IEC/TR 61000-1-6

Edition 1.0   2012-07

TECHNICAL
REPORT

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inside

BASIC EMC PUBLICATION

Electromagnetic compatibility (EMC) –
Part 1-6: General – Guide to the assessment of measurement uncertainty

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INTERNATIONAL
ELECTROTECHNICAL
COMMISSION

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

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ELECTROMAGNETIC COMPATIBILITY (EMC) –

Part 1-6: General – Guide to the assessment of measurement uncertainty

FOREWORD

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IEC 61000-1-6, which is a technical report, has been prepared by the IEC technical committee 77: Electromagnetic compatibility in corporation with CISPR (International Special Committee on Radio Interference).

It forms Part 1-6 of IEC 61000. It has the status of a basic EMC publication in accordance with IEC Guide 107, *Electromagnetic compatibility – Guide to the drafting of electromagnetic compatibility publications*.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
77/397/DTR	77/409/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

A list of all the parts of the IEC 61000 series, published under the general title *Electromagnetic compatibility* (*EMC*) can be found on the IEC website.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

• reconfirmed,
• withdrawn,
• replaced by a revised edition, or
• amended.

A bilingual version of this publication may be issued at a later date.

The contents of the corrigendum of October 2014 have been included in this copy.

IMPORTANT – The 'colour inside' logo on the cover page of this publication indicates that it contains colours which are considered to be useful for the correct understanding of its contents. Users should therefore print this document using a colour printer.

INTRODUCTION

IEC 61000 is published in separate parts, according to the following structure:

Part 1: General

General considerations (introduction, fundamental principles)

Definitions, terminology

Part 2: Environment

Description of the environment

Classification of the environment

Compatibility levels

Part 3: Limits

Emission limits

Immunity limits (in so far as they do not fall under the responsibility of the product committees)

Part 4: Testing and measurement techniques

Measurement techniques

Testing techniques

Part 5: Installation and mitigation guidelines

Installation guidelines

Mitigation methods and devices

Part 6: Generic standards

Part 9: Miscellaneous

Each part is further subdivided into several parts, published either as international standards or as technical specifications or technical reports, some of which have already been published as sections. Others will be published with the part number followed by a dash and a second number identifying the subdivision (example: IEC 61000-6-1).

ELECTROMAGNETIC COMPATIBILITY (EMC) –

Part 1-6: General –
Guide to the assessment of measurement uncertainty

1   Scope

This part of IEC 61000 provides methods and background information for the assessment of measurement uncertainty. It gives guidance to cover general measurement uncertainty considerations within the IEC 61000 series.

The objectives of this Technical Report are to give advice to technical committees, product committees and conformity assessment bodies on the development of measurement uncertainty budgets; to allow the comparison of these budgets between laboratories that have similar influence quantities; and to align the treatment of measurement uncertainty across the EMC committees of the IEC.

Any contributing factor to measurement uncertainty that is mentioned within this Technical Report shall be treated as an example: the technical committee responsible for the preparation of a basic immunity standard is responsible for identifying the factors that contribute to the measurement uncertainty of their basic test method.

It gives a description for

–   a method for the assessment of measurement uncertainty (MU),

–   mathematical formulas for probability density functions,

–   analytical assessment of statistical evaluations,

–   correction of measured data,

–   documentation.

This Technical Report is not intended to summarize all measurement uncertainty influence quantities nor is it intended to define how measurement uncertainty is to be taken into account in determining compliance with an EMC requirement.

NOTE   Some of the examples given in this report are taken from IEC publications other than the IEC 61000 series that have already implemented the evaluation procedure presented here. These examples are used to illustrate the principles.

2   Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-161, *International Electrotechnical Vocabulary (IEV) – Chapter 161: Electromagnetic compatibility*

CISPR 16-1-1, *Specification for radio disturbance and immunity measuring apparatus and methods – Part 1-1: Radio disturbance and immunity measuring apparatus – Measuring apparatus*

CISPR 16-4-2, *Specification for radio disturbance and immunity measuring apparatus and methods – Part 4-2: Uncertainties, statistics and limit modelling – Uncertainty in EMC measurements*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*, corrected 1st edition, 2008

3 Terms, definitions, symbols and abbreviations

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-161, as well as the following apply.

NOTE   Several of the most relevant terms and definitions from IEC 60050-161 are included among the terms and definitions below.

3.1.1
combined standard uncertainty
standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model

[SOURCE: ISO/IEC Guide 99:2007, definition 2.31, modified – Admitted term became the preferred (and only) term.]

3.1.2
confidence level
probability, generally expressed as a percentage, that the true value of a statistically estimated quantity falls within a pre-established interval about the estimated value

[SOURCE: IEC 60050-393:2003, 393-18-31]

3.1.3
coverage factor
numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty

[SOURCE: ISO/IEC Guide 98-3:2008, definition 2.3.6, modified – NOTE was deleted.]

3.1.4
coverage interval
interval containing the set of quantity values of a measurand with a stated probability, based on the information available

[SOURCE: ISO/IEC Guide 99:2007, definition 2.36, modified – True quantity values was changed to quantity values.]

3.1.5
coverage probability
probability that the set of quantity values of a measurand is contained within a specified coverage interval

[SOURCE: ISO/IEC Guide 99:2007, definition 2.37, modified – True quantity values was changed to quantity values.]

3.1.6
distribution function
function giving, for every value ξ, the probability that the random variable X be less than or equal to ξ:

$$G(\xi) = \Pr(X \leq \xi)$$

[SOURCE: ISO/IEC Guide 98-3, Supplement 1:2008, definition 3.2]

3.1.7
error
measured quantity value minus a reference quantity value

[SOURCE: ISO/IEC Guide 99:2007, definition 2.16, modified – Second admitted term became the preferred (and only) term.]

3.1.8
expanded uncertainty
quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand

[SOURCE: ISO/IEC Guide 98-3:2008, definition 2.3.5, modified – Notes 1 to 3 were deleted.]

3.1.9
electromagnetic compatibility
EMC
ability of an equipment or system to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to anything in that environment

[SOURCE: IEC 60050-161:1990, 161-01-07]

3.1.10
emission
phenomenon by which electromagnetic energy emanates from a source

[SOURCE: IEC 60050-161:1990, 161-01-08, modified – The addition "electromagnetic" in the term was deleted.]

3.1.11
emission level
emission level of a disturbing source
level of a given electromagnetic disturbance emitted from a particular device, equipment or system

[SOURCE: IEC 60050-161:1990, 161-03-11]

3.1.12
emission limit
emission limit from a disturbing source
specified maximum emission level of a source of electromagnetic disturbance

[SOURCE: IEC 60050-161:1990, 161-03-12]

3.1.13
immunity
immunity to a disturbance
ability of a device, equipment or system to perform without degradation in the presence of an electromagnetic disturbance

[SOURCE: IEC 60050-161:1990, 161-01-20]

3.1.14
immunity limit
specified minimum immunity level

[SOURCE: IEC 60050-161:1990, 161-03-15]

3.1.15
immunity test level
level of a test signal used to simulate an electromagnetic disturbance when performing an immunity test

[SOURCE: IEC 60050-161:1990, 161-04-41]

3.1.16
indication
quantity value provided by a measuring instrument or a measuring system

[SOURCE: ISO/IEC Guide 99:2007, definition 4.1, modified – Notes 1 and 2 were deleted.]

3.1.17
influence quantity
quantity that is not the measurand but that affects the result of the measurement

[SOURCE: IEC 60050-394:2007, 394-40-27, modified – Note was deleted.]

3.1.18
instrumentation uncertainty
IU
measurement instrumentation uncertainty
MIU
parameter, associated with the disturbance quantity generated during an emission measurement or applied during an immunity test that characterizes the dispersion of the values that could reasonably be attributed to the measurand, induced by all relevant influence quantities that are related to the measurement instrumentation and the test facility

Note 1 to entry: This term is intended to be applicable to both emission measurements and immunity tests. The CISPR 16 series of documents also employs the term 'measurement instrumentation uncertainty' (MIU).

Note 2 to entry: Based on IEC 60359:2001, definition 3.1.4.

3.1.19
intrinsic uncertainty of the measurand
minimum uncertainty that can be assigned in the description of a measured quantity

Note 1 to entry: In theory, the intrinsic uncertainty of the measurand would be obtained if the measurand was measured using a measurement system having a negligible measurement instrumentation uncertainty.

Note 2 to entry: No quantity can be measured with continually lower uncertainty, inasmuch as any given quantity is defined or identified at a given level of detail. If one tries to measure a given quantity at an uncertainty lower than its own intrinsic uncertainty one is compelled to redefine it with higher detail, so that one is actually measuring another quantity. See also ISO/IEC Guide 98-3:2008, D.1.1.

Note 3 to entry: The result of a measurement carried out with the intrinsic uncertainty of the measurand may be called the best measurement of the quantity in question.

[SOURCE: IEC 60359:2001, definition 3.1.11, modified – An additional explanation has been added, i.e. Note 1 to entry.]

3.1.20
level
level of a time varying quantity
value of a quantity, such as a power or a field quantity, measured and/or evaluated in a specified manner during a specified time interval

[SOURCE: IEC 60050-161:1990, 161-03-01, modified – The NOTE was deleted.]

3.1.21
limits of error of a measuring instrument
extreme value of measurement error, with respect to a known reference quantity value, permitted by specifications or regulations for a given measurement, measuring instrument, or measuring system

[SOURCE: ISO/IEC Guide 99:2007, definition 4.26, modified – The term has been clarified and Notes 1 and 2 have been deleted.]

3.1.22
measurand
particular quantity subject to measurement

[SOURCE: IEC 60050-311:2001, 311-01-03]

3.1.23
measurement accuracy
accuracy of measurement
DEPRECATED: precision of measurement
closeness of agreement between a measured quantity value and the true quantity value of a measurand

Note 1 to entry: 'accuracy' is a qualitative concept.

[SOURCE: IEC 60050-311:2001, 311-06-08, modified – The term has been changed and replaced by two terms, Note 1 has been deleted and Note 2 replaced by an explanation.]

3.1.24
measurement precision
closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions

[SOURCE: ISO/IEC Guide 99:2007, definition 2.15, modified – Notes 1 to 4 have been deleted.]

3.1.25
measurement result
set of quantity values being attributed to a measurand together with any other available relevant information

[SOURCE: IEC 60050-311:2001, 311-01-01, modified – The term has been clarified and the definition extended. Notes 1 to 5 have been deleted.]

3.1.26
measuring system
complete set of measuring instruments and other equipment assembled to carry out specified measurements

[SOURCE: IEC 60050-311:2001, 311-03-06]

3.1.27
measurement trueness
closeness of agreement between the average of an infinite number of replicate measured quantity values and the reference quantity value

[SOURCE: ISO/IEC Guide 99:2007, definition 2.14, modified – Only preferred term is given and Notes 1 to 3 have been deleted.]

3.1.28
measurement uncertainty
MU
non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used

[SOURCE: ISO/IEC Guide 99:2007, definition 2.26, modified – Only preferred term is given and Notes 1 to 4 have been deleted.]

3.1.29
probability density function
PDF
derivative, when it exists, of the distribution function

$$g(\xi) = \frac{\mathrm{d}G(\xi)}{\mathrm{d}\xi}$$

Note 1 to entry: $g(\xi)\mathrm{d}\xi$ is the 'probability element'; $g(\xi)\mathrm{d}\xi = \mathrm{Pr}(\xi < X < \xi + \mathrm{d}\xi)$

[SOURCE: ISO/IEC Guide 98-3:2008, definition 3.3, modified – Equation has been changed.]

3.1.30
random error
difference between a measurement and the mean that would result from an infinitely large number of measurements of the same measurand carried out under repeatability conditions

[SOURCE: IEC 60050-394:2007, 394-40-33, modified – Definition was changed and Notes 1 and 2 have been deleted.]

3.1.31
repeatability
repeatability of results of measurements
closeness of agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement, i.e.:

- by the same measurement procedure,

- by the same observer,

- with the same measuring instruments, used under the same conditions,

- in the same laboratory,

- at relatively short intervals of time

[SOURCE: IEC 60050-311:2001, 311-06-06, modified – Note has been deleted.]

3.1.32
reproducibility of measurements
closeness of agreement between the results of measurements of the same value of a quantity, when the individual measurements are made under different conditions of measurement:

- principle of measurement,

- method of measurement,

- observer,

- measuring instruments,

- reference standards,

- laboratory,

- under conditions of use of the instruments, different from those customarily used,

- after intervals of time relatively long compared with the duration of a single measurement.

Note 1 to entry: The term 'reproducibility' also applies to the instance where only certain of the above conditions are taken into account, provided that these are stated.

[SOURCE: IEC 60050-311:2001, 311-06-07, modified – Note 1 has been deleted and Note 2 has been renumbered Note 1 to entry.]

3.1.33
sensitivity coefficient
relationship between a change in an output estimate, y, for a corresponding change in an input estimate, x_i.

3.1.34
standard deviation of a single measurement in a series of measurements
parameter characterising the dispersion of the result obtained in a series of n measurements of the same measurand

$$s\left(q_j\right) = \sqrt{\frac{1}{(n-1)}\sum_{j=1}^{n}\left(q_j - \overline{q}\right)^2}$$

where $\overline{q}$ is the mean value of n measurements

[SOURCE: ISO/IEC Guide 98-3:2008, definition B.2.17, modified – Term, definition and equation have been modified and Notes 1 to 4 have been deleted.]

3.1.35
standard deviation of the arithmetic mean of a series of measurements
parameter characterising the dispersion of the arithmetic mean of a series of independent measurements of the same value of a measured quantity, given by the formula:

$$s\left(\overline{q}\right) = \sqrt{\frac{1}{n\cdot(n-1)}\sum_{j=1}^{n}\left(q_j - \overline{q}\right)^2}$$

Note 1 to entry: $s\left(\overline{q}\right)$ is the standard uncertainty for type A evaluation (see 5.3), if $\overline{q}$ is used as the estimate.

3.1.36
standard uncertainty
measurement uncertainty expressed as a standard deviation

[SOURCE: ISO/IEC Guide 99:2007, definition 2.30, modified – Admitted term became the preferred (and only) term.]

3.1.37
systematic error
difference between the arithmetic mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions and the true value of the measurand.

[SOURCE: IEC 60050-394:2007, 394-40-32, modified – Definition was changed and the Note has been deleted.]

3.1.38
tolerance
maximum variation of a value permitted by specifications, regulations, etc. for a given specified influence quantity

3.1.39
true value
actual value of the quantity being measured

Note 1 to entry: This can never be known absolutely but can be approximated (within the bounds of uncertainty) by traceability to national standards.

[SOURCE: IEC 60050-311:2001, 311-01-04, modified – Complement to term was deleted, definition has been changed, Notes 1 to 4 have been deleted and Note 1 to entry has been added.]

3.1.40
type A evaluation
evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions

[SOURCE: ISO/IEC Guide 99:2007, definition 2.28, modified – Admitted term became the preferred (and only) term and Notes 1 to 3 have been deleted.]

3.1.41
type B evaluation
evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty

[SOURCE: ISO/IEC Guide 99:2007, definition 2.29, modified – Admitted term became the preferred (and only) term and Examples and Note have been deleted.]

3.2   Symbols

X Generic quantity

a^+ Upper bound of quantity X

a^- Lower bound of quantity X

d Number of axes of field probe

N Number of repeated indications

v Number of degrees of freedom, $v = N - 1$

v_X Coefficient of variation of X

M Number of samples of N repeated indications

P Coverage probability

p Probability value, $p = (1 - P) / 2$

Q_i Random indication

$\overline{Q}$ Mean of a sample of N indications

$\overline{Q_j}$ Mean of the *j*-th sample of *N* indications

$\overline{\overline{Q}}$ Mean of *M* samples of *N* indications

$s(Q_i)$ Experimental standard deviation

$s(\overline{Q})$ Experimental standard deviation of the mean, $s(\overline{Q}) = s(Q_i)/\sqrt{N}$

$u(Q_i)$ Type A (evaluation of the) standard uncertainty, $u(Q_i) = \eta(v) \cdot s(Q_i)$

$\eta(v)$ Coefficient which transforms the experimental standard deviation to the Type A standard uncertainty

$u(\overline{Q})$ Type A (evaluation of the) standard uncertainty of the mean, $u(\overline{Q}) = u(Q_i)/\sqrt{N}$

$t_p(v)$ Upper critical value of the Student's *t* PDF with *v* degrees of freedom corresponding to probability *p* in one tail

$X_{\min}$ Lower value of a (specification, tolerance, coverage) interval for quantity *X*

$X_{\max}$ Upper value of a (specification, tolerance, coverage) interval for quantity *X*

$G(X)$ Distribution function of quantity *X*, $G(\hat{X}) = \Pr(X \le \hat{X})$, where $\Pr(\cdot)$ stands for "probability that"

$g(X)$ Probability density function (PDF) of quantity *X*, $g(X) = \mathrm{d}G(X)/\mathrm{d}X$

$\langle X \rangle$ Expected value of quantity *X*, $\langle X \rangle = \int X \cdot g(X)\mathrm{d}X$

x Best estimate of quantity *X*, $x = \langle X \rangle$

σ_X^2 Variance of quantity *X*, $\sigma_X^2 = \langle (X - \langle X \rangle)^2 \rangle = \int (X - x)^2 \cdot g(X)\mathrm{d}X$

$u(x)$ Type B (evaluation of the) standard uncertainty, $u(x) = \sigma_X$

X_i Influence quantity to a mathematical measurement model

x_i Best estimate of the influence quantity to a mathematical measurement model

δX_i Correction for influence quantity X_i

Y Output quantity from a mathematical measurement model

y Best estimate of the measurand, corrected for all recognized and significant systematic effects

c_i Sensitivity coefficient, partial derivative, with respect to X_i, of the measurement model, evaluated at the best estimates x_i of the input quantities X_i

$u(x_i)$ Standard uncertainty of the best estimate of the influence quantity X_i

$u_c(y)$ Combined standard uncertainty of the best estimate of the measurand

k Coverage factor

$U(y)$ Expanded uncertainty of the best estimate of the measurand, $U(y) = k \cdot u_c(y)$

3.3 Abbreviations

CLT   Central Limit Theorem

EM   Electromagnetic

EMC   Electromagnetic Compatibility

EME   Electromagnetic Environment

EUT   Equipment Under Test

FAR   Fully Anechoic Room

GUM   Guide to the expression of Uncertainty in Measurement

IEC   International Electrotechnical Commission

IFU   Intrinsic Field Uncertainty

IUM   Intrinsic Uncertainty of the Measurand

LPU   Law of Propagation of Uncertainty

MIU   Measurement Instrumentation Uncertainty

MU   Measurement Uncertainty

OATS   Open Area Test Site

PDF   Probability Density Function

RSS   Root of the Sum of Squares

SAC   Semi-Anechoic Chamber

SCU   Standards Compliance Uncertainty

VSWR   Voltage Standing Wave Ratio

4   General

4.1   Overview

This Technical Report presents background material on the principles of Measurement Uncertainty (MU) and guidelines on the calculation and application of MU values. The Technical Report is intended as an aid to those preparing EMC standards under the IEC 61000 series.

4.2   Classification of uncertainty contributions

An estimated value of an electrical or electromagnetic (EM) quantity becomes more meaningful when a quantitative statement of uncertainty and confidence is reported together with this value. Further discussion is focused here on the experimental evaluation of uncertainty, rather than evaluation through numerical calculation (e.g. Monte Carlo methods or simulation) that may provide an alternative or additional method for the evaluation of uncertainty.

MU can be subdivided into different components (see Figure 1).
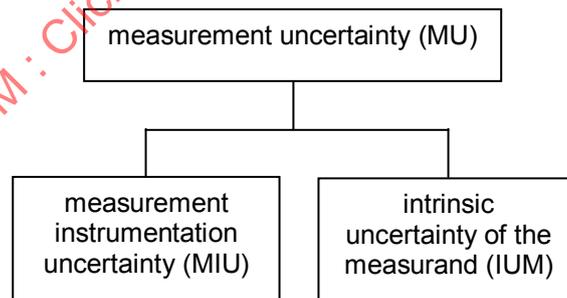
IEC   1303/12

**Figure 1 – Classification of uncertainty components associated with
the experimental evaluation of uncertainty in EMC testing and measurement**
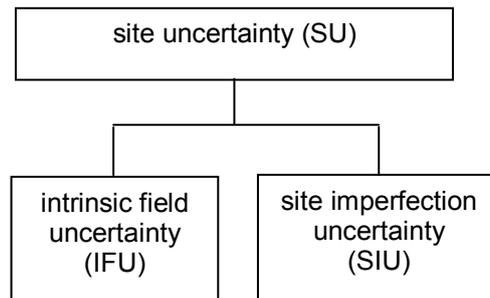
IEC   1304/12

**Figure 2 – Classification of uncertainty components associated
with site uncertainty (e.g. reverberation chambers)**

Figure 1 shows a classification of contributions to the MU, which consists of two components:

a) measurement instrumentation uncertainty (MIU), which represents the contribution by the instrumentation (e.g., antennas/probes, analyzers, cables, test facility) and

b) the intrinsic uncertainty of the measurand (IUM), which represents the contribution by the EUT (e.g. instability, setup, lack of definition of the setup).

Figure 2 shows a classification of contributions to the site uncertainty for e.g. reverberation chambers, which consists of two components:

c) intrinsic field uncertainty (IFU), which represents the contribution inherent to the complexity (IFU for radiated phenomena, where applicable) and

d) imperfections of the test site (SIU).

NOTE   Site uncertainty and site imperfection uncertainty are the same in the case of a semi-anechoic chamber or a fully anechoic room.

MU thus contributes only if a process of measurement (i.e. quantification of an EM quantity) actually takes place. By contrast, SU is always present whenever an EM excitation has been generated because the EM quantity of interest is then physically existent and fluctuating, irrespective of whether or not a process of measurement takes place. For example, the wall reflections generated by an ideal calibrated reference radiator placed in a test site produce random spatial fluctuations of the field included in the SU. These reflections can be residual (as e.g. in a FAR) or intentional (as e.g. in a reverberation chamber) and are present irrespective whether any additional monitoring antenna or probe is present or not.

The process of calibration/validation of the test site verifies that the level of site imperfections is within acceptable bounds, but it does neither influence nor eliminate the contribution of the SU. Measurement uncertainty may consist of a MIU contribution and the site imperfection contribution (e.g. NSA measurement in a FAR).

NOTE 1   In this classification, the term "instrumentation" is more restricted than in other documents, e.g., CISPR 16-4-1. In a FAR, the site uncertainty is caused solely by site imperfections and is incorporated within MIU in CISPR 16-4-1 and CISPR 16-4-2. For other test sites (including multi-path EM environments, e.g., reverberation chambers and more general fading channels), even the idealized site may exhibit inherent field uncertainty as an additional component to site imperfections. An explicit model and example of IFU is described in 5.2.3.4.

NOTE 2   In an anechoic environment, the MU of the complete test is also known as the standards compliance uncertainty (SCU) in CISPR 16-4-1. In ISO/IEC Guide 99:2007, the IUM is referred to as the definitional uncertainty (DU).

4.3   Limitations of the GUM

The "Guide to the expression of uncertainty in measurement" (GUM), see ISO/IEC 98-3:2008, provides the theoretical framework within which this Technical Report was developed. The GUM uncertainty approach has, however, fundamental limitations. If these limitations are

exceeded the results produced are no longer valid. Essentially, the theoretical framework of the GUM is based on (see [1]):

a) the Law of Propagation of Uncertainties (LPU), and

b) the Central Limit Theorem (CLT).

To insure that an uncertainty evaluation made according to the procedure described in ISO/IEC 98-3 may be correct, the assumptions required for the validity of both LPU and CLT shall be satisfied. The Supplement 1 to ISO/IEC 98-3 describes a numerical technique aimed at extending the validity of the uncertainty evaluations to cases where the application of ISO/IEC 98-3 does not produce reliable results.

CLT applies when

a) the measurement model is linear or quasi-linear, that is, it should be verified, at least to an approximation consistent with fitness for purpose, that the measurand can be expressed as

$$Y = c_0 + c_1 X_1 + c_2 X_2 + \ldots + c_n X_n$$

b) input quantities are independent,

c) $|c_i u(x_i)|$ have comparable magnitude,

d) n is sufficiently large (say $n \geq 3$).

If the requirements a) through d) are satisfied then Y approximately follows a normal PDF having an expected value y and a standard uncertainty $u(y)$, where

$$y = c_0 + c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

and

$$u(y) = [(c_1 u(x_1))^2 + (c_2 u(x_2))^2 + \ldots (c_n u(x_n))^2]^{1/2}$$

4.4 Principles

When performing either an emission measurement or an immunity test, a measurement instrumentation chain is required. The MIU is a fundamental property of this instrumentation chain.

At the most fundamental level, the act of measurement involves the acquisition of the numerical value of some measurand. The true value of the measurand is written hereafter simply as x.

To perform a measurement, some form of measurement instrumentation chain (forming the measurement system) is required. A measurement system will return a numerical value for the measurand, written hereafter simply as x', that is referred to as the *estimate* of the *true value* of the measurand, x, because the two parameters are related according to the following equation:

$$\Pr(x' < X < x' + dx') = g(x')dx' \tag{1}$$

where the function of the true value, $g(x')$ is a fundamental characteristic of the measurement system. The function $g(x')$ is formally referred to as the PDF of the measurement system. The function $g(x')$ is fundamentally statistical in nature: that is, the function $g(x')$ defines the probability that a given value of x' will be returned by the measurement system for a given true value, x, of the measurand.

An example of the form of $g(x)$ that is typical for a complex measurement system is presented below (see Figure 3). Note the form displayed: It is Gaussian. This means that the measurement system is most likely to return an estimate of the value of the measurand, x', that is equal to the true value of the measurand, x. However, the value returned by the measurement system, x', can differ from the true value, x, according to the deterministic properties of $g(x)$.
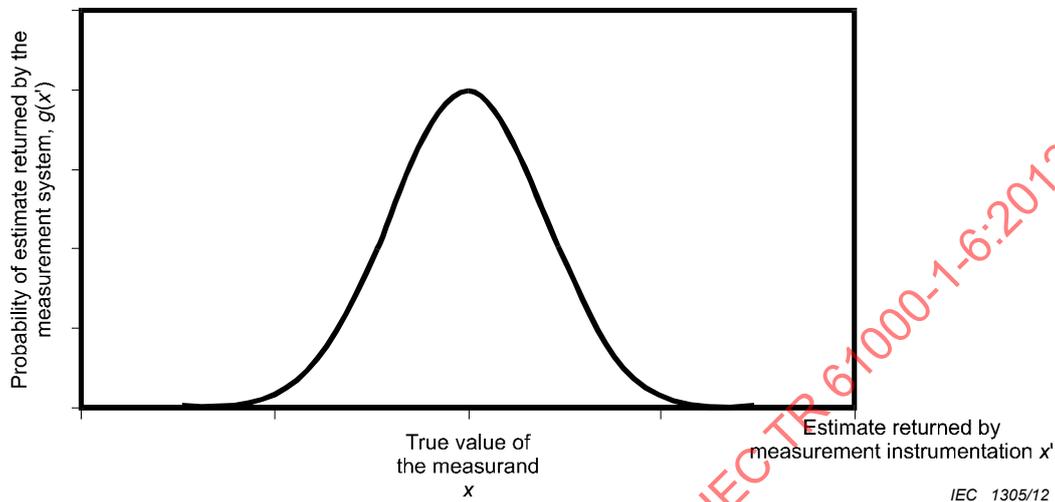
Figure 3 – Example of $g(x')$

The function, $g(x)$, of a complex measurement system has a fundamental impact upon the interpretation of the estimate returned by the measurement system. This is described with the aid of Figure 4 and the consideration of the five points (A, B, C, D and E) displayed.

The fundamental point is that when a measurement system returns an estimate, x', of the true value of the measurand, x, no knowledge exists regarding the specific position of the estimate within the range of $g(x)$.
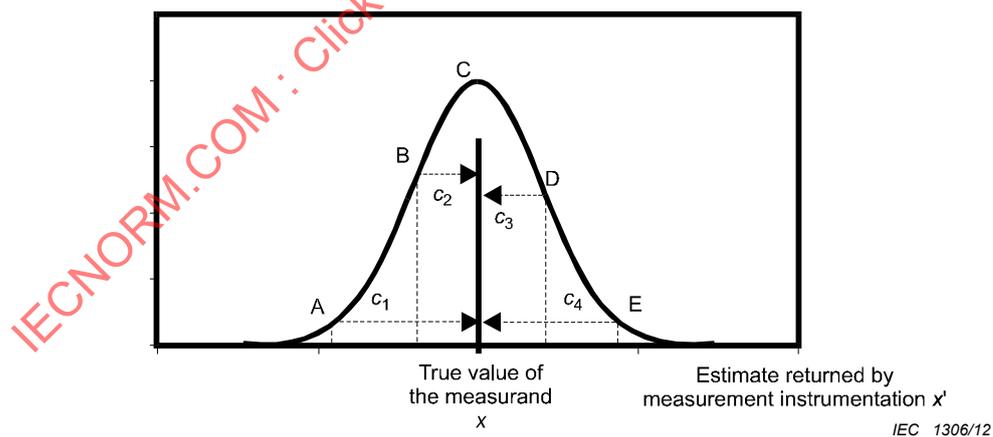
Figure 4 – Impact of $g(x)$ on interpretation of x'

Imagine that a measurand of true value, x, is subjected to a measurement using a measurement system with known function, $g(x)$, and the estimate, x', is returned.

It is possible that, at the time of measurement, the relationship between the true value of the measurand, x, and the estimate returned by the measurement system, x', was that

- shown as point A: in this case, the estimate, x', has significantly underestimated the true value, x, and it would be necessary to make the correction c_1 to x' to find x;

- shown as point B: in this case, the estimate, x', has again underestimated the true value, x (although not as much as the previous case), and it would be necessary to make the correction c_2 to x' to find x;

- shown as point C: in this case, the estimate, x', has correctly reported the true value, x, and it is not necessary to correct x' to find x;

- shown as point D: in this case, the estimate, x', has overestimated the true value, x, and it would be necessary to make the correction c_3 to x' to find x;

- shown as point E: in this case, the estimate, x' has significantly overestimated the true value, x, and it would be necessary to make the correction c_4 to x' to find x.

This means that the true value of the measurand could exist about the estimate returned by the measurement system in accordance with the PDF presented (see Figure 5).

Estimate returned by
measurement instrumentation
x'

True value of
the measurand
x

IEC   1307/12
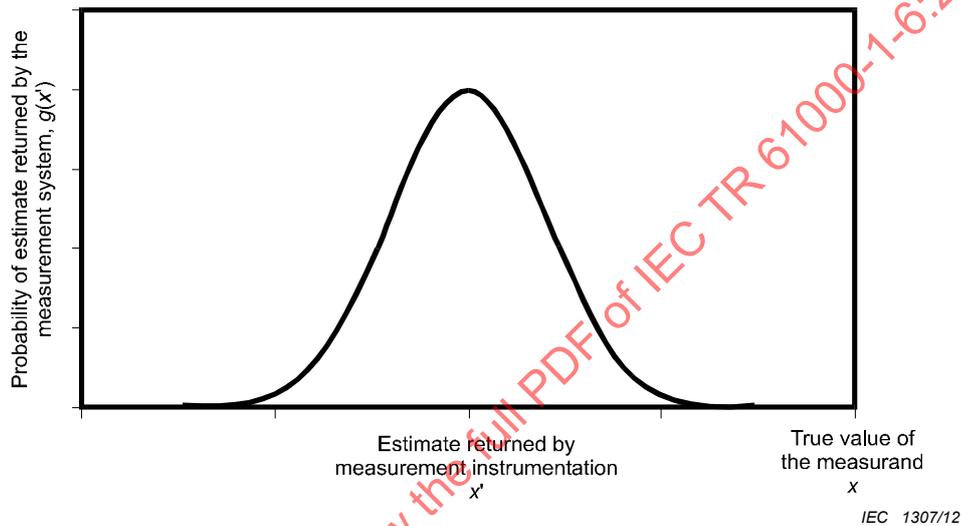
Figure 5 – Estimate returned by the measurement system

5   Measurement uncertainty budget development

5.1   Basic steps

Table 1 summarizes the steps for calculating the MU.

Table 1 – Basic steps for calculating MU

Step	Action	Test lab skill	Statistical tools
1	To write down an exact definition of the measurand (i.e. the quantity to be measured or injected)	Y	
2	To gather the input quantities X_i to MU (e.g. fishbone/Ishikawa diagram) Define the model equation	Y	
3	To provide the best estimate x_i and the PDF of the input quantities. All assumptions should be documented.	Y	
4	To calculate the standard uncertainty $u(x_i)$ of each influence quantity (using either Type A evaluation of uncertainty or Type B using simple division related to specific divisors for certain PDFs).		Y
5	To evaluate the sensitivity coefficients c_i of the input quantities.	Y	
6	To obtain the individual contributions to the standard uncertainty $u_i = c_i \cdot u(x_i)$ of each influence quantity.		Y

Step	Action	Test lab skill	Statistical tools
7	To combine the individual contributions to obtain the "combined standard uncertainty" u_c, i.e. using the root-sum of the squares (RSS) rule.		Y
8	To obtain the expanded uncertainty, U, for a given level of confidence $U = k \cdot u_c$, where k is the coverage factor for the required level of confidence.		Y

Comments on the steps:

Step 1

As an example in emission measurements on an open area test site, the measurand is not just the field strength at the location of the receiving antenna, but the "maximum electric field strength, in dB(μV/m), emitted by the EUT in horizontal and vertical polarisations at the specified horizontal distance from the EUT at a height of between 1 m and 4 m above a reflecting ground plane, with the EUT rotated 360° in azimuth". A detailed definition of the measurand will also help to identify the input quantities. A definition similarly detailed may apply to a quantity to be injected on the EUT in an immunity test.

Step 2

A model equation will show how the measurand is calculated including all possible correction factors. An example taken from CISPR 16-4-2 for the measurement of the disturbance current

$$I = V_r + A_c + Y_T + \delta V_{sw} + \delta V_{pa} + \delta V_{pr} + \delta V_{nf} + \delta M + \delta Z_{cp} + \delta D_{AE} + \delta Z_{AE} + \delta V_{env},$$

where

V_r is the input voltage to the measuring receiver,

A_c is the attenuation of the connecting cable,

Y_T is the current probe transfer admittance,

$\delta V_{sw} + \delta V_{pa} + \delta V_{pr} + \delta V_{nf}$ are the receiver corrections (see CISPR 16-4-2),

δM is the mismatch correction,

δZ_{cp} is the correction for the current probe insertion impedance,

δD_{AE} is the correction for errors caused by disturbance from the auxiliary equipment (AE),

δZ_{AE} is the correction for errors due to the deviation of the AE impedance from the assumed ideal termination impedance, and

δV_{env} is the correction for the effect of the environment on the test setup.

All quantities in this example are given in logarithmic units.

A correction is the compensation for a systematic error. A correction may be known from calibration reports or from internally documented evaluations of the test laboratory. A correction with unknown magnitude that is considered to be equally likely to be positive or negative, is taken to be zero. All known corrections are assumed to have been applied, in accordance with the model. This is expressed with the model equation. Every correction (even a zero correction) also serves as an influence quantity having an associated uncertainty.

Step 3

The list should be written in the form of a table.

Step 4

The standard uncertainty $u(x_i)$ is calculated by dividing a confidence interval for x_i by a factor that depends on the PDF for x_i and on the level of confidence associated with the interval. For a symmetric U-shaped, rectangular or triangular PDF, where X_i is estimated to lie between $(x_i - a)$ and $(x_i + a)$ with a level of confidence of 100 %, $u(x_i)$ is taken as $a/\sqrt{2}$, $a/\sqrt{3}$ or $a/\sqrt{6}$ respectively, where $a = (a^+ + a^-)/2$ is the half-width of the interval. For a normal PDF, the divisor is 2 if the confidence interval for x_i has a level of confidence of 95 % (the value is twice the experimental standard deviation), or 1 if the confidence interval for x_i has a level of confidence of 68 % (the value is the experimental standard deviation). In case of a symmetrical or non-symmetrical PDF, the expected value of the correction is δx_i. In the case where the expected value cannot be calculated from the PDF, then $\delta x_i = c_i(a^+ - a^-)/2$ should be considered to be applied for a correction of the measurement value. If this is insignificant (i.e. very small compared with the standard uncertainty), it is acceptable to use the average of the two boundaries.

Table 2 – Expressions used to obtain standard uncertainty

PDF	Expression for standard uncertainty	Graph
Rectangular	$u(x_i) = a/\sqrt{3}$	
Triangular	$u(x_i) = a/\sqrt{6}$	
U-shaped	$u(x_i) = a/\sqrt{2}$	
Normal (from repeatability evaluation $k = 1$)	$u(x_i) = a$	
Normal (from calibration report $k = 2$)	$u(x_i) = a/2$	
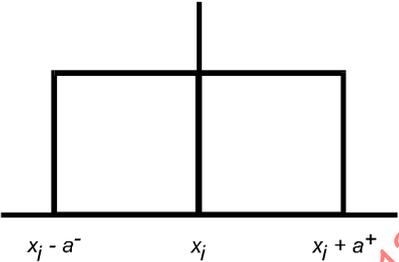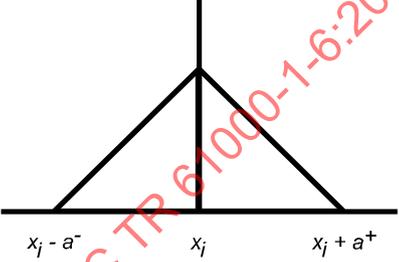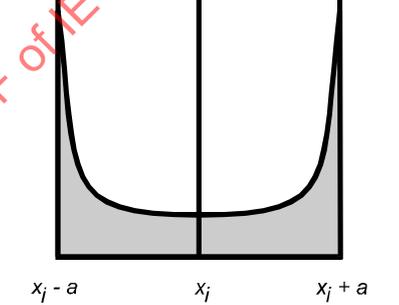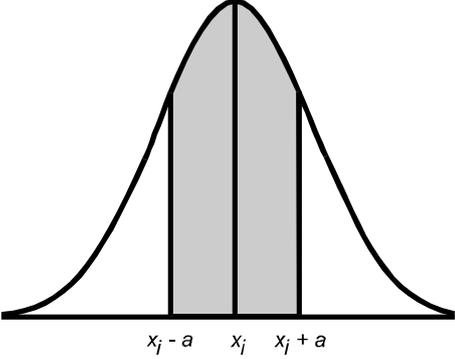
Step 5

The sensitivity coefficients are partial derivatives of the model functions for the measurands with respect to the varying influence quantity. If the model functions are linear when expressed in logarithmic units all sensitivity coefficients c_i become 1 or 1 (c_i = 1 or −1) and therefore need not be listed in the table.

Step 6

If all c_i = 1, then all $u_i = u(x_i)$

Step 7

The combined standard uncertainty is calculated in case of all input quantities being uncorrelated using $u_c(y) = \sqrt{\sum_i c_i^2\, u^2(x_i)}$

Step 8

The expanded uncertainty is calculated using $U(y) = k\, u_c(y)$. Examples of k: If one needs to describe that the uncertainty is lower than or equal to $U(y)$ with a level of confidence of 95 %, then k should be 2 (more exactly 1,96). If a measurement result needs to be below a threshold with a level of confidence of 95 %, then k should be 1,64 for comparison of the measurement result R with the threshold L ($R + U \leq L$) [2].

5.2   Probability density functions

5.2.1   Rectangular

5.2.1.1   Overview

The rectangular PDF applies to quantities that have the following characteristics:

- they are known to exist within a finite interval [a^-, a^+]

- no knowledge is available regarding the likelihood that the quantity will adopt a given value within the known finite interval [a^-, a^+];

- it is assumed that the quantity is equally likely to adopt any value within the known finite interval [a^-, a^+].

A rectangular PDF is also known as a uniform PDF.

5.2.1.2   Application

The rectangular PDF is applied to quantities whose values are known to be limited to some finite interval but for which no information is available regarding the likelihood of the quantity adopting a given value within this known, finite interval.

An example for the use of the rectangular PDF is the manufacturer's or standard's stated tolerance to estimate an uncertainty contribution. This tolerance defines the finite interval over which the value of the parameter may vary, but supplies no information regarding the likelihood of the quantity adopting any given value within this interval.

5.2.1.3   Diagram

The rectangular PDF across an interval [a^-, a^+] ($a^- < a^+$) is expressed as:

$$\begin{cases} g(x) = \dfrac{1}{a^+ - a^-} & \text{for } a^- \leq x \leq a^+ \\ g(x) = 0 & x < a^-, a^+ < x \end{cases} \tag{2}$$

where a, b denote the lower and upper boundaries of the interval.

A rectangular PDF is illustrated in Figure 6. Its height is $1 / (a^+ - a^-)$, because the area under any PDF is normalized to unity.

$1/(a^+-a^-)$

a^- a^+ X

Figure 6 – Rectangular PDF

The mean value of $g(y)$ is

$$\mu_X = \int_{a^-}^{a^+} xg(x)dx = \frac{a^- + a^+}{2} \tag{3}$$

The variance of $g(y)$ is the square of the standard deviation σ which is given by:

$$\sigma_x^2 = \int_{a^-}^{a^+} (x - \mu)g(x)dx = \frac{(a^+ - a^-)^2}{12} \tag{4}$$

whence the standard deviation

$$\sigma_X = \frac{\left|a^+ - a^-\right|}{2\sqrt{3}} \tag{5}$$

The standard uncertainty associated with the estimate $(a^- + a^+)/2$ of X is this standard deviation.

Coverage factors for 95 % , 99 % and 100 % confidence intervals of X are 1,65 (i.e. 95/100 • $\sqrt{3}$), 1,71 and $\sqrt{3}$, respectively.

5.2.1.4   Examples of application of the rectangular PDF

5.2.1.4.1   Digital displays

Use of the rectangular PDF arises naturally when a measuring instrument is fitted with a digital read-out display. Such a display is naturally limited to reporting the estimated value of the measurand to a finite number of decimal figures, with the result that the instrument rounds the estimated value of the measurand. The instrument is assumed to display a value in dB.

The interval containing the value of the measurand in this case is equal to ± half the value of the least significant digit reported; hence, if the display reports a value to one decimal place accuracy, that interval is ±0,05 dB; similarly, if the display reports a value to two decimal place accuracy, that interval is ±0,005 dB.

For example, the standard uncertainty applicable to a digital display accurate to within one decimal place is:

$$u = \sigma = \frac{0,05 - (-0,05)}{2\sqrt{3}} dB = 0,03 \quad dB \tag{6}$$

whereas the standard uncertainty applicable to a digital display accuracy within two decimal places is:

$$u = \sigma = \frac{0,005 - (-0,005)}{2\sqrt{3}} \, dB = 0,003 \quad dB \tag{7}$$

where the standard uncertainty value in formula (6) is reported within two decimal places and the standard uncertainty value in formula (7) is reported within three decimal places.

5.2.1.4.2   Electric-field display

Consider an electric field that is measured by an electric-field probe with an electric-field display using optical fibers. The measurands fluctuate between a high and a low value, and no other knowledge is available. If, for example, the highest and the lowest values are 6,64 V/m and 6,38 V/m, respectively, then the standard deviation is calculated from (5) as 0,075 V/m. If the expected value is 6,51 V/m, the relative measurement uncertainty is calculated as:

$$20 \lg\left(\frac{6,51 + 0,075}{6,51}\right) = 0,10 \quad dB \tag{8}$$

5.2.1.4.3   Thermal-drift range of a signal generator

When a thermal-drift interval of a signal generator provided by a manufacturer is $\pm 0,01$ dB/°C, and the ambient temperature is measured as (20 ± 2) °C, then the interval between highest and the lowest values are expected as $\pm 0,02$ dB. In this case, the standard deviation (uncertainty) is calculated from Equation (5) as 0,011 5 dB.

5.2.1.4.4   Antenna height

When a measuring scale with subdivisions down to millimetres is printed on an antenna mast, the interval for the highest and the lowest values of one reading is $\pm 0,5$ mm. If the values are converted to the electric field as $\pm 0,01$ dB (measured) at the observation point, the standard deviation (uncertainty) is calculated from Equation (5) as 0,006 dB.

5.2.2   Triangular

5.2.2.1   Overview

The triangular PDF applies to quantities that have the following characteristics:

- they are known to exist within a finite interval [a^-, a^+];
- knowledge is available regarding the highest probability that exists at a certain value c within the known finite interval [a^-, a^+];
- it is assumed, that the probability from a^- to c and c to a^+ changes linearly.

5.2.2.2   Application

The triangular PDF is applied to quantities whose values are known to be limited to some finite interval and for which information is available that the highest probability exists at a certain value c within the known finite interval. The symmetric triangular PDF is associated to the sum of two quantities having the same rectangular PDF. For example, the PDF appears in the sum of the scores on casting two fair six-sided dice, when each dice has the rectangular PDF.

An example of where use of the triangular PDF arises naturally is where a difference between or a sum of two measurands displayed in digital should be obtained. In this case, each measurand has the rectangular PDF.

5.2.2.3   Diagram

The triangular PDF across an interval $[a^-, a^+]$ ($a^- < a^+$) is expressed as:

$$g(x) = \begin{cases} \dfrac{2(x - a^-)}{(a^+ - a^-)(c - a^-)} & \text{for } a^- \leq x \leq c \\[2ex] \dfrac{2(a^+ - x)}{(a^+ - a^-)(a^+ - c)} & \text{for } c \leq x \leq a^+ \\[2ex] 0 & \text{otherwise} \end{cases} \tag{9}$$

where a^-, a^+ denote the lower and upper boundaries of the interval, and c denotes the mode, respectively.

A triangular PDF is illustrated in Figure 7. Its shape is triangular, and the triangle vertex is $2 / (a^+ - a^-)$.

IEC 1309/12
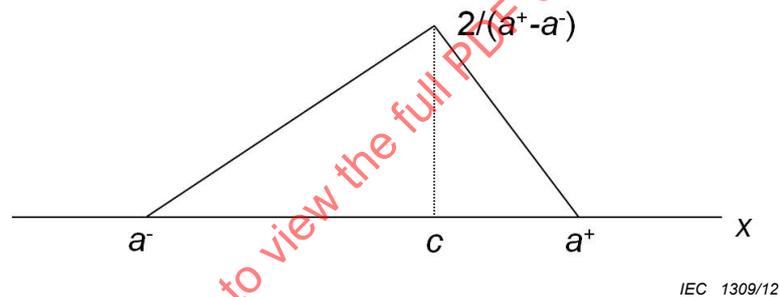
Figure 7 – Triangular PDF

The mean value of $g(x)$ is

$$\mu_x = \int_{a^-}^{a^+} x g(x) dx = \frac{a^- + a^+ + c}{3} \tag{10}$$

The variance of $g(x)$ is the square of the standard deviation σ and is given by:

$$\sigma_x^2 = \int_{a^-}^{a^+} (x - \mu) g(x) dx = \frac{(a^-)^2 + (a^+)^2 + c^2 - a^- \cdot a^+ - a^- \cdot c - a^+ \cdot c}{18} \tag{11}$$

Hence the standard deviation

$$\sigma_x = \frac{\sqrt{\left|(a^-)^2 + (a^+)^2 + c^2 - a^- \cdot a^+ - a^- \cdot c - a^+ \cdot c\right|}}{3\sqrt{2}} \tag{12}$$

The standard uncertainty of the triangular PDF is this standard deviation.

If the PDF is symmetrical, i.e., mode *c* is center of the interval [*a*⁻, *a*⁺], the standard uncertainty is

$$\sigma_X = \frac{\left| a^+ - a^- \right|}{2\sqrt{6}} \tag{13}$$

In this case, coverage factors for 95 %, 99 % and 100 % confidence intervals of a triangular PDF are 2,32 (i.e., 95/100 × $\sqrt{6}$), 2,42, and 2,45, respectively.

The triangular PDF is often used in place of the normal (Gaussian) PDF because of its simpler expression and easier mathematical treatment.

5.2.2.4   Example of application of the triangular PDF

Application: Sum of digital display readings

The use of the triangular PDF arises naturally when a sum of two readings using two measuring instruments fitted with a digital read-out display is obtained. Such a display is naturally limited to reporting the estimated value of the measurand to a finite number of decimal figures, with the result that the instrument rounds the estimated value of the measurand.

In a case, the total antenna height should be obtained from two vertically-jointed antenna masts that show readings with a digital display. When each display reports a value to one decimal place accuracy, e.g. ±0,05 mm, the interval of uncertainty is within ±0,1 mm from the sum of two readings. In the case, since mode *c* is the centre of the PDF, the shape of the PDF is symmetric.

The standard uncertainty applicable to the total height using triangular PDF is:

$$u = \sigma = \frac{\left| 0,1 - (-0,1) \right|}{2\sqrt{6}} = 0,041 \text{ mm} \tag{14}$$

where this standard uncertainty value is reported to within three decimal places.

5.2.3   Gaussian

5.2.3.1   Overview

The Gaussian (or Normal) PDF applies to a continuously (as opposed to discretely) fluctuating quantity *X*. Therefore, it is suitable for modelling statistical properties of physical quantities in the area of EMC.

A Gaussian PDF *g*(*x*) is symmetric, unimodal, and is characterized by two parameters. These parameters are given, for example, by the mean <*X*> and standard deviation σ. With this choice,

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x - \langle X \rangle)^2}{2\sigma^2} \right], \qquad -\infty < X < +\infty \tag{15}$$

This PDF is a solution of the differential equation

$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} + \frac{x - \langle X \rangle}{\sigma^2} g(x) = 0 \tag{16}$$

with initial value $g(x = 0) = 1/(\sigma\sqrt{2\pi})$. In the area of EMC, Equation (16) is useful to represent measurands that are characterized by random fluctuations.

The expression of the Gaussian distribution function $G(x)$ requires the use of special functions (i.e., functions that cannot be expressed as finite additions, multiplications and root extractions of other functions) and is given by

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{(x' - \langle X \rangle)^2}{2\sigma^2}\right]\mathrm{d}x' = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x - \langle X \rangle}{\sigma\sqrt{2}}\right)\right] \tag{17}$$

where erf() is the error function, which can be obtained from tables or through numerical computation.

Although the normal PDF has a doubly-infinite domain [-∞,+∞], the relatively small and rapidly decreasing values of its PDF for $|x - <X>|/\sigma \gg 1$ (i.e., in its `tails') ensure that this PDF can often also be used, at least to good approximation, to describe distributions of physical quantities that can take only finite and/or positive values, provided the value of $<X>/\sigma$ is sufficiently large (typically, 5 or more) . For many practical purposes, the values of $(x - <X>)/\sigma$ can then be restricted to the interval [−5, +5].

The coverage factors k for two-sided confidence intervals for 95 %, 99 % and 99,5 % levels of confidence are 1,960, 2,576 and 2,807. For one-sided intervals, the corresponding values are 1,645, 1,960 and 2,576.

5.2.3.2   Diagram

IEC   1310/12

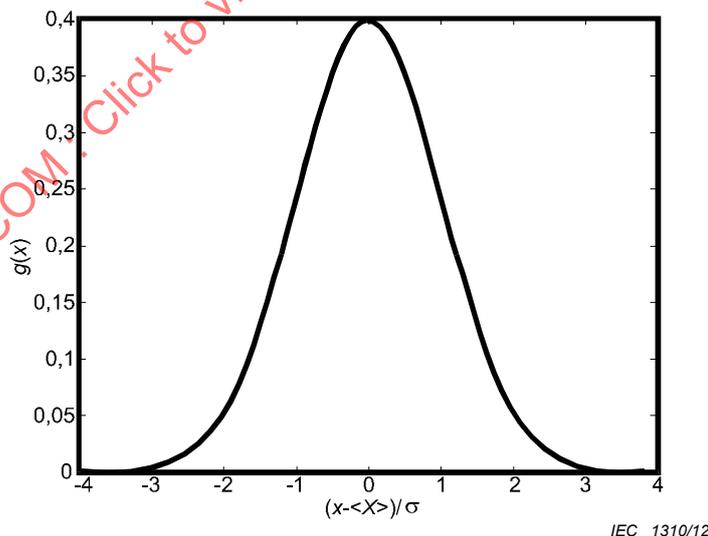**Figure 8 – Normal PDF for standardized *X***

5.2.3.3   Applications

5.2.3.3.1   Relationship to central limit theorem

The practical importance of the normal distribution lies in its fundamental role in statistical inference and, where applicable, the central limit theorem. This theorem states that the sample mean associated with sets of *N* independent random variables with any of a wide

range of PDFs (possibly asymmetric and/or discrete) having a finite population mean and finite population standard deviation σ tends to the normal PDF as the number of variables tends to infinity. The limit PDF has the same mean value but a standard deviation $\sigma/\sqrt{N}$, which is also known as the standard error. In addition, if the population is itself normally distributed, then its sample mean has a normal PDF for any (not just merely asymptotic large) value of *N*.

Furthermore, the normal PDF serves as an approximation of the discrete binomial PDF for large sample sizes. (An adequate correction may have to be applied for small sample sizes.)

The advantage of any linear system lies in the fact that for any random input quantity exhibiting a given distribution (e.g. normal distribution) yields an output quantity having the same distribution (e.g. in this case also normal).

Normality of a population distribution is often an essential condition for many statistical procedures on sets of sample data to be valid. Whether and to what extent a given measurand or field variable *X* has or can be assumed to have a normal PDF should be tested using an appropriate goodness-of-fit test. When the assumption of a normal distribution is not valid, then the following alternative options for applying statistical procedures are available:

- apply a nonparametric procedure;
- apply a transformation of *X* (e.g., square root, logarithm, etc.) to achieve approximate normality;
- apply another procedure that makes use of more general distributions than the normal distribution (e.g., *t*-distribution).

5.2.3.3.2 Estimation

The values of the two population parameters <*X*> and σ are usually unknown in practice, in which case their values shall be estimated from the data as the sample mean $\overline{X}$ and sample standard deviation *s*. Unbiased efficient estimators are thereby preferred, to allow for consistency when comparing results obtained from tests or laboratories based on different sample sizes. For normal distributions, expressions for the sampling distributions, mean and standard deviation of $\overline{X}$ and *s* can be obtained explicitly [3], because of the fact that only for normal distributions are $\overline{X}$ and *s* statistically independent [4]. For unknown <*X*> and σ, the distribution of $\overline{X}$ has a Student *t*-distribution.

5.2.3.3.3 PDFs of sum, difference, product, ratio, squared values and roots of quantities with normal PDF

Occasionally, the distribution and uncertainty are required for random quantities that are obtained via elementary operations (addition, subtraction, multiplication or division) between two normal random variables. For example,

- the impedance can be calculated from the ratio of randomly fluctuating electric and magnetic fields
- energy, intensity and power of fluctuating fields are all proportional to the squared field (in the time domain) or the squared magnitude of the complex field (frequency domain); field magnitude is proportional to the square root of the field intensity.

For normally distributed and statistically independent *X* and *Y*, the PDF of their sum, difference, product and ratio can be expressed in closed form. The sum or difference has again a normal PDF with mean <*X*> $\pm$ <*Y*> and variance $\sigma_X^2 + \sigma_Y^2$. If *X* and *Y* are correlated, then a term proportional to the correlation coefficient shall be added. The product of *X* and *Y* has a MacDonald PDF [5] (special case of a Bessel K PDF), while their ratio exhibits a Cauchy PDF [6]. The square of a real-valued *X* exhibits a chi-square PDF with one degree of freedom. On this basis, extensions to PDFs of single- and multi-axis (vectorial) fields (real- or complex-valued) and associated sampling distributions can be obtained. For example, the

amplitude distribution for a measurand *X* with zero mean value is the one for the root-sum-square of in-phase and quadrature components, i.e. a Rayleigh or chi-square PDF with two degrees of freedom.

By extension, the amplitude of a field or other measurand with a d.c. component (e.g. common-mode signal) that is not negligible compared to the a.c. component (differential-mode components) ($<X> \neq 0$) exhibit a Nakagami-Rice PDF, although its individual in-phase and quadrature components still have a normal PDF.

5.2.3.4   Example

Consider the CW electric field, *E*, received by a receiving antenna under test at a fixed separation distance and fixed relative orientation with respect to a transmitting reference antenna. If the transmit and receive antennas are connected to a Vector Network Analyzer, then the effects of phase shift can be understood. The measured electric field can then be viewed as a phasor having a real (*E'*) and imaginary (*E''*) part. Moving the two antennas to different locations inside a fully anechoic room will cause random fluctuations of the complex-valued electric field because of residual reflections by the walls of the room, which can be quantified by the normalized site attenuation (NSA). The real (*E'*) and imaginary (*E''*) components of the field exhibit normal fluctuations with respect to their average values. Typically, their mean values will be different whereas their standard deviations are equal ($\sigma_{E'} = \sigma_{E''} = \sigma$). For example, for the real part,

$$g(e') = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(e'-\langle E'\rangle)^2}{2\sigma^2}\right], \quad -\infty < e' < +\infty \tag{18}$$

As an example, consider biased field fluctuations with a signal-to-noise ratio (SNR) equal to 20 dB and 10 dB with respect to the average values $<E'>$ and $<E''>$, i.e. $\sigma = 0,1 <E'> = 0,3 <E''>$. A symmetric 95 % confidence interval for *E'* is then given by

$[<E'> - 1,960\ \sigma, <E'> + 1,960\ \sigma] = [0,804\ <E'>, 1,196\ <E'>]$. The electric field magnitude $A = |E| = \sqrt{(E'^2 + E''^2)}$ then has the PDF

$$f_A(a) = \frac{a}{\sigma^2} \exp\left(-\frac{a_0^2 + a^2}{2\sigma^2}\right) I_0\left(\frac{a_0 a}{\sigma^2}\right), \quad a \geq 0 \tag{19}$$

where

$a_0 = \sqrt{(\langle E'\rangle^2 + \langle E''\rangle^2)} = 10,54\ \sigma$ is the magnitude of the average field and

I_0 is the modified Bessel function of the first kind, zero-order.

The corresponding asymmetric 95 % confidence interval for *A* is then [8,63 σ, 12,54 σ].

For smaller SNRs, the asymmetry of the confidence with respect to a_0 or σ is larger. For a field with zero mean value, *A* is Rayleigh distributed with a 95 % confidence interval given by [0,225 σ, 2,715 σ]. This represents, for example, the intrinsic field uncertainty in an ideal reverberation chamber in the absence of direct illumination of the EUT.

5.2.4   U-Shape

5.2.4.1   Overview

In order to introduce the U-shaped PDF it is convenient to assume a very simple situation where a generator, whose output reflection coefficient is Γ_e, is directly connected to a power meter, having an input reflection coefficient Γ_r. Even in an ideal case where both the generator and the power meter were perfectly calibrated, the reading of the power meter, P_M, won't be equal to the output power setting of the generator, P_G. This is due to the mismatch at both generator output and power meter input. The relation between P_M and P_G is

$$P_G/P_M = |1 - \Gamma_e \Gamma_r|^2$$

Since it is not usually convenient to evaluate the correction term $|1 - \Gamma_e \Gamma_r|^2$ because:

a) both the magnitude and phase of $\Gamma_e\Gamma_r$ should be known over the frequency range of interest, and

b) the magnitude of $\Gamma_e \Gamma_r$ is small, then the correction is dealt with in a statistical sense.

If the maximum magnitude of $\Gamma_e \Gamma_r$ over the frequency range of interest is known, $K = |\Gamma_e\Gamma_r|_{MAX}$ and the phase φ of $\Gamma_e\Gamma_r$ is assumed to be uniform within 0 and 2π, then a random variable X can be associated to the mismatch correction, where

$$X = |1 - Ke^{j\varphi}|^2$$

5.2.4.2   Diagram

The PDF of X is

$$g(X) = \frac{1}{\pi\sqrt{\left[X - (1-K)^2\right]\left[(1+K)^2 - X\right]}} \tag{20}$$

for $(1 - K)^2 < X < (1 + K)^2$ and $g(X) = 0$ otherwise. The PDF is symmetric around the expected value $x = 1 + K^2$ and the standard deviation is $u(x) = \sqrt{2}K$. The U-shape PDF is illustrated as shown in Figure 9.

Usually in EMC logarithmic units are adopted, hence the random variable is 10 lg (X). The PDF of 10 lg (X) is still U-shaped but asymmetric within the bounds $\delta X^{\pm} = 20$ lg $(1 \pm K)$. The expected value of 10 lg (X) is zero (0 dB) and the standard deviation is approximately

$$\sigma = K \cdot \frac{20\lg(e)}{\sqrt{2}} = 6{,}14 \cdot K \tag{21}$$

This approximation is valid for any practical value of K between 0 and 1.

$g(X)$

$1/2\pi K$

$(1 - K)^2$ $1 + K^2$ $(1 + K)^2$

IEC 1311/12
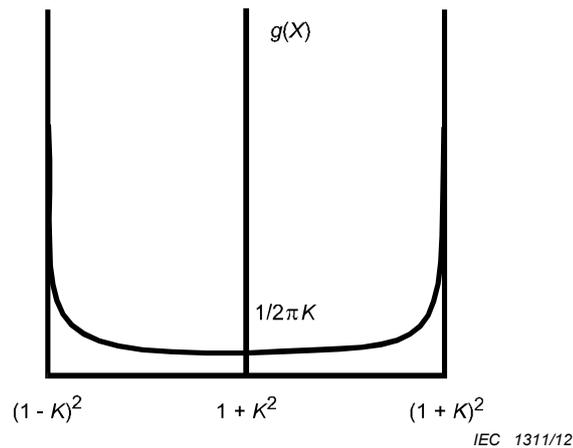
Figure 9 – U-shaped PDF

5.2.4.3 Application

The simplest practical arrangement sees two devices connected together by an electrical cable. A signal will pass from one device, the 'source', to the other, the 'load', via the cable.

In a radiated emissions measurement, the simplest practical arrangement involves a measurement antenna connected to a measuring receiver via a cable with a certain length.

In a radiated immunity test, the simplest practical arrangement involves the output of a power amplifier being connected to a test antenna via a cable with a certain length.

EMC emission measurements and immunity tests typically involve a number of equipment items that perform separate functions that are required to be located at different places and hence require cables to connect them together. So, in a radiated emission measurement, the measurement antenna is located above the ground-plane of an open area test site (OATS) while the measurement receiver is typically located some distance away. Similarly, in a radiated immunity test, the antenna is located within the Semi-Anechoic Chamber (SAC) or Fully Anechoic Room (FAR) while the signal generator and associated power amplifier are located some distance away, typically outside the Chamber/Room.

It is possible to measure the VSWR of all terminals with respect to a common reference impedance.

#1 $\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ #2

Γ_e Γ_r

IEC 1312/12
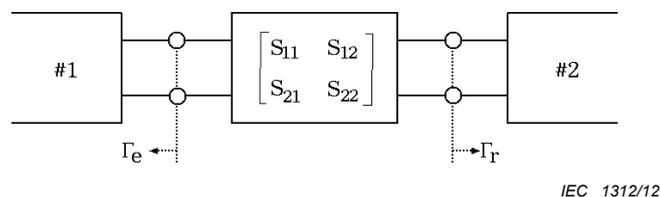
Figure 10 – Example of a circuit

Figure 10 shows an example of a part of an immunity testing apparatus. The circuits #1 and #2 are assumed to be a signal generator and a power amplifier respectively. Circuits #1 and #2 are connected by the central circuit that is expressed in terms of its S-parameters.

The centre circuit may, in practice, consist of

- a single cable that extends between circuits #1 and #2,
- a number of cables and required connectors that extend between circuits #1 and #2,
- a complex cascade of cable(s), with/without connectors and with/without attenuators.

Any signal passing between circuits #1 and #2 will undergo reflection and transmission at any impedance mismatch. The mismatch occurs:

- at the boundary between circuit #1 and the central circuit
- at the boundary between the central circuit and circuit #2.

A correction factor for the mismatch, δM, is expressed as:

$$\delta M = 20\lg\left|(1-\Gamma_e S_{11})(1-\Gamma_r S_{22}) - S_{21}^2 \Gamma_e \Gamma_r\right| \tag{22}$$

where

Γ_e is the reflection coefficient of the output port of the generator;

Γ_r is the reflection coefficient of the input port of the amplifier;

S_{11} is the reflection coefficient of the input port of the central circuit;

S_{22} is the reflection coefficient of the output port of the central circuit;

S_{21} is the transmission coefficient of the central circuit;

Each reflection coefficient is complex. However, only the amplitudes are known because the specifications prepared by manufacturers are only VSWR. Therefore, the radiated electric-field strength or the net power cannot be calibrated by using the precise correction factors, δM. The mismatched uncertainty occurs due to the lack of the phase information of the reflection coefficients. Therefore, the worst case uncertainty is formulized by using the maximum and the minimum of δM, which are expressed by the following equation using Equation (22).

$$\delta M^{\pm} = 20\lg\left[1\pm\left(|\Gamma_e||S_{11}|+|\Gamma_r||S_{22}|+|\Gamma_e||\Gamma_r||S_{11}||S_{22}|+|\Gamma_e||\Gamma_r||S_{21}|^2\right)\right] \tag{23}$$

The PDF of δM describes a U-shape. The standard deviation of the PDF is obtained as:

$$\sigma = \frac{\delta M^+ - \delta M^-}{2\sqrt{2}} \tag{24}$$

Then the standard deviation is the standard uncertainty of the impedance mismatch.

5.2.4.4   Example of uncertainty calculation

When the circuit parameters are given by the specifications of a manufacturer, as shown in Case 1 of Table 3, the maximum and the minimum of δM are calculated as 0,626 dB and -0,675 dB respectively by using Equation (23). Then, the standard uncertainty is calculated by Equation (24) as 0,46 dB. When the circuit parameters are given by the specifications of a manufacturer as shown in Case 2 of Table 3 as another case, the maximum and the minimum of δM are calculated as 1,71 dB and −2,12 dB, respectively. The standard uncertainty is also calculated as 1,35 dB. The difference between the absolute value of the maximum δM and of the minimum δM is small if the values of the reflection coefficients are small. The standard uncertainty $u(x_i)$ is approximated as the following equation:

$$u(x_i) = \frac{|\delta M^-|}{\sqrt{2}} \qquad \text{for } K < 0,3 \tag{25}$$

Table 3 – Examples of circuit parameters

Parameters	Case 1	Case 2
Reflection coefficient, Γ_e	0,2 (VSWR = 1,5)	0,333 (VSWR = 2,0)
Reflection coefficient, Γ_r	0,333 (VSWR = 2,0)	0,5 (VSWR = 3,0)
S-parameter, S_{11}	0,056 (−25 dB)	0,1 (−20 dB)
S-parameter, S_{22}	0,032 (−32 dB)	0,1 (−20 dB)
S-parameter, S_{12}	0,89 (−1,0 dB)	0,89 (−1,0 dB)
S-parameter, S_{21}	0,89 (−1,0 dB)	0,89 (−1,0 dB)

If the reflection coefficient of the power amplifier is measured inclusive of a cable that connects with the input port of the amplifier, the center circuit can be ignored. In this case, Equation (23) is simplified to the following equation because the S_{11} and S_{22} of the center circuit are zero, and S_{12} and S_{21} are 1.

$$\delta M^{\pm} = 20\lg\left[1 \pm |\Gamma_e\|\Gamma_r|\right] \tag{26}$$

5.3 Concept of Type A and Type B evaluation of uncertainty

5.3.1 General considerations

The evaluation of standard uncertainty is classified as Type A or Type B, depending on the way the evaluation is done. Type A evaluation is carried out through the statistical analysis of measured quantity values, Type B evaluation is carried out by means other than a Type A evaluation. It is worth noting that Type A and Type B evaluations are mutually exclusive. Therefore, when filling out an uncertainty budget, it is immediately clear if an uncertainty contribution results from one or the other type of evaluations. The classification scheme of Type A and Type B is not problematic for the development of standards.

Another important classification deals with the character of a measurement error: an error that, in replicate measurements remains constant or varies in a predictable manner, is named systematic error; on the contrary an error that, in replicate measurements varies in an unpredictable manner, is named random error. The classification systematic random, quite the opposite of Type A, Type B, is suggestive, since it refers to anyone's experience of the physical world.

Alternative definitions of systematic and random errors can be given according to the way they are evaluated (i.e. they are operative definitions); the systematic error is the mean that would result from an infinite number of measurements of the same measurand minus a reference value of the measurand. The random error is the measured value minus the mean that would result from an infinite number of measurements of the same measurand. In both cases the measurements shall be carried out using the same measurement procedure, the same operators, the same measuring system, the same operating conditions, the same location, and making replicate measurements on the same or similar objects over a short period of time (repeatability conditions).

The evaluation of the systematic error requires the availability of a reference value whose deviation from the unknown value of the measurand is known a-priori to be small with respect to the systematic error under evaluation (see Note 1). This is the usual practice in instruments calibration. Consider the case where a standard RF voltage source is used to calibrate a receiver. The reference value of the voltage amplitude shall deviate from the actual value generated by the source by less than the measurement accuracy of the receiver (e.g. by one order of magnitude) stated in the instrument specifications.

In principle, the evaluation of systematic and random errors would require the computation of the mean of an infinite number of measurements. In practice the number of measurements, *N*, shall be large enough so that the deviation between the mean of a sample of *N* measurements and that of infinite measurements is small with respect to the systematic and random errors that are evaluated (see Note 1). This deviation can be quantified by the experimental standard deviation of the mean, which decreases proportionally to $1/\sqrt{N}$ when *N* increases.

NOTE 1   The repetition of the argument is inherent in the evaluation of both systematic and random errors.

The following text deals with uncertainties rather than with errors and talks about uncertainties originating from systematic or random effects rather than systematic or random errors. This is because uncertainty is connected to probability while error is connected to determinism. It is much more convenient to adopt a probabilistic approach to the evaluation of the reliability of a measurement rather than a deterministic one. However, thinking in terms of errors is often useful as they are a link to common experience. Further, the specifications of measuring instruments are given in terms of their measurement accuracy which is the combination of the measurement trueness, closely related to systematic errors, and the measurement precision, closely related to random errors.

Sometimes the same error can be classified as systematic or random, depending on the circumstance. For example, the amplitude error of a receiver at a fixed frequency can be classified as systematic since the indication does not change by repeating the measurement at that frequency (see Note 2). However, the same amplitude error appears as random in the whole frequency range of the receiver, since the error randomly varies from one frequency to another. The manufacturer of the receiver then specifies a limit of error within which the random frequency dependent error is confined over the frequency range of the receiver. The ambiguity of the systematic-random classification is one of the reasons for the introduction of the Type A - Type B classification. Uncertainties are therefore classified according to the way they are evaluated rather than to their nature.

NOTE 2   This is true if the source is stable and provides a signal whose amplitude is much greater than the receiver noise level.

5.3.2   Type A evaluation of standard uncertainty

The Type A evaluation consists of determining the best estimate *q* of a quantity *Q* and the standard uncertainty through the statistical analysis of a sample of *N* repeated indications Q_i, where *i* = 1, 2, …, *N*. The best estimate is given by the arithmetic mean $\overline{Q}$ of the indications,

$$\overline{Q} = \frac{1}{N}\sum_{i=1}^{N} Q_i \tag{27}$$

then $q = \overline{Q}$.

Note that an upper case letter, such as *Q*, is used here to denote at the same time

a) the name of a quantity,

b) the unique, although unknown, value of that quantity,

c) any possible random value associated to that quantity.

The meaning will be different depending on the context. A lower case letter is used for the best estimate of a quantity, hence *q* is the best estimate of *Q*.

A measure of the dispersion of the random values Q_i around $\overline{Q}$ is given by the experimental standard deviation $s(Q_i)$:

$$s(Q_i) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(Q_i - \overline{Q})^2} \qquad (28)$$

The experimental standard deviation is the root-mean-square deviation of the values Q_i from $\overline{Q}$. It is assumed that the indications are independent; that is we have no reason to suspect that a significant relationship exists among them. The mean of the squares is calculated by dividing by $N – 1$ instead of N. This choice can be justified in several ways. Firstly, the dispersion cannot be estimated from a single indication. As a consequence, if trying to evaluate Equation (28) in the case $N = 1$ the 0/0 undetermined form results, instead of the misleading zero value obtained substituting N in place of $N – 1$ in Equation (28). Secondly, the number of independent deviations is $N – 1$ since, see Equation (27), $\sum_{i=1}^{N}\left(Q_i - \overline{Q}\right) = 0$.

Therefore, dividing by N a rather optimistic estimate of the dispersion can be obtained. The number $\nu = N – 1$ is named number of degrees of freedom. One degree of freedom is frozen by the choice of the arithmetic mean as the center value for the calculation of the deviation. In general, the number of degrees of freedom is the number of independent indications minus the number of parameters derived from these indications. Note that if the dispersion is calculated around a value that is not obtained combining the sample of indications (e.g. the value derived from averaging a historical collection of data or a reference or conventional value) it is more appropriate to divide by N instead of $N – 1$ because no degree of freedom is frozen by the choice of the center value, and $\nu = N$.

The arithmetic mean of a sample of N random values is a random value too. This means that if the arithmetic mean of each of M samples made of N random indications is calculated, M values of $\overline{Q}_j$ are obtained, with $j = 1, 2, …, M$, which vary at random. It is obvious that the larger N the smaller the deviation (in the average) among the $\overline{Q}_j$ values. In order to evaluate the dispersion of the values $\overline{Q}_j$ around the grand total average $\overline{\overline{Q}} = \frac{1}{M}\sum_{i=1}^{M}\overline{Q}_j$ we can use Equation (28) to obtain

$$s(\overline{Q}_j) = \sqrt{\frac{1}{M-1}\sum_{j=1}^{M}(Q_j - \overline{\overline{Q}})^2} \qquad (29)$$

It can be shown (see [7]) that $s(\overline{Q}_j) \gg s(\overline{Q})$ where $s(\overline{Q})$ is the experimental standard deviation of the mean

$$s(\overline{Q}) = \sqrt{\frac{1}{N(N-1)}\sum_{i=1}^{N}(Q_i - \overline{Q})^2} \qquad (30)$$

or

$$s(\overline{Q}) = \frac{s(Q_i)}{\sqrt{N}} \qquad (31)$$

where $s(\overline{Q})$ provides a quantitative description of the spread of the values $\overline{Q}_i$ around $\overline{\overline{Q}}$.

If M is large, as it is tacitly assumed in Equation (29), the experimental standard deviation of the mean provides an estimate of the deviation between the mean of a sample of N indications and that of infinite number of indications. Equation (30) is very powerful, because it states that in order to evaluate the reliability of the mean of a sample of N indications we do

not need *M* samples of *N* indications, as in Equation (29), but we can rely upon a single sample.

Equation (31) requires some comments. As *N* increases the estimate of the dispersion of the random indications, $s(Q_i)$, which is a random number too, tends to become more and more reliable. It can be demonstrated (see [7] and ISO/IEC Guide 98-3, Annex E) that the relative dispersion of $s(Q_i)$ is nearly $\sqrt{2(N-1)}$ when $N \geq 3^2$ (see Note). Hence $s(Q_i)$ tends to a constant value when *N* tends to infinity, and $s(\overline{Q})$ tends to zero. In conclusion, the experimental standard deviation of the mean can be reduced by increasing the number of measurements. This is a rather obvious result due to the fact that the mean of *N* indications tends to a constant value when *N* increases. What is less obvious is that the dispersion of the mean decreases proportionally to $1/\sqrt{N}$.

NOTE 1   The same is true for $s(\overline{Q})$, due to Equation (28). Also, the relative dispersion is 76 % when *N* = 2 (see ISO/IEC Guide 98-3, Annex E, Table E.1).

An important recommendation: Do not confuse the experimental standard deviation with the experimental standard deviation of the mean. They are distinct quantities used to answer two completely different questions. The experimental standard deviation can be adopted to evaluate the **measurement repeatability** or the **measurement reproducibility** since they are quantities aimed at describing the capability of a measurement setup and/or method to provide results in close agreement. Measurement repeatability or measurement reproducibility shall not depend on the number *N* of repeated measurements that is chosen to evaluate them, as it would be if the experimental standard deviation of the mean is used. In particular they are not expected to decrease with increasing *N*. On the contrary, the uncertainty of the arithmetic mean of a series of indications generally decreases with increasing the length of the series, and it is appropriately described by the experimental standard deviation of the mean. In general, the choice between the experimental standard deviation or the experimental standard deviation of the mean is dictated by the measurement model that is adopted.

The experimental standard deviation, or the experimental standard deviation of the mean, is a necessary step to but do not constitute the final result of the Type A uncertainty evaluation. Indeed, we discussed about the unreliability of the experimental standard deviation, especially when the number of indications *N* is low. For example, when *N* = 10, i.e. a relatively large number of repeated measurements, the relative dispersion of this parameter is about 24 %. It is far preferable to have a value of the uncertainty which can be considered *exact* for all its subsequent uses. Indeed, uncertainty interpretation is more direct, calculations are easier (see Note 1) and the choice is coherent with the result of the Type B evaluation, where the result of uncertainty calculation is not uncertain. The price for the exactness is a somewhat larger value of uncertainty. If *Q* follows a normal PDF, then both the experimental standard deviation and the experimental standard deviation of the mean are expanded by an appropriate coefficient $\eta(v) > 1$ to provide the **Type A (evaluation of the) standard uncertainty** $u(Q_i)$

$$u(Q_i) = \eta(v)s(Q_i) \qquad (32)$$

and the Type A (evaluation of the) standard uncertainty of the mean $u(\overline{Q})$

$$u(\overline{Q}) = \frac{u(Q_i)}{\sqrt{N}} \qquad (33)$$

where

$$\eta(v) = \begin{cases} 6{,}48 & v = 1 \\ 2{,}20 & v = 2 \\ \sqrt{\dfrac{v}{v-2}} & v \geq 3 \end{cases} \tag{34}$$

A selection of values of $\eta(v)$ for $1 \leq v \leq 99$ (that is $2 \leq N \leq 100$) is reported in Table 4. $\eta(1)$ and $\eta(2)$ are numerical values obtained as $t_{0,025}(1) / t_{0,025}(\infty)$ and $t_{0,025}(2) / t_{0,025}(\infty)$, respectively. $t_p(v)$ is the upper critical value of the Student's t-PDF with η degrees of freedom, corresponding to probability p in one tail. Indeed $t_{0,025}(1) = 12{,}71$ and $t_{0,025}(\infty) = 1{,}96$, hence $t_{0,025}(1) / t_{0,025}(\infty) = 6{,}48$. Further $t_{0,025}(2) = 4{,}30$ therefore $t_{0,025}(2) / t_{0,025}(\infty) = 2{,}20$. The value of the probability p is related to the coverage probability P that we will adopt for the final statement of uncertainty, that is $p = \dfrac{1-P}{2}$ (e.g. $P = 0{,}95$ implies $p = 0{,}025$). When $N \geq 3$ we have $t_p(v)/t_p(\infty) \approx \sqrt{v/(v-2)}$, irrespective of the value of p.

NOTE 2 This approach permits to get rid of the concept of effective degrees of freedom and the use of the Welch-Satterthwaite formula (see ISO/IEC Guide 98-3, Annex G).

Table 4 – Values of the expansion coefficient $\eta(v)$ which transforms the standard deviation to the Type A standard uncertainty

v	1	2	3	4	5	6	7	8	9
$\eta(v)$	6,48	2,20	1,73	1,41	1,29	1,22	1,18	1,15	1,13

v	10	11	12	13	14	19	29	49	99
$\eta(v)$	1,12	1,11	1,10	1,09	1,08	1,06	1,04	1,02	1,01

Type A standard uncertainty and Type A standard uncertainty of the mean are the results of Type A evaluation of standard uncertainty.

Example 1: Evaluation and check of measurement repeatability. Evaluation phase: Non-repeatability is evaluated (e.g. yearly) through Equation (32), by collecting a relatively large number N of independent indications (e.g. $N = 10$) to obtain $u(Q_i)$. Check phase: Repeatability can be rapidly checked (e.g. monthly) through a couple of indications Q_1 and Q_2 obtained by using the same measurement procedure, setup and instruments used for the determination of $u(Q_i)$. If $|Q_1 - Q_2| > 1{,}96 \cdot \sqrt{2} \cdot u(Q_i)$, then one can state, with probability less than 5 % to be in error, that the measurement was not carried out under the same repeatability conditions leading to the previously evaluated dispersion $u(Q_i)$. The subject of this example can be chosen for current injected by using a Bulk Current probe, peak value of ESD, rise time of the surge short-circuit current, etc.

NOTE 3 The difference $Q_1 - Q_2$ follows a normal PDF, whose expected value is zero and the standard deviation is $\sqrt{2} \cdot u(Q_i)$

Example 2: Evaluation of the field uniformity over the Uniform Field Area (UFA) in the radiated immunity test described in IEC 61000-4-3. The field non-uniformity can be evaluated through Equation (32). Since the field magnitude is a positive quantity with a large dispersion, the result of the calculation is generally expressed in the uncertainty budget using logarithmic units. It is convenient to transform the field meter indications from V/m to dB(V/m), then to calculate the dispersion of the dB(V/m) values to obtain a result in dB.

5.3.3 Type B evaluation of standard uncertainty

In many cases the best estimate and the standard uncertainty of a quantity are not obtained through the statistical analysis of a sample of indications but from the available information about the values, which can reasonably be achieved by that quantity. The available information may consist of a measuring instrument manufacturer specification, calibration reports, technical and application notes, scientific literature, previous experience (including data records).

For example, it is known that the sine-wave voltage measurement accuracy of a receiver compliant with the requirements in CISPR 16-1-1 shall be within ±2 dB. Differently stated, the limits of error (or tolerance) of the receiver shall be ±2 dB. We do not know the actual measurement error at a specific frequency and for a specific setting of the receiver, but we know that it is definitely bound by the tolerance interval. Type B uncertainty evaluation is noteworthy in that the measurement error is thought to be a random variable E, achieving values in the tolerance interval (E_{min}, E_{max}). In the example, E_{min} = −2 dB and E_{max} = +2 dB. Good sense suggests that it is more probable that E is close to the centre of the tolerance interval rather than to its limiting values. A more conservative assumption suggests that E is evenly distributed in the tolerance interval. This assumption is chosen more frequently, at least if there is no evidence to the contrary. In general terms, the possible values of E are distributed within the tolerance interval in accordance with a statistical distribution, which corresponds to the available information. In the receiver example, if the conservative choice is adopted, the PDF of the error is constant and different from zero inside the tolerance interval while is zero outside. This PDF is named rectangular or uniform. Otherwise a triangular PDF can be chosen if there is convincing evidence that error values are concentrated around the centre of the interval.

These concepts are expressed in the following mathematical formulae. The statistical error distribution is described by the PDF $g(E)$ which is defined as follows

$$\Pr\!\left(\hat{E} < E < \hat{E} + \mathrm{d}E\right) = g\!\left(\hat{E}\right)\mathrm{d}E \tag{35}$$

where $\Pr\!\left(\hat{E} < E < \hat{E} + \mathrm{d}E\right)$ is the probability that E lies between $\hat{E}$ and $\hat{E} + \mathrm{d}E$, and $\mathrm{d}E$ is an infinitesimal increment. The function $g(E)$ is a density because it is the probability per infinitesimal increment of the error. Then the measurement unit of the PDF is the reciprocal of the measurement unit of E (probability is dimensionless). The PDF has the following general properties:

$$g(E) \geq 0, \text{ for any } E \tag{36}$$

$$\int g(E)\mathrm{d}E = 1, \text{ integral over all the possible values of } E \tag{37}$$

In our example $\int_{E_{min}}^{E_{max}} g(E)\mathrm{d}E = 1$. In general terms the interval (E_{min}, E_{max}) encompasses a large portion p of the possible values of E, where usually p = 1 or p = 0,95, hence

$$\int_{E_{min}}^{E_{max}} g(E)\mathrm{d}E = p \tag{38}$$

Property (37) permits to derive the mathematical expression of the rectangular PDF. Indeed, since the area of the rectangle is unity and the width of the base is $E_{max} - E_{min}$, its height is $1 / (E_{max} - E_{min})$. In the case of the triangular PDF again the width of the base is $E_{max} - E_{min}$ but the height is, in this case, $2 / (E_{max} - E_{min})$.

Knowing the PDF we can derive the best estimate E of the measurement error and the Type B (evaluation of the) standard uncertainty. Consider the value $E \cdot g(E)\mathrm{d}E$. This value is the product of a particular error value E, and the probability of achieving error values in the narrow neighborhood around E. If we sum up $E \cdot g(E)\mathrm{d}E$ for all the possible values of E we obtain the weighted average of E, where the weights are represented by the probabilities $g(E)\mathrm{d}E$. This type of weighted average is called expected value of E, or briefly $\langle E \rangle$, where

$$\langle E \rangle = \int E \cdot g(E)\mathrm{d}E \tag{39}$$

the integral being over all the possible values of E. It can be easily shown that $\langle E \rangle$ is the limit value of the arithmetic mean of an infinitely large sample of possible values of E. This property can be demonstrated by approximating the probability in Equation (35) with the ratio between the number of sample values falling in the interval $\left(\hat{E}, \hat{E} + \Delta E\right)$ and the total number of sample values. ΔE represents the width of an interval containing a significant number of sample values. In conclusion, in the Type B evaluation scheme the expected value is the counterpart of the arithmetic mean, which is the best estimate in the Type A evaluation scheme. Hence, the best estimate of the measurement error is $e = \langle E \rangle$. Thus, the integration in Equation (39) may be simplified for symmetric PDFs. For symmetric distributions, such as rectangular and triangular PDFs, then $e = 1 / 2 \cdot (E_{min} + E_{max})$.

Considering now the quantity $E - \langle E \rangle$, which corresponds to the deviation between a particular error value and the best estimate of the error, the weighted average of the squared deviation can be calculated as

$$\left\langle \left(E - \langle E \rangle\right)^2 \right\rangle = \int \left(E - \langle E \rangle\right)^2 g(E)\mathrm{d}E \tag{40}$$

(again, integration is over all possible values of E).

Note that the term on the left of Equation (40) is the expected value of the squared deviation $\left(E - \langle E \rangle\right)^2$ and is called a variance of E, usually indicated as σ_E^2. It can be demonstrated, just following the line of reasoning in the previous paragraph, that the variance is the limit value of the square of the experimental standard deviation of an infinitely large sample of possible values of E. The conclusion is that the Type B standard uncertainty is the square root of the variance

$$u(e) = \sqrt{\int \left(E - \langle E \rangle\right)^2 g(E)\mathrm{d}E} \tag{41}$$

where, for homogeneity of notation with the Type A standard uncertainty, we set $u(e) = \sigma_E$. Solving the integral in Equation (41) we find in the case of the rectangular PDF

$$u(e) = \frac{E_{max} - E_{min}}{\sqrt{12}} \tag{42}$$

while in the case of the triangular PDF we have

$$u(e) = \frac{E_{max} - E_{min}}{\sqrt{24}} \tag{43}$$

Returning to an earlier example, consider the amplitude error of a receiver with E_{min} = −2dB and E_{max} = +2dB. If the PDF is assumed to be rectangular, $u(e)$ = 1,2 dB. If the PDF is assumed to be triangular, $u(e)$ = 0,8 dB. In both cases, due to symmetry, the best estimate of the error is e = 0 dB.

Example 1: Type B standard uncertainty derived from a calibration report (e.g. of the gain of an EMC antenna) stating the best estimate and the coverage interval corresponding to 95 % coverage probability, normal PDF. Note that the value of the effective degrees of freedom is irrelevant (may be stated in the calibration report: In any case, divide $E_{max} - E_{min}$ by 2 • 1,96 = 3,92 to obtain $u(e)$).

NOTE   The symbol E is used in place of the more general symbol X because in the example dealt with here E refers to a measurement error. The results derived are however general.

5.4   Sampling statistics

5.4.1   General considerations

In several practical situations in EMC testing and measurement, measurement uncertainty (MU) is estimated with reference to a *relatively small* set of measured data. A few examples are:

- in laboratory comparisons, the number of measured values of the measurand (this number being equal to the number of laboratories involved) is typically of the order of ten or less, rather than one hundred or more;

- in angular scans of an EUT during shielding, susceptibility, or emissions testing, the number of aspect angles for an EUT placed on a turntable in a FAR is severely limited for economical reasons;

- in height scans of emissions measurements in an anechoic environment, the number of heights and orientations of the EUT is limited;

- the number of paddle wheel steps in a reverberation chamber testing is typically of the order of a few tens per rotation (minimum 12), for example when operation is near the lowest usable frequency.

The restricted size of the data set is caused by economical constraints or due to more fundamental physical limitations.

While *ensemble* statistics and PDFs (e.g., normal PDF) yield asymptotically exact estimates of the expected value and confidence intervals for infinitely large sample data sets, MU may dramatically increase for "small" sample sets, typically for N = 40 statistically independent samples or fewer. [The value of N is influenced by the statistical independence between the sample values, which shall be investigated separately.] Ensemble statistics and ensemble distributions correspond to the idealized case where N approaches infinity.

The spread around the mean value (standard deviation, confidence limits, etc.) is strongly affected by the sample size. Only test results for which N is constant are directly comparable.

Strictly, the sample statistics (e.g., sample standard deviation) calculated from a given data set apply to that set only. However, especially when N is not exceedingly small, the calculated sample statistics serve as good approximations for the sample statistics of other (repeated) data sets related to the same measurand, measured under identical conditions of the measurement.

5.4.2   Sample mean and sample standard deviation

The sample mean of a measured (or otherwise evaluated) quantity X (e.g., complex field, field magnitude, power or energy density, etc.) is

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{N} \tag{44}$$

where

X_i is the i-th sample value and

$\overline{X}$ is an estimate for the ensemble value $<X>$.

The sample experimental standard deviation of X is defined as

$$s_X = \sqrt{\frac{\sum_{i=1}^{N}\left(X_i - \overline{X}\right)^2}{N-1}} . \tag{45}$$

The sample standard deviation of $\overline{X}$ (also known as the standard error of $\overline{X}$) is defined as

$$s_{\overline{X}} = \frac{s_X}{\sqrt{N}} \tag{46}$$

NOTE The definition of $s_{\overline{X}}$ requires the N samples to be statistically independent, whereas for $\overline{X}$ no such restriction applies.

5.4.3   Sample coefficient of variation

For non-central PDFs of X (which is the usual case in EMC), the coefficient of variation $v_X = \sigma_X/\mu_X$ serves as measure of relative uncertainty of X: it provides information on relative (as opposed to absolute) levels of fluctuation of X. In general terms, this concept is particularly useful if the PDF of X is unknown. If the data set represents repeated measurements of a quantity X that is subject to random fluctuations (e.g. noise), then $1/v_X$ represents the signal-to-noise ratio for the data set.

NOTE   The coefficient of variation is only applicable to linear and not to logarithmic quantities.

In practice, the ensemble coefficient of variation and its sample value have to be estimated based on sampled values of the mean and standard deviation. The mean value of the sample coefficient of variation is estimated as the ratio of sample mean to sample standard deviation:

$$n_X = \frac{\overline{X}}{s_X} \tag{47}$$

Since $\overline{X}$ and s_X are random quantities (see 5.3.2), n_X is itself a random quantity, that varies from sample set to sample set. For a normal PDF of X, the sample standard deviation of the coefficient of variation is $v_x\sqrt{\left(1+2v_x^2\right)}/\sqrt{(2N)}$.

5.4.4   Limits of sample-statistical confidence intervals

Just like any other sample statistic, the value (i.e., position) of the upper (ξ^+) and lower (ξ) boundaries of an η % confidence interval $\xi^{\pm}_{(1\pm\eta/100)/2}$ fluctuates and, hence, acquires uncertainty when the sample size N is finite. This occurs because the sample-to-sample variations of the empirical sampling distribution between sample sets implies that the probability that the measurand does not exceed a specified value fluctuates. Only the

underlying ensemble PDF has confidence limits that have deterministic values (i.e., fixed locations), whereas the boundaries for sampling varies and sampling distributions are random variables. The random confidence limits and their sample values are represented as $\Xi^{\pm}_{(1\pm\eta/100)/2}$ and $\xi^{\pm}_{(1\pm\eta/100)/2}$, respectively.

For a symmetric 95 % confidence interval for a general distribution of X with PDF $g(X)$ and distribution function $G(X)$, the general expression of the standard errors of the upper and lower boundaries and covariance of $\Xi^{\pm}_{(1\pm\eta/100)/2}$ are

$$\sigma(\Xi^{+}_{0,975}) = \sqrt{\frac{0,025 \times 0,975}{\left[g\left(G^{-1}(0,975)\right)\right]^2 N}}, \qquad \sigma(\Xi^{-}_{0,025}) = \sqrt{\frac{0,025 \times 0,975}{\left[g\left(G^{-1}(0,025)\right)\right]^2 N}} \tag{48}$$

For a standard normal PDF for X,

$$\sigma(\Xi^{+}_{0,975}) = \sigma(\Xi^{-}_{0,025}) \cong \frac{2,671}{\sqrt{N}}\sigma_X = \frac{1,363}{\sqrt{N}}\xi^{+}_{0,975} = -\frac{1,363}{\sqrt{N}}\xi^{-}_{0,025} \tag{49}$$

Corresponding values for other confidence levels and for the power or field strength of random electromagnetic fields (total and rectangular components) are given in [3].

For example, for a sample of N = 10, 100 or 1 000 independent measured values of a normally distributed field quantity, the empirically determined upper and lower boundaries of the symmetric 95 % confidence interval for this quantity each have a standard deviation of 43,1 %, 13,6 %, and 4,3 %, respectively. By comparison, for a 99 % confidence level, the standard deviations are instead 59,9 %, 18,9 %, and 5,9 %, respectively. Thus, the higher the chosen level of confidence for a measurand, the larger the uncertainty of the location of the boundaries of the sample-statistical confidence interval for this measurand for a given number of sample values. Stated differently, for a chosen level of confidence, confidence intervals become wider and the location of their boundaries of the associated confidence interval becomes simultaneously fuzzier when the sample size decreases. Sharp boundaries can only occur for parameters of infinitely large sample sets (populations). Therefore, determining confidence intervals based on very small data sets has only limited practical value. In practice, acceptable values for σ_{Ξ} do not only depend on σ_X but rather on the relative uncertainty σ_X/μ_X that is deemed acceptable. The sample size can be estimated to account for boundary fuzziness, e.g., subtracting $2\sigma_{\Xi}$ from the lower boundary and adding $2\sigma_{\Xi}$ to the upper boundary of the percentiles of the population distribution that define a confidence interval.

5.4.5 Sampling distribution and sampling statistics of mean value

5.4.5.1 General

More complete statistical information and characterization of sample values is given by their sampling *distribution* [3].

For a random X whose ensemble distribution is known or assumed to be the normal PDF, the sampling distribution of X is a Student's t PDF or Bessel K PDF, depending on whether X is considered in its (dimensionless) sample-standardized or (dimensioned) non-standardized form, respectively [8]. The former case is important for comparing measured data of X with theoretical PDFs, while the latter is relevant in the estimate of the actual value of a physical test quantity (e.g., field strength in V/m rather than in dB with respect to the mean as a reference value).

5.4.5.2 Complex-valued field or current

For normally distributed X, the sampling distributions of $\overline{X}$ (when considered as a random variable with respect to all possible data sets of N values; not to be confused with the sample

standard deviation, which is a single number) is also normal, with mean value *<X>* but standard deviation $\sigma_X / \sqrt{N}$:

$$g(\bar{x};N) = \sqrt{\frac{N}{2\pi\sigma^2}} \exp\left[-\frac{N(\bar{x} - \langle X \rangle)^2}{2\pi\sigma^2} \right]$$
(50)

If *X* has a PDF different from a normal PDF, then the sampling PDFs of *X*, sample mean value and sample standard deviation are usually difficult to compute. The situation is often complicated further by the fact that only for normal *X* are the sample mean and sample standard deviation statistically independent quantities. Therefore, when the ensemble field quantity is (or can be traced back to) only a normal PDF, rigorous results can be derived. For example, the amplitude or energy density of an unbiased ideal random field (rectilinear component or vector field) can still be calculated [3]. In most other cases, the sample-statistical characterization is necessarily restricted to the sample standard deviation and/or sample confidence intervals, for which general expressions remain valid.

5.4.5.3 Power (energy density, intensity)

The PDF of the sampling average of the power or intensity *W* for an *unbiased* field ($\langle E \rangle$=0) is

$$g(\bar{w};N) = \frac{(dN)^{dN/2}}{\Gamma(dN)\sigma_{\overline{W}}} \left(\frac{\bar{w}}{\sigma_{\overline{W}}} \right)^{dN-1} \exp\left(-\sqrt{dN}\,\frac{\bar{w}}{\sigma_{\overline{W}}} \right)$$
(51)

where $\Gamma(dN)$ represents the (complete) gamma function. Here, *d* refers to the number of axes of the antenna or field probe. For example, for a wire antenna or single-axis probe, *d* = 1; for a helical antenna supporting left- or right-hand circular polarisation, *d* = 2; for a three-axis field probe with internal combination, *d* = 3.

The sampling mean and sampling standard deviation are given by

$$\langle \overline{W} \rangle = d\sigma_E^2 = 2d\sigma^2 \qquad \sigma_{\overline{W}} = 2\sqrt{\frac{d}{N}}\sigma^2$$
(52)

where $\sigma_E^2 = 2\sigma^2$ is the variance of a circular complex-value (analytic) field *E*, with σ representing the standard deviation of the in-phase or quadrature component of *E*.

Symmetric confidence intervals for the η % confidence level are obtained from the distribution function by analytical or numerical solution of

$$\frac{\gamma\left(dN, \sqrt{dN}\,\dfrac{\xi_{(1\pm\eta/100)/2}}{\sigma_{\overline{W}}} \right)}{\Gamma(dN)} = \frac{1 \pm \eta/100}{2}$$
(53)

where $\gamma(dN)$ represents the incomplete gamma function.

5.4.5.4 Field amplitude

For the sampling average of the field strength $\overline{A}$ for an *unbiased* field ($\langle E \rangle$ = 0),

$$g(\overline{a};N) = \frac{2\left[d - \left(\dfrac{\Gamma\left(dN + \dfrac{1}{2}\right)}{\Gamma(dN)}\right)^2\right]^{dN}}{\Gamma(dN)\sigma_{\overline{A}}}\left(\frac{\overline{a}}{\sigma_{\overline{A}}}\right)^{2dN-1}\exp\left\{-\left[dN - \left(\frac{\Gamma\left(dN + \dfrac{1}{2}\right)}{\Gamma(dN)}\right)^2\right]\left(\frac{\overline{a}}{\sigma_{\overline{A}}}\right)^2\right\} \quad (54)$$

with sampling mean and sampling standard deviation given by

$$\langle\overline{A}\rangle = \sqrt{2}\,\frac{\Gamma\left(dN + \dfrac{1}{2}\right)}{\Gamma(dN)}\sigma \qquad \sigma_{\overline{A}} = \sqrt{\frac{2}{N}\left[dN - \left(\frac{\Gamma\left(dN + \dfrac{1}{2}\right)}{\Gamma(dN)}\right)^2\right]}\,\sigma \quad (55)$$

Confidence intervals for the η % confidence level are obtained from the distribution function of $\overline{A}$ by analytical or numerical solution of the Equation (56)

$$\frac{\gamma\left(dN, \left[dN - \left(\dfrac{\Gamma\left(dN + \dfrac{1}{2}\right)}{\Gamma(dN)}\right)^2\right]\left(\dfrac{\xi_{(1\pm\eta/100)/2}}{\sigma_{\overline{A}}}\right)^2\right)}{\Gamma(dN)} = \frac{1\pm\eta/100}{2} \quad (56)$$

Figure 11 and Figure 12 show the limits of 95 %, 99 % and 99,5 % confidence intervals for $\overline{W}$ and $\overline{A}$ as a function of N for measurements using a wire antenna or single-axis probe (d = 1).
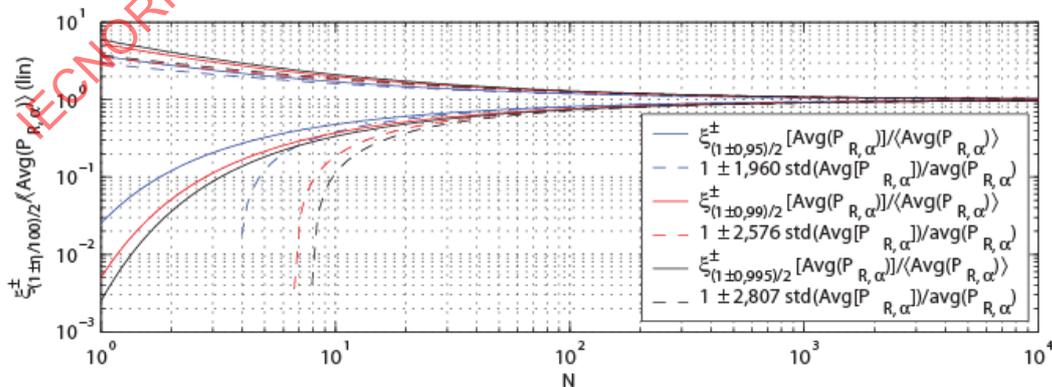
Figure 11 – Limits of 95 %, 99 % and 99,5 % confidence intervals for $\overline{W}$ as a function of N for measurements using a rectilinear antenna or single-axis probe

IEC 1314/12
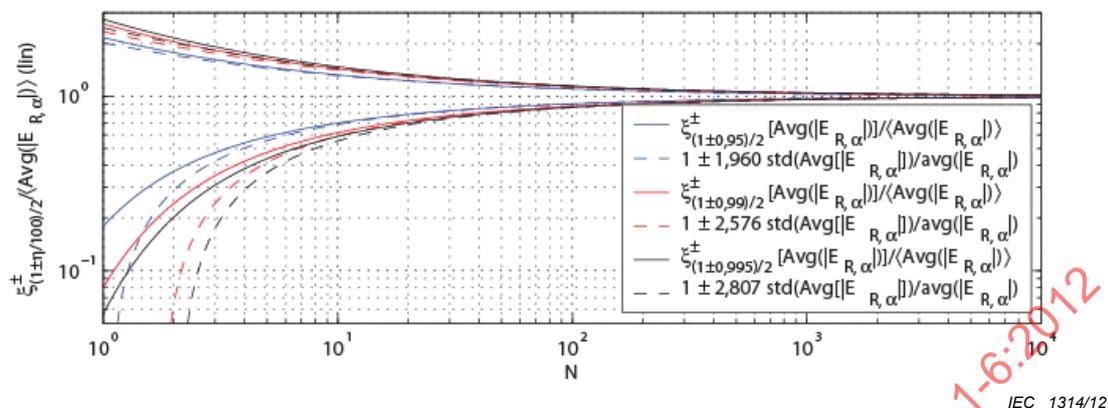
Figure 12 – Limits of 95 %, 99 % and 99,5 % confidence intervals for $\overline{A}$ as a function of *N* for measurements using a rectilinear antenna or single-axis probe

5.4.6 Sampling distribution and sampling statistics of standard deviation

5.4.6.1 General

The sample standard deviation is a random variable, varying from sample to sample for a given (fixed) sample size *N*, hence, it is meaningful to characterize its sampling PDF and sampling statistics.

5.4.6.2 Complex-valued field or current

For the sampling standard deviation of a field or current with circular Gauss normal PDF, the sampling standard deviation has a χ PDF with *N* degrees of freedom, i.e.,

$$g(s_x;N) = \frac{2}{\Gamma\left(\frac{N-1}{2}\right)\sigma_{s_x}} \left[\frac{N-1}{2} - \left(\frac{\Gamma\left(\frac{N}{2}\right)}{\Gamma\left(\frac{N-1}{2}\right)}\right)^2\right]^{\frac{N-1}{2}} \left(\frac{s_x}{\sigma_{s_x}}\right)^{N-2} \exp\left[-\left[\frac{N-1}{2} - \left(\frac{\Gamma\left(\frac{N}{2}\right)}{\Gamma\left(\frac{N-1}{2}\right)}\right)^2\right]\left(\frac{s_x}{\sigma_{s_x}}\right)^2\right] \quad (57)$$

with sampling mean and sampling standard deviation given by

$$\langle S_x \rangle = \sqrt{\frac{2}{N-1}} \frac{\Gamma\left(\frac{N}{2}\right)}{\Gamma\left(\frac{N-1}{2}\right)} \sigma_x \qquad \sigma_{s_x} = \sqrt{1 - \frac{2}{N-1}\left(\frac{\Gamma\left(\frac{N}{2}\right)}{\Gamma\left(\frac{N-1}{2}\right)}\right)^2} \sigma_x \qquad (58)$$

5.4.6.3 Power (energy density, intensity)

For the power or intensity *W* for an unbiased field ($\langle E \rangle = 0$),

$$g(s_W) = \frac{\left(dN - \frac{1}{2}\right)^{\frac{1}{2}\left(dN - \frac{1}{2}\right)}}{\Gamma\left(dN - \frac{1}{2}\right)\sigma_{S_W}} \left(\frac{s_W}{\sigma_{S_W}}\right)^{\left(dN - \frac{3}{2}\right)} \exp\left(-\sqrt{dN - \frac{1}{2}} \frac{s_W}{\sigma_{S_W}}\right) \tag{59}$$

with sampling mean and sampling standard deviation

$$\langle S_W \rangle = \sigma_W \qquad \sigma_{S_W} = \frac{\sigma_W}{\sqrt{dN - \frac{1}{2}}} \tag{60}$$

5.4.6.4 Field amplitude

For the field strength, again for an unbiased field, the PDF of S_A is

$$g(s_A; N) = \frac{2\left[dN - \frac{1}{2} - \left(\frac{\Gamma(dN)}{\Gamma\left(pN - \frac{1}{2}\right)}\right)^2\right]^{dN - \frac{1}{2}}}{\Gamma\left(dN - \frac{1}{2}\right)\sigma_{S_A}} \left(\frac{s_A}{\sigma_{S_A}}\right)^{2(dN-1)} \exp\left(-\left[dN - \frac{1}{2} - \left(\frac{\Gamma(dN)}{\Gamma\left(dN - \frac{1}{2}\right)}\right)^2\right]\left(\frac{s_W}{\sigma_{S_W}}\right)^2\right) \tag{61}$$

with sampling mean and sampling standard deviation given by

$$\langle S_A \rangle = \frac{1}{\sqrt{dN - \frac{1}{2}}} \frac{\Gamma(dN)}{\Gamma\left(dN - \frac{1}{2}\right)} \sigma_A \qquad \sigma_{S_A} = \sqrt{1 - \frac{1}{dN - \frac{1}{2}}\left(\frac{\Gamma(dN)}{\Gamma\left(dN - \frac{1}{2}\right)}\right)^2} \, \sigma_A \tag{62}$$

Figure 13 show the limits of 95 %-confidence intervals for S_X as a function of N for measurements using a rectilinear antenna ($d = 1$), together with its normal approximation.
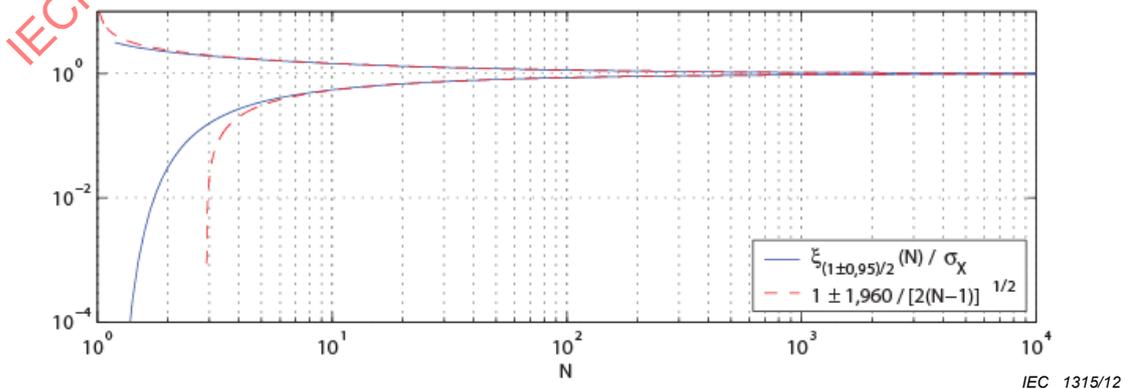
**Figure 13 – 95 % confidence intervals for S_X as a function of N
for measurements using a single-axis detector**

5.5 Conversion from linear quantities to decibel and vice versa

5.5.1 General considerations

The conversion from linear units (e.g., μV/m, mW, etc.) to logarithmic units, i.e., decibel (e.g., dB(μV/m), dB(mW), etc.), or *vice versa*, is a nonlinear transformation [9]. Therefore, the PDF of electromagnetic measurands and physical quantities such as field strength, power, etc., when expressed for units dB, is in general different from the one applicable to the same quantities but expressed for linear units.

The determination of the PDF is fundamental to the calculation of the propagation of uncertainty. From the transformed PDF, estimates for dispersion and confidence intervals follow in a straightforward manner.

5.5.2 Normally distributed fluctuations

5.5.2.1 Complex field

In general, the probability for obtaining negative values of the in-phase or quadrature components is not negligible. Hence, in strict terms, a conversion to dB is in this case not possible. However, if the PDF is sufficiently well concentrated around its central value (implying $\sigma/\mu \ll 1$), then the probability for obtaining negative values can be neglected and a transformation is then possible.

For

$$F = 20\lg(E) \tag{63}$$

with

$$g_E(E) = \frac{\exp\left[-\dfrac{(E-\mu)^2}{2\sigma^2}\right]}{\sqrt{2\pi}\,\sigma} \tag{64}$$

the PDF of F is

$$g_F(F) = \frac{\exp\left[-\dfrac{(F-20\lg(\mu))^2}{2\left(\dfrac{20\,\sigma}{\ln 10}\right)^2}\right]}{\sqrt{2\pi}\,\dfrac{20\,\sigma}{\ln 10}} \tag{65}$$

This transformed PDF can also be used in cases where the original PDF is approximately Gauss normal, on account of the central limit theorem, e.g., for maximum-value PDFs of field strength or power.

5.5.2.2 Field strength (χ-distributed linear quantity)

For field strength, the transformation of variants is specified by

$$B = 20\lg(A) \tag{66}$$

and yields the transformed PDF

$$g_B(B) = C\frac{\ln 10}{20} 10^{\frac{B}{20}} g_A\left(A = 10^{\frac{B}{20}}\right) \tag{67}$$

in which C is a normalization constant. The mean value of B is

$$\langle B \rangle = \int_{-\infty}^{+\infty} 20\lg(X) g_A(A = X)dX \tag{68}$$

The inverse transformation from units dBV to linear units V is

$$g_A(A) = \frac{20}{A \cdot \ln(10)} g_B(B = 20\lg(A)) \tag{69}$$

with mean value

$$\langle A \rangle = \int_{-\infty}^{+\infty} \exp\left(\frac{X \cdot \ln(10)}{20}\right) g_B(B = X)dX \tag{70}$$

In particular, for a Rayleigh distributed A, i.e., for

$$g_A(A) = \frac{A \cdot \exp\left(-\dfrac{A^2}{2\sigma^2}\right)}{\sigma^2} \tag{71}$$

we obtain

$$g_B(B) = C\frac{\ln 10}{20\sigma^2} 10^{\frac{B}{20}} \exp\left(-\frac{10^{\frac{B}{20}}}{2\sigma^2}\right) \tag{72}$$

where $C = e$ for $\sigma = 1/\sqrt{2}$.

Figure 14 shows the PDF of B for a Rayleigh distributed A at selected values of σ.
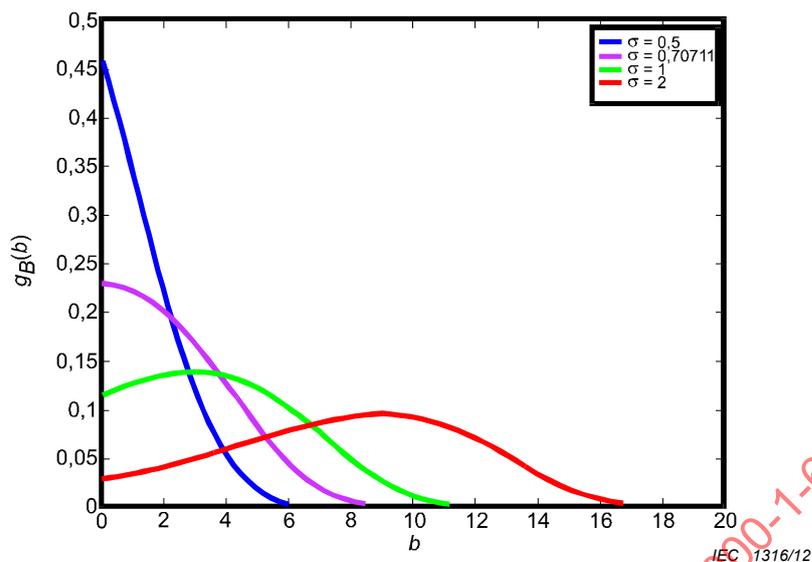
Figure 14 – PDF of _B_ for a Rayleigh distributed _A_ at selected values of σ

5.5.2.3   Power, intensity, energy density ($\mathcal{X}^2$-distributed linear quantity)

For field intensity, energy density or power, a transformation of variants specified by

$$V = 10\lg(R) \tag{73}$$

yields

$$g_V(V) = \frac{\ln 10}{10} 10^{\frac{V}{10}} \cdot g_R\!\left(R = 10^{\frac{V}{10}}\right) \tag{74}$$

with mean value

$$\langle V \rangle = \int_{-\infty}^{+\infty} 10\lg(X) g_R(R = X)\,\mathrm{d}X \tag{75}$$

The inverse transformation from units dBW to linear units W is

$$g_R(R) = \frac{10}{R \cdot \ln(10)} g_V(V = 10\lg(R)) \tag{76}$$

with mean value

$$\langle R \rangle = \int_{-\infty}^{+\infty} \exp\!\left(\frac{X \cdot \ln(10)}{10}\right) g_V(V = X)\,\mathrm{d}X \tag{77}$$

For an exponentially distributed _R_, i.e., for

$$g_R(R) = \frac{\exp\!\left(-\dfrac{R}{2\sigma^2}\right)}{2\sigma^2} \tag{78}$$

we obtain

$$g_V(V) = C \frac{\ln 10}{20\sigma^2} 10^{\frac{V}{10}} \exp\left(-\frac{10^{\frac{V}{10}}}{2\sigma^2}\right) \tag{79}$$

which coincides with the result for a Rayleigh distributed field strength in dBV.

NOTE   It can be shown that this coincidence between field and power PDFs for dB units is true for any pair of PDFs of A and R; not just for the pair of Rayleigh and exponential PDFs.

The mean value is

$$\langle V \rangle = 10 \lg(2\sigma^2) - 2,507 \tag{80}$$

which shows that averaging of logarithmic values (e.g., when performing measurements using a network analyzer for time-varying conditions with video averaging) gives rise to an average value that is about 2,5 dB lower than when the values are linearly averaged.

5.5.3   Uniformly distributed fluctuations

The previous results for a signal-plus-noise, in which the signal is constant and the noise has a normal PDF, can be generalized to other PDFs, e.g., for the case where the fluctuations around a constant value are uniformly distributed.

Specifically, if X has a uniform PDF with interval [a, b] in linear units (i.e. centred around the value $(a + b)/2$ with half width $(b - a)/2$), then its conversion to dBV, calculated as $Y = 20 \lg_{10}(X)$, has the PDF

$$g_Y(Y) = \frac{\ln 10}{20(b-a)} 10^{\frac{Y}{20}} \tag{81}$$

whereas its conversion to dBW calculated as $Z = 10 \lg_{10}(X)$ has the PDF

$$g_Z(Z) = \frac{\ln 10}{10(b-a)} 10^{\frac{Z}{10}} \tag{82}$$

For the inverse transformation from a uniform PDF in units dBV to linear units for fields or voltages, the PDF is

$$g_X(X) = \frac{20}{(b-a)\ln(10)} \cdot \frac{1}{X} \tag{83}$$

6   Applicability of measurement uncertainty

Some parameters of the disturbance quantity defined in the applicable basic standard[1] may have an associated tolerance. If there is no tolerance requirement for a quantity q in the applicable basic standard, the measurement uncertainty assigned to this quantity shall be

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[1]   For example the IEC 61000-4 series of standards.

derived both from the calibration results on the instrument(s) used to measure this quantity and other input quantities (e.g. temperature drift, long-term drift). In general, the difference between the indicated value during calibration q_i and the value realized by the calibration laboratory q_c is a systematic error, which can be corrected, when the instrument is used. In that case, this difference will not be part of the total measurement uncertainty budget. The flow chart in Figure 15 shows:
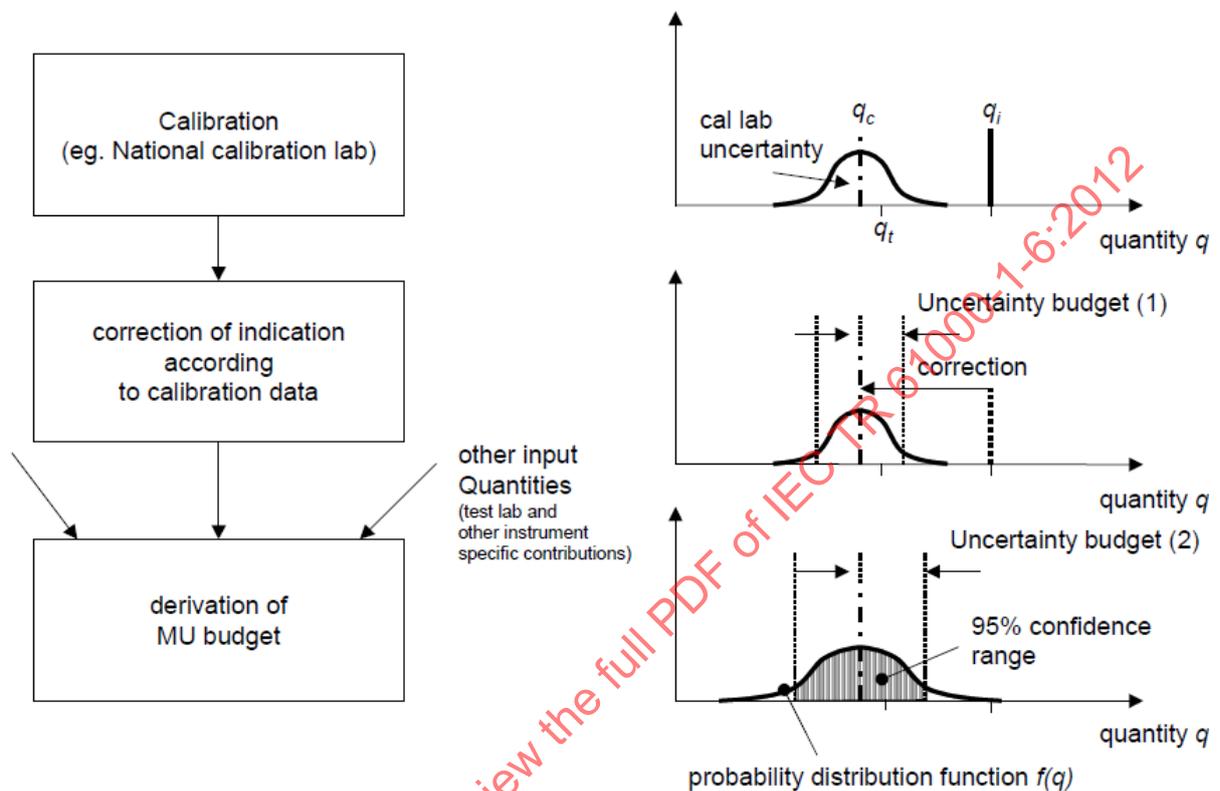
Figure 15 – Measurement uncertainty budget for a quantity to be realized in the test laboratory

where q_i, q_t, and q_c are values as indicated by the instrument to be calibrated, the true value (however, the true value is unknown in most cases), and the value as realized by the calibration laboratory, respectively.

During calibration in the calibration laboratory the instrument to be calibrated will give an indicated value q_i. The instruments used for calibration by the calibration laboratory will give a value q_c. Since the instrumentation used by the calibration laboratory has a measurement uncertainty, the true value q_t will be in an interval around q_c.

By correction of the indication shown by the instrument by the correction factor c, q_i and q_c will become the same value. The uncertainty budget (1) at this stage will only include the uncertainty of the calibration laboratory as stated in the calibration certificate.

Since the instrument may be affected by other influence quantities, which were not present during calibration, these other input quantities shall be considered in the uncertainty budget (2).

Measurement uncertainties need also to be considered, if the applicable basic standard requires that a certain quantity be within a tolerance band specified by a lower and an upper tolerance limit as shown in Figure 15. In principle, the procedure shown above is extended by a process step, where the fulfillment of the tolerance requirements is checked.
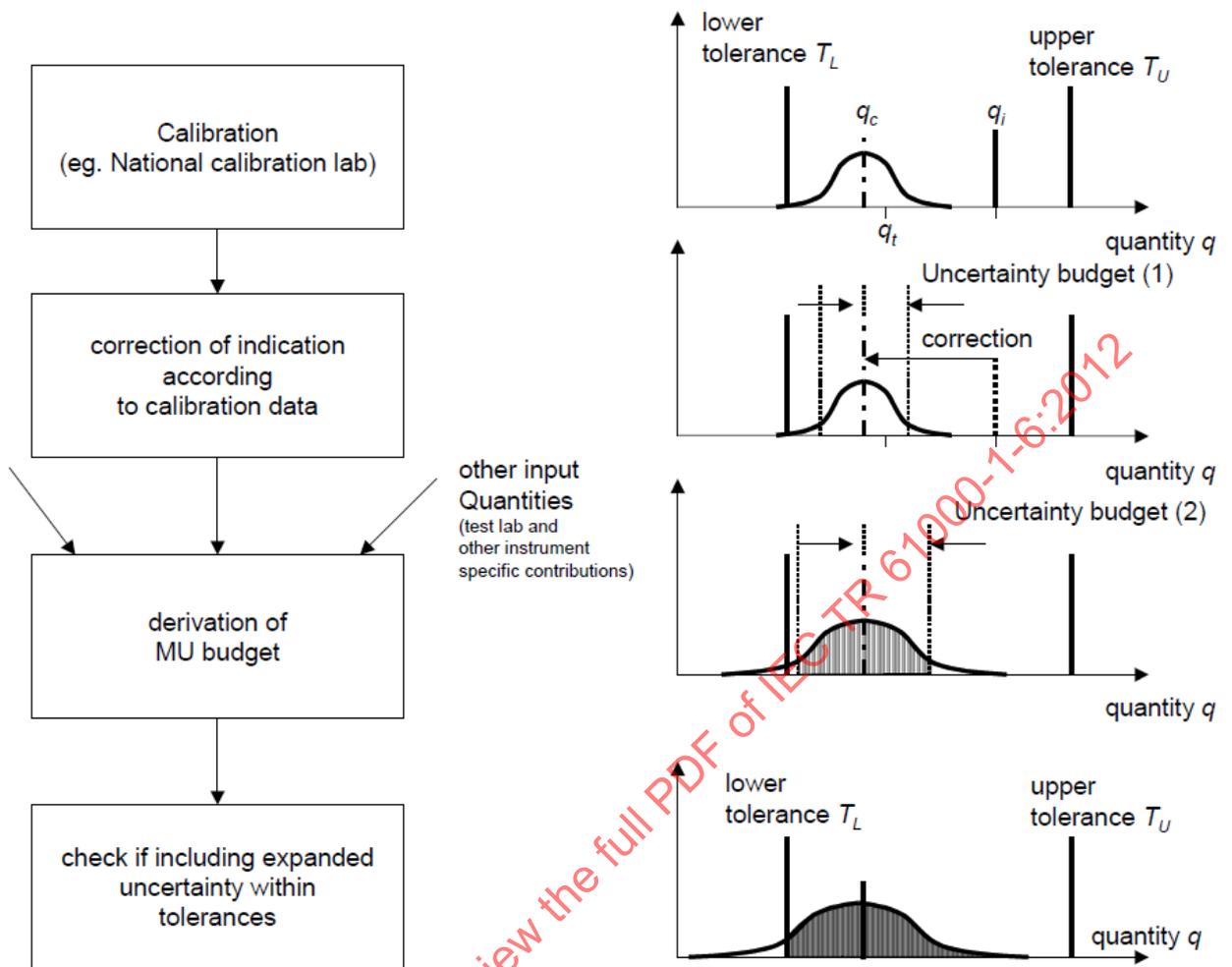
IEC 1318/12

Figure 16 – Relationship between measurement uncertainty budgets for a quantity to be realized in the test laboratory and tolerances given for this quantity in the applicable basic standard

In Figure 16, T_L and T_U are the lower and upper limits of tolerance required by the applicable basic standard, respectively.

Where the value as indicated by the instrument is already within the tolerance band without correction, then a correction is not needed. Figure 17 shows some situations, where and how an instrument is suitable for tests / measurements as specified in the applicable basic standard with tolerances.

Indication of the instrument is already in the tolerance band. Corrections are not applied. The instrument can be used for the tests.	lower tolerance T_L ... upper tolerance T_U ... q_c ... q_i ... quantity q
Indication of the instrument is already in the tolerance band. Corrections are applied to increase the quality of the measurements. The instrument can be used for the tests.	lower tolerance T_L ... upper tolerance T_U ... q_c correction q_i ... quantity q
Indication of the instrument is outside the tolerance band. Corrections are applied. After correction (derived from calibration results) the corrected value is within the tolerance band. The instrument can be used for the tests.	lower tolerance T_L ... upper tolerance T_U ... q_c correction q_i ... quantity q
Indication of the instrument is inside or outside the tolerance band. After correction (derived from calibration results) the corrected value is outside the tolerance band. The instrument cannot be used for the tests.	lower tolerance T_L ... upper tolerance T_U ... q_c correction q_i ... quantity q
Indication of the instrument is inside or outside the tolerance band. With corrections it is inside the tolerance band, but in the grey zone (either q_c-measurement uncertainty$<T_L$ or q_c+measurement uncertainty$>T_U$), i.e. statistically it cannot be justified, if the true value is inside or outside the tolerance band. The instrument can be used.[1]	lower tolerance T_L ... upper tolerance T_U ... q_c correction q_i ... quantity q
Indication of the instrument is inside or outside the tolerance band. With corrections it is outside the tolerance band, but in the grey zone (either q_c+measurement uncertainty$>T_L$ or q_c-measurement uncertainty$<T_U$), i.e. statistically it cannot be justified, if the true value is inside or outside the tolerance band. The instrument should not be used. [1]	lower tolerance T_L ... upper tolerance T_U ... q_c correction q_i ... quantity q

[1] This approach deviates from other concepts found in literature, because the tolerances given in the standards of the 61000-4-x series once were defined without taking into account uncertainties. Therefore uncertainties should not be used to demonstrate compliance with tolerances.

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Figure 17 – Situations, where and how an instrument is suitable for tests and/or measurements as specified in the applicable basic standard with tolerances

7   Documentation of measurement uncertainty calculation

An assessment of measurement uncertainty according to this guide shall be documented in a report. The report shall include all elements presented in Table 1. The report shall contain all the information necessary to reproduce the uncertainty budget. In particular, the following shall be recorded:

–   identification of the measurand;

–   identification of the influence quantities applicable to the measurement chain;

–   identification of the category for each influence quantity, i.e. Type A or type B;

–   identification of the applicable PDF used for each influence quantity;

–   identification of the applicable mathematical formula used for each PDF;

–   the calculated standard uncertainty for each contribution;

–   sensitivity coefficient c_i;

–   the calculated combined standard uncertainty;

–   the coverage factor k, usually $k = 2$ (for 95 % confidence);

–   the calculated expanded uncertainty.

The values of uncertainty can be expressed in dB or in %. All uncertainty values shall be expressed with not more than three significant digits (e.g. 2,35 dB or 12,4 %). Examples of uncertainty budgets are given in Annex A and Annex B. Note that uncertainty values shall not be "zero", i.e. zero values shall be avoided.

NOTE   The transformation between dB and % can be calculated by $10^{\frac{(unit\_in\_dB)}{20}} \times 100$.

Annex A
(informative)

Example of MU assessment for emission measurements

A.1 Symbols

A.1.1 General symbols

X_i Influence quantity

x_i Estimate of X_i

δX_i Correction for influence quantity

$u(x_i)$ Standard uncertainty of x_i

c_i Sensitivity coefficient

y Result of a measurement, (the estimate of the measurand), corrected for all recognised significant systematic effects

$u_c(y)$ (Combined) Standard uncertainty of y

$U(y)$ Expanded uncertainty of y

k Coverage factor

a^+ Upper abscissa of a PDF

a^- Lower abscissa of a PDF

A.1.2 Symbol and definition of the measurand in the example

E Maximum electric field strength, in dB(μV/m), in horizontal and vertical polarisations at the applicable height scan range and at the specified horizontal distance from the EUT that is rotated 360° in azimuth

A.1.3 Symbols for input quantities common to all disturbance measurements

a_c Attenuation of the connection between the receiver and the antenna, in dB

δM Correction for the error caused by mismatch, in dB

V_r Receiver voltage reading, in dB(μV)

δV_{sw} Correction for receiver sine wave voltage inaccuracy, in dB

δV_{nf} Correction for the effect of the receiver noise floor, in dB

A.1.4 Symbols of input quantities specific for radiated disturbance measurements

G_p Preamplifier gain

δG_p Correction for instability of preamplifier gain in dB

F_a Antenna factor in dB(V/m)

$\delta F_{a\,f}$ Correction for antenna factor interpolation error in dB

$\delta F_{a\,dir}$ Correction for antenna directivity in dB

$\delta F_{a\,ph}$ Correction for antenna phase centre location in dB

$\delta F_{a\,cp}$ Correction for antenna cross-polarisation response in dB

δS_{VSWR} Correction for imperfect site voltage standing wave ratio (S_{VSWR}) in dB

δA_{NT} Correction for the effect of setup table material on measurement results in dB

δd Correction for imperfect antenna distance in dB