{\omega C}\alpha\right) - j\left(\alpha - \frac{G}{\omega C}\beta\right)\right]}{1 + \frac{G^2}{\omega^2 C^2}} \quad (12)$$

4.2.3.2 Equations useful at high frequencies

After substituting Equations (7) and (8) into Equation (12), the real and imaginary parts of the characteristic impedance are obtained as given in Equations (13) and (14) respectively. These are well suited for simplification (see 4.3) at high frequencies:

$$Re\ Z_C = \frac{\sqrt{\frac{L}{C}}\left[\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}\right]}{\left(1 + \frac{G^2}{\omega^2 C^2}\right)\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1 + \frac{R^2}{\omega^2 L^2}\right)\left(1 + \frac{G^2}{\omega^2 C^2}\right)}}} \quad (13)$$

$$-Im\ Z_C = \frac{\dfrac{R}{2\omega\sqrt{LC}} + \dfrac{G}{2\omega C}\sqrt{\dfrac{L}{C}} - \dfrac{G}{\omega C}\sqrt{\dfrac{L}{C}}\left[\dfrac{1}{2}\left(1 - \dfrac{R}{\omega L}\dfrac{G}{\omega C}\right) + \dfrac{1}{2}\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}\right]}{\left(1+\dfrac{G^2}{\omega^2 C^2}\right)\sqrt{\dfrac{1}{2}\left(1 - \dfrac{R}{\omega L}\dfrac{G}{\omega C}\right) + \dfrac{1}{2}\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}} \tag{14}$$

4.2.3.3 Equations useful at low frequencies

On the other hand, by substituting Equations (9) and (10) into Equation (12), the real and imaginary parts given in Equations (15) and (16) respectively are obtained. These are useful for simplification in the low frequency range:

$$Re\ Z_C = \frac{\sqrt{\dfrac{R}{2\omega C}}\left[\sqrt{\dfrac{\omega L}{R} - \dfrac{G}{\omega C} + \sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}} + \dfrac{G}{\omega C}\sqrt{\dfrac{G}{\omega C} - \dfrac{\omega L}{R} + \sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right]}{\left(1+\dfrac{G^2}{\omega^2 C^2}\right)} \tag{15}$$

$$-Im\ Z_C = \frac{\sqrt{\dfrac{R}{2\omega C}}\left[\sqrt{\dfrac{G}{\omega C} - \dfrac{\omega L}{R} + \sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}} - \dfrac{G}{\omega C}\sqrt{\dfrac{\omega L}{R} - \dfrac{G}{\omega C} + \sqrt{\left(1+\dfrac{\omega^2 L^2}{R^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right]}{\left(1+\dfrac{G^2}{\omega^2 C^2}\right)} \tag{16}$$

4.2.4 Phase and group velocity

The phase propagation time (per unit length) is:

$$\tau_P = \frac{\beta}{\omega} \tag{17}$$

By introducing β from Equations (8) and (10), we obtain:

$$\tau_P = \sqrt{LC}\sqrt{\frac{1}{2}\left(1 - \frac{R}{\omega L}\frac{G}{\omega C}\right) + \frac{1}{2}\sqrt{\left(1+\frac{R^2}{\omega^2 L^2}\right)\left(1+\frac{G^2}{\omega^2 C^2}\right)}} \tag{18}$$

and

$$\tau_P = \sqrt{\frac{RC}{2\,\omega}}\sqrt{\left(\frac{\omega L}{R} - \frac{G}{\omega C}\right) + \sqrt{\left(1+\frac{\omega^2 L^2}{R^2}\right)\left(1+\frac{G^2}{\omega^2 C^2}\right)}} \tag{19}$$

The group propagation time (per unit length) is:

$$\tau_G = \frac{d\beta}{d\omega} \tag{20}$$

$$\tau_G = \frac{\beta}{\omega} + \frac{1}{2}\left(\frac{L'}{L} + \frac{C'}{C}\right)\beta + \frac{\omega^2 LC}{4\beta}\left[\left(-\frac{G}{\omega C} + \frac{\dfrac{R}{\omega L}\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}{\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right)\frac{d\left(\dfrac{R}{\omega L}\right)}{d\omega}\right]\left[+\left(-\frac{R}{\omega L} + \frac{\dfrac{G}{\omega C}\left(1+\dfrac{R^2}{\omega^2 L^2}\right)}{\sqrt{\left(1+\dfrac{R^2}{\omega^2 L^2}\right)\left(1+\dfrac{G^2}{\omega^2 C^2}\right)}}\right)\frac{d\left(\dfrac{G}{\omega C}\right)}{d\omega}\right] \tag{21}$$

The phase and group velocities are, respectively,

$$v_{\mathrm{P}} = \frac{1}{\tau_{\mathrm{P}}} \tag{22}$$

$$v_{\mathrm{G}} = \frac{1}{\tau_{\mathrm{G}}} \tag{23}$$

The above expressions are accurate and valid within the whole frequency range. If C and $G/(\omega C)$ can be regarded as frequency independent coefficients, then we obtain:

$$\tau_{\mathrm{G}} = \frac{\beta}{\omega} + \frac{\beta}{2}\frac{L'}{L} + \frac{C}{4\beta}\left[-\frac{G}{\omega C} + \frac{\frac{R}{\omega L}\left(1+\frac{G^2}{\omega^2 C^2}\right)}{\sqrt{\left(1+\frac{R^2}{\omega^2 L^2}\right)\left(1+\frac{G^2}{\omega^2 C^2}\right)}} \right]\left(-R + R'\omega - \frac{L'R}{L}\omega \right) \tag{24}$$

The above expressions, which are valid within the entire frequency range, can be simplified into approximate expressions, which are valid at high or low frequencies only.

4.3 High frequency representation of secondary parameters

The high frequency representations of the formulas are useful over a broad range of frequencies extending from voice frequency on up because of the range of values for the dissipation factor. $G/(\omega C) = \tan\delta < 0{,}03$ (< 3 %) even for PVC insulated cables up to 1,5 MHz and for the polyethylene (PE), insulation is very small at about 0,000 1 (0,01 %). This results in approximations, which in practice are valid for the whole frequency range as follows:

$$Re\, Z_{\mathrm{C}} \approx \sqrt{\frac{L}{C}}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{R^2}{\omega^2 L^2}}} \tag{25}$$

$$-Im\, Z_{\mathrm{C}} \approx \frac{R}{2\omega C\ Re\, Z_{\mathrm{C}}} - \frac{G}{\omega C} Re\, Z_{\mathrm{C}} + \frac{\frac{G}{2\omega C}\frac{L}{C}}{Re\, Z_{\mathrm{C}}} \tag{26}$$

$$\alpha \approx \frac{R}{2\ Re\, Z_{\mathrm{C}}} + \frac{G\left(\sqrt{\frac{L}{C}}\right)^2}{2\ Re\, Z_{\mathrm{C}}} \tag{27}$$

$$\beta \approx \omega C\ Re\, Z_{\mathrm{C}} \tag{28}$$

$$\tau_{\mathrm{P}} \approx \sqrt{LC} \tag{29}$$

$$\tau_{\mathrm{G}} \approx \frac{\beta}{\omega} + \frac{\beta}{2}\frac{L'}{L} + \frac{C}{4\beta}\left(-\frac{G}{\omega C} + \frac{\frac{R}{\omega L}}{\sqrt{1+\frac{R^2}{\omega^2 L^2}}} \right)\left(-R + R'\omega - \frac{L'R}{L}\omega \right) \tag{30}$$

when also $R/(\omega L) < 0{,}1$, which is true for high frequencies ($f > 1$ MHz for 0,5 mm wire), the formulas holding better than about 1 % accuracy can be further simplified as shown below.

$$Re\, Z_C \approx \sqrt{\frac{L}{C}} \tag{31}$$

$$-Im\, Z_C \approx \frac{R}{2\omega C\, Re\, Z_C} - \frac{G}{\omega C} Re\, Z_C \approx \sqrt{\frac{L}{C}}\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right) \tag{32}$$

$$\alpha \approx \frac{R}{2\ Re\, Z_C} + \frac{G}{2}\ Re\, Z_C \approx \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \tag{33}$$

$$\beta \approx \omega C\ \ Re\, Z_C \approx \omega\sqrt{LC} \tag{34}$$

$$\tau_P \approx \sqrt{LC} \tag{35}$$

$$\tau_G \approx \tau_P + \frac{\beta}{2}\frac{L'}{L} + \frac{C}{4\beta}\left(-\frac{G}{\omega C} + \frac{R}{\omega L}\right)\left(-R + R'\omega - \frac{L'R}{L}\omega\right) \tag{36}$$

4.4 Frequency dependence of the primary and secondary parameters

4.4.1 Resistance

The high frequency resistance (surface resistance) of a solid round wire for frequencies where the wire radius r is greater than twice the skin depth δ can be regarded as consisting of two parts where one is constant and the other $f^{0,5}$ dependent.

$$R = R_C + R_S = R_C + \rho\sqrt{\omega} \approx R_0\left(\frac{1}{4} + \frac{r}{2\delta}\right) \tag{37}$$

$$\rho = \frac{R_S}{\sqrt{\omega}} = \frac{R_0\, r}{4}\sqrt{2\mu\sigma} \tag{38}$$

The above is true for a solid wire alone. In a pair, the proximity effects and the presence of other pairs and possible screen contribute both to the resistance and inductance. These effects can increase the R by about 15 % at 1 MHz and follow also approximately the square-root of frequency law. Also, the constant component of resistance while often neglected, is about 15 % of the frequency dependent component at 1 MHz for a 0,5 mm diameter copper pair.

4.4.2 Inductance

The total inductance consists also of two main components such that

$$L \approx L_E + L_I = L_E + \frac{R_S}{\omega} = L_E + \frac{\rho}{\sqrt{\omega}} \tag{39}$$

The external free space inductance is reduced by the proximity effect of the pair and the free space limiting effects of the nearby shield and/or other pairs. These inductive components are negative and fairly frequency independent at high frequencies.

4.4.3 Characteristic impedance

The characteristic impedance high frequency asymptotic value Z_∞ is given by Equation (40).

$$Z_\infty = \sqrt{\frac{L_E}{C}} \tag{40}$$

The high frequency impedance formulas are given by Equations (41) and (42):

$$Re\, Z_C \approx \sqrt{\frac{L}{C}} \approx Z_\infty \left(1 + \frac{R_S}{2\,\omega L_E}\right) = Z_\infty + \frac{\rho}{2\sqrt{L_E C}\sqrt{\omega}} \tag{41}$$

$$\begin{aligned} -Im\; Z_C &\approx \sqrt{\frac{L}{C}} \left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right) \\ &\approx \frac{R_C + \rho\sqrt{\omega}}{2\omega\sqrt{L_E C}\left(1 + \frac{\rho}{2\,L_E\sqrt{\omega}}\right)} - \sqrt{\frac{L_E}{C}}\left(1 + \frac{\rho}{2\,L_E\sqrt{\omega}}\right)\frac{\tan\delta}{2} \\ &\approx \frac{R_C}{2\omega\sqrt{L_E C}} + \frac{\rho}{2\sqrt{L_E C}\sqrt{\omega}} - \frac{Z_\infty}{2}\left(1 + \frac{L_I}{L_E}\right)\tan\delta \\ &\approx \frac{\rho}{2\sqrt{L_E C}\sqrt{\omega}} - \frac{Z_\infty}{2}\tan\delta \end{aligned} \tag{42}$$

4.4.4 Attenuation coefficient

Using the above approximations with Equations (31) through (36) results in the remaining equations of this subclause:

$$\alpha \approx \frac{\left(R_C - \frac{\rho^2}{2\,L_E}\right)}{2\,Z_\infty} + \frac{\rho\sqrt{\omega}}{2Z_\infty} + \frac{\rho\sqrt{\omega}\;\tan\delta}{4\,Z_\infty} + \frac{\omega\sqrt{L_E C}\;\tan\delta}{2} \tag{43}$$

which is of the form:

$$\alpha \approx A + B\sqrt{\omega} + C\omega \tag{44}$$

where A, B and C are constants.

The first term of Equation (44) indicates that at the low end of the high frequency range the attenuation increases a little more slowly than the square-root-law. The first $\omega^{0,5}$ term in Equation (43) which is dominant in the high frequency attenuation formula also appears in the phase coefficient, Equation (45).

$$\beta \approx \omega\sqrt{LC} \approx \omega\sqrt{L_E C}\left(1 + \frac{R}{2\,\omega L_E}\right) \approx \omega\sqrt{L_E C} + \frac{\rho\sqrt{\omega}}{2\,Z_\infty} \tag{45}$$

4.4.5 Phase delay and group delay

The phase and group delay are given in Equations (46) and (47) respectively:

$$\tau_{\mathrm{P}} \approx \sqrt{LC} = \sqrt{L_{\mathrm{E}} C}\left(1+\frac{R}{2\omega L_{\mathrm{E}}}\right) \approx \sqrt{L_{\mathrm{E}} C} + \frac{\rho}{2 Z_{\infty}\sqrt{\omega}} \qquad (46)$$

$$\begin{aligned}
\tau_{\mathrm{G}} &\approx \tau_{\mathrm{P}} + \frac{\beta}{2}\frac{L'}{L} + \frac{C}{4\beta}\left(-\frac{G}{\omega C}+\frac{R}{\omega L}\right)\left(-R + R'\omega - \frac{L'R}{L}\omega\right) \\
&\approx \left(1-\frac{R}{4\omega L}\right) - \frac{R}{8\omega L}\left(\frac{R}{\omega L}-\frac{G}{\omega C}\right) \\
&\approx \tau_{\mathrm{P}}\left(1-\frac{R}{4\omega L}\right) \qquad (47) \\
&\approx \sqrt{L_{\mathrm{E}} C}\left(1+\frac{R}{4\omega L_{\mathrm{E}}}\right) \\
&\approx \sqrt{L_{\mathrm{E}} C} + \frac{\rho}{4\sqrt{\omega}\, Z_{\infty}}
\end{aligned}$$

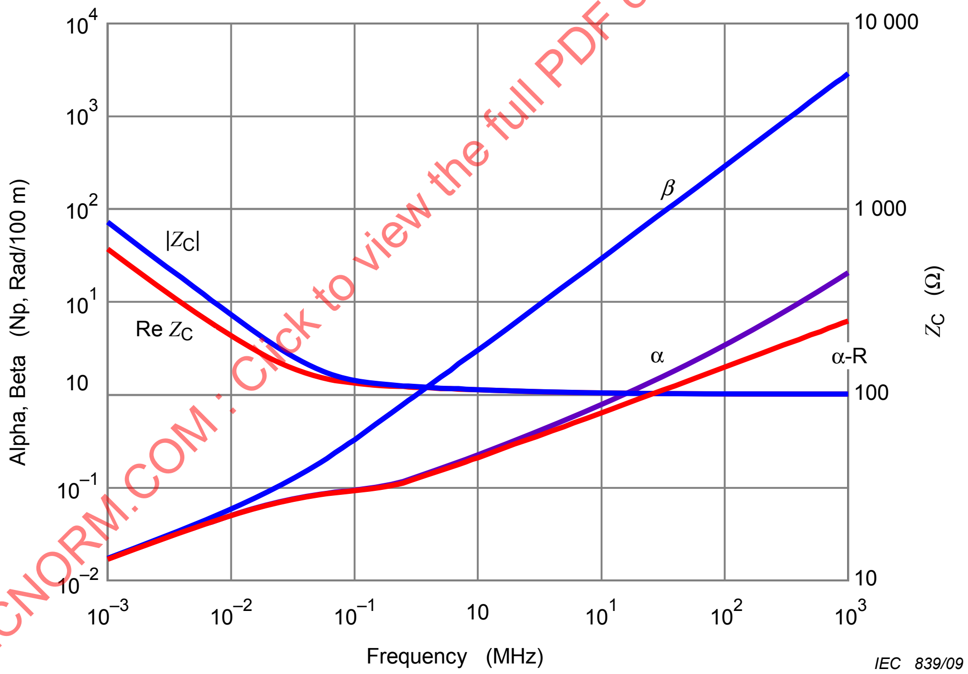


Figure 1 – Secondary parameters extending from 1 kHz to 1 GHz

Figure 1 shows the secondary parameters of a UTP pair with 0,5 mm conductors versus frequency. At voice frequencies, the attenuation and phase coefficients are substantially equal. At these frequencies, the absolute value of the characteristic impedance and the real part of the characteristic impedance differ by the square-root of 2. At frequencies above 100 kHz, attenuation is much less than the phase coefficient on the Nepers and radians scale, and the characteristic impedance is mostly real. The total attenuation (Alpha) differs from the conductor attenuation (Alpha-R) by the dielectric component of attenuation for this example, where the dissipation factor is assumed to be 0,01.

5 Measurement of characteristic impedance

5.1 General

The characteristic impedance Z_C of a homogeneous cable pair is defined as the quotient of a voltage wave and current wave which are propagating in the same direction, either forwards (f) or backwards (r). For homogeneous cables (with no structural variations), the characteristic impedance can be measured directly as the quotient of voltage U and current I at the cable ends.

$$Z_C = \frac{U_f}{I_f} = \frac{U_r}{I_r} \tag{48}$$

A number of methods for obtaining characteristic impedance are described. Some of these methods offer convenience (perhaps at the cost of accuracy in portions of the frequency range). Others offer capability beyond what is currently needed for routine product inspection but are useful in laboratory evaluation where measurement throughput is not as critical.

The open/short circuit single-ended impedance measurement made with a balun in 5.2 is viewed as the reference method for obtaining the data. Alternative methods are listed below:

a) characteristic impedance determined from phase coefficient and capacitance measurements (see 5.4);
b) terminated cable impedance measurements (see 5.5);
c) extended open/short impedance measurements excluding balun performance (see 5.6);
d) extended open/short impedance measurements made without a balun (see 5.7);
e) open/short impedance measurements at low frequencies with a balun (see 5.8;
f) impedance measurements obtained by modal decomposition technique (see 5.9).

It is intended that impedance measurements will be performed using sufficiently closely spaced frequencies so that impedance variation is adequately represented. Either a linear sweep or a logarithmic sweep may be used depending on whether the high end or low end, respectively, of the desired frequency range is to be more fully represented. Typically, several hundred points (such as the available 401 points) are required depending on frequency range and cable length.

The balun used for connecting the symmetric cable pair to the coaxial port on the test instrument shall have a pass-band frequency range adequate for the desired measurement range. It shall be capable of transforming from the instrument port impedance to the nominal pair impedance. The three step impedance measurement calibration is performed at the secondary (pair side) of the balun.

Function fitting (discussed in 5.3) of the impedance data is useful for separating structural effects from the characteristic impedance when such effects are substantial. Where function fitting is used, the concept is that measurements from nearby frequencies aid in the interpretation of the values obtained at a particular frequency. Function fitting of the impedance magnitude or real part results in high values (typically 0,5 Ω or less) because of the positive and negative deviations not being symmetrical on the impedance scale. Function fitting can be carried out on the S-parameter values, which are linear responses, if more rigorous results (both impedance and SRL) are desired.

5.2 Open/short circuit single-ended impedance measurement made with a balun (reference method)

5.2.1 Principle

Open and short circuit measurements made with a balun from one end of a symmetric cable pair is the reference method for obtaining characteristic impedance values. The characteristic impedance is the geometric mean of the product of the open and short circuit measured values and is defined as:

$$Z_{C} = \sqrt{Z_{OC}\, Z_{SC}} \tag{49}$$

When the cable is not homogenous, an impedance inclusive of structural effects is obtained:

$$Z_{CM} = \sqrt{Z_{OC}\, Z_{SC}} \tag{50}$$

where Z_{CM} is the complex characteristic impedance together with structure (input impedance), expressed in ohms (Ω).

Equation (49) represents the characteristic impedance, Z_{C}, when structural effects are negligible. The fitting of the open/short impedance data with a characteristic impedance such as function of frequency can be employed to obtain Z_{C} from the input impedance, Z_{CM}, Equation (50) when structural effects are substantial. Equations (49) and (50) (and this measurement technique) are valid for frequencies extending from low values, where the cable length is only a fraction of a wavelength, to high frequencies where cable length represents many wavelengths.

5.2.2 Test equipment

A network analyser (together with an S-parameter unit) or an impedance meter can be used to obtain the data. Figure 2 shows the main components of an impedance measurement circuit where the generator and receiver are parts of the network analyser. An S-parameter unit, where the key component is the reflection bridge, is used with a network analyser to separate the reflected signal from the incident signal. A balun with the appropriate frequency range, impedance (such as 50 Ω to 100 Ω for 50 Ω equipment and 100 Ω pair) and balanced at least as well as the pair under test facilitates making measurements on symmetric pairs under balanced conditions. Three terminating conditions, open, short and the nominal load resistance, are used as appropriate for the type of measurement being made (open, short or terminated).

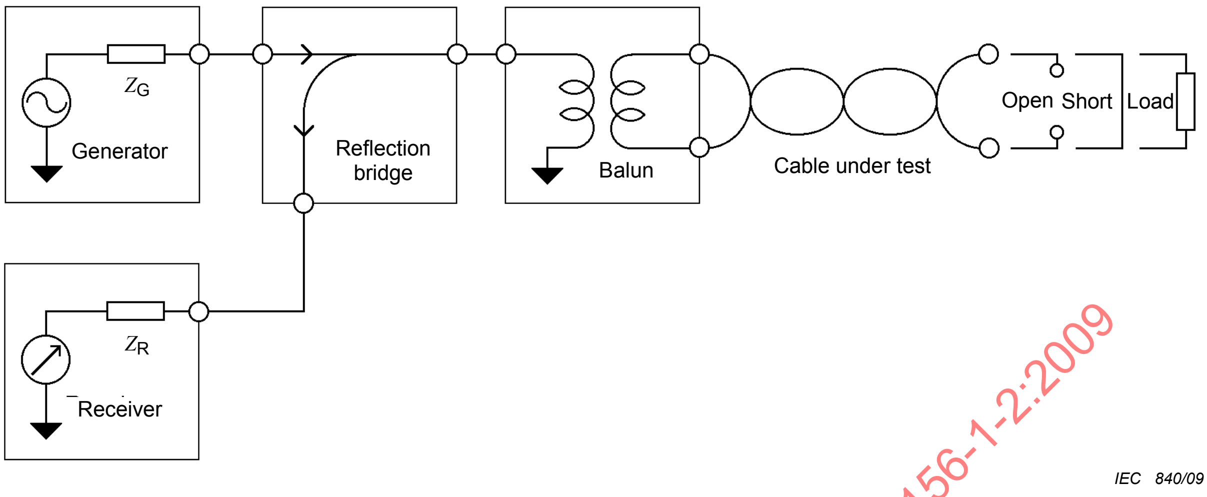


Figure 2 – Diagram of cable pair measurement circuit

5.2.3 Procedure

A three step calibration procedure using the same open, short and load terminations as used for the actual measurements is carried out at the secondary of the balun with the cable pair disconnected. Upon completing the 3-step calibration procedure at the secondary of the balun, the network analyser is capable of measuring directly the complex reflection coefficient (S-parameter) or impedance of a cable pair. An internal 3-step calibration procedure including calculations is provided by most network analysers when an S-parameter unit is used. The method presented in 5.6 covers a similar 3-step calibration procedure by using the F-matrix principle where all the quantities are stated as impedances. This method is useful when the network analyser is not suitably equipped, in which case the computations can be accomplished external to the analyser.

The measured impedance (open or short) is computed from the reflection coefficient measurements S_{11} by means of Equation (51) either by the network analyser or by a computer (on acquired data):

$$Z_{\mathrm{MEAS}} = Z_{\mathrm{R}} \frac{1+S_{11}}{1-S_{11}} \tag{51}$$

5.2.4 Expression of results

Conceptually, several approaches are possible. On the one hand, the input impedance consisting of the combined characteristic impedance and structural effects can be viewed as needing to meet a broader single requirement (such as the 85 Ω to 115 Ω range) over the specified frequency range. Alternatively, a narrower range (such as a 95 Ω to 105 Ω range) can be viewed as being a requirement for the asymptotic component of function fitted characteristic impedance. In this case, RL or SRL specifications are used to control structural effects. The advantage of a broad single requirement in many instances is measurement simplification.

The advantage of separating the two effects is that of obtaining quantitative information for the two effects. The requirements for the impedance and structural effects are given in the relevant cable specification.

5.3 Function fitting the impedance magnitude and angle

5.3.1 General

Function fitting of the impedance data is useful for separating structural effects from the characteristic impedance when such effects are substantial. Where function fitting is used, the concept is that measurements from nearby frequencies aid in the interpretation of the values obtained at a particular frequency. Function fitting of the impedance magnitude or real part results in high values (typically 0,5 Ω or less) ,because of the positive and negative deviations not being symmetrical on the impedance scale. Function fitting can be carried out on the S-parameter values, which are linear responses, if more rigorous results (both impedance and SRL) are desired.

5.3.2 Impedance magnitude

5.3.2.1 Function fitting the magnitude of the characteristic impedance

While function fitting can be applied to the real and imaginary components of Z_C, the usual situation is that interest in the magnitude is greater than interest in the two separate components or the angle. The impedance magnitude tracks the real component closely at high frequencies where the imaginary component is small.

Function fitting of the impedance magnitude or real part results in fairly high values (typically 0,5 Ω or less), because of the positive and negative deviations not being symmetric on the impedance scale. Function fitting can be carried out on the S-parameter values, which is a linear response scale, if more rigorous results (both impedance and SRL) are desired.

This method differs from smoothing in that a characteristic impedance like function (based on transmission theory) is used to fit the measured data (obtained from Equation (50) or terminated impedance data). The function is stated as follows.

The fitted characteristic impedance magnitude is calculated with a least squares curve fit to Z_C, based on Equation (52):

$$|Z_C| = K_0 + \frac{K_1}{f^{1/2}} + \frac{K_2}{f} + \frac{K_3}{f^{3/2}} \tag{52}$$

NOTE Where terminated cable impedance data is used instead of open/short data, round-trip loss of measured length should be sufficiently large (in the 10 dB to 20 dB range for desired accuracies in the 5 Ω to 1,5 Ω range respectively when maximum deviation is 15 Ω – see 5.5).

Discreet point data equally spaced according to the log of frequency is advantageous for function fitting in that it results in appropriate weighting of the lower and upper ends of a multi-decade frequency sweep. Linear frequency spacing with logarithmic weighting may be used in the calculations when log of frequency spacing leads to concern about undersampling at high frequencies. Plotting the data versus the log of frequency is helpful here (as it is in network theory). The function fitting for individual data sets can readily be accomplished by importing ASCII format data obtained from the network analyser directly into a spreadsheet program and using the built-in regression procedures. Optimized software for analyzing numerous data sets is desirable for use in a production setting.

The terms of the right hand side of Equation (52) generally diminish in importance from left to right. The first two terms have strong theoretical basis. The constant term has the strongest basis in that it represents the space (external) inductance (largest component of inductance) and the capacitance of the pair (see Clause 4). The second term is significant in that it represents the component of characteristic impedance resulting from the internal inductance. The last two terms are supplied to provide for second order effects such as the capacitance decreasing with frequency, as with polar insulation materials or the effects of a shield. In the latter case, the low frequency end function fitting range is limited to frequencies where slope is increasing with frequency (2nd derivative positive).

The fit coefficients are calculated from Equation (53) where all summations are performed over N data points.

$$\begin{bmatrix} \sum_{i=1}^{N} |Z_{CM}| \\ \sum_{i=1}^{N} \frac{|Z_{CM}|}{\sqrt{f_i}} \\ \sum_{i=1}^{N} \frac{|Z_{CM}|}{f_i} \\ \sum_{i=1}^{N} \frac{|Z_{CM}|}{f_i^{3/2}} \end{bmatrix} = \begin{bmatrix} N & \sum_{i=1}^{N} \frac{1}{\sqrt{f_i}} & \sum_{i=1}^{N} \frac{1}{f_i} & \sum_{i=1}^{N} \frac{1}{f_i^{3/2}} \\ \sum_{i=1}^{N} \frac{1}{\sqrt{f_i}} & \sum_{i=1}^{N} \frac{1}{f_i} & \sum_{i=1}^{N} \frac{1}{f_i^{3/2}} & \sum_{i=1}^{N} \frac{1}{f_i^{2}} \\ \sum_{i=1}^{N} \frac{1}{f_i} & \sum_{i=1}^{N} \frac{1}{f_i^{3/2}} & \sum_{i=1}^{N} \frac{1}{f_i^{2}} & \sum_{i=1}^{N} \frac{1}{f_i^{5/2}} \\ \sum_{i=1}^{N} \frac{1}{f_i^{3/2}} & \sum_{i=1}^{N} \frac{1}{f_i^{2}} & \sum_{i=1}^{N} \frac{1}{f_i^{5/2}} & \sum_{i=1}^{N} \frac{1}{f_i^{3}} \end{bmatrix} \times \begin{bmatrix} K_0 \\ K_1 \\ K_2 \\ K_3 \end{bmatrix} \quad (53)$$

5.3.2.2 Obtaining log spaced data

Choose to acquire equally spaced data points on a log frequency basis when possible. This approach provides better weighting emphasis for data spanning several decades. Most network analysers offer this type of sweep. Convert the data being fitted to log spacing by interpolation, when it is equally spaced on a linear frequency scale. Alternatively, use $1/f$ weighting (this means weighting a 10 MHz data point by 0,1 when a 1 MHz data point is weighted by 1) in performing the summations to simulate log frequency spaced data points. The 4th order system of equations and unknowns is solved by the computer, by using determinants or matrix inversion techniques.

5.3.2.3 Fewer terms

Depending on the measurement frequency range and the amount of structural variation, usage of one or more of the higher order terms may not be justifiable. The contributions from the higher order terms are intended to be second order. Where the data spans one decade or less, only the first two terms (or perhaps only the constant term) may be justified. The resultant function fit is considered valid if it has a negative slope at low frequencies, is asymptotic at higher frequencies and is free of oscillation with frequency.

Two or three terms may be sufficient when the data spans only one or two decades of frequency. This is accomplished by discarding one or more lower rows of Equation (53) and the same number of rightmost columns of the square matrix. While a four term fit is indicated by Equations (52) and (53), in some cases fewer terms may suffice. It is shown in 4.4.2 that just associating the inductance variation of a cable pair with frequency, calls for the first two terms of Equation (52). This is particularly true when the low frequency range of the data being fitted extends below about 3 MHz. If the capacitance is changing with frequency as it does when polar dielectric material is present, more terms are generally justified.

Four criteria indicate use of fewer terms – check or have the computer program determine if the fitted function obtained by solving Equation (53) meets the following set of four criteria.

a) The fitted function, except when it is only a constant, has negative slope for frequencies below 3 MHz.

b) The 10 MHz fitted value is within the impedance range of +5 to –2 of the high frequency asymptote (fitted constant value).

c) The area under the fitted function supplied by the frequency dependent terms on a log frequency basis, exclusive of the constant area, is positive (constant component is not above the data).

d) The sum of the negative areas (those due to negative coefficients) is less than the total area due to the frequency dependent terms.

If all four criteria are not met, the number of terms in the function (Equation (52)) shall be reduced by one by omitting the highest order term. Otherwise, data spanning a wider range of frequencies and generally resulting in a better fit must be obtained and fitted. The fit for impedance magnitude shall have a monotonic downward behaviour with increasing frequency and approach a high frequency asymptote to a reasonable extent.

5.3.2.4 Compute and plot fitted results

Compute values for the magnitude of the characteristic impedance, according to coefficients obtained from the fit at the desired frequencies, and plot the results and/or tabulate the fitted results at specification frequencies as desired.

5.3.3 Function fitting the angle of the characteristic impedance

This is useful when the characteristic impedance is to be specified as a complex quantity. The fitting equation for the angle of the characteristic impedance, $\angle Z_C$, is given in Equation (54). The equation should contain the same powers of frequency as those being used for the magnitude of the characteristic impedance.

$$\angle Z_C = L_0 + \frac{L_1}{f^{1/2}} + \frac{L_2}{f} + \frac{L_3}{f^{3/2}} \tag{54}$$

The coefficients for the impedance angle can be calculated with Equation (53).

Plot the results as desired.

NOTE This procedure is necessary only if the angle of the characteristic impedance is of interest or if structural return loss (SRL) is being calculated at frequencies low enough to result in a significant angle (degrees).

5.4 Characteristic impedance determined from measured phase coefficient and capacitance

5.4.1 General

The mean characteristic impedance (homogeneous line) at any frequency can be obtained from the ratio of propagation coefficient to shunt admittance. At high frequencies, the real part of Z_C can be obtained by dividing delay by capacitance. This method is expedient for dielectric materials which do not change with frequency (non-polar) permitting a readily obtained low frequency value of capacitance to represent the high frequency range but is more difficult to apply when the capacitance changes with frequency as it does for polar dielectric materials. It results in characteristic impedance values free of structural effects. Justification for this method is supplied in Clause 4.

5.4.2 Equations for all frequencies case and for high frequencies

Characteristic impedance Z_C may be expressed as the propagation coefficient divided by the shunt admittance as given in Equation (55). This relationship holds at any frequency. Characteristic impedance is readily separated into the real and imaginary components when $G << \omega C$.

$$Z_C = \frac{\alpha + j\,\beta}{G + j\,\omega C} \approx \frac{\beta}{\omega C} - \frac{j\,\alpha}{\omega C} \tag{55}$$

At high frequencies, where the imaginary component of impedance is small, and the real component and magnitude are substantially the same, Equation (55) can be written as:

$$Z_C = \frac{\beta}{\omega C} = \frac{\tau_p}{C} \tag{56}$$

$$-Im\ Z_C \approx \frac{\alpha}{\omega C} \approx Re\ Z_C - Z_\infty = \frac{\beta}{\omega C} - Z_\infty \tag{57}$$

$$Z_\infty = \sqrt{\frac{L_E}{C}} \tag{58}$$

5.4.3 Procedure for the measurement of the phase coefficient

5.4.3.1 General

The phase coefficient measurement procedure, in the situation where the complex characteristic impedance is desired, is similar to that outlined for attenuation measurement in 6.3.3 of IEC 61156-1, Edition 3 (2007).

5.4.3.2 Phase coefficient

The phase coefficient of a pair of conductors is a measure of the phase shift incurred by a sinusoidal signal as it propagates over a length of pair and is affected by the materials and geometry of the insulated conductors.

The phase coefficient, β, relates to the measurements as:

$$\beta = \angle(V_{1F}) - \angle(V_{1N}) + 2\pi k \tag{59}$$

The phase coefficient can be obtained as a result of the same measurement procedure used to obtain the attenuation (see 6.3.3 of IEC 61156-1:2007 (Edition 3)) by using a network analyser (which measures vector quantities). For balanced pairs, the transmit and receive ports of the measurement instrument shall supply balanced voltage with respect to ground and balanced currents (commonly accomplished with a balun). Pairs under test shall be terminated in their nominal impedance ±1 %.

5.4.3.3 Determining multiplier *k*

The multiplier k in Equation (59) may be determined either by examining the analyser display or numerically with the aid of a computer.

5.4.3.4 Determining k by examination

To determine the multiplier k, examine the analyser display and interpret the acquired data over the range of frequencies as appropriate. The phase meter or network analyser normally yields only the difference between the first and second terms shown on the right hand side of Equation (59). Figure 3 shows the total phase and the sawtooth representation obtained from a network analyser. When a network analyser is used, a trace of the phase coefficient cycling through the 2π radians (360°) range is generally displayed on a CRT display, facilitating the determination of k. A frequently used technique in the interactive mode is to start at a low frequency where $k = 0$, by counting the number of 2π to 0π traversals to obtain the value for k.

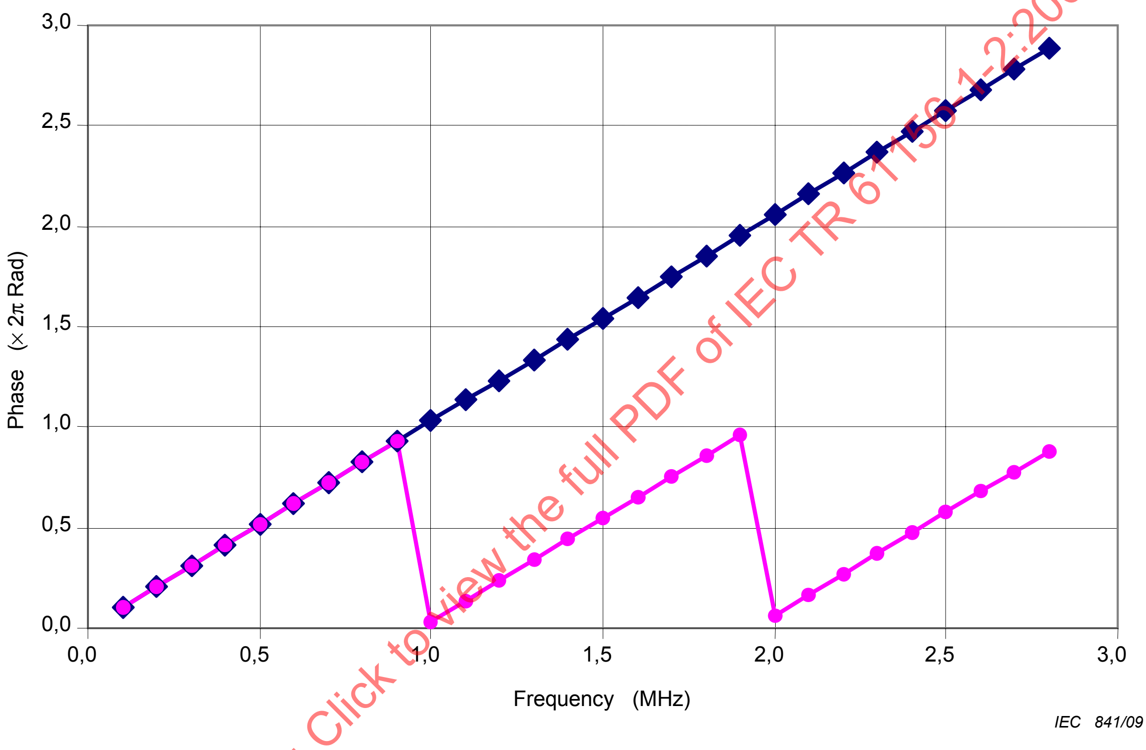


Figure 3 – Determining the multiplier of 2π radians to add to the phase measurement

5.4.3.5 Obtaining k numerically

Determine k numerically by acquiring the phase information obtained with the network analyser digitally using an interface with a digital computer as was done with the points plotted in Figure 3. Follow the data acquisition with a program procedure which starts by establishing a starting slope from several points in the $k = 0$ (multiple of 2π radian) frequency region. Let the program continue by examining each remaining point in succession. If the point is not within 2π radians of the continuous phase line being established, increment k until it is. This approach works even when intermediate values of k are passed over, once the correct starting slope is established.

5.4.3.6 Obtaining total phase from the length function

To obtain the total phase, use the procedure called the "length" function, which is built into many network analysers. This internal procedure subtracts the specified length, which can be expressed as seconds of delay (actually a constant time frequency), from the internally established total delay and displays it. The phase trace is conveniently kept within the $0\ \pi$ to 2π (or alternately $-\pi$ to $+\pi$) range over the whole frequency range by supplying the appropriate length value to the analyser.

5.4.4 Phase delay

Phase delay is a measure of the amount of time a simple sinusoidal signal is delayed when propagating through the length of a pair or cable. As with the phase coefficient, it is affected by the materials and geometry of the insulated conductors.

Equation (60) is used to calculate the phase delay τ_P, as a function of frequency from the phase coefficient β, measured in 5.4.3.2.

$$\tau_P = \frac{\beta}{\omega} \tag{60}$$

5.4.5 Phase velocity

Phase velocity (reciprocal of phase delay) is a measure of the velocity with which a sinusoidal signal propagates through a cable and is normally reported in units of distance per second such as m/s.

Equation (61) is used to calculate the phase velocity v_P, as a function of frequency from the phase coefficient β, measured in 5.4.3.2.

$$v_P = \frac{\omega}{\beta} \tag{61}$$

NOTE Phase velocity is sometimes reported as a ratio consisting of the phase velocity divided by the velocity of light in a vacuum (c). It is then reported as, for example, 0,71 c, meaning 0,71 × speed of light. A variation is to report it as a percentage such as 71 %.

5.4.6 Procedure for the measurement of the capacitance

The capacitance of the same length as that measured for the phase coefficient (delay) shall be measured between the two conductors of the pair in accordance with 6.2.5 of IEC 61156-1, Edition 3 (2007).

5.5 Determination of characteristic impedance using the terminated measurement method

A single terminated impedance measurement can be made in place of the open and short circuit measurements when the terminating impedance is sufficiently similar to the impedance being measured (within 15 Ω) and when the roundtrip loss of the measured length is sufficiently large (at least 10 dB). This measurement is useful when the convenience of using the network analyser in a stand-alone mode is desired. Use of this method is with the understanding that the open and short circuit method is the reference method.

Understanding the difference between the measured terminated impedance and the open/short circuit impedance is facilitated by the following equations. The equation for the terminated input impedance Z_T is:

$$Z_T = Z_C \frac{1+\varsigma e^{-2\gamma l}}{1-\varsigma e^{-2\gamma l}} \tag{62}$$

where the reflection coefficient ς is given by:

$$\varsigma = \frac{Z_R - Z_C}{Z_R + Z_C} \tag{63}$$

Z_R and Z_C are the terminating impedance (usually a resistance) and the actual characteristic impedance respectively. Having a closely matched termination or sufficient roundtrip attenuation is adequate for making the terminated measurement yield results close to those obtained by the open and short circuit method.

Equation (62) can be restated as follows:

$$Z_T - Z_C = (Z_R - Z_C)\ e^{-2\gamma l} \left(\frac{Z_T + Z_C}{Z_R + Z_C} \right) \tag{64}$$

Equation (62) indicates that a 15 Ω difference between the termination resistor and the cable impedance is reduced to a maximum error of approximately 5 Ω with a round trip loss of 10 dB. A 20 dB round trip loss insures that a 15 Ω impedance difference is reduced to a rather minimal 1,5 Ω error.

5.6 Extended open/short circuit method using a balun but excluding the balun performance

5.6.1 Test equipment and cable-end preparation

The equipment required for the impedance and S-parameter measurement is that defined in 5.2. For this balanced form of measurement, the termination condition for other pairs and a shield, if present, is of little consequence. These conductors are close to ground even when permitted to float because of the pair twist of the pair under test. Letting these conductors float is acceptable.

5.6.2 Basic equations

Characteristic impedance and the propagation coefficient are expressed in Equation (65) and Equation (66) respectively:

$$Z_C = \sqrt{Z_R^2 \left(\frac{Z_{itr} - Z_{itf}}{Z_{itr} - Z_{its}} \right)^2 \left(\frac{Z_{itcf} - Z_{its}}{Z_{itcf} - Z_{itf}} \right) \left(\frac{Z_{itcs} - Z_{its}}{Z_{itcs} - Z_{itf}} \right)} \tag{65}$$

$$\gamma = \alpha + j\beta = \frac{1}{l} \tanh^{-1} \sqrt{\left(\frac{Z_{itcf} - Z_{its}}{Z_{itcf} - Z_{itf}} \right) \left(\frac{Z_{itcs} - Z_{its}}{Z_{itcs} - Z_{itf}} \right)} \tag{66}$$

where

Z_{itf} is the input impedance measured by leaving the balanced output of the balun open (Ω);

Z_{its} is the input impedance measured by shorting the balanced output of the balun (Ω);

Z_{itr} is the input impedance measured by terminating the balanced output of the balun in a non-inductive, resistive load (Z_R Ω) which value is balanced to ±1 % (Ω);

Z_{itcf} is the input impedance measured by connecting the balanced output of the balun in a twisted pair with far end of the pair open (Ω);

Z_{itcs} is the input impedance measured by connecting the balanced output of the balun in a twisted pair with far end of the pair shorted (Ω).

5.6.3 Measurement principle

Extended single end, open/short circuit method using a balun, but excluding the balun performance. The input impedance measurements are implemented by means of an impedance bridge or network analyzer and S-parameter test set (see Figure 4 and Figure 5).

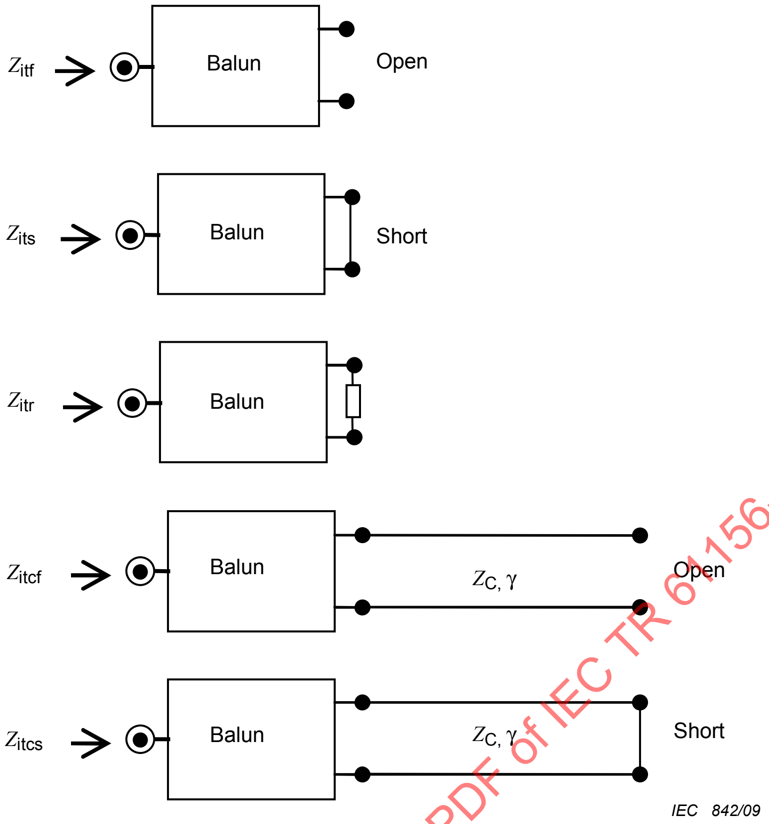


Figure 4 – Measurement configurations

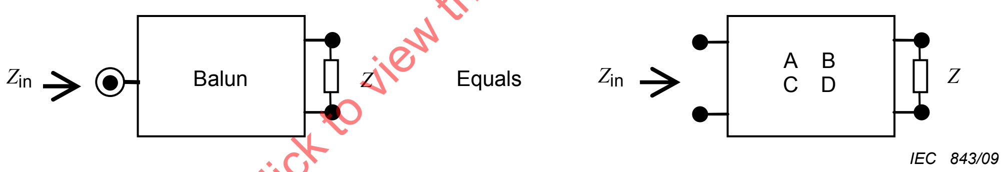


Figure 5 – Measurement principle with four terminal network theory

$$Z_{\text{in}} = \frac{A\,Z + B}{C\,Z + D} \qquad (67)$$

where

Z_{in} is the input impedance;

Z is the load impedance such as open, short, termination, cable pair open or cable pair shorted.

$$Z_{\text{itf}} = Z_{\text{in}}\big|_{Z=\infty} = A/C,\;\; A = Z_{\text{itf}}\; C \qquad (68)$$

$$Z_{\text{its}} = Z_{\text{in}}\big|_{Z=0} = B/D,\;\; B = Z_{\text{its}}\; D \qquad (69)$$

$$Z_{\text{itr}} = Z_{\text{in}}\big|_{Z=R} = \frac{AR + B}{CR + D} \qquad (70)$$

$$Z_{\text{itcf}} = Z_{\text{in}}\big|_{Z=Z_{\text{if}}} = \frac{AZ_{\text{if}} + B}{CZ_{\text{if}} + D} \qquad (71)$$

$$Z_{\text{itcs}} = Z_{\text{in}}\Big|_{Z=Z_{\text{is}}} = \frac{AZ_{\text{is}} + B}{CZ_{\text{is}} + D} \tag{72}$$

where

Z_{if} is the impedance presented by cable pair with far end open (Ω);

Z_{is} is the impedance presented by cable pair with far end shorted (Ω).

Substituting Equation (68) and Equation (69) into Equation (70),

$$\frac{D}{C} = \frac{R(Z_{\text{itf}} - Z_{\text{itr}})}{Z_{\text{itr}} - Z_{\text{its}}} \tag{73}$$

From Equation (71),

$$Z_{\text{if}} = \frac{B - Z_{\text{itcf}}\ D}{Z_{\text{itcf}}\ C - A} \tag{74}$$

From Equation (72),

$$Z_{\text{is}} = \frac{B - Z_{\text{itcs}}\ D}{Z_{\text{itcs}}\ C - A} \tag{75}$$

Finally:

$$\begin{aligned}
Z_{\text{C}}^2 &= Z_{\text{if}}\ Z_{\text{is}} = \left(\frac{B - Z_{\text{itcf}}\ D}{Z_{\text{itcf}}\ C - A}\right)\left(\frac{B - Z_{\text{itcs}}\ D}{Z_{\text{itcs}}\ C - A}\right) \\
&= \left(\frac{D}{C}\right)^2 \left(\frac{Z_{\text{itcf}} - Z_{\text{its}}}{Z_{\text{itcf}} - Z_{\text{itf}}}\right)\left(\frac{Z_{\text{itcs}} - Z_{\text{its}}}{Z_{\text{itcs}} - Z_{\text{itf}}}\right) \\
&= R^2 \left(\frac{Z_{\text{itr}} - Z_{\text{itf}}}{Z_{\text{itr}} - Z_{\text{its}}}\right)^2 \left(\frac{Z_{\text{itcf}} - Z_{\text{its}}}{Z_{\text{itcf}} - Z_{\text{itf}}}\right)\left(\frac{Z_{\text{itcs}} - Z_{\text{its}}}{Z_{\text{itcs}} - Z_{\text{itf}}}\right) \\
\tanh^2 \gamma l = \frac{Z_{\text{is}}}{Z_{\text{if}}} &= \left(\frac{Z_{\text{itcf}} - Z_{\text{itf}}}{Z_{\text{itcf}} - Z_{\text{its}}}\right)\left(\frac{Z_{\text{itcs}} - Z_{\text{its}}}{Z_{\text{itcs}} - Z_{\text{itf}}}\right)
\end{aligned}$$

5.7 Extended open/short circuit method without using a balun

5.7.1 Basic equations and circuit diagrams

Characteristic impedance and the propagation coefficient are defined by Equation (76) and Equation (77) respectively:

$$\frac{1}{Z_{\text{C}}} = \sqrt{\left(Y_{\text{ff}} - \frac{1}{4}Y_{\text{uf}}\right)\left(Y_{\text{fs}} - \frac{1}{4}Y_{\text{us}}\right)} \tag{76}$$

$$\gamma = \alpha + j\beta = \frac{1}{l}\ \tanh^{-1}\sqrt{\frac{\left(Y_{\text{ff}} - \frac{1}{4}Y_{\text{uf}}\right)}{\left(Y_{\text{fs}} - \frac{1}{4}Y_{\text{us}}\right)}} \tag{77}$$

where

Y_{ff} is the admittance measured with measurement mode a (S);

Y_{fs} is the admittance measured with measurement mode b (S);

Y_{uf} is the admittance measured with measurement mode c (S);

Y_{us} is the admittance measured with measurement mode d (S).

The measurement configurations are given in Figure 6.

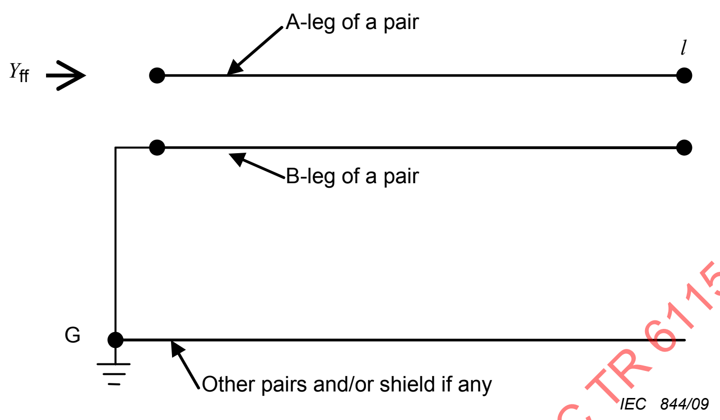


Figure 6a – Measurement mode a: Y_{ff}

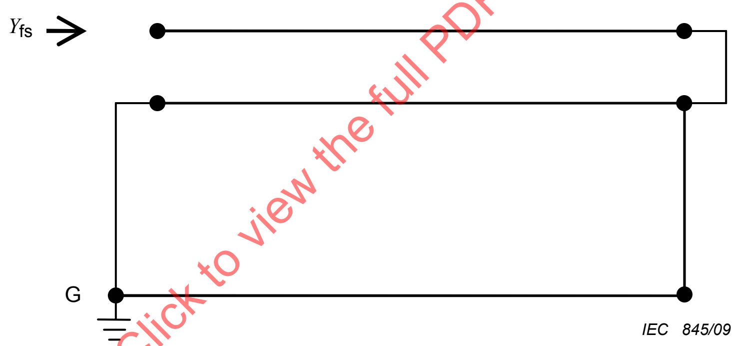


Figure 6b – Measurement mode b: Y_{fs}

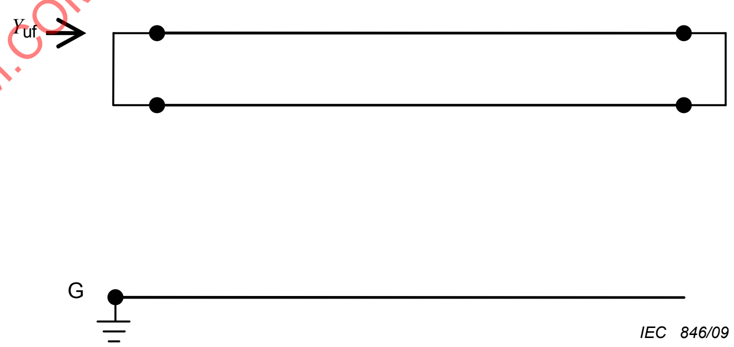


Figure 6c – Measurement mode c: Y_{uf}

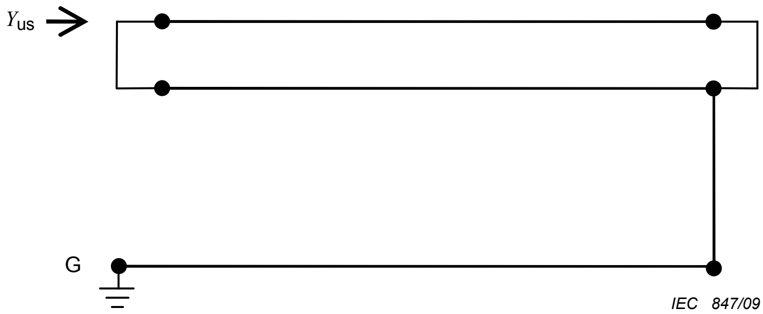


Figure 6d – Measurement mode d: Y_{us}

Key

→ connecting inner conductor of unbalanced type measuring equipment

G connecting outer conductor of unbalanced type measuring equipment

The above set of four admittance measurement configurations assumes the pair is perfectly balanced. Generally, some degree of unbalance is present. This method can be used without additional measurements if the pair unbalance is less than 1 %.

Figure 6 – Admittance measurement configurations

5.7.2 Measurement principle

The measurement principle is given in Figure 7. The input admittance measurements are implemented by means of an impedance bridge or network analyzer and *S*-parameter test set.

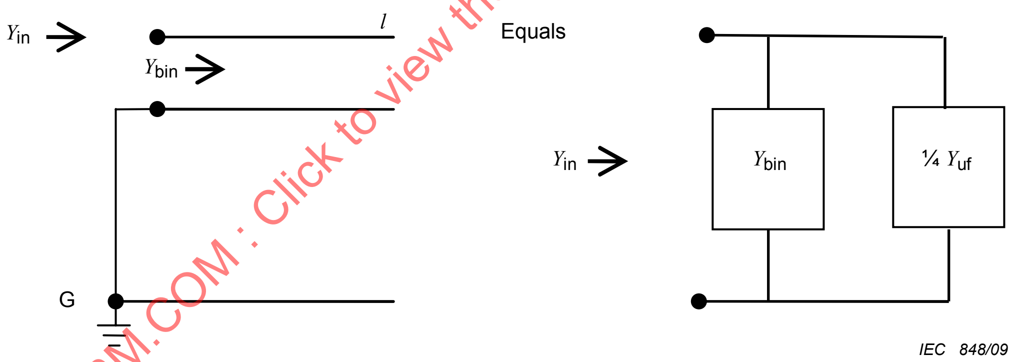


Figure 7 – Admittance measurement principle

For the open circuit case, the measured admittance is given by:

$$Y_{in} = Y_{bin} + \frac{1}{4} Y_u \tanh \gamma_u l = Y_{bin} + \frac{1}{4} Y_{uf} \tag{78}$$

where

γ_u is the unbalanced (common mode) propagation coefficient;

Y_u is the unbalanced (common mode) characteristic admittance;

Y_{bin} is the input admittance of the balanced circuit (open or short).

$$Y_{ff} = Y_{in}|_{Y_{bin}=Y_f} = Y_f + \frac{1}{4}\ Y_{uf} \tag{79}$$

$$Y_{fs} = Y_{in}|_{Y_{bin}=Y_s} = Y_s + \frac{1}{4}\ Y_{us} \tag{80}$$

where

Y_f is the balanced open circuit admittance;

Y_s is the balanced short circuit admittance.

From Equation (79),

$$Y_f = \frac{1}{Z_f} = Y_{ff} - \frac{1}{4}\ Y_{uf} \tag{81}$$

From Equation (80),

$$Y_s = \frac{1}{Z_s} = Y_{fs} - \frac{1}{4}\ Y_{us} \tag{82}$$

$$\frac{1}{Z_C} = Y_C = \sqrt{Y_f\ Y_s} = \sqrt{\left(Y_{ff} - \frac{1}{4} Y_{uf}\right)\left(Y_{fs} - \frac{1}{4} Y_{us}\right)} \tag{83}$$

$$\gamma = \alpha + j\beta = \frac{1}{l} \tanh^{-1} \sqrt{\frac{\left(Y_{ff} - \frac{1}{4} Y_{uf}\right)}{\left(Y_{fs} - \frac{1}{4} Y_{us}\right)}} \tag{84}$$

5.8 Open/short impedance measurements at low frequencies with a balun

For the measurement of the characteristic impedance of a cable, the open/short-circuit method can be applied, especially in the frequency range up to 1 MHz. An impedance measuring set with an accuracy of ±2 % is recommended.

The measurement is carried out at the relevant frequency by connecting the pair (or one side of the quad) at one end through a balun to the test set. At the other end, the conductors should be isolated (open-circuited) or short-circuited.

In the open-circuited condition:

$$Z_{CO} = R_L\ e^{j\Psi_L} \tag{85}$$

In the short-circuited condition

$$Z_{CC} = R_K\ e^{j\Psi_K} \tag{86}$$

The modulus of the characteristic impedance is:

$$|Z| = [\ R_L \times R_K]^{1/2} \tag{87}$$

$$\text{Arg}\,|Z| = 1/2\,(\Psi_L + \Psi_K) \qquad (88)$$

The attenuation constant is derived from:

$$\alpha = \frac{8{,}686}{2\,l} \times \text{arctan}\,h \left[\frac{2\sqrt{\frac{R_K}{R_L}}}{1+\frac{R_K}{R_L}} \times \cos\left[1/2\,(\Phi_K - \Phi_L)\right] \right] \quad (\text{dB/km}) \qquad (89)$$

where l is the length of the cable under measurement (km).

The phase constant is derived from:

$$\beta = \frac{1}{2\,l} \left[\text{arctan}\,h \left[\frac{2\sqrt{\frac{R_K}{R_L}}}{1-\frac{R_K}{R_L}} \times \sin\left[1/2\,(\Phi_K - \Phi_L)\right] \right] + n \times \pi \right] \quad (\text{rad/km}) \qquad (90)$$

As the function arctan is ambiguous, the value of n has to be determined. In practice, the following formula gives, in most cases, the exact value of n:

$$n = \text{integer}\left[\,|(1/\pi)\,(b - 2\pi f Z_c C_3/500)| + 0{,}2\right] \qquad (91)$$

where

C_3 is the mutual capacitance of the test specimen (nF).

$$\beta = \text{arctan} \left[\frac{2\sqrt{\frac{R_K}{R_L}}}{1-\frac{R_K}{R_L}} \times \sin\left[1/2\,(\Phi_K - \Phi_L)\right] \right] \qquad (92)$$

The phase velocity is derived from:

$$v = 2\pi f / \beta \qquad (93)$$

5.9 Characteristic impedance and propagation coefficient obtained from modal decomposition technique

5.9.1 General

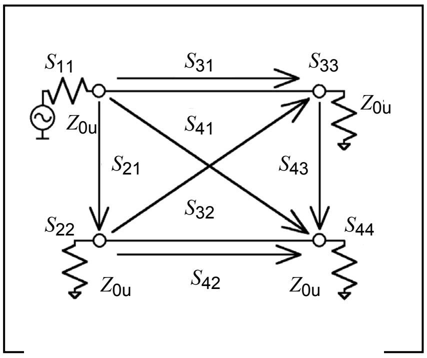
This more involved method results in data for the characteristic impedance and propagation coefficient if desired. Furthermore, it yields data for the unbalanced (common) mode as well as cross modal coupling. All combinations of S-parameters are measured using a conventional unbalanced instrument without the use of baluns, with other conductor ends terminated. The balanced- and unbalanced-mode components (impedance element of the matrix) are derived from the measured S-matrix by a mathematical operation ("mathematical balun").

5.9.2 Procedure

The procedure is as follows.

a) Calibrate the network analyser system. Full-2-port calibration is recommended.

b) Measure each element of the S-matrix of the Equation (94), e.g. S_{11}, S_{31} (S_{31}), and S_{33} are measured by connecting the one end of the conductor of the pair to the other port of the network analyser. All the rest of the ends of the conductors of the twisted pair, which may be terminated to the receptacle of the standard connectors respectively, should be terminated with the standard dummy of the network analyser.



$$\begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{22} & S_{32} & S_{42} \\ S_{31} & S_{32} & S_{33} & S_{43} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (94)$$

c) Transform the S – matrix into the Z – matrix (Y – matrix) using the following equations.

$$Z = z_{0u}\,[E + S][E - S\,]^{-1} \quad (95)$$

$$Y = \frac{1}{z_{0u}}\,[E - S][E + S\,]^{-1} \quad (96)$$

where

E is the unit matrix of 4 × 4;

Z_{0u} is the system impedance of a scalar value.

d) Once the impedance matrix is obtained, the characteristic impedance and the propagation coefficient for the balanced mode are calculated by the following equations:

$$Z_C = 2\sqrt{\frac{Z_{11} - 2Z_{21} + Z_{22}}{Y_{11} - 2Y_{21} + Y_{22}}} \quad (97)$$

$$\gamma = \frac{1}{2l}\ln\left(\frac{\frac{1}{2}\sqrt{(Z_{11} - 2Z_{21} + Z_{22})(Y_{11} - 2Y_{21} + Y_{22})} + 1}{\frac{1}{2}\sqrt{(Z_{11} - 2Z_{21} + Z_{22})(Y_{11} - 2Y_{21} + Y_{22})} - 1}\right) \quad (98)$$

5.9.3 Measurement principle

This method utilizes the modal decomposition theory, which has been established in the field of analyzing a multi-conductor system.

Notation of secondary coefficient: The secondary coefficient is expressed using an impedance matrix Z and an admittance matrix Y. The transmission line system illustrated in Figure 8 is presumed linear and symmetrical to show simple expression.

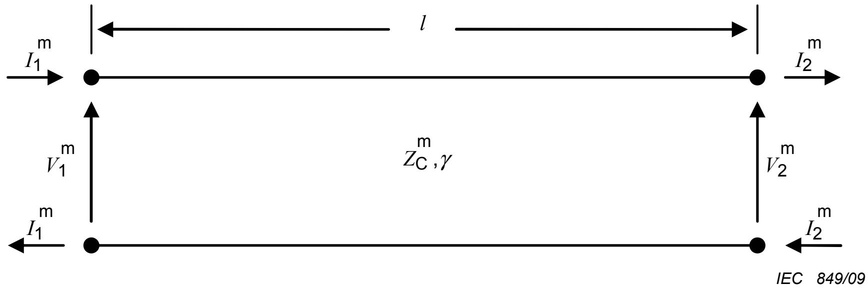


Figure 8 – Transmission line system

$$\begin{bmatrix} V_1^m \\ I_1^m \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_C^m \sinh(\gamma l) \\ \sinh(\gamma l)/Z_C^m & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_2^m \\ I_2^m \end{bmatrix} \tag{99}$$

When modified, the second part of the matrix equation is:

$$V_2^m = \frac{Z_C^m}{\sinh(\gamma l)} I_1^m - Z_C^m \coth(\gamma l)\, I_2^m \tag{100}$$

Substituting this into Equation (100), the following impedance matrix is derived:

$$\begin{bmatrix} V_1^m \\ V_2^m \end{bmatrix} = \begin{bmatrix} Z_C^m \coth(\gamma l) & Z_C^m/\sinh(\gamma l) \\ Z_C^m/\sinh(\gamma l) & Z_C^m \coth(\gamma l) \end{bmatrix} \begin{bmatrix} I_1^m \\ -I_2^m \end{bmatrix} = \begin{bmatrix} Z_{11}^m & Z_{21}^m \\ Z_{21}^m & Z_{11}^m \end{bmatrix} \begin{bmatrix} I_1^m \\ -I_2^m \end{bmatrix} \tag{101}$$

Similarly, the admittance expression is derived:

$$\begin{bmatrix} I_1^m \\ -I_2^m \end{bmatrix} = \begin{bmatrix} \coth(\gamma l)/Z_C^m & -1/Z_C^m \sinh(\gamma l) \\ -1/Z_C^m \sinh(\gamma l) & \coth(\gamma l)/Z_C^m \end{bmatrix} \begin{bmatrix} V_1^m \\ V_2^m \end{bmatrix} = \begin{bmatrix} Y_{11}^m & Y_{21}^m \\ Y_{21}^m & Y_{11}^m \end{bmatrix} \begin{bmatrix} V_1^m \\ V_2^m \end{bmatrix} \tag{102}$$

Thus we can get the secondary constants $Z_C{}^m$ and γ as:

$$Z_C^m = \sqrt{\frac{Z_{11}^m}{Y_{11}^m}}, \quad \gamma = \frac{1}{l} \coth^{-1} \sqrt{Z_{11}^m Y_{11}^m} = \frac{1}{2l} \ln\left(\frac{\sqrt{Z_{11}^m Y_{11}^m} + 1}{\sqrt{Z_{11}^m Y_{11}^m} - 1} \right) \tag{103}$$

Because Z_{11}^m can be obtained by measuring the ratio of V_1^m to I_{11}^m with the other terminal opened, that is, by letting $I_2^m = 0$,

$$Z_{11}^m = \left. \frac{V_1^m}{I_1^m} \right|_{I_2^m = 0} = Z_C^m \coth(\gamma l), \quad Y_{11}^m = \left. \frac{I_1^m}{V_1^m} \right|_{V_2^m = 0} = \frac{1}{Z_C^m} \coth(\gamma l) \tag{104}$$

thus, $Z_{11}^{m} = Z_{open}^{m}$ and $Y_{11}^{m} = Y_{short}^{m}$. This shows that Equations (103) are identical to those which are well known to us as equations for the open/short method.

For the case of a twisted pair cable, the impedance and the admittance matrix in the modal domain shall be derived.

5.9.4 Scattering matrix to impedance matrix

5.9.4.1 General

The impedance and admittance matrices of the modal domain of the balanced mode can calculate the secondary constants of the pair.

The following three steps are required:

a) measure the scattering parameters of multi-conductor circuit;

b) calculate the impedance and admittance matrix (Z-matrix and Y-matrix respectively) from the scattering matrix (S-matrix); and

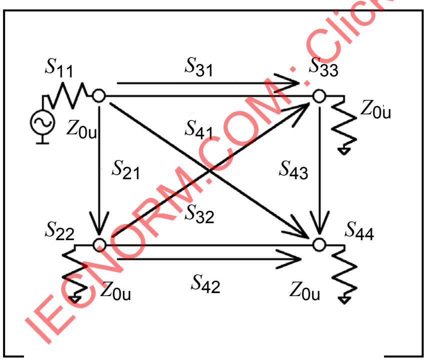
c) calculate the impedance and admittance of the balanced mode according to the modal decomposition theory.

5.9.4.2 Step 1: *S*-matrix measurement

The measurement is as follows.

a) Calibrate the network analyser system. Full 2-port calibration is recommended.

b) Measure each element of the S-matrix of the Equation (105), e.g. S_{11}, S_{31} (S_{31}), and S_{33} are measured by connecting the one end of the conductor of the pair to the other port of the network analyser. All the rest of the ends of the conductors of the twisted pair, which may be terminated to the receptacle of the standard connectors respectively, should be terminated with the standard dummy of the network analyser.



$$\begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{22} & S_{32} & S_{42} \\ S_{31} & S_{32} & S_{33} & S_{43} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (105)$$

5.9.4.3 Step 2: Transform *S*–matrix into *Z*–matrix

Transform the S – matrix into the Z – matrix (Y – matrix) using the following equations:

$$Z = z_{0u}\,[E+S][E-S\,]^{-1},\quad Y = \frac{1}{z_{0u}}\,[E-S][E+S\,]^{-1} \quad (106)$$

where E is a unit matrix of 4 × 4, z_0 is the system impedance of a measuring equipment and is defined as a scalar value (typically 50 Ω system).

5.9.4.4 Step 3: Modal decomposition

According to the modal decomposition theory, the impedance matrix Z^{m} and the admittance matrix Y^{m} for a twisted pair cable can be obtained from the multi-conductor line circuit impedance (Z) and admittance (Y) as follows.

$$Z^{\mathrm{m}} = P^{-1} Z Q, \; Y^{\mathrm{m}} = Q^{-1} Y P \tag{107}$$

where the diagonalizing matrices P and Q are 4×4 real matrices and given as follows:

$$P = \begin{bmatrix} \frac{1}{2} & 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix}, \; Q = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ -1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix} \tag{108}$$

When the line circuit is assumed to be linear, the matrices are symmetrical and their expressions become:

$$Z^{\mathrm{m}} = \begin{bmatrix} Z_{11} - 2Z_{21} + Z_{22} & \frac{Z_{11} - Z_{22}}{2} & Z_{31} - Z_{41} - Z_{32} + Z_{42} & \frac{Z_{31} + Z_{41} - Z_{32} - Z_{42}}{2} \\ \frac{Z_{11} - Z_{22}}{2} & \frac{Z_{11} + 2Z_{21} + Z_{22}}{4} & \frac{Z_{31} - Z_{41} + Z_{32} - Z_{42}}{2} & \frac{Z_{31} + Z_{41} + Z_{32} + Z_{42}}{4} \\ Z_{31} - Z_{32} - Z_{41} + Z_{42} & \frac{Z_{31} + Z_{32} - Z_{41} - Z_{42}}{2} & Z_{33} - 2Z_{43} + Z_{44} & \frac{Z_{33} - Z_{44}}{2} \\ \frac{Z_{31} - Z_{32} + Z_{41} - Z_{42}}{2} & \frac{Z_{31} + Z_{32} + Z_{41} + Z_{42}}{4} & \frac{Z_{33} - Z_{44}}{2} & \frac{Z_{33} + 2Z_{43} + Z_{44}}{4} \end{bmatrix} \tag{109}$$

$$Z_{11}^{\mathrm{m}} = Z_{11} - 2Z_{21} + Z_{22} \tag{110}$$

$$Y^{\mathrm{m}} = \begin{bmatrix} \frac{Y_{11} - 2Y_{21} + Y_{22}}{4} & \frac{Y_{11} - Y_{22}}{2} & \frac{Y_{31} - Y_{41} - Y_{32} + Y_{42}}{4} & \frac{Y_{31} + Y_{41} - Y_{32} - Y_{42}}{2} \\ \frac{Y_{11} - Y_{22}}{2} & Y_{11} + 2Y_{21} + Y_{22} & \frac{Y_{31} - Y_{41} + Y_{32} - Y_{42}}{2} & Y_{31} + Y_{41} + Y_{32} + Y_{42} \\ \frac{Y_{31} - Y_{32} - Y_{41} + Y_{42}}{4} & \frac{Y_{31} + Y_{32} - Y_{41} - Y_{42}}{2} & \frac{Y_{33} - 2Y_{43} + Y_{44}}{4} & \frac{Y_{33} - Y_{44}}{2} \\ \frac{Y_{31} - Y_{32} + Y_{41} - Y_{42}}{2} & Y_{31} + Y_{32} + Y_{41} + Y_{42} & \frac{Y_{33} - Y_{44}}{2} & Y_{33} + 2Y_{43} + Y_{44} \end{bmatrix} \tag{111}$$

$$Y_{11}^{\mathrm{m}} = \frac{Y_{11} - 2Y_{21} + Y_{22}}{4} \tag{112}$$

The following equations are derived from Equations (103).

$$Z_{\mathrm{C}}^{\mathrm{m}} = \sqrt{\frac{Z_{11}^{\mathrm{m}}}{Y_{11}^{\mathrm{m}}}} = 2\sqrt{\frac{Z_{11} - 2Z_{21} + Z_{22}}{Y_{11} - 2Y_{21} + Y_{22}}} \tag{113}$$

$$\gamma = \frac{1}{l}\coth^{-1}\sqrt{(Z_{11}^{m}\,Y_{11}^{m})} = \frac{1}{2l}\ln\left(\frac{\sqrt{Z_{11}^{m}Y_{11}^{m}}+1}{\sqrt{Z_{11}^{m}Y_{11}^{m}}-1}\right)$$
$$= \frac{1}{l}\coth^{-1}\left\{(Z_{11}-2Z_{21}+Z_{22})\left(\frac{Y_{11}-2Y_{21}+Y_{22}}{4}\right)\right\}^{1/2} \quad (114)$$
$$= \frac{1}{2l}\ln\left(\frac{\frac{1}{2}\sqrt{(Z_{11}-2Z_{21}+Z_{22})(Y_{11}-2Y_{21}+Y_{22})}+1}{\frac{1}{2}\sqrt{(Z_{11}-2Z_{21}+Z_{22})(Y_{11}-2Y_{21}+Y_{22})}-1}\right)$$

5.9.5 Expression of results

When the secondary transmission parameters deal with frequency domain data and show that the data varies substantially versus frequency, the least squares function fit method is used to extract the secondary transmission parameters as theoretic ideal parameters of the transmission line.

6 Measurement of return loss and structural return loss

6.1 General

Return loss and SRL are both useful for quantifying the level (amount) of the reflected signal. Return loss combines the effects of reflections due to both the deviation from the nominal impedance (such as 100 Ω) and structural effects. It is specified when system performance is the primary interest.

While return loss characterizes the performance of the channel or link, SRL is used to represent the structural effects of the cable medium itself relative to Z_C and is useful for cable evaluation.

6.2 Principle

The same measurement principles apply as in 5.2. Many network analysers yield return loss in a direct manner as a menu item. The circuit given in Figure 5 is suitable for the RL and SRL measurements. Where calibration of the network analyser and S-parameter unit is performed relative to the reference impedance, the return loss, RL, is given by Equation (115):

$$RL = -20\log\left|S_{11}\right| \quad (115)$$

Stated in terms of the impedances the return loss, RL, is given by Equation (116):

$$RL = -20\log\left|\frac{Z_T - Z_R}{Z_T + Z_R}\right| \quad (116)$$

NOTE Open/short circuit data is not appropriate for return loss since both ends of the circuit must be terminated with the reference impedance. The difference between the Z_T used here and the Z_C used for SRL is obviously small when roundtrip loss is large enough to render the distant-end reflection negligible.

The SRL is obtained by Equation (117), where Z_C is the fitted characteristic impedance being used as the reference value.

$$SRL = -20 \log \left| \frac{Z_{CM} - Z_C}{Z_{CM} + Z_C} \right| \quad (117)$$

7 Propagation coefficient effects due to periodic structural variation related to the effects appearing in the structural return loss

7.1 General

The characteristic impedance Z_C of a cable is defined as the quotient of a voltage wave (U) and current wave (I) which are propagating in the same direction, forwards (f) or backwards (r). For homogeneous cables with no structural variations, the characteristic impedance can be measured directly as the quotient of voltage and current at the cable ends.

$$Z_C = U_f \; / \; I_f = U_r \; / \; I_r \quad (118)$$

The other characteristics which are important for a cabling system are the input and output impedances and the corresponding return losses and the structural return loss of the cable. These characteristics include structural variation in the cable. They are measured by the S_{11} and S_{22} parameters of the cable, as described in the following.

Important cable-related parameters, which for their part describe the quality of the cable as a transmission medium, are the characteristic impedance Z_C and the structural return loss SRL.

System-related parameters are the input impedance and the return loss at the input and output of the cable, which are related to the scattering parameters S_{11} and S_{22}. The insertion loss is also a system-related parameter which is denoted by S_{21}.

The transmission (propagation) coefficient:

$$\gamma = \alpha + j\beta \quad (119)$$

is only cable-related. It has already been discussed in Clause 4.

7.2 Equation for the forward echoes caused by periodic structural inhomogeneities

The reflected signals down the line have normally little direct effect on the transmission but through double reflections they influence the forward transmission causing forward echoes at resonant spike frequencies.

With periodic inhomogeneities extending throughout the line, the forward echo coefficient q can be calculated from Equation (120) when the measured periodic structural return loss $PSRL$ coefficient is p at a resonant frequency.

$$|q|_{max} = K |p|^2_{max} \quad (120)$$

where

$$K = \frac{2\alpha l - 1 + e^{-2\alpha l}}{(1 - e^{-2\alpha l})^2} \quad (121)$$

When $2\alpha l >> 1$ (Np):

$$K \approx 2\alpha l - 1 \quad (122)$$

The above is only cable- and cable length-related.