INTERNATIONAL STANDARD

IEC 62539

First edition
2007-07

IEEE 930

Guide for the statistical analysis of electrical insulation breakdown data

Reference number
IEC 62539(E):2007
IEEE Std 930-2004

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GUIDE FOR THE STATISTICAL ANALYSIS OF ELECTRICAL INSULATION BREAKDOWN DATA

FOREWORD

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The text of this standard is based on the following documents:

IEEE Std	FDIS	Report on voting
930 (2004)	112/59/FDIS	112/69A/RVD

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IEEE Guide for the Statistical Analysis of Electrical Insulation Breakdown Data

Sponsor

Statistical Technical Committee
of the
IEEE Dielectrics and Electrical Insulation Society

Approved 29 March 2005

American National Standards Institute

Approved 23 September 2004

IEEE-SA Standards Board

Abstract: This guide describes, with examples, statistical methods to analyze times to break down and breakdown voltage data obtained from electrical testing of solid insulating materials, for purposes including characterization of the system, comparison with another insulator system, and prediction of the probability of breakdown at given times or voltages.
Keywords: breakdown voltage and time, Gumbel, Lognormal distributions, statistical methods, statistical confidence limits, Weibull

IEEE Introduction

This introduction in not part of IEEE Std 930-2004, IEEE Guide for the Statistical Analysis of Electrical Insulation Breakdown Data.

Endurance and strength of insulation systems and materials subjected to electrical stress may be tested using constant stress tests in which times to breakdown are measured for a number of test specimens, and progressive stress tests in which breakdown voltages may be measured. In either case it will be found that a different result is obtained for each specimen and that, for given test conditions, the data obtained may be represented by a statistical distribution.

Failure of solid insulation can be mostly described by extreme-value statistics, such as the Weibull and Gumbel distributions, but, historically, also the lognormal function has been used. Methods for determining whether data fit to either of these distributions, graphical and computer-based techniques for estimating the most likely parameters of the distributions, computer-based techniques for estimating statistical confidence intervals, and techniques for comparing data sets and some case studies are addressed in this guide.

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GUIDE FOR THE STATISTICAL ANALYSIS OF ELECTRICAL INSULATION BREAKDOWN DATA

1. Scope

Electrical insulation systems and materials may be tested using constant stress tests in which times to breakdown are measured for a number of test specimens, and progressive stress tests in which breakdown voltages may be measured. In either case, it will be found that a different result is obtained for each specimen and that, for given test conditions, the data obtained may be represented by a statistical distribution. This guide describes, with examples, statistical methods to analyze such data.

The purpose of this guide is to define statistical methods to analyze times to breakdown and breakdown voltage data obtained from electrical testing of solid insulating materials, for purposes including characterization of the system, comparison with another insulator system, and prediction of the probability of breakdown at given times or voltages.

Methods are given for analyzing complete data sets and also censored data sets in which not all the specimens broke down. The guide includes methods, with examples, for determining whether the data is a good fit to the distribution, graphical and computer-based techniques for estimating the most likely parameters of the distribution, computer-based techniques for estimating statistical confidence intervals, and techniques for comparing data sets and some case studies. The methods of analysis are fully described for the Weibull distribution. Some methods are also presented for the Gumbel and lognormal distributions. All the examples of computer-based techniques used in this guide may be downloaded from the following web site "http://grouper.ieee.org/groups/930/IEEEGuide.xls." Methods to ascertain the short time withstand voltage or operating voltage of an insulation system are not presented in this guide. Mathematical techniques contained in this guide may not apply directly to the estimation of equipment life.

2. References

The following publications may be used when applicable in conjunction with this guide. When the following standards are superseded by an approved revision, the revision shall apply.

ASTM D149-97a(2004) Standard Test Method for Dielectric Breakdown Voltage and Dielectric Strength of Solid Electrical Insulating Materials at Commercial Power Frequencies.[1]

[1]ASTM publications are available from the American Society for Testing and Materials, 100 Barr Harbor Drive, West Conshohocken, PA 19428-2959, USA (http://www.astm.org/).

BS 2918-2, Methods of test for electric strength of solid insulating materials.[2]

IEC 60243 series, Electrical strength of insulating materials—Test Methods—Part 1: Tests at power frequencies. [3]

3. Steps required for analysis of breakdown data

3.1 Data acquisition

3.1.1 Commonly used testing techniques

There are two commonly used breakdown tests for electrical insulation: *constant stress* tests and *progressive stress* tests. In these tests a number of identical specimens are subjected to identical test regimes intended to cause electrical breakdown. In constant stress tests the same voltage is applied to each specimen (they are often tested in parallel) and the times to breakdown are measured. The times to breakdown may be widely distributed with the longest time often being more than two orders of magnitude that of the shortest. In progressive stress tests an increasing voltage is applied to each specimen, usually breakdown voltages are measured. The voltage may be increased continuously with time or in small steps. Other protocols, for example impulse testing, may also be used. Breakdown voltages may be much less widely distributed with the highest voltage sometimes only being 2% more than the lowest voltage.

Various international standards, e.g., BS 2918-2 and IEC 60243 series, give appropriate experimental procedures for constant and progressive stress tests. This guide is intended to provide a more rigorous treatment for the breakdown data obtained in this way.

3.1.2 Other data

Breakdown data may also be available from other sources; for example, times to breakdown of the insulation in service may be available. Such data is generally much more difficult to analyze since the history of each failed insulator may not be the same (see 3.1.4), particularly as units that failed will have been replaced. It may also be unclear how many such insulation systems are in service and hence what proportion of them have failed. The techniques described in this guide are, nevertheless, appropriate for such data provided sufficient care is exercised in their application.

3.1.3 Data requirements

The number of data points required depends upon the number of parameters that describes the distribution and the confidence demanded in the results. If possible, failure data on at least ten specimens should be obtained; serious errors may result with less than five specimens (see also 3.2.2).

If all the specimens break down, the data is referred to as complete. In some cases, not all the specimens break down, the data is then referred to as censored. Censored data may be encountered in constant stress tests where the data are analyzed or the test is terminated before all the specimens break down. Censored data can also occur with progressive stress tests where the power supply has insufficient voltage capability to break down all the samples. In these cases, the data associated with a single group of specimens, those with the highest strength, are not known and the data set is said to be singly censored.[4] Data may also be progressively censored. In this case, specimens may be withdrawn (or their data discounted) at any time or

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[2]Bristish Standards are available from IHS Engineering/IHS International, 15 Iverness Way East, Englewood, CO 80112, USA.

[3]IEC publications are available from the Sales Department of the International Electrotechnical Commission, Case Postale 131, 3, rue de Varembé, CH-1211, Genève 20, Switzerland/Suisse (http://www.iec.ch/). IEC publications are also available in the United States from the Sales Department, American National Standards Institute, 25 West 43rd Street, 4th Floor, New York, NY 10036, USA (http://www.ansi.org/).

voltage; such data are often referred to as "suspended." This may be the case where specimen breakdown is due to a spurious mechanism such as termination failure or flashover or where the specimen is deliberately withdrawn for alternative analysis. Censoring can occur by plan or by accident in many insulation tests and it is essential that this is taken into account in the data analysis. Less confidence can be placed in the analysis of a censored data set than in a complete set of data with the same number of specimens. If possible censored data sets should include at least ten (non-censored) data points and at least 30% of the specimens should have broken down.

3.1.4 Practical precautions in data capture

Specimens should, as far as possible, be identical, have the same history prior to testing, and be tested under the same conditions. In measuring the breakdown characteristics of materials it should be noted that the breakdown field (kilovolt per millimeter)[5] is usually dependent upon the rate of voltage rise, specimen thickness, electrode material, configuration and method of attachment, temperature, area, and frequency if an alternating voltage is applied. Other factors such as humidity and specimen age may also be important. With insulating systems such as cables and bushings, surface and interfacial partial discharges must be minimized and stress enhancements due to protrusions, contaminants and voids are likely to reduce breakdown strengths considerably.

The scope of this guide is limited to ac voltage testing, but the techniques may be applied to other failure tests (such as impulse or dc testing) with care. Knowledge of the failure mechanism may be required in order to establish the appropriate parameters to be measured. In pulse energized dc systems, for example, it may be more appropriate to measure the number of pulses to breakdown than the dc time to failure. Precautions in data capture are described more fully elsewhere, e.g., Abernethy [B1].[6]

3.2 Characterizing data using a probability function

3.2.1 Types of failure distribution

Failure data, such as that described in breakdown of electrical insulation, may be represented in a histogram form as numbers of specimens failed in consecutive periods. For example, the times to breakdown of polymer coated wires subject to constant ac stress are shown in Figure A.1 as a histogram. The mean and standard deviation of this data set is easily found using a scientific calculator and the corresponding Normal probability density function can be superimposed on the histogram. Whilst the Normal is probably the best known and its parameters (the mean and standard deviation) are easily calculated; it is not usually appropriate to electrical breakdown data. For example, it can be seen in Figure A.1 that its shape is rather different to the histogram. In particular the Normal distribution has a finite probability of failure at (physically impossible) negative times. An important step in analyzing breakdown data is the selection of an appropriate distribution.

Distributions for electrical breakdown include the Weibull, Gumbel, and lognormal. The most common for solid insulation is the Weibull and is the main distribution described in this guide. It is found to have wide applicability and is a type of extreme value distribution in which the system fails when the weakest link fails. The Gumbel distribution, another extreme value distribution, may have applicability in breakdown involving percolation, in liquids and in cases where fault sites such as voids are exponentially distributed. The effect of the size of test specimens (thickness, area, volume) on life or breakdown voltage can be modeled using extreme value distributions. The lognormal distribution may be useful where specimens break

[4] This is also known as "right" censored data since specimens beyond a certain time or voltage are not tested. It is possible to have "left" censored data but this does not usually occur in electrical breakdown testing. In this guide, "singly" censored data always refers to "right" censoring.

[5] To convert this unit value from kV/mm to kV/inch multiply the value in kV/mm value by 25.4.

[6] The numbers in brackets correspond to those of the bibliography in Annex B.

down due to unrelated causes or mechanisms. The lognormal distribution may be closely approximated by the Weibull distribution.

The previous distributions may be described in terms of two parameters (as the normal distribution is described in terms of the mean and standard deviation). To give more generality, however, a third parameter may be included corresponding to a time before, or a voltage below, which a specimen will not break down.

In some cases two or more mechanisms may be operative, this may necessitate combining two or more distributions functions.

Mathematical descriptions of these distributions are given in Clause 4.

3.2.2 Testing the adequacy of a distribution

Having chosen a distribution to represent a set of breakdown data, it is necessary to check that the distribution is adequate for this purpose. It was seen in 3.2.1 that, although the parameters of a Normal distribution could be found for a given set of data, this did not imply that the distribution was an adequate representation (e.g., Figure A.1). The most common technique to test the adequacy of the distribution is to plot data points on special probability paper associated with the distribution in question. Such paper is available for all the distributions thus far mentioned. A good fit to a distribution will result in a straight line plot (5.1 and 5.2). Statistical techniques are also available for assessing the adequacy of a distribution; a simple technique is given in 5.4.

3.2.3 Estimating parameters and confidence limits

Probability plots can also be used for graphical estimation of the parameters of the distribution (Clause 6) but this is not recommended; more accurate computation techniques are readily available (Clause 7).

The parameters obtained from all such techniques are only estimates because the measured data points are randomly distributed according to a given failure mechanism. For example, if 100 experiments were performed each with ten specimens, the analysis of each of the 100 experiments would give 100 estimates for the parameters of the probability distribution each of which are slightly different. In such a case, it may be possible to state with (for example) 90% confidence that the true value of the given parameter lies between the fifth largest and fifth smallest value obtained. It is common to calculate (9.1), for each parameter estimate, a statistical confidence interval that encloses the true parameter with high probability. In general, the more specimens tested, the narrower the confidence interval. Enough specimens should be tested so as to obtain sufficiently narrow confidence intervals for practical purposes. If the confidence intervals are calculated to be adequate before all the specimens have failed, the test could be aborted.

If an experiment is poorly performed, for example, if the applied voltage is not held constant in a constant stress test, the statistical confidence intervals are inaccurate. Statistical confidence intervals are valid therefore only for identically tested specimens. If the variation in testing conditions is known it may be possible to estimate confidence intervals, but this is beyond the scope of this guide.

3.3 Hypothesis testing

The estimation of the parameters (and confidence intervals) of the distribution describing an insulating specimen or system may be required for a number of reasons, including:

— Reporting the characteristics of the insulating system following a manufacturing development.
— Testing of a batch of insulating systems and comparing them to another batch for quality control or for development.

— Estimating whether early failures in the system are due to a mechanism likely to cause failure in the remaining parts of the system.

— Estimating equipment life.

— Establishing operating conditions.

Examples of some of these processes are given as case studies in this guide (Clause 11).

4. Probability distributions for failure data

A brief introduction to these distributions has been given in this clause.

4.1 The Weibull distribution

The expression for the cumulative density function for the two-parameter Weibull distribution is shown in Equation (1):

$$F(t;\alpha,\beta) = 1 - \exp\left\{-\left(\frac{t}{\alpha}\right)^{\beta}\right\} \quad (1)$$

where:

- t is the measured variable, usually time to break down or the breakdown voltage,
- $F(t)$ is the probability of failure at a voltage or time less than or equal to t. For tests with large numbers of specimens, this is approximately the proportion of specimens broken down by time or voltage, t.
- α is the scale parameter and is positive, and
- β is the shape parameter and is positive.

The probability of failure $F(t)$ is zero at $t = 0$. The probability of failure rises continuously as t increases. As the time or voltage increases, the probability of failure approaches certainty, that is, $F(\infty) = 1$.

The scale parameter α represents the time (or voltage) for which the failure probability is 0.632 (that is $1 - 1/e$ where e is the exponential constant). It is analogous to the mean of the Normal distribution (e.g., Cochran and Snedecor [B2]). The units of α are the same as t, that is, voltage, electric stress, time, number of cycles to failure etc.

The shape parameter β is a measure of the range of the failure times or voltages. The larger β is, the smaller is the range of breakdown voltages or times. It is analogous to the inverse of the standard deviation of the Normal distribution, Cochran and Snedecor [B2].

The two-parameter Weibull distribution of Equation (1) is a special case of the three-parameter Weibull distribution that has the cumulative distribution function shown in Equation (2).

$$F(t) = 1 - \exp\left\{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}\right\};t \geq \gamma \quad (2)$$

$$0;t < \gamma$$

The additional term γ is called the location parameter. $F(t) = 0$ for $t = \gamma$, that is the probability of failure for $t < \gamma$ is zero.

4.2 The Gumbel distribution

A cumulative Gumbel distribution function is given by Equation (3).

$$F_G(t) = 1 - \exp\left[-\exp\left\{\frac{t-u}{b}\right\}\right]; -\infty \le t + \infty \tag{3}$$

where:

u is the location parameter and may have any value, and

b is the scale parameter and is positive.

The Gumbel distribution is asymmetrical and can have a physically impossible finite probability of breakdown for $t < 0$. This distribution is also called the smallest extreme-value (that is, weakest link) distribution. If t is voltage, then the units of u and b are also voltage.

The Gumbel distribution is closely related to the Weibull distribution. That is, if t has a Weibull distribution then $y = \ln(t)$ has a Gumbel distribution where: $u = \ln(\alpha)$ and $b = 1/\beta$. Estimation techniques for one distribution (Gumbel or Weibull) apply to the other if this transformation is utilized.

4.3 The lognormal distribution

The lognormal distribution has sometimes been used to represent failure data from insulation systems, but it has not been used nearly as often as the extreme-value distributions in 4.1 and 4.2. However, since this probability distribution is a simple logarithmic transformation of the well-known Normal distribution, methods for data analysis are available in all standard statistical references. The probability density function of the lognormal distribution is shown in Equation (4).

$$f_{\ln}(z) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(z-\mu)^2}{2\sigma^2}\right\} \tag{4}$$

where:

$z = \log(t)$,

μ = logarithmic mean, and

σ = logarithmic standard deviation.

The cumulative density function is the integral of the above. There is no closed-form equation for the integral. Values of the distribution are in Cochran and Snedecor [B2] and Natrella [B12] or can be obtained from statistical calculators or computer programs.

4.4 Mixed distributions

It is not uncommon to find that more than one breakdown mechanism is operative in a given specimen. The probability that such a specimen survives to a given value or voltage or time t is $1 - F(t)$. If the probability of failure due to mechanism 1 is $F_1(t)$ and due to mechanism 2 is $F_2(t)$, then the probability of survival is

$$1 - F(t) = [1 - F_1(t)] \times [1 - F_2(t)] \tag{5}$$

If both may be described by the two-parameter Weibull distribution, then we have

$$F(t) = 1 - \exp\left\{-\left[\left(\frac{t}{\alpha_1}\right)^{\beta_1} + \left(\frac{t}{\alpha_2}\right)^{\beta_2}\right]\right\} \tag{6}$$

Other forms of mixed distributions are also possible. A more detailed description can be found in Fischer [B5].

4.5 Other terminology

In this guide, true values of parameters are represented by symbols (e.g., α), estimated values by "hatted" symbols (e.g., $\hat{\alpha}$), and upper and lower confidence bounds by subscripts u and l (e.g., α_u and α_l). Cumulative density functions are in upper case [e.g., *F(t)*] whereas probability density functions are in lower case [e.g., *f(t)*]. The number of specimens is designated as n with the number broken down as r (r is less than n for censored tests, $r = n$ for complete tests).

5. Testing the adequacy of a distribution

5.1 Weibull probability data

Data distributed according to the two-parameter Weibull function should form a reasonably straight line when plotted on Weibull probability paper. A sample probability paper is shown in Figure A.2 (the data plotted on this paper is referred to in Clause 6). The measured data is plotted on the horizontal axis, which is scaled logarithmically. The probability of breakdown is plotted on the vertical axis, which is also highly non-linear.[7]

5.1.1 Estimating plotting positions for complete data

To use this probability paper, place the n breakdown times or voltages in order from smallest to largest and assign them a rank from $i = 1$ to $i = n$. An example of this from progressive stress testing of latex film is shown in Table A.1.

A good, simple, approximation for the most likely probability of failure is found in Ross [B14]:

$$F(i,n) \approx \frac{i - 0.44}{n + 0.25} \times 100\% \tag{7}$$

The Weibull example data in Table A.1 are plotted in Figure A.3. In this case, there were ten specimens (n = 10) and all of them broke down so the data is "complete." The data follows a reasonably straight line and it is therefore reasonable to assume that they are distributed according to the Weibull function. (The line representing the Weibull relationship was plotted using the procedure in Clause 7.)

Some random deviations from a straight line may be expected. If, however, there is a consistent departure from a straight line (for example curvature or a cusp) then another distribution may fit the data better (see 5.3). Probability papers for the Gumbel and lognormal distributions are also available. The probability of failure for these graphs is estimated in exactly the same way.

[7]The plotting position on the horizontal axis, X_i, of the i^{th} data point, x_i, is such that X_i α $\log x_i$. The plotting position on the vertical axis, Y_i, of the probability of failure corresponding to the i^{th} data point, $F(x_i)$, is such that $Y_i = \log\{-\ln[1 - F(x_i)]\}$.

5.1.2 Estimating plotting positions for singly censored data

Table A.2 presents an example of singly censored data from constant stress tests on epoxy resin specimens; these are plotted in Figure A.4. (Again, the line representing the Weibull relationship was plotted using the procedure in Clause 7.) The test was stopped at 144.9 hours and so only seven of the nine specimens broke down; the final 2 had still not broken down and so they were "suspended." Since all the previous specimens had broken down the data set is "singly censored". In such tests, r is the number of specimens that broke down and $r < n$. Place the r breakdown times or voltages in order from smallest to largest and assign them a rank from $i = 1$ to $i = r$. The same formula as for complete data [Equation (7)] should be used for calculating the probability of breakdown.

5.1.3 Estimating plotting positions for progressively censored data

Table A.3 presents an example of progressively censored data in which 7 of the 17 specimens were suspended. For progressively censored data, a modified procedure is required for assigning cumulative probabilities of failure, $F(i,n)$. The rank $i = [1,...,r]$ is substituted for a rank function $I(i)$ given by Equation (8):

$$I(i) = I(i-1) + \frac{n+1-I(i-1)}{n+2-C_i} \qquad (8)$$

for $i > 0$ and $I(0) = 0$. C_i is the total number of broken-down and suspended specimens when the i^{th} breakdown occurs. This expression can then be inserted into a modified form of Equation (7), which is shown in Equation (9):

$$F(i,n) \approx \frac{I(i) - 0.44}{n + 0.25} \times 100\% \qquad (9)$$

This data is shown as a Weibull plot in Figure A.5. [The data points do not form a very straight line and it is possible that they are distributed according to a mixed Weibull distribution; see 4.4, Equation (6).]

5.2 Use of probability paper for the three-parameter Weibull distribution

Table A.4 presents data that, plotted on Weibull paper (Figure A.6), appears to show a downward curvature of the lower percentiles. If this data actually corresponds to a three-parameter Weibull distribution with a location parameter γ, then it should be possible to subtract γ from each data point and re-plot this as a straight line on a Weibull plot. If this is the case, then γ may be found using a trial-and error method (an analytical technique for finding γ is given in Dissado and Fothergill l [B3], page 446). The breakdown mechanisms operative in this case were caused by electrical trees, which take a finite time to grow through the specimen. It can be seen from Figure A.6 that the plot of the original data bends down at approximately 230 hours (the curve is convex looking from the top) indicating that the probability of breakdown before this time tends to zero. This corresponds to a minimum time for a tree to cross the specimen. With this information it is reasonable to hypothesize that the distribution of times to breakdown may be represented by a three-parameter Weibull distribution with location parameter $\gamma = 230$ hours. If this value were accurate then subtracting 230 hours from the original data would result in a new set of data distributed according to the two-parameter Weibull distribution giving a straight line on the Weibull plot. This is shown in column (c) in Table A.4 and as "1st iteration" in Figure A.6. It can be seen that this plot is not quite straight but that it curves up slightly at low values (the curve is concave); this indicates that the first estimate of γ is slightly too large. A successive iteration process may be adopted until an optimum estimate $\hat{\gamma}$ results in a reasonably straight Weibull plot. This is shown in column (d) in Table A.4 in which a second guess of $\gamma = 200$ hours is tried. It can be seen from Figure A.6 that this gives a reasonably straight line; this suggests that the data may follow a three-parameter Weibull distribution with $\gamma = 200$ hours. If, after such an iteration process, a straight line cannot be obtained, then it is reasonable to assume that the data may not be described by a three-parameter Weibull distribution.

5.3 The shape of a distribution plotted on Weibull probability paper

Data that does not result in a reasonably straight line on Weibull probability paper may not be distributed according to the two-parameter Weibull distribution. Figure A.7 shows data distributed according to other functions plotted on Weibull probability paper. The three-parameter Weibull distribution (with a positive value of location parameter, $\gamma > 0$) results in a characteristic convex knee (i.e., turning down at low value) described in 5.2. The Normal and Gumbel distributions both result in concave curves but are difficult to distinguish. Mixed distributions, of the type described by Equation (6), result in two straight lines but these are not always easily distinguishable.

5.4 A simple technique for testing the adequacy of the Weibull distribution

This technique is adapted from Abernethy [B1]. Various techniques exist for checking the adequacy of a two-parameter Weibull distribution. In many cases a check *by eye* using a Weibull plot is sufficient. An alternative technique is to find the correlation coefficient and to check that this is greater than the critical value given in Figure A.8 for the number of specimens broken down (r). The correlation coefficient is found using the method of least squares regression (Annex A), a statistical function that is normally available on commercial spreadsheet programs.

To check for goodness of fit of a set of breakdown times or voltages, place them in order from smallest to largest and assign them a breakdown probability as described in 5.1. For each breakdown time or voltage, t_i, assign a value:

$$Y_i = \ln(t_i) \tag{10}$$

where
$\ln\{t_i\}$ is the natural logarithm or $\log_e\{t_i\}$

For each probability of failure, $F(i,n)$, expressed as a percentage, assign a value:

$$X_i = \ln\left(-\ln\left(1 - \frac{F(i,n)}{100}\right)\right) \tag{11}$$

Use a least squares regression technique[8] to find the correlation coefficient. Use Figure A.8 to establish whether the data points are a good fit to a two-parameter Weibull distribution.

Data for time-to-breakdown for an insulating fluid has been reported by Nelson [B13]. The data was singly censored with 10 of the 12 specimens breaking down (i.e., $r = 10$, $n = 12$). This data set was entered into a spreadsheet, Figure A.9, and the probabilities of breakdown calculated using Equation (7). Values of X and Y were calculated using Equation (11) and Equation (10). The correlation coefficient was calculated using the spreadsheet's built-in function "CORREL." The spreadsheet formulae are shown in Figure A.10. The correlation coefficient is found to be 0.970. From Figure A.8, it is found that the critical correlation coefficient for $r = 10$ is 0.918, which is <0.970. The data is, therefore, a good fit to the two-parameter Weibull distribution.

The example presented here maybe downloaded as a Microsoft® Excel® 97[9] spreadsheet from http://grouper.ieee.org/groups/930/IEEEGuide.xls as example 1 and may be adapted for use as required.

[8]Whilst the failure times or voltages are plotted on the horizontal axis, these have been associated with the Y variable and the failure probability with the X variable. This follows the suggestion of Abernethy [B1] that the failure variable should be regressed against the probability variable and not the other way around. Although this makes no difference when calculating the correlation coefficient, it is important if this technique is used for calculating the Weibull parameters (see 7.1).

[9]Microsoft and Excel are registered trademarks of Microsoft Corporation in the United States and/or other countries.

6. Graphical estimates of Weibull parameters

The principal uses of Weibull probability graph paper are to test the adequacy of the Weibull distribution in describing a data set and to present breakdown data in publications etc. It is possible to use such "Weibull plots" to estimate the parameters α and β. Such estimates are the least accurate means to estimate α and β since different data points should be weighted differently and this is not possible to do *by eye*. The technique may be useful where only rough estimates are required or where there are a large number of data points falling on a good straight line with very limited censoring. It should be noted that plotting the data on a Weibull plot is nevertheless recommended so that the adequacy of the distribution may be assessed.

In order to obtain graphical estimates of the parameters, plot the test data on Weibull probability paper as described in Clause 5. Fit a straight line *by eye* to the data points. The estimate for the scale parameter α, denoted by $\hat{\alpha}$, is the time or voltage corresponding to $F(t) = 63.2\%$. Most commercially available Weibull probability graph papers indicate this probability value by a dashed line so as to facilitate estimation of α. The value of the shape parameter β is estimated from the slope of the line; indeed on normalized axes it is equal to the slope of the line. Commercially available Weibull probability graph papers usually have a technique for estimating β directly.[10] If it is not possible to use this technique then an estimate for β may be found as follows. From the Weibull plot, estimate the times or voltages corresponding to $F(t) = 10\%$ and $F(t) = 90\%$ denoted $\hat{t}_{10}$ and $\hat{t}_{90}$ respectively. An estimate for β is then given by Equation (12).

$$\hat{\beta} = \frac{1.340}{\log_{10}(\hat{t}_{90}/\hat{t}_{10})} \qquad (12)$$

An example of the graphical estimation of Weibull parameters is shown for the data given in Table A.5, which is plotted in Figure A.2. From the Weibull plot, graphical estimates of $\hat{\alpha} = 70$ kV/mm and $\hat{\beta} = 3.8$ may be made. (More accurate estimates using the computational technique described in Clause 7 are $\hat{\alpha} = 70.7$ kV/mm and $\hat{\beta} = 3.62$.)

7. Computational techniques for Weibull parameter estimation

Various computational techniques are available for estimating the Weibull parameters. The 1987 version of this guide recommended the use of the maximum likelihood technique but this has been found to give biased estimates of the parameters, especially for small data sets. The technique recommended here was developed by White [B16] and has been found to be the optimum technique for complete, singly censored and progressively censored data, Montanari *et al* [B9], [B10], [B11]. However, for large data sets least-squares linear regression and maximum likelihood techniques are adequate.

7.1 Larger data sets

For larger data sets, typically with more than 20 breakdowns, the following least-squares regression technique may be used. Place the breakdown times or voltages in order from smallest to largest and assign them a breakdown probability as described in subclause 5.1. Assign values of Y_i based on the each data point, t_i, using Equation (10). Similarly assign values of X_i based on each probability, $F(i,n)$, using Equation (11). Use a least-squares regression technique to find the intercept, $\hat{c}$, and slope, $\hat{m}$. Least squares regression is available on most spreadsheet programs and is also described in Annex A. The estimates of the location parameter, α, and shape parameter, β, are given by Equation (13) and Equation (14).

$$\hat{\alpha} = \exp(c) \qquad (13)$$

[10]On the Weibull paper shown in Figure A.2 a line is constructed through the "estimation point" and at right angles to the Weibull plot. The value of estimated value of shape parameter can be read from where this construction crosses the $\hat{\beta}$ scale.

$$\hat{\beta} = \frac{1}{m} \tag{14}$$

An example of the analysis of a complete data set containing 24 values is shown in the spreadsheet output in Figure A.11(a). The values of $F(i,n)$, X_i, Y_i, and the correlation coefficient are calculated as in the example in 5.4 shown in Figure A.9 and Figure A.10. The values of intercept, $\hat{c}$, slope, $\hat{m}$, are calculated using the spreadsheet's built-in functions "INTERCEPT" and "SLOPE" as shown in Figure A.11(b). Also shown are the calculations for the estimates, $\hat{\alpha}$ and $\hat{\beta}$.

The example presented here may be downloaded as a Microsoft Excel 97 spreadsheet from http://grouper.ieee.org/groups/930/IEEEGuide.xls as "example 2" and may be adapted for use as required

7.2 Smaller data sets

Very small data sets, typically with less than 5 breakdowns, can give rise to erroneous parameter estimates and the best approach, wherever possible, is to obtain more data. Only if more data cannot be obtained should such an analysis, using the White method [B16]be carried out on very small data sets.

For small data sets, typically with less than 15–20 breakdowns, it can be inaccurate to use the standard least-squares regression technique since different points plotted on the Weibull plot need to be allocated different weightings. Place the breakdown times or voltages in order from smallest to largest and assign them a breakdown probability as described in 5.1. Assign values of Y_i based on the each data point, t_i, using Equation (10). Similarly assign values of X_i based on each probability, $F(i,n)$, using Equation (11). Look up the weightings for each data point, w_i, given in Table A.6. This table is also provided in the spreadsheet available for download from "http://grouper.ieee.org/groups/930/IEEEGuide.xls."

If the data is progressively censored then find values of $I(i)$ using Equation (8). Since these are not necessarily integers, use values of i closest to $I(i)$ when assigning weights w_i.

Calculate the weighted averages of X_i and Y_i as shown in Equation (15) and Equation (16).

$$\bar{X} = \frac{\sum_{i=1}^{r} [w_i X_i]}{\sum_{i=1}^{r} [w_i]} \tag{15}$$

$$\bar{Y} = \frac{\sum_{i=1}^{r} [w_i Y_i]}{\sum_{i=1}^{r} [w_i]} \tag{16}$$

Use Equation (17) and Equation (18) to estimate the shape parameter, β, and scale parameter, α.

$$\hat{\beta} = \frac{\sum_{i=1}^{r} [w_i (X_i - \bar{X})^2]}{\sum_{i=1}^{r} w_i (X_i - \bar{X})(Y_i - \bar{Y})} \tag{17}$$

$$\hat{\alpha} = \exp\left\{\bar{Y} - \frac{\bar{X}}{\hat{\beta}}\right\} \tag{18}$$

An example of data from a singly-censored progressive-stress test on miniature XLPE cables is shown as a spreadsheet calculation in Figure A.12. In this case the data is singly censored with seven of the ten specimens having broken down. The weighting factors are taken from the first seven rows of the column headed "n = 10" in Table A.6. These weighting factors, w_i, form the column in the spreadsheet next to the Y column. This column is summed (to give 23.868) and used for calculating the denominator of Equation (15) and Equation (16). The next two columns headed "wX" and "wY" are used to calculate the numerators of Equation (15) and Equation (16) (–14.149 and 74.634 respectively). Hence $\bar{X}$ and $\bar{Y}$ are calculated (as –0.593 and 3.127 respectively). The final two columns in the spreadsheet table are used to calculate the numerator and denominator for estimating β, Equation (17) and are found to be 9.677 and 1.236. Thus, β is estimated to be 7.83 and the estimate for α is 24.6 kV/mm. The spreadsheet formulae are shown in Figure A.13.

The example presented here maybe downloaded as a Microsoft Excel 97 spreadsheet from http://grouper.ieee.org/groups/930/IEEEGuide.xls as "example 3"and may be adapted for use as required.

8. Estimation of Weibull percentiles

It is often useful to estimate the time, voltage or stress for which there is a given probability of failure p%; this is known as the p^{th} percentile. In constant stress life tests, these percentiles are sometimes referred to as "B lives." For example, the "B10 life" is the age at which 10% of the components will fail at a given voltage stress. The p^{th} percentile, t_p, may be estimated by using Equation (19):

$$\hat{t}_p = \hat{\alpha}\left[-\ln\left(1 - \frac{p}{100}\right)\right]^{1/\hat{\beta}} \tag{19}$$

where p is expressed as a percentage.

For example the 0.1, 1.0, 10, and 99 percentiles for the example given in 7.2 are 10.2, 13.7, 18.5, and 29.9 kV/mm, respectively.

9. Estimation of confidence intervals for the Weibull function

If the same experiment involving the testing of many specimens is performed a number of times, the values of the parameter and percentile estimates, $\hat{\alpha}$, $\hat{\beta}$, and $\hat{t}_p$ from each experiment differ. This variation in estimates results from the statistical nature of insulation breakdown, e.g., Dissado and Fothergill [B3]. Therefore, any parameter estimate differs from the *true* parameter value that is obtained from an experiment involving an infinitely large number of specimens. Hence, it is common to give with each parameter estimate a *confidence interval* that encloses the true parameter value with high probability. In general, the more specimens tested, the narrower the confidence interval.

There are various methods of estimating confidence intervals for Weibull parameters, e.g., Lawless [B8] and Nelson [B13]. Many computer programs are available (see for example Abernethy [B1]) although some of these may not be accurate if used with small sample sizes. The exact values of the statistical confidence intervals depend on the method used to estimate the parameters. Many of the methods relate to the maximum likelihood estimation technique or least-squares regression in which the probability of failure has been regressed on to the breakdown variable (time, voltage, etc.). These methods are not appropriate to the estimation techniques described in this guide and may give inaccurate results.

This guide provides a simplified procedure for estimating the bilateral 90% confidence intervals[11] for sample sizes from $n = 4$ to $n = 100$. The technique is applicable to complete and singly-censored data; it is not applicable to progressively-censored data. The technique may be used with up to 50% of the specimens being censored. Since it would be unwieldy to cater for all sample sizes between 4 and 100, a reasonable selection has been included. For sample sizes in this range that are not included, interpolation can be used. Confidence interval tables have been calculated and are included in the spreadsheet available at "http://grouper.ieee.org/groups/930/IEEEGuide.xls". For simplicity in this guide, these tables are represented as curves in Figure A.14 to Figure A.29. The straight lines connecting points on these curves are merely aids to the eye and should not necessarily be taken as appropriate interpolations between the points plotted. These curves have been calculated using a Monte-Carlo method and are estimated to be accurate to within 1% for $4 < n < 20$ and within 4% for $20 < n < 100$. The curves have been especially calculated for this guide. They assume that

a) The data adequately fits the two-parameter Weibull distribution using the simple test described in 5.4.

b) Least squares regression (described in 7.1) has been used for larger data sets with $n \geq 20$ and the White method (described in 7.2) has been used for smaller data sets with $n \leq 20$.

9.1 Graphical procedure for complete and censored data

9.1.1 Confidence intervals for the shape parameter, β

Figures A.14 and A.15 are used to obtain the factors W_l and W_u for the bounds of the 90% confidence intervals for the shape parameter β:

$$\hat{\beta}_l = W_l\hat{\beta} \tag{20}$$

$$\hat{\beta}_u = W_u\hat{\beta}$$

where $\hat{\beta}_l$ and $\hat{\beta}_u$ are the lower and upper limits, respectively for the interval. For the Weibull data in Figure A.12, $n = 10$ and $r = 7$ and so from Figures A.14 and A.15 it can be seen that $W_l = 0.59$and $W_u = 2.10$. Since $\hat{\beta} = 7.83$ (7.2, Figure A.12) the confidence limits for $\hat{\beta}$ are as follows:

$$\hat{\beta}_l = 0.59 \times 7.83 = 4.62 \text{ and } \hat{\beta}_u = 2.10 \times 7.83 = 16.4$$

Thus there is a 90% probability that the true value of β lies between 4.61 and 16.4.

9.1.2 Confidence intervals for the location parameter, α

Figure A.16 and Figue A.17 are used to obtain the factors Z_l and Z_u for the bounds of the 90% confidence interval for the shape parameter α:

$$\hat{\alpha}_l = \hat{\alpha}\exp\{Z_l/\hat{\beta}\} \tag{21}$$

$$\hat{\alpha}_u = \hat{\alpha}\exp\{Z_u/\hat{\beta}\}$$

[11] Bilateral 90% confidence intervals exclude the highest 5% and lowest 5% of the distribution of the variable estimated from many different sets of breakdown data.

where $\hat{\alpha}_l$ and $\hat{\alpha}_u$ are the lower and upper limits, respectively, for the interval. For the Weibull data in Figure A.12, $n = 10$ and $r = 7$. From Figure A.16 and A.17 it can be seen that $Z_l = -0.710$ and $Z_u = 0.645$. Since $\hat{\alpha} = 24.6$ kV/mm and $\hat{\beta} = 7.83$ (7.2, Figure A.12) the confidence limits for α are:

$$\hat{\alpha}_l = 24.6 \times \exp\{-0.710 / 7.83\} = 22.5 \text{ kV/mm} \text{ and } \hat{\alpha}_u = 24.6 \times \exp\{0.645 / 7.83\} = 26.7 \text{ kV/mm}$$

Thus, there is a 90% probability that the true value of α lies between 22.5 kV/mm and 26.7 kV/mm.

9.1.3 Confidence intervals for the Weibull percentiles

Figure A.18 through Figure A.29 are used to obtain the factors $Z_l(p)$ and $Z_u(p)$ for the bounds of the 90% confidence limits for the percentiles $p = 0.1\%$, 1.0%, 5.0%, 10%, 30%, and 95%. The figure numbers and the values of the parameters obtained for the Weibull data in Figure A.12 ($n = 10$, $r = 7$) are shown in Table A.8.

The expressions given in Equation (22) are used to obtain the bounds of the 90% confidence intervals for the p^{th} percentile:

$$\hat{t}_l = \hat{\alpha} \exp\{Z_l(p) / \hat{\beta}\} \qquad (22)$$

$$\hat{t}_u = \hat{\alpha} \exp\{Z_u(p) / \hat{\beta}\}$$

where $\hat{t}_l(p)$ and $\hat{t}_u(p)$ are the lower and upper bounds of the confidence interval for the p^{th} percentile. Using the Weibull data shown in Figure A.12, ($\hat{\alpha} = 24.6$ kV/mm, $\hat{\beta} = 7.83$, and $n = 10$, $r = 7$) the bounds of the percentiles have been calculated using Equation (22) and are shown in Table A.8. Also included in the table, for the sake of completeness, are the bounds for α as this is simply a special case of the 63.21 percentile.

For example, for the 1.0 percentile, it is found that $Z_l(1.0) = -7.70$ and $Z_u(1.0) = -2.25$. Using Equation (22) leads to $\hat{t}_l(1.0) = 9.20$ kV/mm and $\hat{t}_u(1.0) = 18.4$ kV/mm.

9.2 Plotting confidence limits

The confidence limits for the percentiles, together with the confidence interval for α (the 63.2 percentile) can be usefully displayed on Weibull probability paper. For the upper limit plot, the calculated limits *(t* values) corresponding to the 0.1, 1.0, 5.0, 10, 30, 63.2 (α), and 95 percentiles on the graph paper. Join these four points with a smooth line. Similarly, draw a line through the plotted lower confidence limits. These confidence limits are shown together with the estimated "best line" in Figure A.30. Such confidence limits enclose any particular percentile of the true population with 90% probability. The greater the number of specimens tested, the closer the upper and lower curves.

10. Estimation of the parameter and their confidence limits of the log-normal function

10.1 Estimation of lognormal parameters

Exact estimates for the lognormal parameters are available if there is no censoring, that is, $r = n$. These estimates are obtained by taking the logarithms of the failure voltages or times and using the transformed data to find the logarithmic mean, μ, and logarithmic standard deviation, σ. These may be calculated using the following well-known formula of the normal distribution [Equation (23)]:

$$\hat{\mu} = \frac{\sum z}{n} \tag{23}$$

$$\hat{\sigma} = \sqrt{\frac{\sum (z-\mu)^2}{(n-1)}}$$

[carrying forward the notation used in Equation (4)]. These statistical functions are also available on many calculators. For small samples see Dixon and Massey [B4].

10.2 Estimation of confidence intervals of log-normal parameters

The confidence intervals for the log mean and log standard deviation are easily found using the Student's *t* and χ^2 distributions, if there is no censoring. The 90% confidence interval for μ has the lower and upper limits:

$$\mu_l = \hat{\mu} - \frac{t_{0.05}\sigma}{\sqrt{n}} \tag{24}$$

$$\mu_u = \hat{\mu} + \frac{t_{0.05}\sigma}{\sqrt{n}}$$

where $t_{0.05}$ is obtained from a Student's *t* table, with $n - 1$ degrees of freedom. Such a table is found in many standard statistics textbooks, e.g., Cochran and Snedecor [B2]. The lower and upper limits for the 90% confidence interval for the log standard deviation are

$$\sigma_l = \sqrt{\frac{(n-1)\sigma^2}{\chi^2_{0.05}}} \tag{25}$$

$$\sigma_u = \sqrt{\frac{(n-1)\sigma^2}{\chi^2_{0.95}}}$$

where χ^2 is from the χ^2 tables, with $n - 1$ degrees of freedom. Such tables are in many standard statistics textbooks e.g., Cochran and Snedecor [B2]. Confidence intervals for the parameters when only censored data are available can be estimated from the methods described by Lawless and Stone [B7] and Schmee, et al [B15].

11. Comparison tests

A common situation involves testing two or more insulation types or groups of specimens to determine which of the two is *better-quality*. It is easiest to compare test data from two types of insulation by plotting data sets on probability paper. However, visual comparison of two plots is subjective. To analyze results from such a test involves testing the hypothesis to verify that there is no difference between the probability distributions of the data for the two types of insulation. Fulton and Abernethy [B6] have suggested a technique in which the likelihood contour plots of the two distributions are examined to see whether or not they overlap; however, this is reasonably complex to implement. Weibull suggested that a useful *hypothesis test* is to examine whether there is overlap in the confidence limits at a given percentile and he suggested the 10^{th} percentile for this purpose. This is a simple technique and is advocated here.

11.1 Simplified method to compare percentiles of Weibull distributions

If two data sets differ convincingly the following method is used to give an approximate assessment. Determine if the confidence intervals for a chosen percentile of the two distributions overlap. If there is no overlap, for example at the 10th percentile, then the two 10th percentiles differ significantly at the selected confidence level. This comparison does not assume that the two shape parameters are equal. The confidence intervals for the percentiles are calculated as described in 9.1.3.

It is always useful to compare sets of test data on Weibull probability paper. Plot the data from the two (or more) tests on the same graph paper. As described in 9.2, plot the 90% confidence bounds for percentiles for each data set. The test data in Lawless and Stone [B7]and Table A.7 are plotted in Figure A.31 with the 90% confidence intervals. Using least-squares regression (7.1) and the procedures in 9.1 the Weibull parameters and 90% confidence intervals may be estimated. For the unscreened cables it is found that $\hat{\alpha} = 48.3$ kV/mm (with a 90% confidence interval from 46.2 to 50.2 kV/mm) and $\hat{\beta} = 9.40$ (6.80 to 13.7). For the screened cables $\hat{\alpha} = 59.2$ kV/mm (56.6 to 61.7 kV/mm) and $\hat{\beta} = 8.92$ (6.45 to 13.0). For percentiles above approximately 10%, the two intervals do not overlap. Therefore, high percentiles of the two insulation systems differ significantly. Note that for low percentiles, the intervals overlap, and those percentiles probably do not differ significantly. In principle, more specimens need to be tested to show if the two distributions are significantly different.

12. Estimating Weibull parameters for a system using data from specimens

It is sometimes required to estimate the Weibull parameters for an insulation system based on results from tests on specimens of the same thickness. For example it may be required to evaluate the Weibull parameters of a 100 km length of cable based on tests of cable specimens that are only 10 m long. In this example the area of insulation in the complete system is 10^4 times greater than that of the specimen. If it is reasonable to assume that the causes of breakdown are similar in both cases, then it is possible to estimate the Weibull parameters for the complete system, $\hat{\alpha}_c$ and $\hat{\beta}_c$ using the parameters from the test specimens, $\hat{\alpha}_t$ and $\hat{\beta}_t$:

$$\hat{\alpha}_c = \frac{\hat{\alpha}_t}{D^{1/\hat{\beta}}} \text{ and } \hat{\beta}_c = \hat{\beta}_t \qquad (26)$$

where D is the ratio (complete system area)/(test specimen area).

For example if data from breakdown tests on 10 m cable specimens indicated that $\hat{\alpha}_t = 60$ kV and $\hat{\beta}_t = 11$ then the values to be expected for the complete 100 km cable are:

$$D = \frac{100 \text{ km}}{10 \text{ m}} = 10000$$

$$\hat{\alpha}_c = \frac{60 \text{ kV}}{10000^{1/11}} = 26 \text{ kV}$$

$$\hat{\beta}_c = \hat{\beta}_t = 11$$

Annex A

(informative)

Least squares regression

The least squares regression technique uses the following equations to find β and α:

$$\beta = \frac{\sum_{i=1}^{r} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{r} (X_i - \bar{X})^2} \tag{27}$$

$$\alpha = \exp\left(\bar{X} - \frac{\bar{Y}}{\beta}\right) \tag{28}$$

where X_i and Y_i are given by Equation (10) and Equation (11) and

$$\bar{X} = \frac{\sum_{i=1}^{r} X_i}{r} \tag{29}$$

$$\bar{Y} = \frac{\sum_{i=1}^{r} Y_i}{r}$$

Table A.1—Progressive stress test results from latex film

Rank, *i*	*F(i,n)* [Equation (7)]	Breakdown voltage (kV)
1	$\frac{1-0.44}{10+0.25} = 0.055 = 5.5\%$	4.71
2	$\frac{2-0.44}{10+0.25} = 15.2\%$	5.39
3	$\frac{3-0.44}{10+0.25} = 25.0\%$	5.4
4	$\frac{4-0.44}{10+0.25} = 34.7\%$	5.53
5	$\frac{5-0.44}{10+0.25} = 44.5\%$	5.71
6	$\frac{6-0.44}{10+0.25} = 54.2\%$	5.78
7	$\frac{7-0.44}{10+0.25} = 64.0\%$	6.14

Table A.1—Progressive stress test results from latex film *(continued)*

Rank, *i*	*F(i,n)* [Equation (7)]	Breakdown voltage (kV)
8	$\frac{8-0.44}{10+0.25} = 73.8\%$	6.43
9	$\frac{9-0.44}{10+0.25} = 83.5\%$	6.44
10	$\frac{10-0.44}{10+0.25} = 93.3\%$	6.48

Table A.2—Singly censored data taken from constant stress tests on epoxy specimens

Rank ("S" = suspended)[a]	*F(i,n)* [Equation (7)]	Breakdown time (h)
1	$\frac{1-0.44}{9+0.25} = 0.061 = 6.1\%$	15.3
2	$\frac{2-0.44}{9+0.25} = 16.9\%$	30.3
3	$\frac{3-0.44}{9+0.25} = 27.7\%$	48.5
4	$\frac{4-0.44}{9+0.25} = 38.5\%$	89.4
5	$\frac{5-0.44}{9+0.25} = 49.3\%$	90.4
6	$\frac{6-0.44}{9+0.25} = 60.1\%$	105.7
7	$\frac{7-0.44}{9+0.25} = 70.9\%$	144.9
S	Suspended	>144.9
S	Suspended	>144.9

[a]Specimens marked "S" were removed at the times shown; other specimens broke down at the times shown.

Table A.3—Progressively censored data from PET films

Rank, i	C_i [Equation (8)]	Calculation for $I(i)$ [Equation (8)]	$F(i, n)$ [Equation (9)]	Time, h
S[a]	1	Suspended ($I(0) = 0$)		55.47
1	2	$I(1) = 0 + \frac{17+1-0}{17+2-2} = 1.059$	$\frac{1.059-0.44}{17+0.25} = 3.6\%$	57.93
2	3	$I(2) = 1.059 + \frac{17+1-1.059}{17+2-3} = 2.118$	$\frac{2.118-0.44}{17+0.25} = 9.7\%$	59.10
3	4	$I(3) = 2.118 + \frac{17+1-2.118}{17+2-4} = 3.176$	$\frac{3.176-0.44}{17+0.25} = 15.9\%$	62.57
S	5	Suspended		66.84
4	6	$I(4) = 3.176 + \frac{17+1-3.176}{17+2-6} = 4.317$	$\frac{4.317-0.44}{17+0.25} = 22.5\%$	69.54
5	7	$I(5) = 4.317 + \frac{17+1-4.317}{17+2-7} = 5.457$	$\frac{5.457-0.44}{17+0.25} = 29.1\%$	74.74
S	8	Suspended		86.94
S	9	Suspended		87.81
S	10	Suspended		89.31
6	11	$I(6) = 5.457 + \frac{17+1-5.457}{17+2-11} = 7.025$	$\frac{7.025-0.44}{17+0.25} = 38.2\%$	97.84
7	12	$I(7) = 7.025 + \frac{17+1-7.025}{17+2-12} = 8.593$	$\frac{8.593-0.44}{17+0.25} = 47.3\%$	111.71
8	13	$I(8) = 8.593 + \frac{17+1-8.593}{17+2-12} = 10.161$	$\frac{10.161-0.44}{17+0.25} = 56.4\%$	115.38
S	14	Suspended		115.92
9	15	$I(9) = 10.161 + \frac{17+1-10.161}{17+2-15} = 12.120$	$\frac{12.120-0.44}{17+0.25} = 67.7\%$	116.20
10	16	$I(10) = 10.161 + \frac{17+1-10.161}{17+2-16} = 14.080$	$\frac{14.080-0.44}{17+0.25} = 79.1\%$	117.82
S	17	Suspended		141.33

[a]Specimens marked “S” were removed at the times shown; other specimens broke down at the times shown.

Table A.4—Life test showing a three-parameter distribution, 9 of the 10 specimens broke down

Column (a) *F(i, n)* [Equation (7)]	Column (b) Original data *t* (h)	Column (c) First iteration *t* – 230	Column (d) Second iteration *t* – 200
5.5%	240	10	40
15.2%	300	70	100
25.0%	340	110	140
34.7%	390	160	190
44.5%	490	260	290
54.2%	530	300	330
64.0%	590	360	390
73.8%	750	520	550
83.5%	900	670	700
S	>900	—	—

Table A.5—Progressive stress test on polyethylene with artificially thickened lamellae (complete data)

Rank	*F(i, n)* [Equation (7)]	Breakdown field (kV/mm)
1	6.8%	29.7
2	18.9%	46.8
3	31.0%	57.5
4	43.2%	60.3
5	55.3%	64.3
6	67.4%	74.8
7	79.5%	79.7
8	91.6%	90.3

Table A.6—Weighting factors for *n* = [1...20], *r* = [1...*n*]

	n = 1	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9	*n* = 10
***i* = 1**	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927
***i* = 2**		1.483618	1.518551	1.534207	1.540658	1.543926	1.540550	1.546985	1.547534	1.548327
***i* = 3**			2.229664	2.406559	2.463158	2.488459	2.501969	2.510036	2.515240	2.518794
***i* = 4**				2.906772	3.241517	3.360034	3.416318	3.447632	3.466897	3.479613
***i* = 5**					3.510441	4.023407	4.219219	4.316695	4.372751	4.408100
***i* = 6**						4.055446	4.757141	5.041186	5.188062	5.274863
***i* = 7**							4.552888	5.448119	5.828097	6.030850
***i* = 8**								5.011059	6.101251	6.582520
***i* = 9**									5.436251	6.720778
***i* = 10**										5.833329

	n = 11	*n* = 12	*n* = 13	*n* = 14	*n* = 15	*n* = 16	*n* = 17	*n* = 18	*n* = 19	*n* = 20
***i* = 1**	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927	0.607927
***i* = 2**	1.548729	1.549031	1.549264	1.549447	1.549593	1.549712	1.550050	1.549892	1.549961	1.550019
***i* = 3**	2.521329	2.523200	2.524622	2.525726	2.526603	2.527308	2.527886	2.528364	2.528765	2.529104
***i* = 4**	3.488453	3.494851	3.499632	3.503301	3.506177	3.508474	3.510337	3.511871	3.513148	3.514222
***i* = 5**	4.431884	4.448676	4.460986	4.470281	4.477478	4.483165	4.487736	4.491467	4.494433	4.497135
***i* = 6**	5.330755	5.368983	5.396342	5.416616	5.432075	5.444138	5.453737	5.461502	5.467872	5.473166
***i* = 7**	6.153475	6.233854	6.289629	6.330015	6.360243	6.383483	6.401749	6.416377	6.428276	6.438089
***i* = 8**	6.849007	7.008927	7.117245	7.193354	7.249040	7.291090	7.323668	7.349439	7.370196	7.387167
***i* = 9**	7.307014	7.635680	7.841956	7.981184	8.080116	8.153175	8.208792	8.252173	8.286700	8.314660
***i* = 10**	7.310326	8.003976	8.400672	8.653461	8.826172	8.950124	9.042442	9.113228	9.168785	9.213253
***i* = 11**	6.206138	7.872974	8.675570	9.142698	9.444413	9.652808	9.803729	9.917005	10.004422	10.073435
***i* = 12**		6.557768	8.411349	9.323757	9.863149	10.215799	10.461772	10.641377	10.777116	10.882518
***i* = 13**			6.890725	8.927711	9.950288	10.563343	10.968556	11.253721	11.463525	11.62311
***i* = 14**				7.207098	9.424023	10.556741	11.244483	11.70361	12.029366	12.270691
***i* = 15**					7.508618	9.901970	11.144520	11.907696	12.421836	12.789359
***i* = 16**						7.796760	10.363038	11.714883	12.553998	13.124031
***i* = 17**							8.072778	10.808532	12.268990	13.184350
***i* = 18**								8.337752	11.239605	12.807836
***i* = 19**									8.592616	11.657281
***i* = 20**										8.838201

Table A.7—Breakdown stresses (kV/mm): polyethylene for two manufacturing processes

Failure number	Process 1 (unscreened)	Process 2 (screened)
1	35	39
2	35	45
3	36	49
4	40	49
5	43	53
6	43	53
7	43	53
8	46	53
9	46	55
10	48	55
11	48	57
12	48	57
13	48	57
14	48	57
15	48	61
16	51	64
17	51	64
18	51	65
19	51	67
20	57	68

Table A.8—Calculation of bounds of 90% confidence intervals for Weibull percentiles

Percentile	Figure numbers		Values obtained for Weibull data from Figure A.12			
P	$Z_l(p)$	$Z_u(p)$	$Z_l(p)$	$Z_u(p)$	$\hat{t}_l(p)$ (kV/mm)	$\hat{t}_u(p)$ (kV/mm)
0.1%	Figure A.18	Figure A.19	–11.6	–3.40	5.60	15.9
1%	Figure A.20	Figure A.21	–7.70	–2.25	9.20	18.4
5%	Figure A.22	Figure A.23	–5.00	–1.45	13.0	20.5
10%	Figure A.24	Figure A.25	–3.8	–1.05	15.2	21.5
30%	Figure A.26	Figure A.27	–1.90	–0.295	19.3	23.7
63.21% (α)	Figure A.16	Figure A.17	–0.710	0.645	22.4	26.7
95%	Figure A.28	Figure A.29	0.085	2.25	24.9	32.7

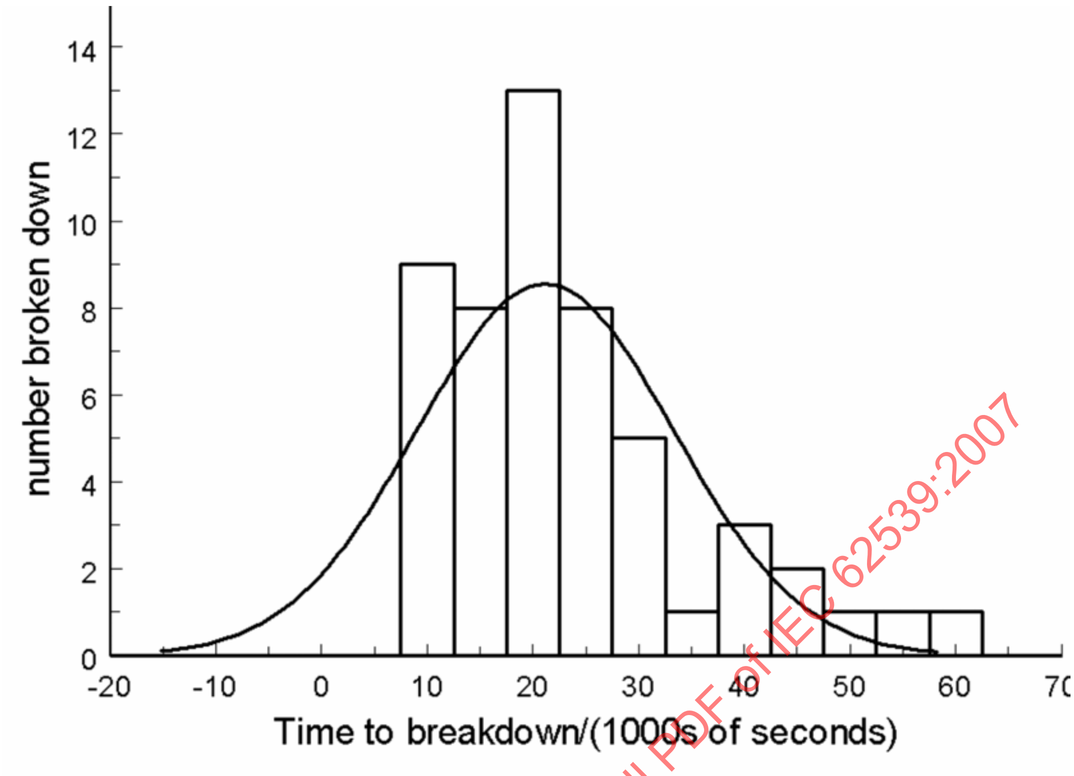

Figure A.1—Constant stress test using wire samples showing histogram of times to breakdown and best fitting Normal distribution

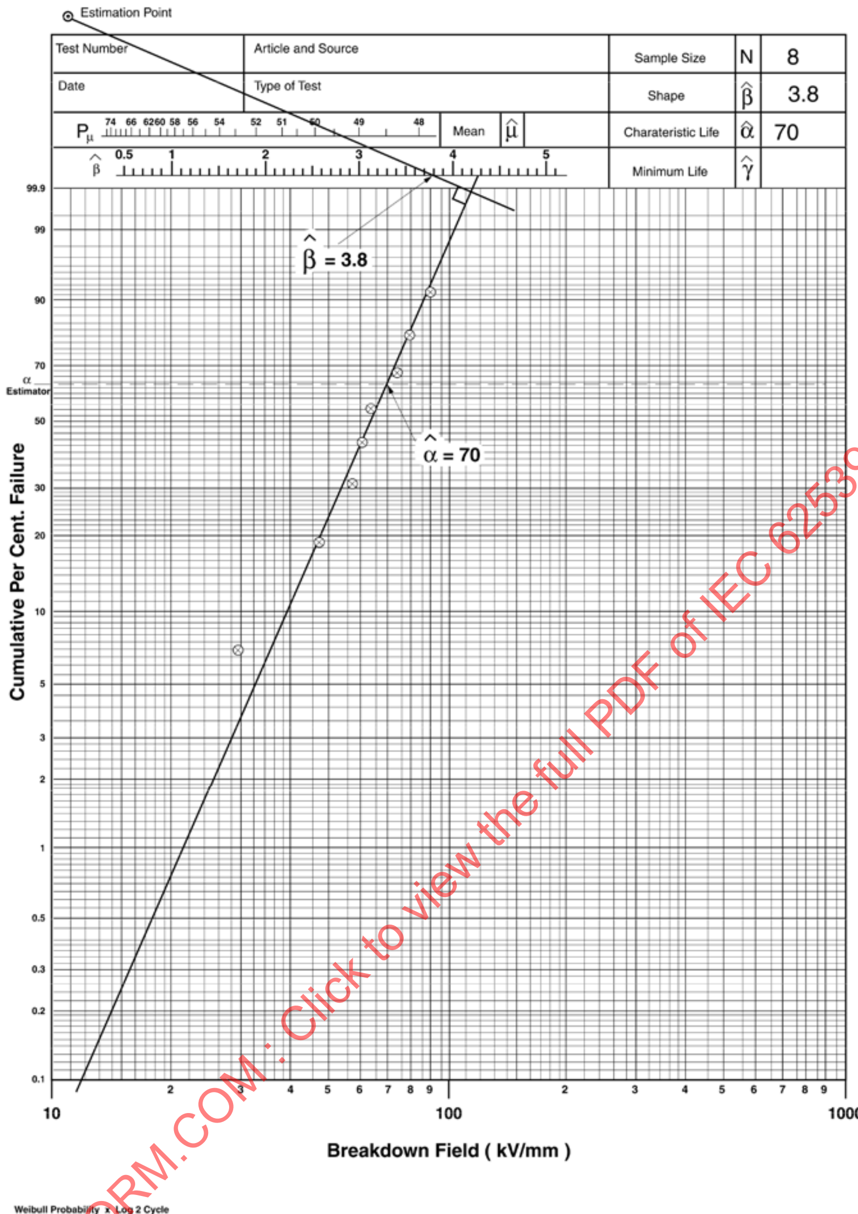

Figure A.2—Weibull probability paper
Data shown is from a progressive stress test (see Table A.5)

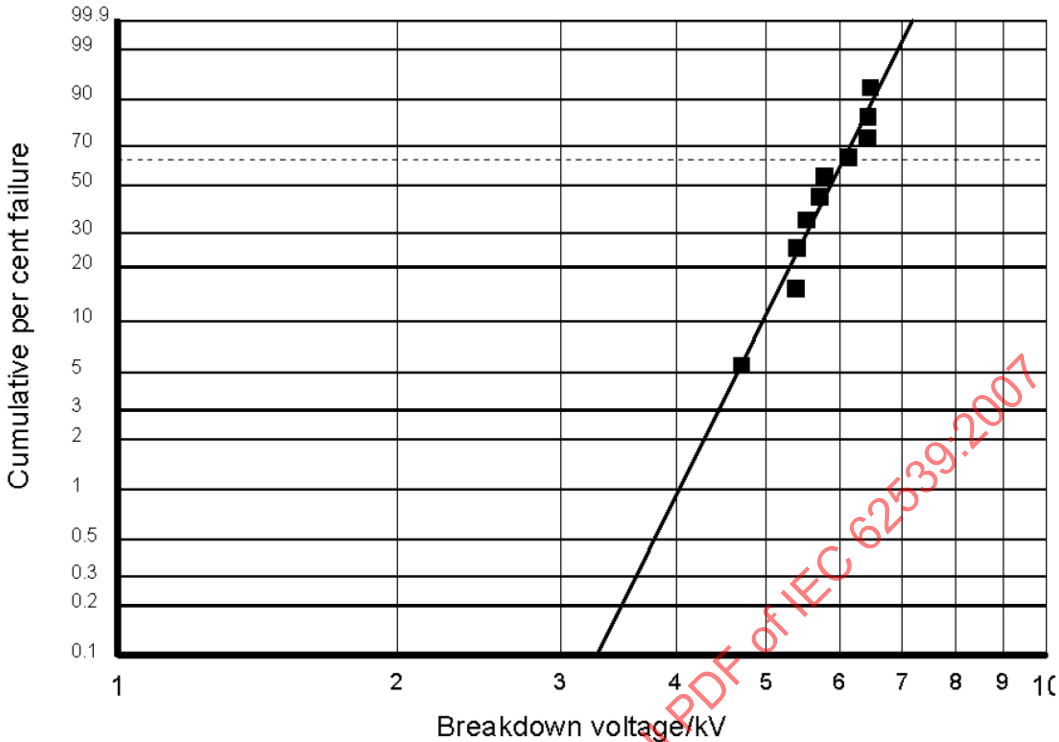

Figure A.3—Weibull plot of data from Table A.1

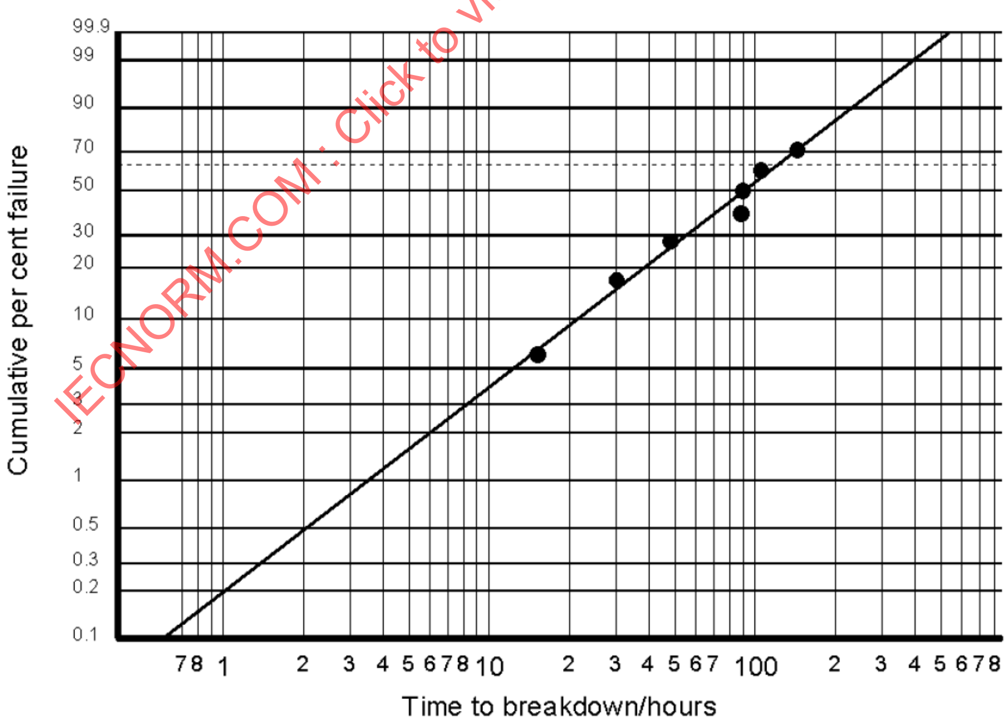

Figure A.4—Weibull plot of data from Table A.2

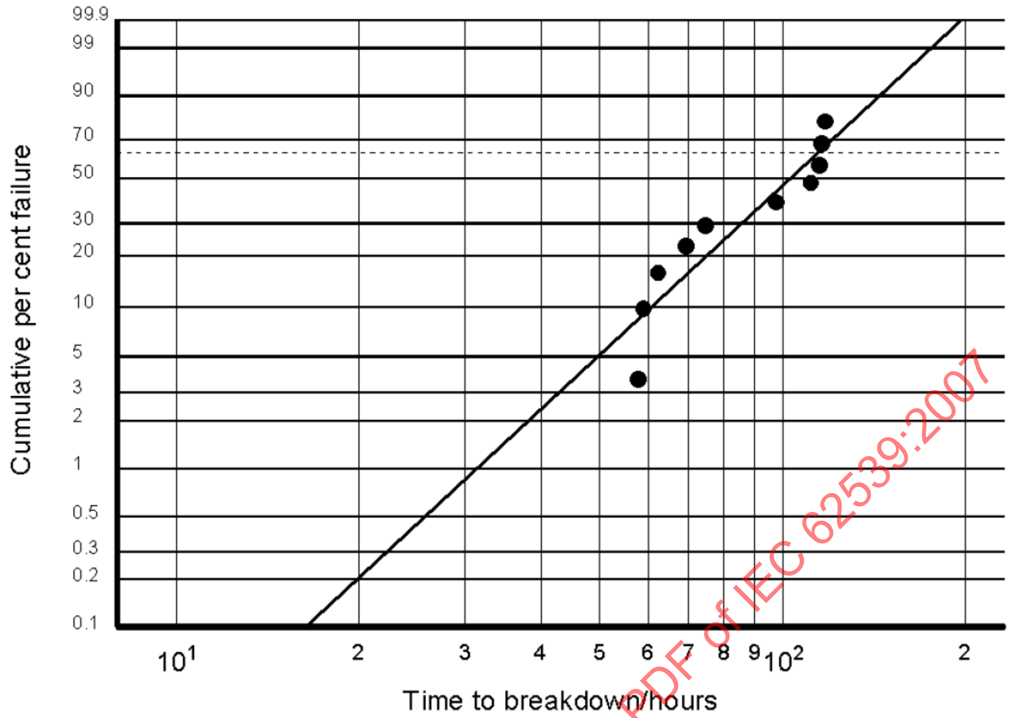

Figure A.5—Weibull plot of PET data presented in Table A.3

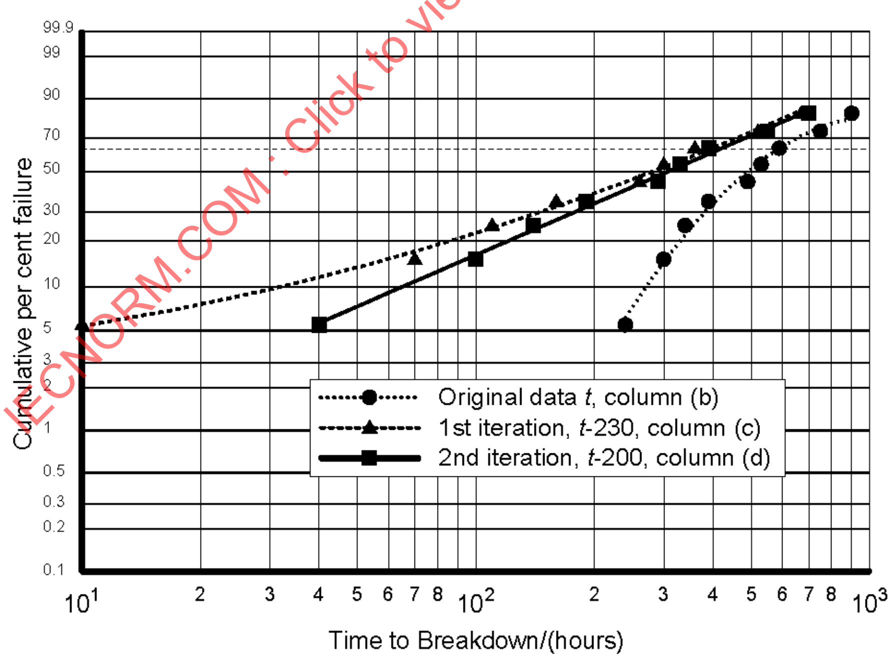

Figure A.6—Trial-and-error method for finding value of location parameter, data based on Table A.4

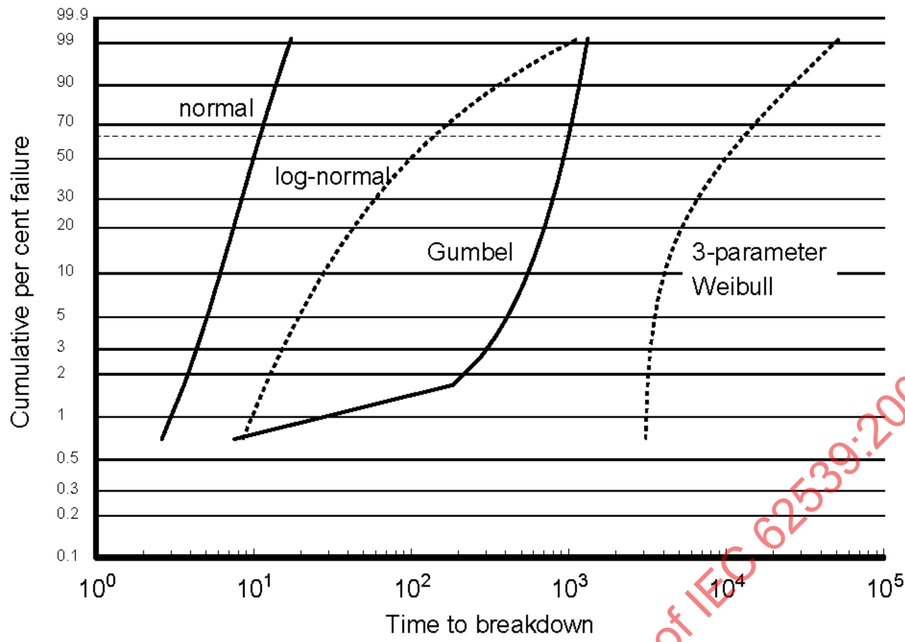

Figure A.7—Plots of other distributions on two-parameter Weibull paper

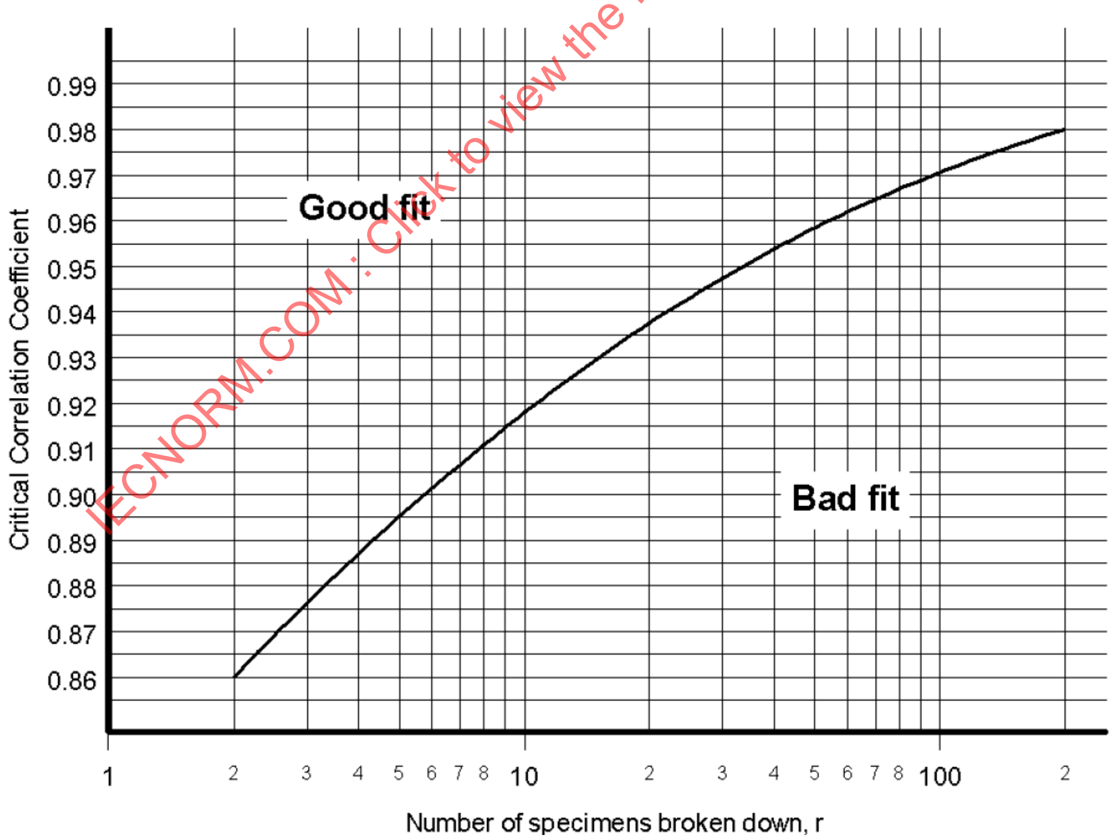

Figure A.8—Plot to check the goodness-of-fit of a two-parameter Weibull distribution

	A	B	C	D	E
1	Insulating Fluid Breakdown				
2					
3	Rank, i	F(i,n)	Breakdown time, t	X	Y
4	1	4.6	50	-3.0620	3.9120
5	2	12.7	134	-1.9935	4.8978
6	3	20.9	187	-1.4506	5.2311
7	4	29.1	882	-1.0690	6.7822
8	5	37.2	1450	-0.7644	7.2793
9	6	45.4	1470	-0.5027	7.2930
10	7	53.6	2290	-0.2655	7.7363
11	8	61.7	2931	-0.0407	7.9831
12	9	69.9	4180	0.1822	8.3381
13	10	78.0	15800	0.4161	9.6678
14	11	S	>20000	---	---
15	12	S	>20000	---	---
16					
17	n	correlation coefficient			
18	12	0.970			

Figure A.9—Using a spreadsheet to find the correlation coefficient of data from a constant stress test on an insulating fluid

	A	B	C	D	E
1	Insulating Fluid Breakdown				
2					
3	Rank, i	F(i,n)	Breakdown time, t	X	Y
4	1	=(A4-0.44)/(A18+0.25)*100	50	=LN(-LN(1-B4/100))	=LN(C4)
5	=A4+1	=(A5-0.44)/(A18+0.25)*100	134	=LN(-LN(1-B5/100))	=LN(C5)
6	=A5+1	=(A6-0.44)/(A18+0.25)*100	187	=LN(-LN(1-B6/100))	=LN(C6)
7	=A6+1	=(A7-0.44)/(A18+0.25)*100	882	=LN(-LN(1-B7/100))	=LN(C7)
8	=A7+1	=(A8-0.44)/(A18+0.25)*100	1450	=LN(-LN(1-B8/100))	=LN(C8)
9	=A8+1	=(A9-0.44)/(A18+0.25)*100	1470	=LN(-LN(1-B9/100))	=LN(C9)
10	=A9+1	=(A10-0.44)/(A18+0.25)*100	2290	=LN(-LN(1-B10/100))	=LN(C10)
11	=A10+1	=(A11-0.44)/(A18+0.25)*100	2931	=LN(-LN(1-B11/100))	=LN(C11)
12	=A11+1	=(A12-0.44)/(A18+0.25)*100	4180	=LN(-LN(1-B12/100))	=LN(C12)
13	=A12+1	=(A13-0.44)/(A18+0.25)*100	15800	=LN(-LN(1-B13/100))	=LN(C13)
14	=A13+1	S	>20000	---	---
15	=A14+1	S	>20000	---	---
16					
17	n	correlation coefficient			
18	12	=CORREL(D4:D13,E4:E13)			

Figure A.10—Spreadsheet of Figure A.9 showing formulae

	A	B	C	D	E	F
2						
3	Rank, i	F(i,n)	Breakdown voltage, t	X	Y	
4	1	2.3	2.65	-3.757	0.9746	
5	2	6.4	2.79	-2.711	1.0260	
6	3	10.6	2.96	-2.193	1.0852	
7	4	14.7	3.02	-1.840	1.1053	
8	5	18.8	3.06	-1.569	1.1184	
9	6	22.9	3.14	-1.345	1.1442	
10	7	27.1	3.33	-1.154	1.2030	
11	8	31.2	3.38	-0.985	1.2179	
12	9	35.3	3.51	-0.832	1.2556	
13	10	39.4	3.56	-0.691	1.2698	
14	11	43.5	3.59	-0.559	1.2782	
15	12	47.7	3.62	-0.434	1.2865	
16	13	51.8	3.67	-0.315	1.3002	
17	14	55.9	3.67	-0.200	1.3002	
18	15	60.0	3.72	-0.086	1.3137	
19	16	64.2	3.74	0.026	1.3191	
20	17	68.3	3.88	0.138	1.3558	
21	18	72.4	3.92	0.253	1.3661	
22	19	76.5	3.98	0.371	1.3813	
23	20	80.7	4.05	0.497	1.3987	
24	21	84.8	4.12	0.633	1.4159	
25	22	88.9	4.15	0.788	1.4231	
26	23	93.0	4.23	0.980	1.4422	
27	24	97.2	4.87	1.270	1.5831	
28						
29	n	Correlation	slope, m	intercept, c	alpha	beta
30	24	0.980	0.116	1.340	3.82	8.64

(a)

	C	D	E	F
29	slope, m	intercept, c	alpha	beta
30	=SLOPE(E4:E27,D4:D27)	=INTERCEPT(E4:E27,D4:D27)	=EXP(D30)	=1/C30

(b)

Figure A.11—(a) Least squares regression (LSR) analysis of large data set using a spreadsheet
(b) Spreadsheet formulae for estimates

	A	B	C	D	E	F	G	H	I	J
1	**Singly-censored XLPE mini-cable data**									
2										
3	**n**	**r**	**X-bar**	**Y-bar**	**beta**	**alpha**				
4	10	7	-0.593	3.127	7.827	24.597				
5										
6	**Rank, i**	**F(i,n)**	**b/down stress, t (kV/mm)**	**X**	**Y**	**weighting factors, w**	**wX**	**wY**	**beta-numerator**	**beta-denomina-tor**
7	1	5.5	17	-2.879	2.833	0.608	-1.750	1.722	3.17781	0.40816
8	2	15.2	19	-1.801	2.944	1.548	-2.789	4.559	2.26079	0.34132
9	3	25.0	21	-1.247	3.045	2.519	-3.141	7.669	1.07806	0.13570
10	4	34.7	23	-0.852	3.135	3.480	-2.964	10.910	0.23333	-0.00777
11	5	44.5	23	-0.530	3.135	4.408	-2.337	13.822	0.01735	0.00239
12	6	54.2	23	-0.246	3.135	5.275	-1.298	16.539	0.63408	0.01577
13	7	64.0	25	0.021	3.219	6.031	0.129	19.413	2.27530	0.34082
14	8	suspended								
15	9	suspended								
16	10	suspended								
17		**TOTALS**				**23.868**	**-14.149**	**74.634**	**9.677**	**1.236**
18										

Figure A.12—Spreadsheet analysis of smaller data set

	A	B	C	D	E	F	G	H	I	J
3	n	r	X-bar	Y-bar	beta	alpha				
4	10	7	=G17/F17	=H17/F17	=I17/J17	=EXP(D4-C4/E4)				
5										
6	Rank, i	F(i,n)	b/down stress, t (kV/mm)	X	Y	weighting factors, w	wX	wY	beta-numerator	beta-denomina-tor
7	1	=(A7-0.44)/(A4+0.25)*100	17	=LN(-LN(1-B7/100))	=LN(C7)	=weightings!K4	=F7*D7	=F7*E7	=F7*(D7-C4)*(D7-C4)	=F7*(D7-C4)*(E7-D4)
8	=A7+1	=(A8-0.44)/(A4+0.25)*100	19	=LN(-LN(1-B8/100))	=LN(C8)	=weightings!K5	=F8*D8	=F8*E8	=F8*(D8-C4)*(D8-C4)	=F8*(D8-C4)*(E8-D4)
9	=A8+1	=(A9-0.44)/(A4+0.25)*100	21	=LN(-LN(1-B9/100))	=LN(C9)	=weightings!K6	=F9*D9	=F9*E9	=F9*(D9-C4)*(D9-C4)	=F9*(D9-C4)*(E9-D4)
10	=A9+1	=(A10-0.44)/(A4+0.25)*100	23	=LN(-LN(1-B10/100))	=LN(C10)	=weightings!K7	=F10*D10	=F10*E10	=F10*(D10-C4)*(D10-C4)	=F10*(D10-C4)*(E10-D4)
11	=A10+1	=(A11-0.44)/(A4+0.25)*100	23	=LN(-LN(1-B11/100))	=LN(C11)	=weightings!K8	=F11*D11	=F11*E11	=F11*(D11-C4)*(D11-C4)	=F11*(D11-C4)*(E11-D4)
12	=A11+1	=(A12-0.44)/(A4+0.25)*100	23	=LN(-LN(1-B12/100))	=LN(C12)	=weightings!K9	=F12*D12	=F12*E12	=F12*(D12-C4)*(D12-C4)	=F12*(D12-C4)*(E12-D4)
13	=A12+1	=(A13-0.44)/(A4+0.25)*100	25	=LN(-LN(1-B13/100))	=LN(C13)	=weightings!K10	=F13*D13	=F13*E13	=F13*(D13-C4)*(D13-C4)	=F13*(D13-C4)*(E13-D4)
14	=A13+1	suspended								
15	=A14+1	suspended								
16	=A15+1	suspended								
17		TOTALS				=SUM(F7:F13)	=SUM(G7:G13)	=SUM(H7:H13)	=SUM(I7:I13)	=SUM(J7:J13)

Figure A.13—Spreadsheet formulae for calculating smaller sample estimates

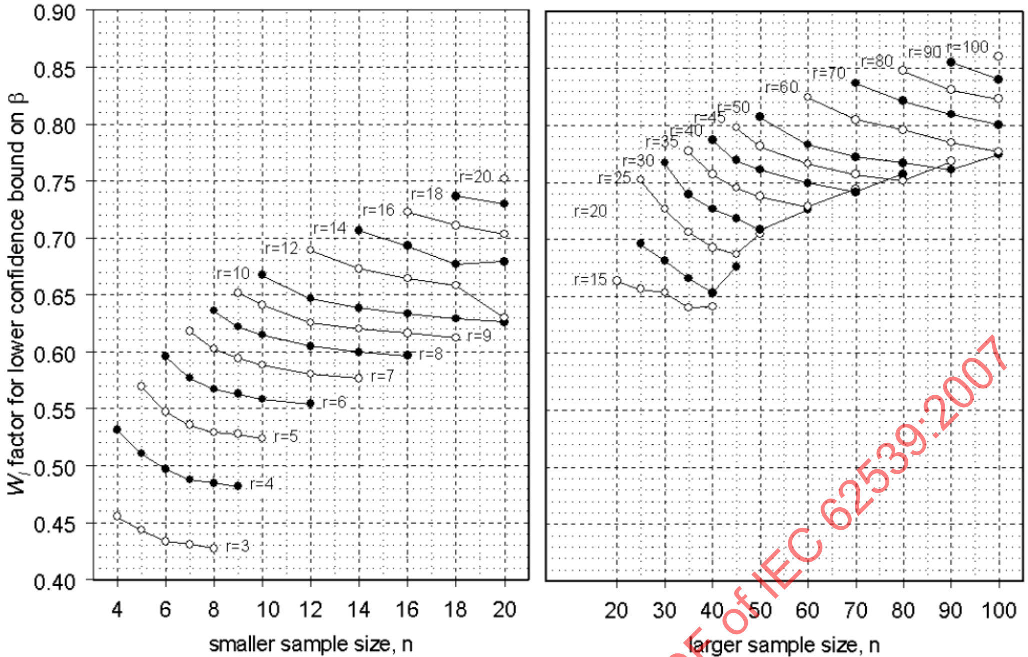

Figure A.14—W_l factor for calculating the lower 90% confidence bound on β

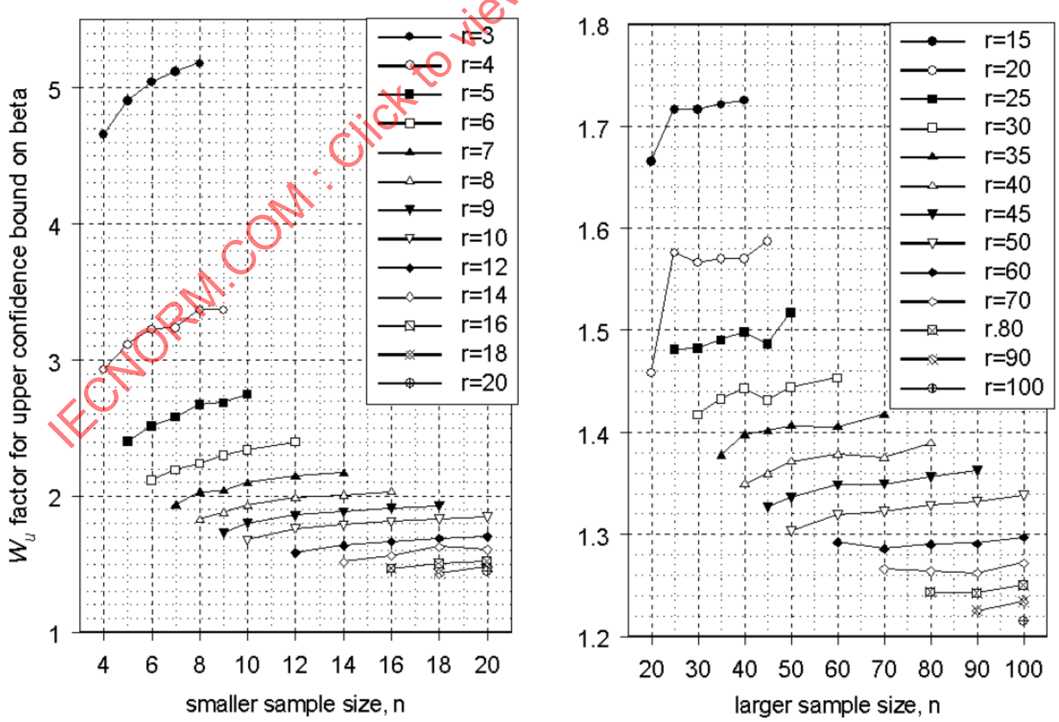

Figure A.15—W_u factor for calculating the upper 90% confidence bound on β